Open Heavy Flavor in Medium

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1.) Heavy Quarks: A “Calibrated” QCD Force

- Vacuum quarkonium spectroscopy well described
- Confinement \( \leftrightarrow \) linear part of potential

**Objective:** Determine medium-modifications of QCD force
\[ \Rightarrow \] deduce transport properties + spectral functions of heavy flavor, probing QGP at varying resolution

**Exploit** \( m_Q \gg \Lambda_{QCD}, T_c, T_{RHIC,LHC} \)
1.2 Quarkonia in Medium

- potential $V(r,T) \rightarrow$ binding energy $E_B(p,T)$
- Dissociation rate $\rightarrow$ width $\Gamma(p,T,E_B)$
- How do heavy quarks within quarkonia interact with the medium?
- Not a good thermometer…

![Graph showing distribution of energy with temperature](image)
• Radiation suppressed ($q_0^2 \sim q^2/2m_Q << q^2$) → Brownian motion via elastic interactions

• Thermalization delayed by $m_Q/T$ → memory in URHICs

• Direct access to transport coefficient (p → 0)
  $D_s (2\pi T)$ (∼ $\eta/s \sim \sigma_{EM}/T$ !?)

• Scattering rates → widths (quantum effects); quasiparticles? ($m_Q >> T$)
  → simple estimate: $D_s(2\pi T)=3 \Rightarrow \Gamma_Q \sim 1\text{GeV}
  \Rightarrow$ Implications for QGP structure?

• Non-perturbative effects (heavy-quark potential!)

• Probe of hadronization ($D_s, B_s, \Lambda_c, \ldots$)
Outline

1.) Introduction

2.) Open Heavy-Flavor Transport
   - Boltzmann vs. Fokker-Planck
   - Transport Coefficients

3.) Heavy-Quark Interactions in QGP
   - Perturbative Interactions
   - Non-perturbative Approach + Bulk Properties

4.) Hadronization
   - Fragmentation vs. Recombination

5.) Phenomenology
   - Basic Data Features + Transport Coefficients

6.) Conclusions
2.1 Heavy-Flavor Transport in URHICs

- initial cond.
  (nPDFs, ...), pre-equil. fields
- c-quark diffusion in QGP liquid
- c-quark hadronization
- D-meson diffusion in hadron liquid

- no “discontinuities” in interaction

⇒ diffusion toward $T_{pc}$ and hadronization same interaction (confining!)

2.2 Transport Approaches

- **Boltzmann equation for HQ phase-space distribution** $f_Q$

$$\left[ \frac{\partial}{\partial t} + \frac{p}{\omega_p} \frac{\partial}{\partial x} + F \frac{\partial}{\partial p} \right] f_Q(t, x, p) = C[f_Q]$$

- explicit simulation of medium (quasi-) particles in collision term
- semi-classical approximation

- **Fokker-Planck equation**

$$\frac{\partial}{\partial t} f_Q(t, p) = \frac{\partial}{\partial p_i} \left\{ A_i(p) f_Q(t, p) + \frac{\partial}{\partial p_j} [B_{ij}(p) f_Q(t, p)] \right\}$$

- follows from Boltzmann with $p^2 \sim m_Q T >> q^2 \sim T^2$; ok for $m_Q/T \geq 5$
- does not require quasi-particle medium
- well suited for strongly coupled medium where $E_{th} \leq \Gamma_{q,Q} < m_Q$
2.3 Transport Coefficients

\[ \frac{\partial}{\partial t} f_Q(t, p) = \gamma \frac{\partial}{\partial p_i} [p_i f_Q(t, p)] + D_p \Delta_p f_Q(t, p) \]

- thermal relaxation time: \( \tau_Q = 1/\gamma \)
- Einstein relation: \( T = D_p / \gamma m_Q \)
- spatial diffusion constant: \( D_s = T / \gamma m_Q \), \( \langle x^2 \rangle - \langle x \rangle^2 = 6 D_s t \)
- relation to bulk medium: \( D_s (2\pi T) \sim \eta / s (4\pi) \)
- Key ingredient: heavy-light scattering amplitude \( T_{Qj} \)
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3.1 Perturbative QCD Approaches

3.1.1 Leading Order

- gluon exchange screened by Debye mass:
  \[
  G(t) = \frac{1}{t} \to \frac{1}{t - \mu_D^2}, \quad \mu_D = gT
  \]

- dominated by forward scattering

- long thermalization time: \( \tau_c \geq 20 \text{ fm/c} \) \((T \leq 300 \text{MeV}, \alpha_s=0.4)\)

3.1.2 Next-to-Leading Order

- bad convergence behavior (even for \( \alpha_s \sim 0.1 \))

[Caron-Huot+Moore ‘08]
3.1.3 “Effective” pQCD with Running Coupling

- \( M_{qj} \sim \frac{\alpha}{t} \to \frac{\alpha_{\text{eff}}(t)}{t - \tilde{\mu}^2} \)

- run \( \alpha_s \) to \( m_D \sim gT \), rather than \( 2\pi T \)
- reduced Debye mass \( \tilde{\mu}^2 = \frac{1}{5} \mu_D^2 \)

- factor \( \sim 10 \) faster thermalization: \( \tau_c \approx 2-3 \text{ fm/c} \)
- perturbative regime? Need to resum large diagrams…

[Deviations from][Peshier ’07, Gossiaux et al ’08, BAMPS ‘14]
3.2 Nonperturbative Approach: T-Matrix

- Scattering equation
  \[ T_{ij} = V_{ij} + \int V_{ij} D_i D_j T_{ij} \]

- **Strong** coupling → resummation:
  \[ D_i = 1 / [\omega - \omega_k - \Sigma_i(\omega, k)] \]

- Thermal parton **propagators**:

- Parton self-energies → self-consistency:

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- In-medium potential \( V \)?

[Manarelli, Cabrera, Riek, Liu,…’05-’18]
3.3.2 Potential Extraction from Lattice Data

- Free Energy
  \[ F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta} \ln \left( \int_{-\infty}^{\infty} d\omega \sigma(\omega, r_1 - r_2) e^{-\beta \omega} \right) \]

- Q\bar{Q} Spectral Function
  \[ \sigma(\omega, r) = \frac{1}{\pi} \frac{(V + \Sigma)_{f}(\omega)}{(\omega - (V + \Sigma)_{R})^2 + (V + \Sigma)_{I}^2(\omega)} \]

- Large imaginary parts $\Rightarrow$ $V > F$
- Remnants of confining force above $T_c$!
3.3.3 QGP Equation of State + Spectral Functions

Thermodynamic Potential

Selfconsistent SFs

\[ \Omega = -\frac{1}{\beta} \sum_n \text{Tr} \{ \ln(-G^{-1}) + (G_0^{-1} - G^{-1})G \} \pm \Phi \]

\[ G = G_0 + G_0 \Sigma G \quad \Sigma = GT \quad T = V + VGGT \]

- Near \( T_c \) light partons melt + broad hadronic resonances emerge

Quark spectral functions

“Meson” \( T \)-matrix

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4.) **Hadronization of Heavy Quarks**

- **Fragmentation**

  \[ c \rightarrow D, D^*, D_s, \Lambda_c, \ldots \]

  determined by empirical fragmentation functions \( D_{c\rightarrow H_c}(z) \);
  in principle universal ("vacuum": \( e^+e^- \) collisions or high \( p_T \))

- **Coalescence / Recombination**

  \[ c + q (s) \rightarrow D (D_s), D^*, \ldots ; \quad c + q + q (s) \rightarrow \Lambda_c (\Xi_c), \ldots \]

  depends on environment (phase space of surrounding anti-\(q\)/quarks)

  -- instantaneous coalescence (based on spatial wave functions)

  -- resonance recombination (momentum space)
4.2 Heavy-Quark Recombination

- **Instantaneous Coalescence Models (ICMs)**
  
  \[
  f_h(p_h') = \int \left[ \prod_i dp_i f_i(p_i) \right] W({\{p_i\}}) \delta(p_h' - \sum_i p_i)
  \]

  \[
  W_s = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 k^2}
  \]

  Wigner function, \(\sigma\) \(\sim\) radius parameter for each hadron \(h\)

  - energy not conserved \(\rightarrow\) challenge for chemical + thermal equilibrium

- **Resonance Recombination Model (RRM)**

  - derived from Boltzmann equation
  
  \[
  f_M(x, \vec{p}) = \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2}{(2\pi)^3} f_q(x, \vec{p}_1) f_{\bar{q}}(x, \vec{p}_2) \sigma_M(s) \nu_{rel}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)
  \]

  \[\sigma_M(s) \nu_{rel} \sim |T_{Qj}|^2:\] resonant heavy-light scattering amplitude

  \(\rightarrow\) direct connection to \(T\)-matrix interactions in QGP near \(T_c\)

  \(\rightarrow\) compatible with equilibrium limits

[Hwa ‘80, Likhoded et al ‘83, … Greco et al + Fries et al ‘03,…]
[Ravagli et al ‘07, He et al ‘12]
4.3 Heavy-Flavor Hadro-Chemistry

- Distinct pattern of charm-hadron production up to $p_T \sim 10$ GeV
- $D_s$ enhanced due to strangeness equilibration in QGP

- Relevance of recombination in $pp$
- sensitive to charm-baryon spectrum (“missing” resonances in PDG)
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5.1 Heavy-Flavor Data at RHIC + LHC

- "Flow bump" in $R_{AA} +$ large $v_2$ $\leftrightarrow$ strong coupling near $T_{pc}$ (recombination)
- High-precision $v_2$: transition from elastic to radiative regime
5.2 The $v_2 - R_{AA}$ Correlation Story

Heavy-Flavor Electrons [PHENIX '07]

- Challenge: Large elliptic flow but not too large suppression in $R_{AA}$
  → better met by non-perturbative than perturbative interactions

[van Hees et al '05] [Moore+Teaney '05, Armesto et al '05]
5.3 Charm Diffusion Coefficient

\[ D_s = \frac{T}{[m_Q \gamma_Q(p=0)]} \]

- suggests minimum of \( D_s(2\pi T) \sim 2-4 \) near \( T_{pc} \)
- scatt. rate: \( \Gamma_{\text{coll}} \sim \frac{3}{D_s} \sim 1 \text{ GeV} \) – no light quasi-particles!
- Importance of \( p \)-dependence (\( D_s \) not the full story)
5.4 Heavy-Flavor vs. Bulk Transport

Heavy-Quark Diffusion and Viscosity

- Strongly coupled: \((2\pi T) D_s \sim (4\pi) \eta/s\)
- Perturbative: \((2\pi T) D_s \sim 5/2 (4\pi) \eta/s\)
- Transition as \(T\) increases

[Liu+RR ‘19]
6.) **Summary**

- Open heavy flavor direct probe of in-medium QCD force ($m_Q \gg T, \Lambda_{QCD}$)
- Interaction strength well beyond perturbative needed
  → utilize lattice-QCD “data” to constrain heavy-quark potential
- Remnants of confining force drive the properties of sQGP as strongly coupled quantum liquid for $T \sim 1-2T_c$
- Quantum many-body theory connects:
  (i) small $D_s, \eta/s$,… (ii) melting light partons (iii) hadron formation
- Future heavy-flavor data (incl. bottom!) will provide ample constraints to quantify the $T$- and $p$-dependent transport and hadronization mechanisms in sQGP
4.6.2 Sensitivity to Heavy-Quark Transport

- Strongly-coupled potential gives max. low-$p_T$ c-quark $v_2$
- Weakly-coupled potential (~ free energy) ruled out

[He et al. ‘18]
4.4 Hamiltonian Approach to QGP

- **In-Medium Hamiltonian** with "bare" 2-body interactions

\[
H = \sum \varepsilon_i(p) \psi_i^\dagger(p) \psi_i(p) + \psi_i^\dagger(\frac{P}{2} - p) \psi_j^\dagger(\frac{P}{2} + p) V_{ij}^a \psi_j(\frac{P}{2} + p') \psi_i(\frac{P}{2} - p')
\]

- effective in-medium mass \(\varepsilon_i(p) = \sqrt{M_i^2 + p^2}\)

- Interaction ansatz: **Cornell potential** with relativistic corrections

\[
V_{ij}^a(p, p') = R_{ij}^C F_a^C V_C(p - p') + R_{ij}^S F_a^S V_S(p - p')
\]

- color-Coulomb and string ("confining") interaction
- decent spectroscopy in vacuum

[\text{Liu+RR '16}]

- **Implement into** Brueckner / Luttinger-Ward-Baym approach
2.1 Quarkonium Transport in Heavy-Ion Collisions

- Inelastic reactions:
  
  \[ \text{detailed balance: } J/\psi + g \leftrightarrow c + \bar{c} + X \]

- Rate equation:
  
  \[ \frac{dN_\psi}{d\tau} = - \Gamma_\psi \left[ N_\psi - N_{\psi}^{eq} \right] \]

- Transport coefficients
  - Equilibrium limit \( N_{\psi}^{eq}(m_\psi, T; N_{cc}) \)
  - Reaction Rate \( \Gamma_\psi (E_B(T)) \)
    - “Strong” binding \( E_B \geq T \)
    - “Weak” binding \( E_B < m_D \)

- gluo-dissociation (“singlet-to-octet”)

- “quasi-free”/ Landau damping
4.3 Quarkonium Spectral Functions + Correlators

**Charmonium**

![Graph showing spectral functions and correlators for Charmonium.]

**Bottomonium**

![Graph showing spectral functions and correlators for Bottomonium.]

- $J/\psi$ / $\Upsilon(1S/2S)$ melting ($300/500/250\text{MeV}$) not inconsistent with pheno.

[S.Liu+RR '17]
2.2 Excitation Functions: SPS - RHIC - LHC

Charmonium

- Gradual increase of total $J/\psi$ $R_{AA}$
- Regeneration and suppression increase
- Regeneration concentrated at low $p_T$!

Bottomonium

- Gradual suppression
- Regeneration ($N_{\Upsilon_{eq}}$) small
- Qualitative difference from $J/\psi$

[data: NA50, PHENIX, STAR, ALICE, CMS]
2.4 Upshot of Quarkonium Phenomenology

Use temperature estimates from hydro/photons/dileptons to infer:

\[ T_0^{\text{SPS}} (~240) < T_{\text{melt}}(J/\psi, \Upsilon') \leq T_0^{\text{RHIC}} (~350) < T_{\text{melt}}(\Upsilon) \leq T_0^{\text{LHC}} (~550) \]

- Remnants of confining force survive at SPS [hold \(J/\psi\) together]
- Confining force screened at RHIC+LHC [“melts” \(J/\psi + \Upsilon(2S)\)]
- Color-Coulomb screening at LHC [\(\Upsilon(1S)\) suppression]
- Thermalizing charm quarks recombine at LHC [large \(J/\psi\) yield]
4.1 Divide out Cold-Nuclear-Matter Effects

\[ R_{AA}^{\text{hot}} = \frac{R_{AA}^{\text{tot}}}{S_{\text{CNM}}} \]

- \( J/\psi \) suppression at SPS mostly from feeddown (\( \sigma_{\psi N} \sim 7.5 \text{mb} \)), melts in the RHIC \( \rightarrow \) LHC regime (not unlike \( \Upsilon(2S) \)).
2.2 In-Medium $\Upsilon$ Suppression

- Binding energies control reaction rates
- $\Upsilon(1S)$ suppression sensitive to $E_B(T)$
- Significant regeneration for $\Upsilon(2S)$
2.1 Quarkonium Transport in URHICs

Production + evolution of $c\bar{c}$ wave pack.

- $\tau_{\text{form}} \sim 1\text{fm/c}$
- $\tau_{c}^{\text{eq}} \sim 5\text{fm/c}$
- $\sim T_{\text{melt}}$: $\psi$ can form
- QGP kinetics $c + \bar{c} \leftrightarrow \psi$
- $\tau_{\psi}^{\text{eq}} \sim 1/\Gamma_{\psi}$
- $\sim T_{\text{pc}}$: $c$ and $\bar{c}$ hadronize
- Hadronic kinetics

3.3 Properties of Charmonium Excess

- excess concentrated at low $p_T$
- systematic softening of $J/\psi$ $p_T$-spectra with increasing $\sqrt{s}$ → nature of source changes
3.3 Heavy-Quark Interactions in QGP

Minimal / Desirable Ingredients and Features

- **Microscopic** description of scattering amplitude
- Realistic in-medium **interaction kernel** (screening)
- **Nonperturbative** interactions (color-Coulomb / pQCD not enough)
- **Resummation** (strong coupling)
- **Hadronization** approaching $T_c$ from above (bound states)
- Elastic + radiative processes (low / high $p_T$)
- Realistic **medium partons**: quasiparticles, widths, parton spectral functions, …? Equation of state?
- Quantitatively rooted in **constraints from lattice QCD**
3.4 Broader Theory Efforts

- EMMI Rapid Reaction Task Force [NPA ‘18 in press]
- HQ-jet working group
- Scrutinize components of open HF phenomenology
  - initial spectra, CNM + pre-equil. effects
  - bulk evolution, implementation of transport
  - $p$-, $T$-dep. transport coeffs. (QGP+hadronic)
  - hadronization mechanisms

- Example:
  use common transport coefficient ($5 \times pQCD$) in different bulk medium evolution and hadronization models

\[ D_s (2\pi T) < 6 \]