Motivations: jets & bulk

1. Currently modelled as two different entities
2. Where the bulk ends and the jet starts?
A unified framework for both jets and bulk?

- Bulk and jet constituents are partons before hadronization.
- They should be described by a unified way in QCD.
For a subclass of observables:

\[
\frac{d\sigma}{dp_T d\eta} = \sum_{abc} f_a \otimes f_b \otimes \hat{\sigma}_{ab \rightarrow c} \otimes J_c^{\text{med}}
\]

Bulk enters.

The Bjorken Picture for bulk matter

1. The valence quarks pass through each other.

2. Produced "wee" partons fill a central plateau in rapidity.

3. The saturation model (CGC): \( k \sim Q_s > \Lambda_{QCD} \)


Let us see what follows from this picture.
Bulk in AA collisions
The bottom-up thermalization

At $\tau \sim 1/Q_s$, produced wee (called "hard") partons go on mass-shell.

$$k \sim Q_s, \quad \text{number density } n_h \sim \frac{1}{\alpha_s}$$

Then, the thermalization proceeds in kinetic theory:

- Stage I: early-time attractor driven by longitudinal expansion
- Stage II: Setting up the stage for thermalization
- Stage III: quenching of "hard" gluons
  
  heating up thermal bath with $T \ll Q_s$

**Common mechanism with jet quenching!**

Isotropization in the bottom-up thermalization

\[
\left( \tau Q_s \right)^{-3/2} \alpha_s^{13} (Q_s \tau)^{5/2} \alpha_s^{-13/5}
\]

<table>
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<tr>
<th>Region</th>
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\( P_L/P_T \) vs. \( \tau Q_s \)
Stage 0: freeing wee gluon from wave functions

1. **Energy density**: $\epsilon \propto \frac{1}{\tau}$ at large $\tau$


2. **Two point function in the limit** $\tau \to \infty$

$$G_{22}^{a\mu,b\nu}(X, p) \to 2\pi \delta(p^2) \delta^{ab} \sum_{\lambda=\pm} \epsilon_{\lambda}(p) \epsilon^{*\nu}(p) f^{cl}(X, p)$$

where $f^{cl}(X, p)$ a longitudinally boost-invariant distribution.

$\delta(p^2)$: classical field does go on mass-shell as $\tau \to \infty$!

Does kinetic theory follows classical fields?

The question:

The answer:

No in $\phi^4$ theory at this order.


QFT has features that complicates the picture in kinetic theory!

Still an open question in QCD!
Modern understanding of Bottom-up thermalization

1. **Stage I: attractor in classical statistical approximation (CSA):**
   

2. **Bottom-up in kinetic theory:**
   

3. **An entire evolution by matching:**
   

4. **Go beyond?**
   
   - Fast isotriupization has been shown in CSA.
     
   
   - Technical difficulty: One has to deal with nonrenormalizability
     

**Another open question!**
Connection to jet quenching

energy loss $\sim Q_s$ at the end of thermalization.
Connection to jet quenching

Some detailed calculations:

1. Radiative energy spectrum can be calculated analytically.

\[
\omega \frac{dl}{d\omega} \sim \alpha_s N_c \sqrt{\hat{q}(\tau) \frac{\tau^2}{\omega}} \quad \text{for } t_f \ll \tau
\]


2. Radiative correction to \(p_T\)-broadening and \(\hat{q}\):

- Leading-logs and resummation

\[
\hat{q}_{\text{resum}}(\tau) = \hat{q}(\tau) \left( \frac{I_1(2\sqrt{\alpha} Y)}{{\sqrt{\alpha}} Y} \right) \quad \text{with } Y = \ln(\tau/\lambda(\tau))
\]

looks like a static medium with \(\hat{q} = \hat{q}(\tau)\) at each time!


- Finite terms may also be important: Zakharov, arXiv:2003.10182 [hep-ph].
Bulk in small systems
Hydro vs non-hydro modes

Two statements that are generally true:

1. QFTs contain hydrodynamics.
2. QFTs go beyond hydrodynamics in different ways.

Examples:

\[ Dk^2 + \ldots \]

AdS/CFT

\[ Dk^2 + \ldots \]

QGP

\[ \theta(x^0) \langle [T^\alpha \beta(x), T^\gamma \delta(0)] \rangle \]

the analytic structure of \( G_{R}^{\alpha \beta, \gamma \delta}(\omega, \vec{k}) = -i \int d^4x e^{ik \cdot x} \theta(x^0) \langle [T^\alpha \beta(x), T^\gamma \delta(0)] \rangle \)
Probes to non-hydro (particle-like) modes

1. **Studying jet quenching**
   
   high $p_T$ jets probe short wavelength

2. **Studying small systems:** $G_R(\tau, k) \sim c_{\text{hyd}} e^{-Dk^2\tau} + \text{non-hydro terms}$

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**Graphical Content:**

- **QGP**
- **Sensitive to non-hydro**
- **Decreasing opacity**

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**Bin Wu**

12/18 Early Time Dynamics and Bulk
Flow in small and/or dilute limit

Flow is a signature for final-state interactions


1. Similar to kinetic theory in relaxation time approximation:


2. Physics depends on only one dimensionless parameter:

\[
\text{Opacity: } \hat{\gamma} = \gamma R^{3/4} (\varepsilon_0 \tau_0)^{1/4} = R/l_{mfp}
\]


3. The transport coefficients: 

\[
\eta_s = \frac{1}{5} \frac{T}{\gamma \varepsilon^{1/4}}.
\]
Features of thermalization in CKT


(cf. talk by Wilke van der Schee)

1. **Commonalities with bottom-up thermalization** (left figure).
   - Early-time attractor
   - Late-time (hydro) attractor

2. **The late-time dynamics is modified by opacity** (right figure).

   A potential probe to hydrodynamization!
Qualification of being a fluid

Criteria:

\[
\text{hydro-like} \Leftrightarrow Q < 0.1
\]

in terms of ”fluid quality” for \( T^{\mu\nu} \) calculated in some model

\[
Q(t,r) = \sqrt{\frac{(T - T_{\text{hyd}})^{\mu\nu}(T - T_{\text{hyd}})_{\mu\nu}}{(T_{\text{id}})^{\mu\nu}(T_{\text{id}})_{\mu\nu}}}
\]

where \( T_{\text{hyd}} \) is the energy-momentum tensor in hydrodynamics.
How much "fluid" is CKT?


Confronting experimental Data

\[ v_2^{\text{energy}} \]

\[ v_2 \text{from 5.02 TeV PbPb, ALICE} \]

\[ \hat{\gamma} > 4 \quad 4 > \hat{\gamma} > 3 \quad 3 > \hat{\gamma} > 2 \quad 2 > \hat{\gamma} \]

\[ \text{Kin. Th. } \epsilon_{\text{eos}} = 0.9, \quad \frac{\eta}{s} = \frac{3}{4\pi} \]

\[ \epsilon_2 \text{ uncertainty: GGMLO} \]

Conclusions

1. For large systems:
   ✓ The bottom-up thermalization is supported by more detailed calculations.
   ✓ The soft gluon radiation from jets with $t_f \lesssim \tau$ look like the static case.
   ✗ There are some technical questions for going beyond

2. For small systems:
   ✓ Non-hydro modes, which dominates at early time, also contribute to flow.
   ✓ Interplay between hydro and non-hydro modes can be studied (in CKT).
   ✗ potential modification of the bottom-up thermalization?
   ✗ jet quenching?

3. A unified framework for jets & bulk?