Open Heavy Flavor in Medium

Ralf Rapp

Cyclotron Institute +
Dept of Phys & Astro
Texas A&M University
College Station, TX, USA

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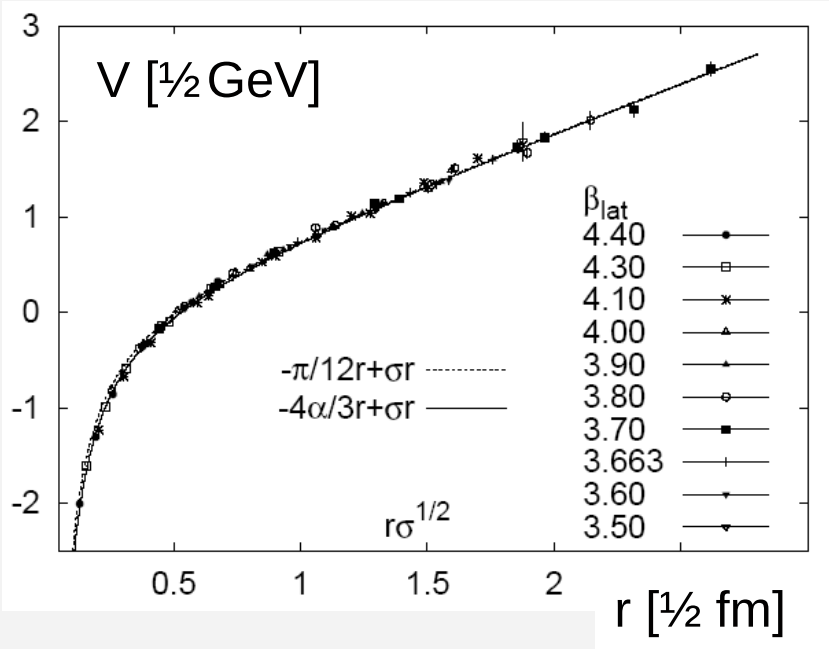
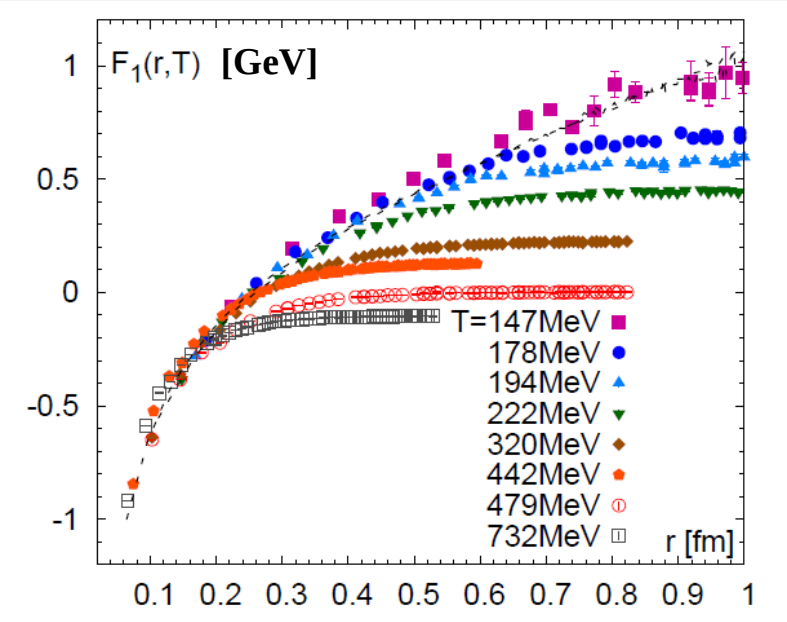
1.) Heavy Quarks: A "Calibrated" QCD Force





[Bazavov et al '13]

- Vacuum quarkonium spectroscopy well described
- Confinement ↔ **linear part of potential**

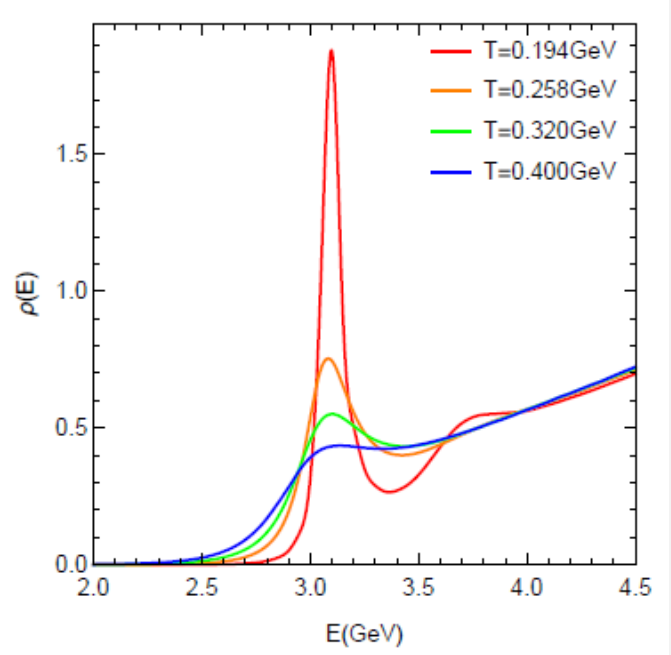
Objective: Determine medium-modifications of QCD force

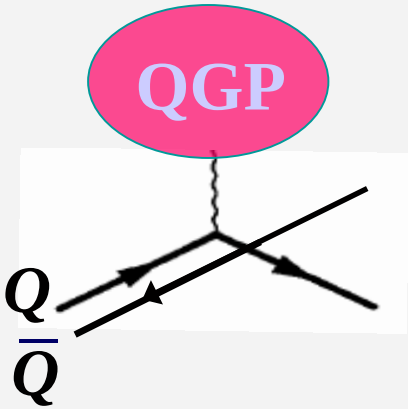
⇒ deduce transport properties + spectral functions of heavy flavor, probing QGP at varying resolution

Exploit $m_Q >> \Lambda_{QCD}$, T_c, $T_{RHIC,LHC}$

1.2 Quarkonia in Medium

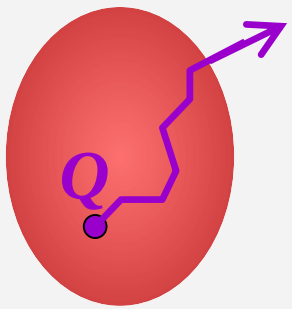
- potential **V(r,T)** → binding energy **E_B(p,T)**
- Dissociation rate → width **Γ(p,T,E_B)**
- **How do heavy quarks within quarkonia interact with the medium?**
- **Not a good thermometer…**





1.3 Heavy Quarks in QGP: A Hard Scale for Soft Physics

- Radiation suppressed ($q_0^2 \sim q^2/2m_Q << q^2$)
  → **Brownian motion via elastic interactions**



- Thermalization **delayed** by m_Q/T
  → memory in URHICs
- Direct access to **transport coefficient** ($p \to 0$)
  $\mathcal{D}_s\,(2\pi T)$ ($\sim \eta/s \sim \sigma_{EM}/T$!?)
- **Scattering rates**
  → widths (**quantum effects**); quasiparticles? ($m_Q >> T$)
  → simple estimate: $\mathcal{D}_s(2\pi T)=3 \Rightarrow \Gamma_Q \sim 1\text{GeV}$
  ⇒ **Implications for QGP structure?**

- **Non-perturbative effects** (heavy-quark potential!)
- Probe of hadronization (D_s, B_s, Λ_c,…)

Outline

1.) **Introduction**

2.) **Open Heavy-Flavor Transport**
- Boltzmann vs. Fokker-Planck
- Transport Coefficients

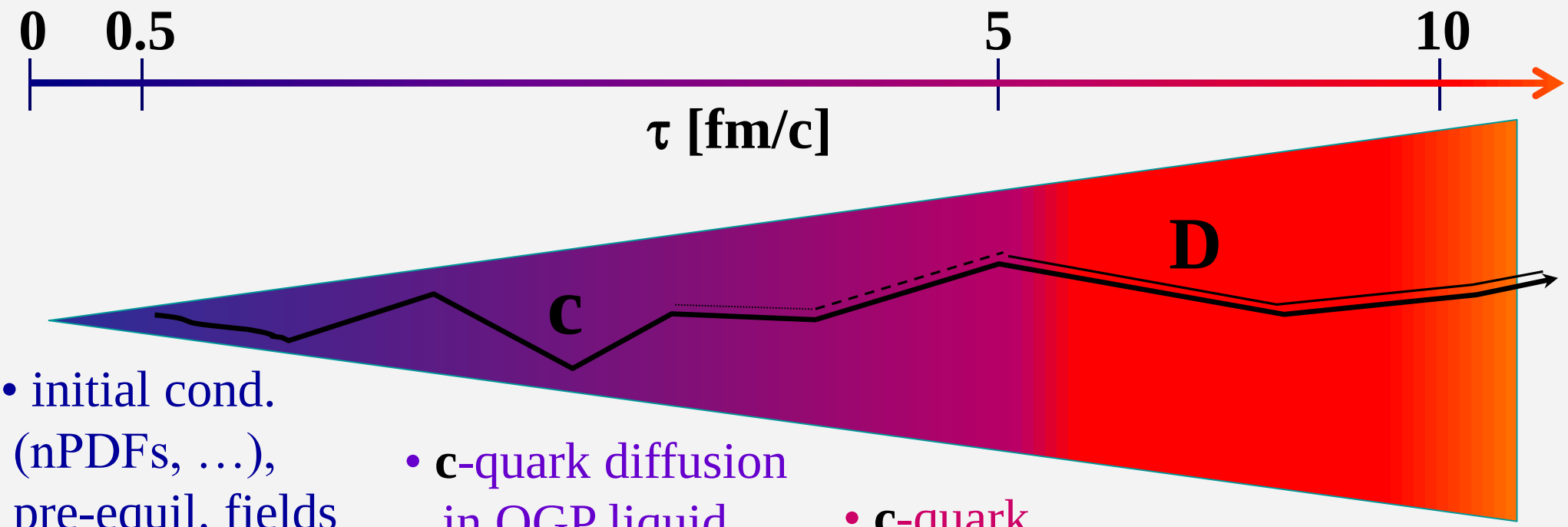
3.) **Heavy-Quark Interactions in QGP**
- Perturbative Interactions
- Non-perturbative Approach + Bulk Properties

4.) **Hadronization**
- Fragmentation vs. Recombination

5.) **Phenomenology**
- Basic Data Features + Transport Coefficients

6.) **Conclusions**

2.1 Heavy-Flavor Transport in URHICs



- initial cond. (nPDFs, …), pre-equil. fields
- **c**-quark diffusion in QGP liquid
- **c**-quark hadronization
- **D**-meson diffusion in hadron liquid

- no "discontinuities" in interaction

⇒ **diffusion toward T_{pc}** and **hadronization** same interaction (**confining**!)

[Moore+Teaney '05, van Hees et al '05, Gossiaux et al '08, Vitev et al '08, Das et al '09, Uphoff et al '10, M.He et al '11, Beraudo et al '11, Cao et al '13, Bratkovskaya et al '14, …]

2.2 Transport Approaches

- **Boltzmann equation for HQ phase-space distribution f_Q**

$$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{p}}{\omega_{\boldsymbol{p}}}\frac{\partial}{\partial \boldsymbol{x}} + \boldsymbol{F}\frac{\partial}{\partial \boldsymbol{p}}\right] f_{\mathrm{Q}}(t,\boldsymbol{x},\boldsymbol{p}) = C[f_{\mathrm{Q}}]$$

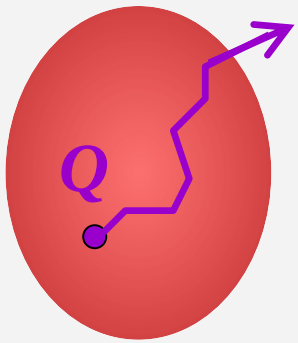
- explicit simulation of medium (quasi-) particles in collision term
- semi-classical approximation

- **Fokker-Planck equation**

$$\frac{\partial}{\partial t} f_Q(t,\boldsymbol{p}) = \frac{\partial}{\partial p_i}\left\{A_i(\boldsymbol{p}) f_Q(t,\boldsymbol{p}) + \frac{\partial}{\partial p_j}[B_{ij}(\boldsymbol{p}) f_Q(t,\boldsymbol{p})]\right\}$$

- follows from Boltzmann with $\mathbf{p^2 \sim m_Q T >> q^2 \sim T^2}$; ok for $\mathbf{m_Q/T \geq 5}$
- **does not require quasi-particle medium**
- well suited for strongly coupled medium where $\mathbf{E_{th} \leq \Gamma_{q,Q} < m_Q}$

2.3 Transport Coefficients



$$\frac{\partial}{\partial t} f_Q(t,p) = \gamma \frac{\partial}{\partial p_i}[p_i f_Q(t,p)] + D_p \Delta_{\vec{p}} f_Q(t,p)$$

Fokker-Planck

thermalization rate

$\gamma p = \int d^3q\, w_Q(q,p)\, q$

$\sim \int |T_{Qj}|^2 (1-\cos\theta)\, f^j$

momentum diffusion coefficient

$D_p = \int d^3q\, w_Q(q,p)\, q^2$

- thermal relaxation time $\tau_Q = 1/\gamma$
- Einstein relation: $T = D_p / \gamma m_Q$
- **spatial diffusion constant**: $\mathcal{D}_s = T/\gamma m_Q$, $\langle x^2\rangle - \langle x\rangle^2 = 6\, \mathcal{D}_s\, t$
- relation to bulk medium: $\mathcal{D}_s\, (2\pi T) \sim \eta/s\, (4\pi)$

- Key ingredient: **heavy-light scattering amplitude** T_{Qj}

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- Non-perturbative Approach + Bulk Properties

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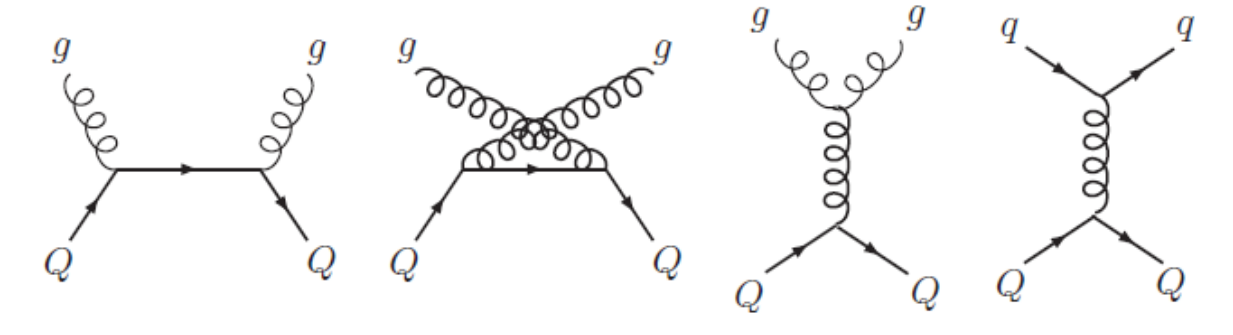
3.1 Perturbative QCD Approaches

3.1.1 Leading Order



[Svetitsky '88, Mustafa et al '98, Molnar et al '04, Zhang et al '04, Hees+RR '04, Teaney+Moore'04]

- gluon exchange screened by Debye mass:

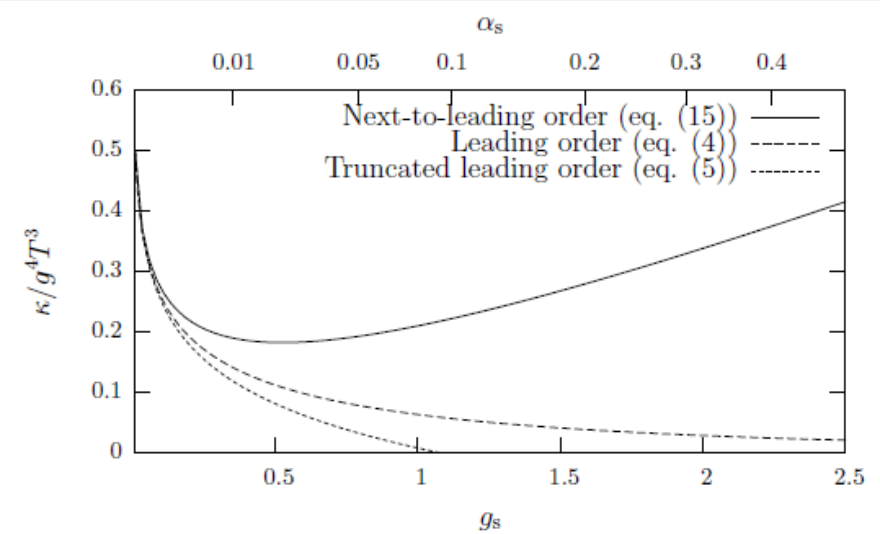
$$G(t) = \frac{1}{t} \rightarrow \frac{1}{t - \mu_D^2}, \quad \mu_D = gT$$

- dominated by forward scattering
- long thermalization time: $\tau_c \geq 20$ fm/c ($T \leq 300$MeV, $\alpha_s = 0.4$)

3.1.2 Next-to-Leading Order

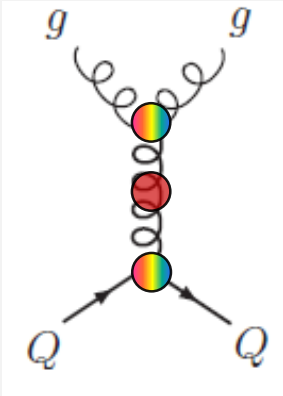
- bad convergence behavior (even for $\alpha_s \sim 0.1$)

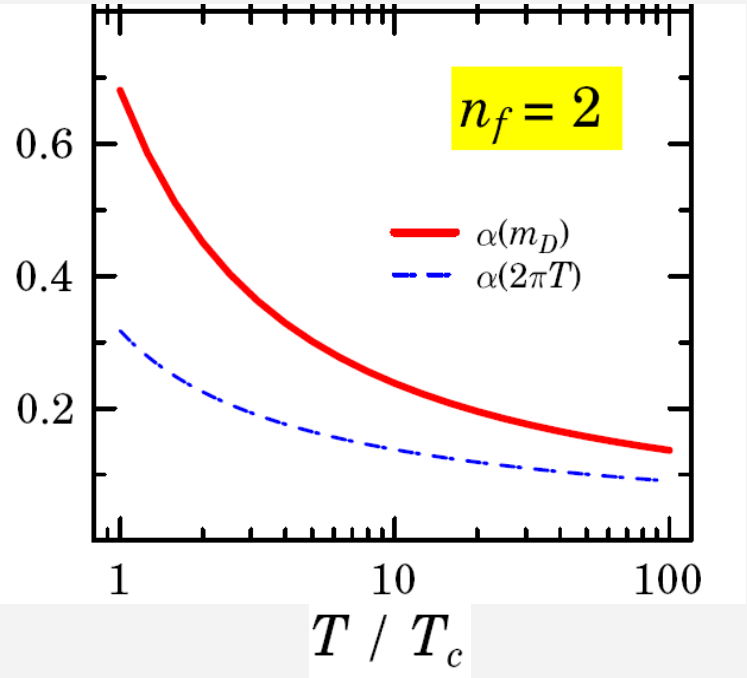
[Caron-Huot+Moore '08]



3.1.3 "Effective" pQCD with Running Coupling

$$\mathcal{M}_{Qj} \sim \frac{\alpha}{t} \rightarrow \frac{\alpha_{\rm eff}(t)}{t - \tilde{\mu}^2}$$





- run α_s to $m_D \sim gT$, rather than $2\pi T$
- reduced Debye mass $\tilde{\mu}^2 = \frac{1}{5}\mu_D^2$

- factor ~10 faster thermalization: $\tau_c \approx$ 2-3 fm/c
- perturbative regime? Need to resum large diagrams…

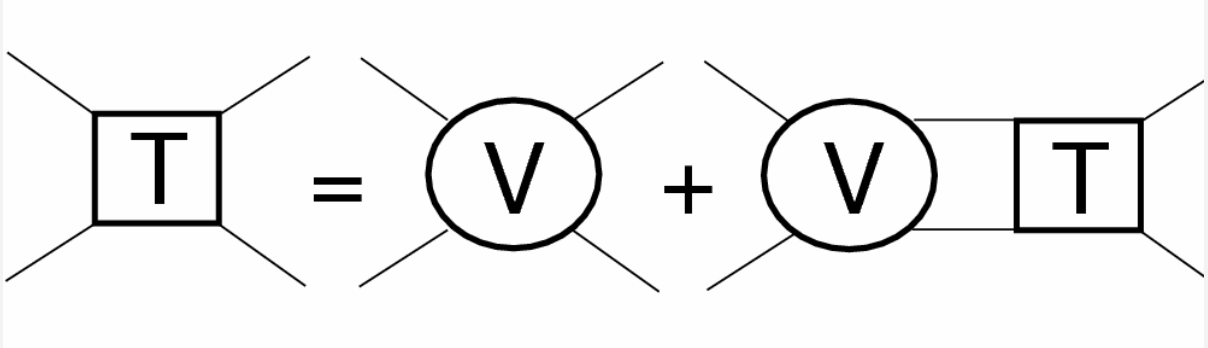
[Peshier '07, Gossiaux et al '08, BAMPS '14]

3.2 Nonperturbative Approach: T-Matrix

- **Scattering equation**

i=Q,q,g

j=Q,q,g

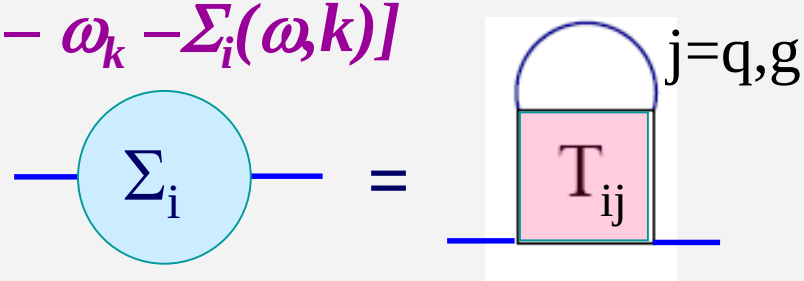


- **Strong** coupling → resummation:

$$T_{ij} = V_{ij} + \int V_{ij} D_i D_j T_{ij}$$

- Thermal parton **propagators**: $D_i = 1 / [\omega - \omega_k - \Sigma_i(\omega,k)]$

- Parton **self-energies** → **self-consistency**:



- $q \sim T << m_Q \Rightarrow q_{(4)}^2 = q_0^2 - q^2 \approx -q^2$

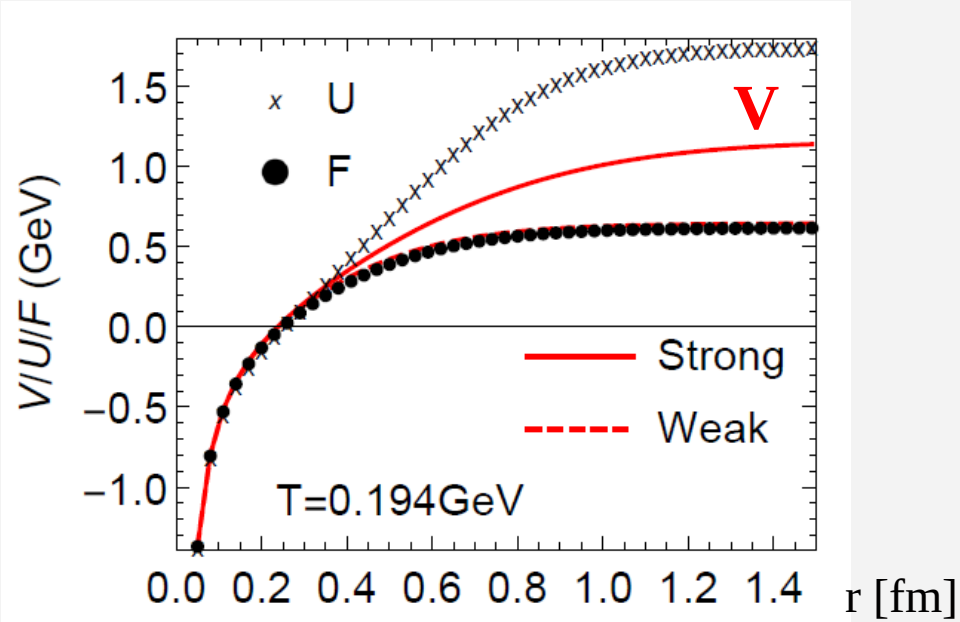
  ⇒ **3D** reduction of Bethe-Salpeter equation

- **In-medium potential V?**

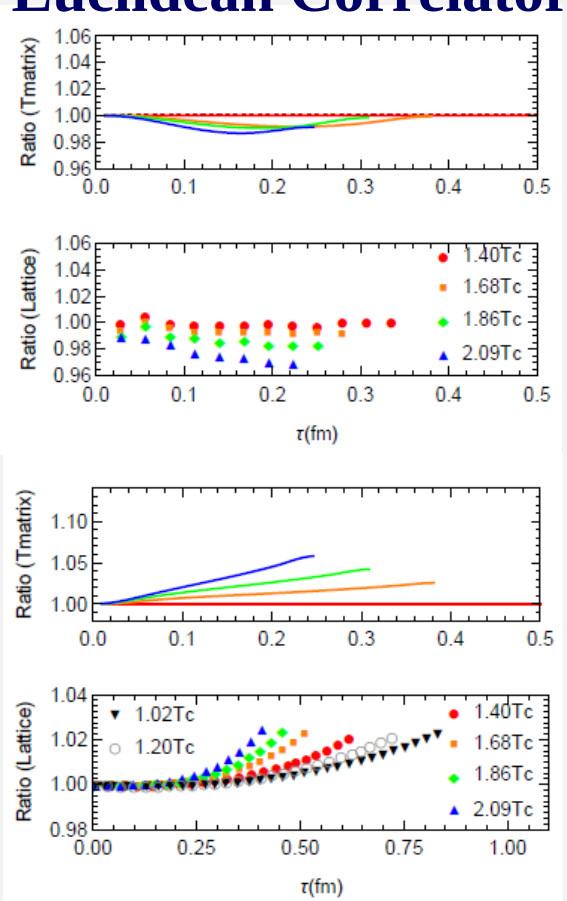
[Mannarelli, Cabrera, Riek, Liu,…'05-'18]

3.3.2 Potential Extraction from Lattice Data

- **Free Energy** $$F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta}\ln\left(\int_{-\infty}^{\infty} d\omega\,\sigma(\omega, r_1 - r_2)\, e^{-\beta\omega}\right)$$
- **$Q\bar{Q}$ Spectral Function** $$\sigma(\omega, r) = \frac{1}{\pi}\frac{(V+\Sigma)_I(\omega)}{(\omega - (V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$$



Euclidean Correlators



- Large imaginary parts ⇒ **V > F**
- **Remnants of confining force above T_c!**

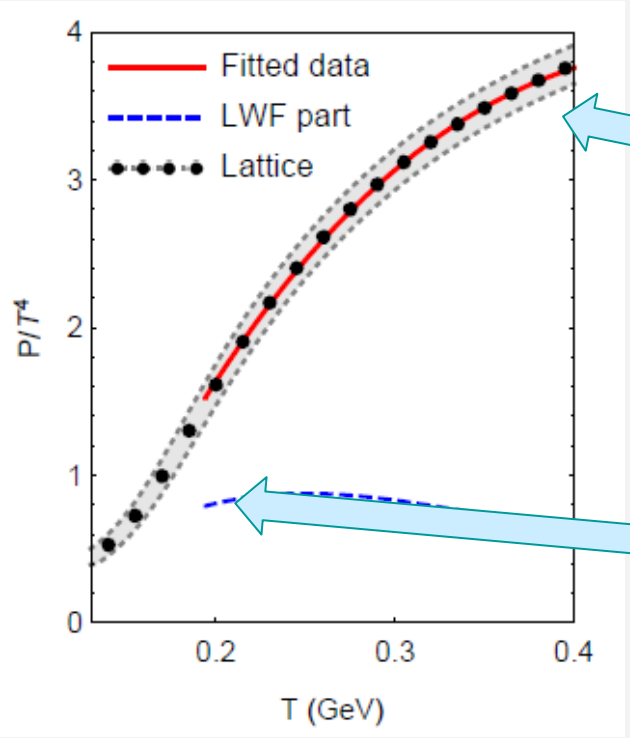
3.3.3 QGP Equation of State + Spectral Functions

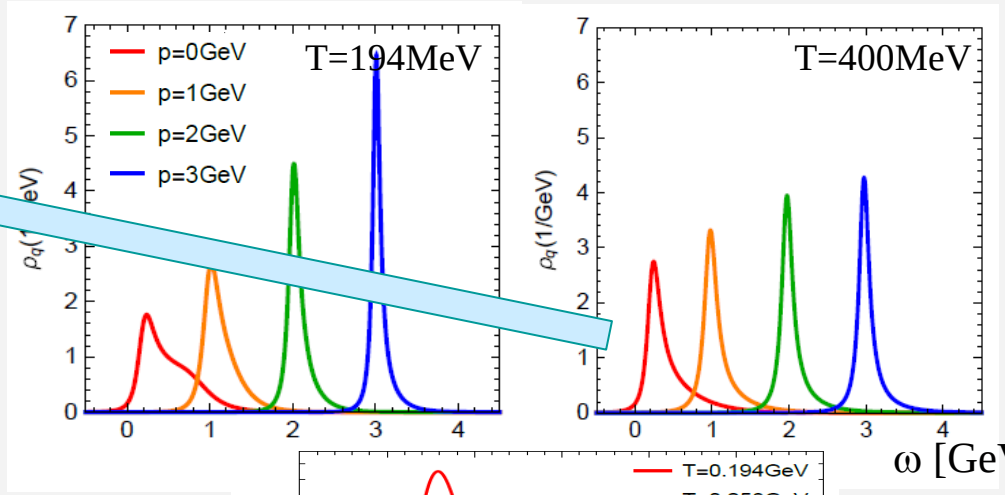
Thermodynamic Potential

$$\Omega = \mp \frac{-1}{\beta} \sum_n \mathrm{Tr}\{\ln(-G^{-1}) + (G_0^{-1} - G^{-1})G\} \pm \Phi$$

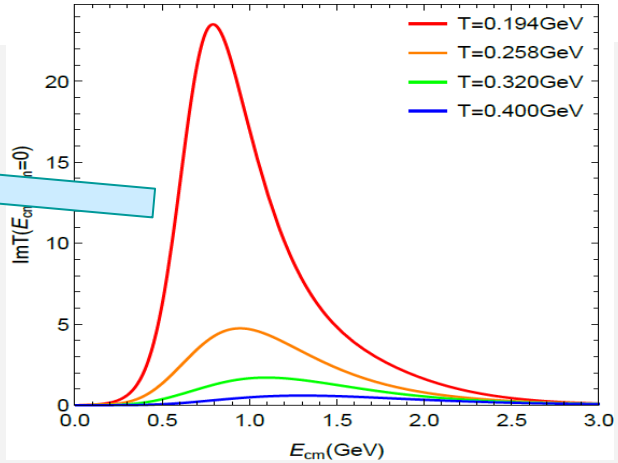
Selfconsistent SFs

$$G = G_0 + G_0 \Sigma G \qquad \Sigma = GT \qquad T = V + VGGT$$





Quark spectral functions



"Meson" **T**-matrix

- **Near T_c light partons melt + broad hadronic resonances emerge**

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4.) Hadronization of Heavy Quarks

- **Fragmentation**

  $\mathbf{c \rightarrow D, D^*, D_s, \Lambda_c, \ldots}$

  determined by empirical fragmentation functions $\mathbf{D_{c \rightarrow H_c}(z)}$;
  in principle universal ("vacuum": $\mathbf{e^+e^-}$ collisions or high $\mathbf{p_T}$)

- **Coalescence / Recombination**

  $\mathbf{c + q\,(s) \rightarrow D\,(D_s), D^*, \ldots}$; $\mathbf{c + q + q\,(s) \rightarrow \Lambda_c\,(\Xi_c), \ldots}$

  depends on environment (phase space of surrounding anti-/quarks)

-- instantaneous coalescence (based on spatial wave functions)

-- resonance recombination (momentum space)

4.2 Heavy-Quark Recombination

- **Instantaneous Coalescence Models (ICMs)** **[Hwa '80, Likhoded et al '83, … Greco et al + Fries et al '03,…]**

$$f_h(\boldsymbol{p}'_h) = \int \Big[\prod_i d\boldsymbol{p}_i f_i(\boldsymbol{p}_i)\Big] W(\{\boldsymbol{p}_i\})\delta(\boldsymbol{p}'_h - \sum_i \boldsymbol{p}_i)$$

$$W_s = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 \boldsymbol{k}^2}$$

Wigner function, **σ** ~ radius parameter for each hadron **h**

- energy not conserved → challenge for chemical + thermal equilibrium

- **Resonance Recombination Model (RRM)** **[Ravagli et al '07, He et al '12]**

- derived from Boltzmann equation

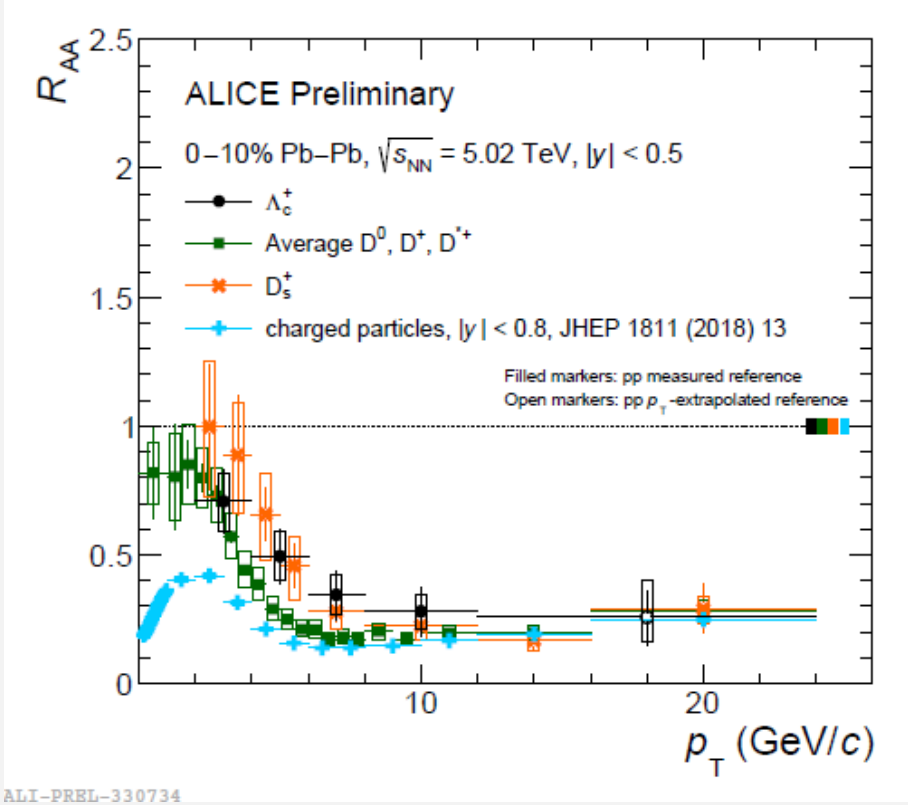
$$f_M(\vec{x},\vec{p}) = \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^3} f_q(\vec{x},\vec{p}_1) f_{\bar{q}}(\vec{x},\vec{p}_2)\ \sigma_M(s) v_{\rm rel}(\vec{p}_1,\vec{p}_2)\delta^3(\vec{p}-\vec{p}_1-\vec{p}_2)$$

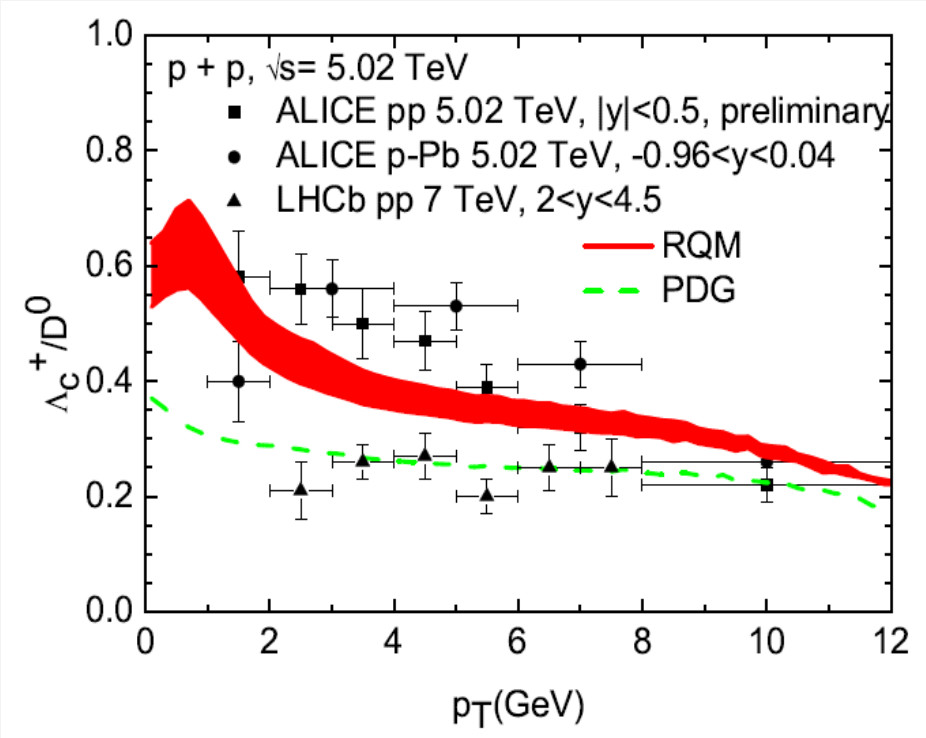
$\sigma_M(s)\, v_{rel} \sim |T_{Qj}|^2$: resonant heavy-light scattering amplitude

→ direct connection to ***T***-matrix interactions in QGP near $\mathbf{T_c}$

→ compatible with equilibrium limits

4.3 Heavy-Flavor Hadro-Chemistry





- Distinct pattern of charm-hadron production up to **p_T ~ 10 GeV**
- **D_s** enhanced due to strangeness equilibration in QGP

- Relevance of recombination in **pp**
- sensitive to charm-baryon spectrum ("missing" resonances in PDG)

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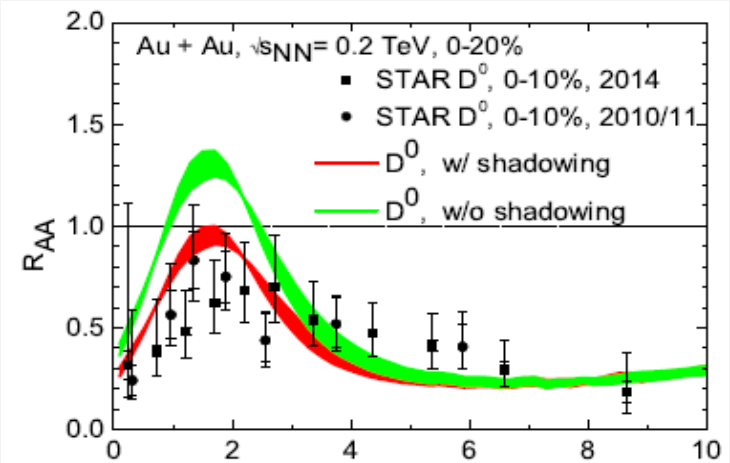
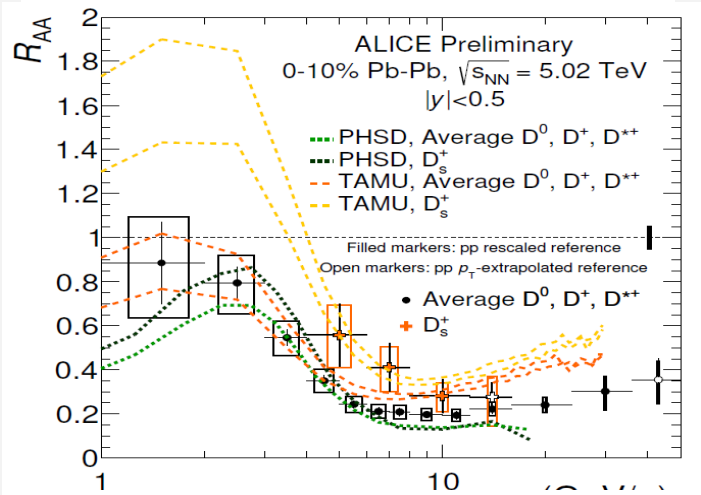
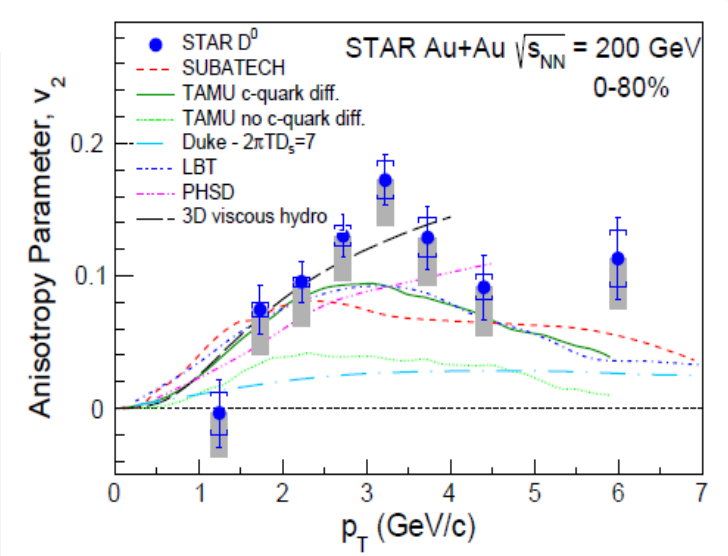
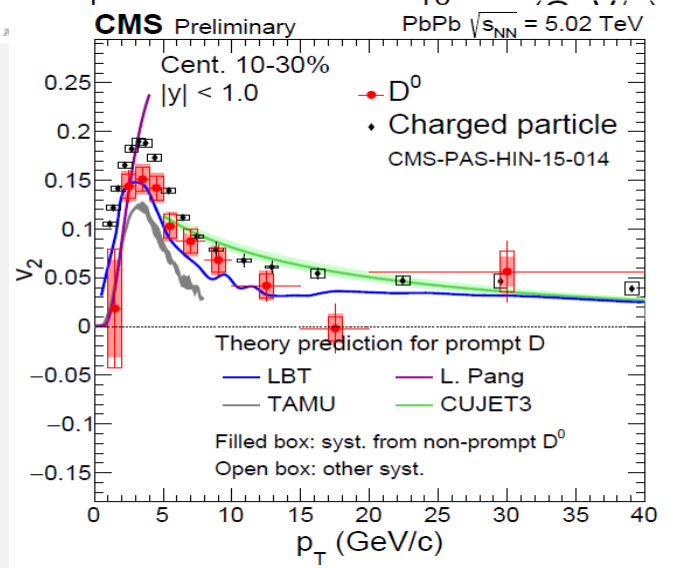
5.1 Heavy-Flavor Data at RHIC + LHC





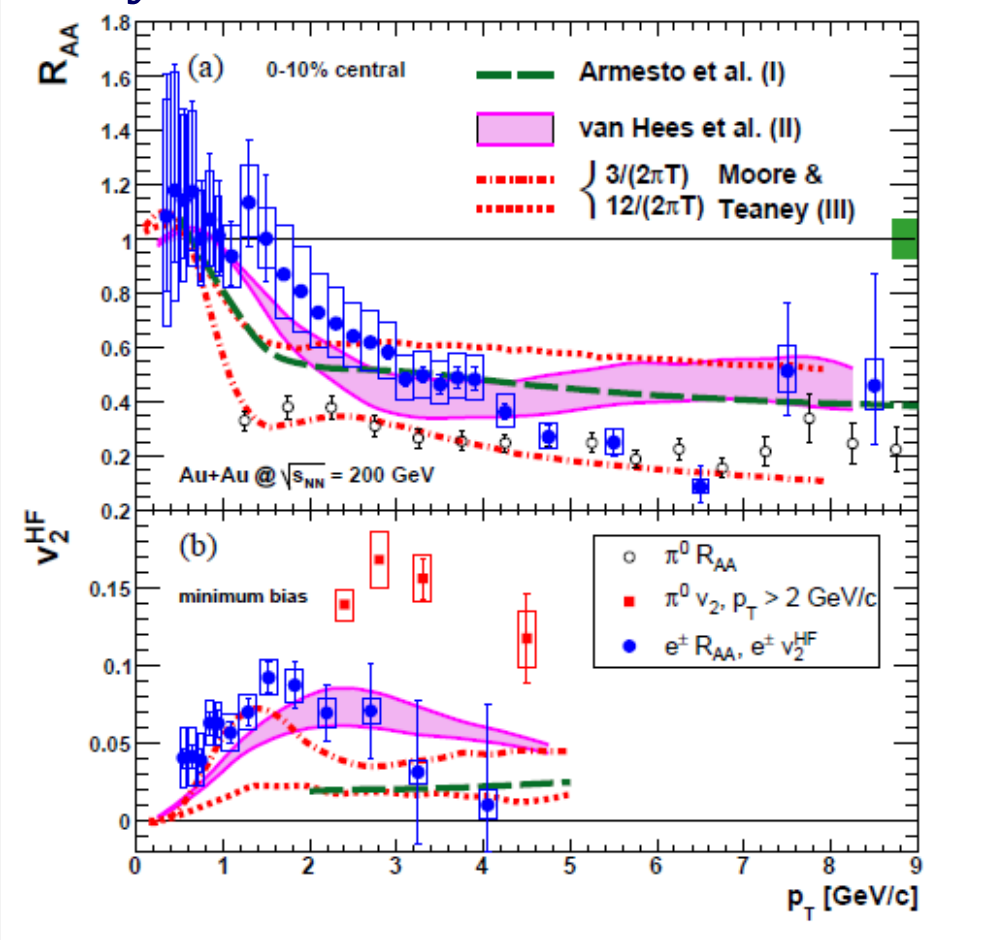




- "Flow bump" in $\mathbf{R_{AA}}$ + large $\mathbf{v_2}$ ↔ strong coupling near $\mathbf{T_{pc}}$ (**recombination**)
- High-precision $\mathbf{v_2}$: transition from elastic to radiative regime

5.2 The v_2 – R_{AA} Correlation Story

Heavy-Flavor Electrons [PHENIX '07]



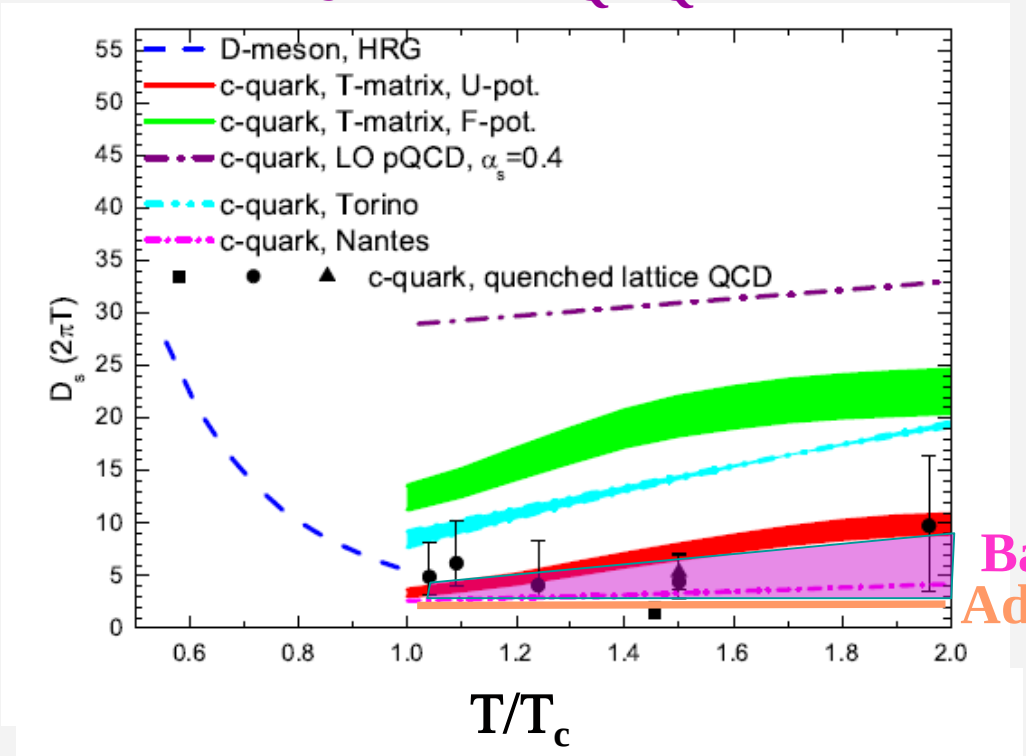
- Challenge: Large elliptic flow but not too large suppression in $\mathbf{R_{AA}}$

  → better met by **non**-perturbative than perturbative interactions

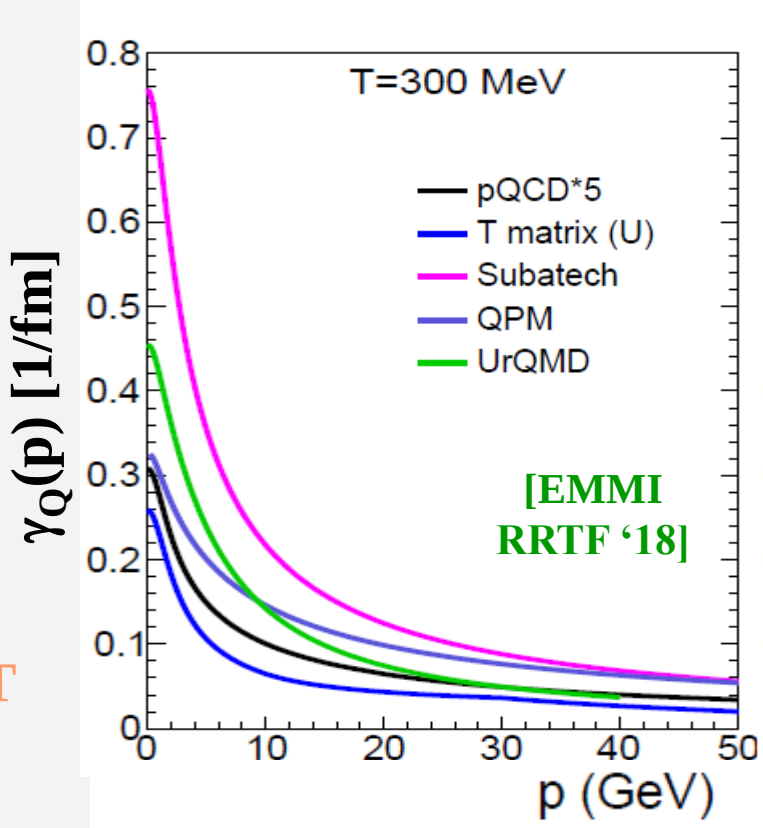
[van Hees et al '05] [Moore+Teaney '05, Armesto et al '05]

5.3 Charm Diffusion Coefficient

$\mathcal{D}_s = T / [m_Q \, \gamma_Q(p=0)]$



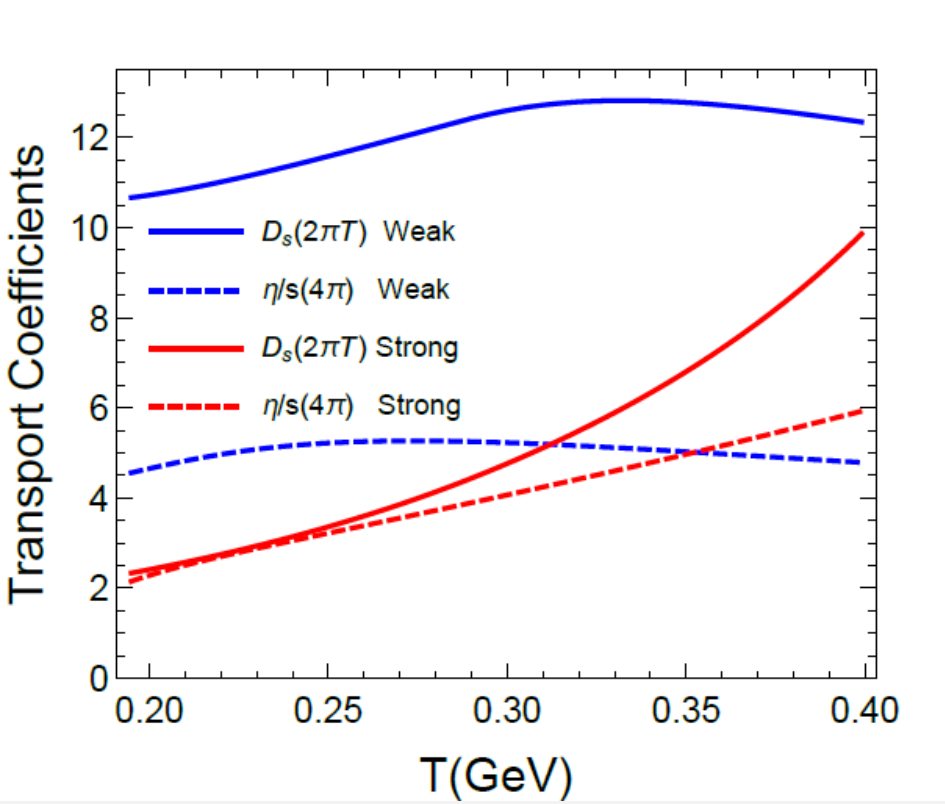
Bayes.
AdS/CFT

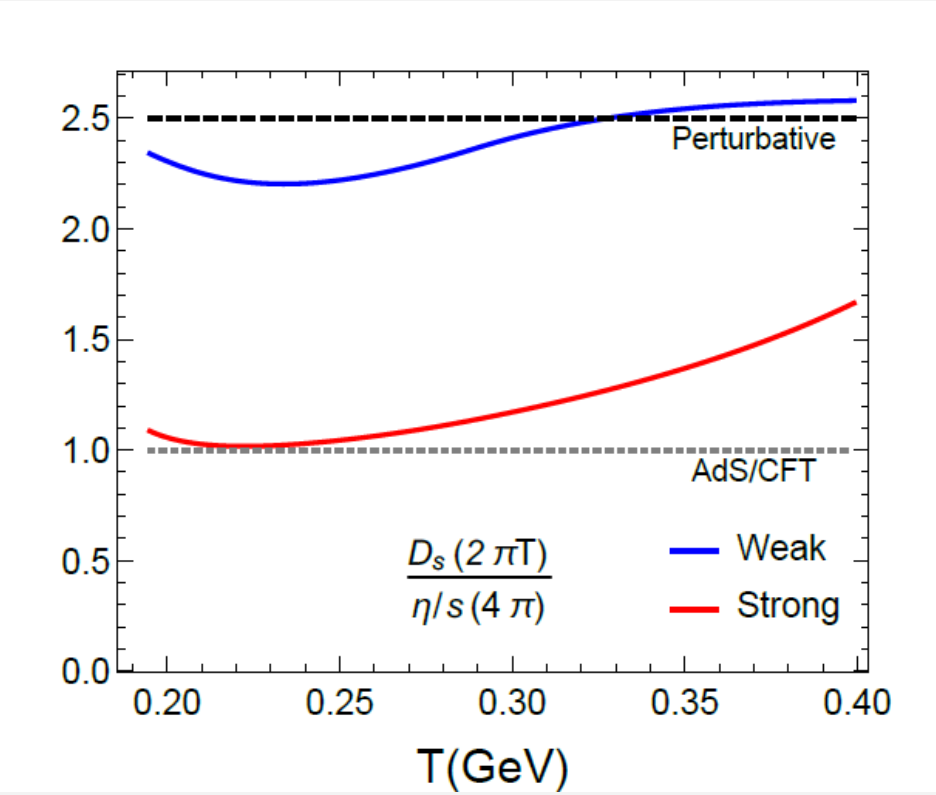


- suggests minimum of $\mathcal{D}_s(2\pi T) \sim 2\text{-}4$ near T_{pc}
- scatt. rate: $\Gamma_{coll} \sim 3/\mathcal{D}_s \sim 1$ GeV – **no light quasi-particles!**
- Importance of **p**-dependence ($\mathcal{D}_s$ not the full story)

5.4 Heavy-Flavor vs. Bulk Transport

Heavy-Quark Diffusion and Viscosity





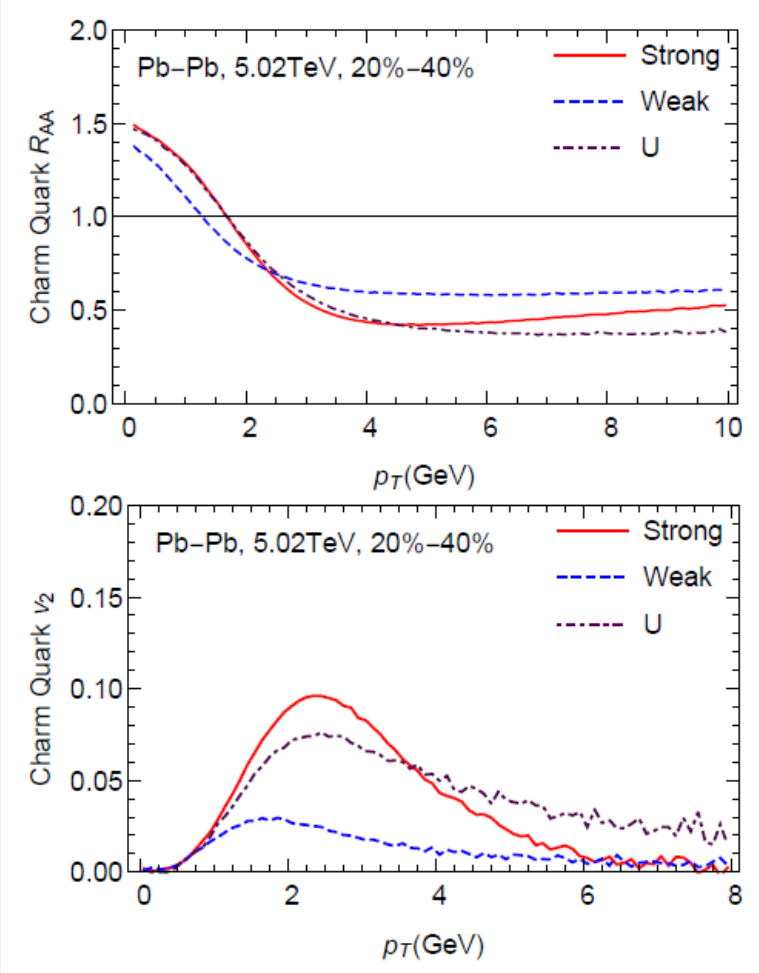
- Strongly coupled: **$(2\pi T)\ D_s \sim (4\pi)\ \eta/s$**
- Perturbative: **$(2\pi T)\ D_s \sim 5/2\ (4\pi)\ \eta/s$**
- Transition as **T** increases

[Liu+RR '19]

6.) Summary

- Open heavy flavor direct probe of in-medium QCD force ($\mathbf{m_Q >> T, \Lambda_{QCD}}$)
- Interaction strength well beyond perturbative needed
  $\rightarrow$ utilize lattice-QCD "data" to constrain heavy-quark potential
- Remnants of confining force drive the properties of sQGP as strongly coupled quantum liquid for $\mathbf{T \sim 1\text{-}2\ T_c}$
- Quantum many-body theory connects:
  (i) small $\mathcal{D}_s$, η/s,… (ii) melting light partons (iii) hadron formation
- Future heavy-flavor data (incl. bottom!) will provide ample constraints to quantify the **T**- and **p**-dependent transport and hadronization mechanisms in sQGP

4.6.2 Sensitivity to Heavy-Quark Transport



[He et al. '18]

- Strongly-coupled potential gives max. low-$\mathbf{p_T}$ **c**-quark $\mathbf{v_2}$
- Weakly-coupled potential (~ free energy) ruled out

4.4 Hamiltonian Approach to QGP

- **In-Medium Hamiltonian** with ``bare" 2-body interactions

$$H = \sum \varepsilon_i(\mathbf{p})\psi_i^\dagger(\mathbf{p})\psi_i(\mathbf{p}) + \psi_i^\dagger(\frac{\mathbf{P}}{2} - \mathbf{p})\psi_j^\dagger(\frac{\mathbf{P}}{2} + \mathbf{p})V_{ij}^a\psi_j(\frac{\mathbf{P}}{2} + \mathbf{p}')\psi_i(\frac{\mathbf{P}}{2} - \mathbf{p}')$$

  - effective in-medium mass $\varepsilon_i(\mathbf{p}) = \sqrt{M_i^2 + \mathbf{p}^2}$

- Interaction ansatz: **Cornell potential** with relativistic corrections

$$V_{ij}^a(\mathbf{p}, \mathbf{p}') = \mathcal{R}_{ij}^{\mathcal{C}}\mathcal{F}_a^{\mathcal{C}}V_{\mathcal{C}}(\mathbf{p} - \mathbf{p}') + \mathcal{R}_{ij}^{\mathcal{S}}\mathcal{F}_a^{\mathcal{S}}V_{\mathcal{S}}(\mathbf{p} - \mathbf{p}')$$

  - **color-Coulomb** and **string** ("**confining**") interaction
  - decent spectroscopy in vacuum

[Liu+RR '16]

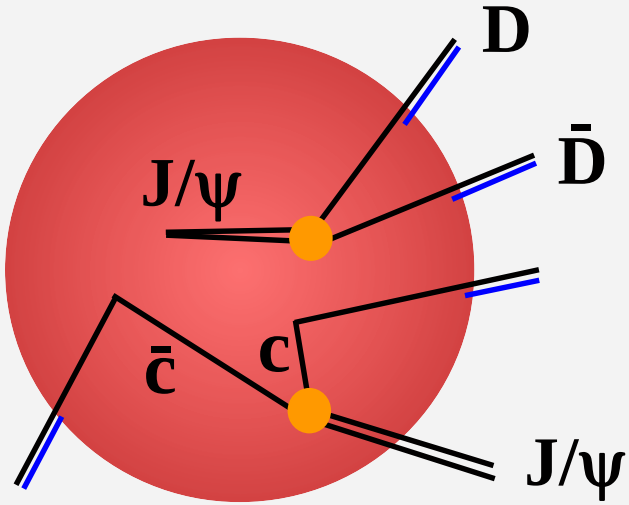
- **Implement into Brueckner / Luttinger-Ward-Baym approach**

2.1 Quarkonium Transport in Heavy-Ion Collisions

[PBM+Stachel '00, Thews et al '01, Grandchamp+RR '01, Gorenstein et al '02, Ko et al '02, Andronic et al '03, Zhuang et al '05, Ferreiro et al '11, …]

- **Inelastic reactions:**

  detailed balance: $J/\psi + g \rightleftarrows c + \bar{c} + X$

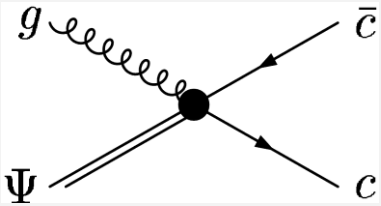


- **Rate equation:**

$$\frac{dN_\psi}{d\tau} = -\Gamma_\psi \left[N_\psi - N_\psi^{eq} \right]$$

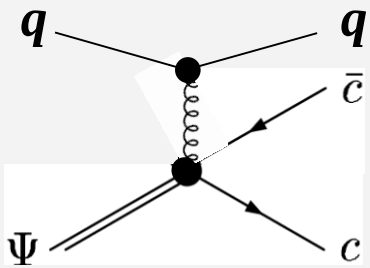
- **Transport coefficients**
  - **Equilibrium limit** $N_\psi^{eq}(m_\psi, T; N_{cc})$
  - **Reaction Rate** $\Gamma_\psi (E_B(T))$

"Strong" binding $E_B \geq T$



- gluo-dissociation ("singlet-to-octet")

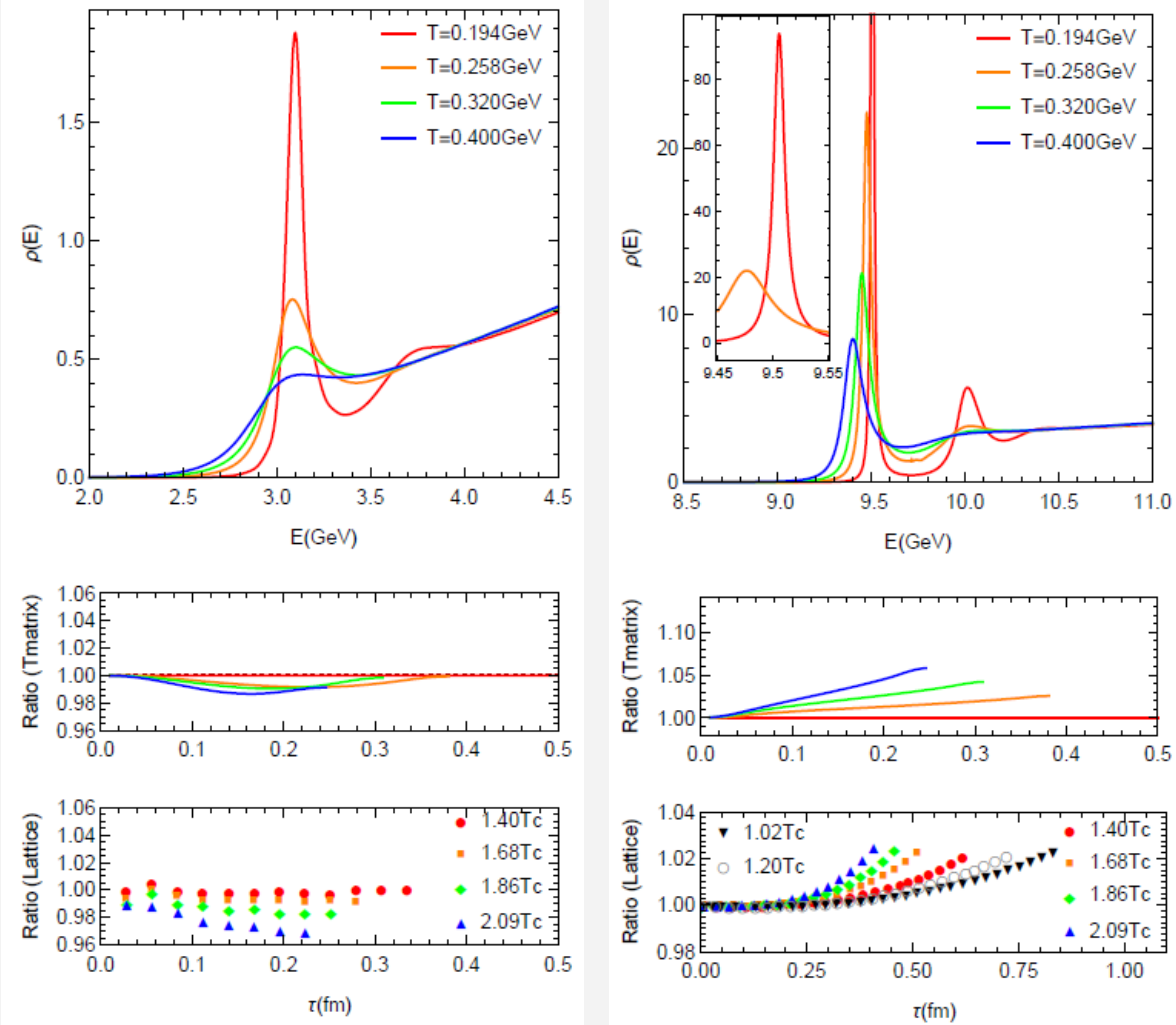
"Weak" binding $E_B < m_D$



- "quasi-free"/ Landau damping

4.3 Quarkonium Spectral Functions + Correlators

Charmonium **Bottomonium**

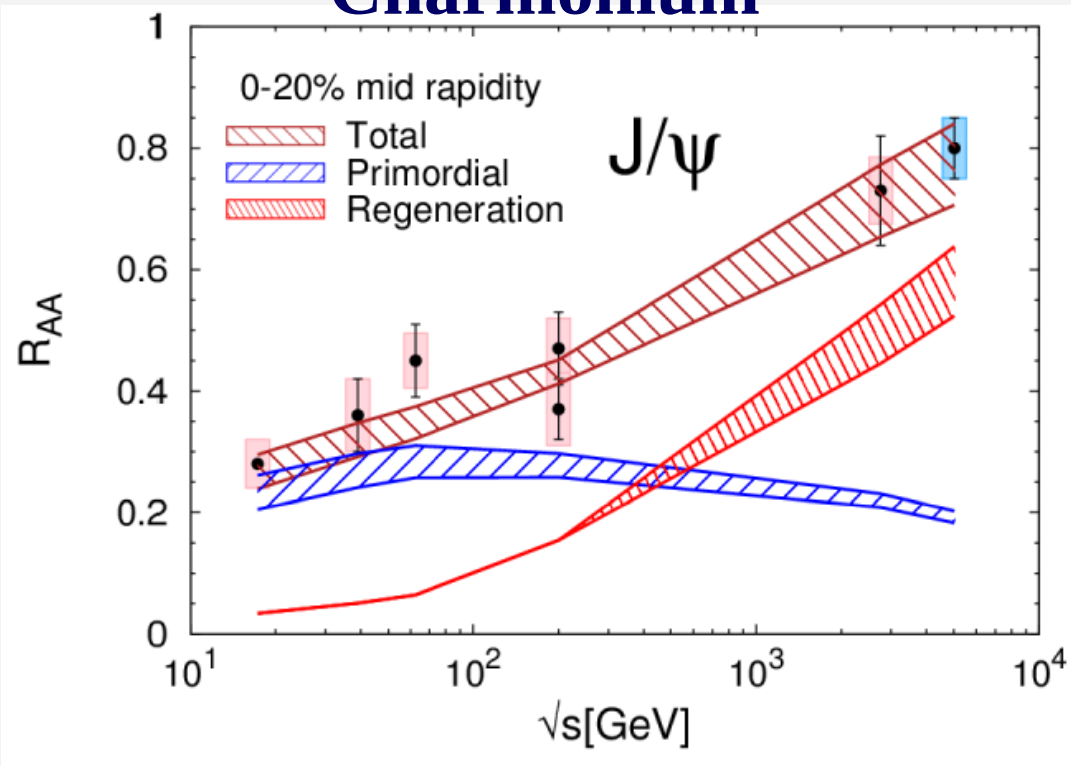


[S.Liu+RR '17]

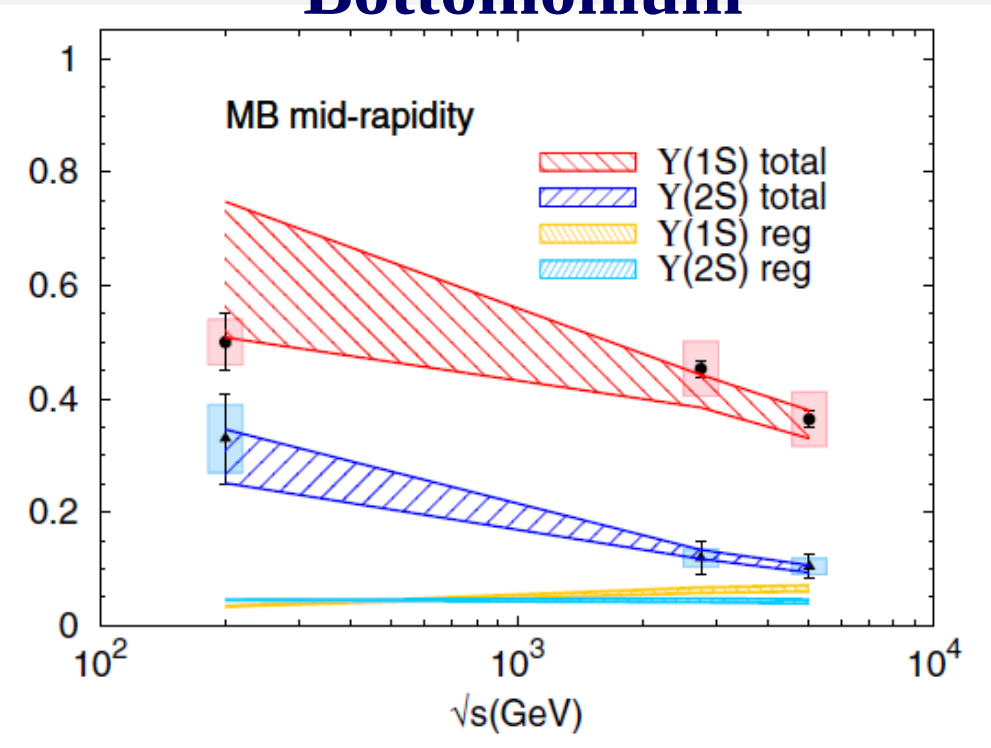
- **J/ψ / ϒ(1S/2S)** melting (**300/500/250MeV**) not inconsistent with pheno.

2.2 Excitation Functions: SPS - RHIC - LHC

Charmonium



Bottomonium



- Gradual **increase** of total **J/ψ** $\mathbf{R_{AA}}$
- Regeneration **and** suppression increase
- Regeneration concentrated at low $\mathbf{p_T}$!

- Gradual **suppression**
- Regeneration ($\mathbf{N_\Upsilon^{eq}}$) small
- Qualitative difference from **J/ψ**

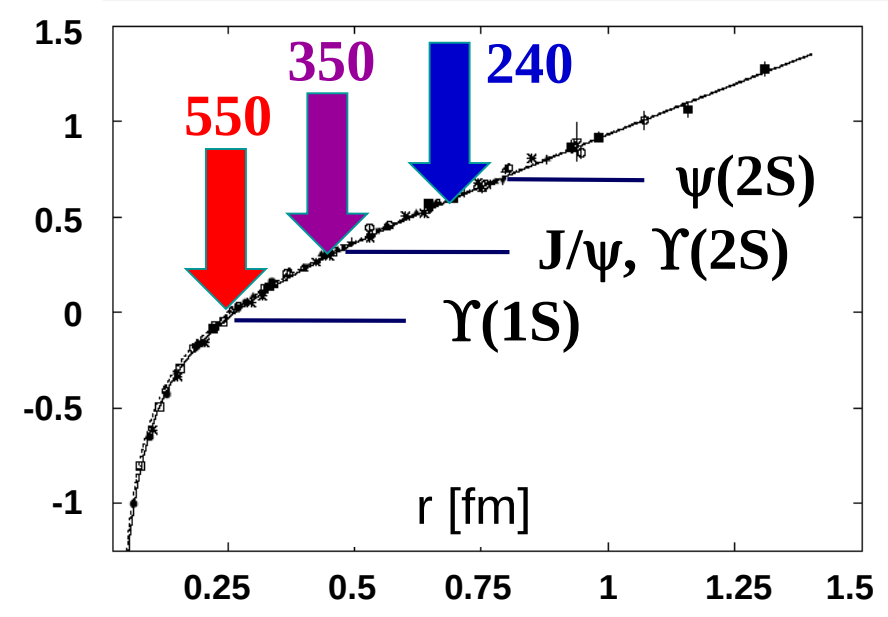
[**data**: **NA50, PHENIX, STAR, ALICE, CMS**]

2.4 Upshot of Quarkonium Phenomenology

Use temperature estimates from hydro/photons/dileptons to infer:

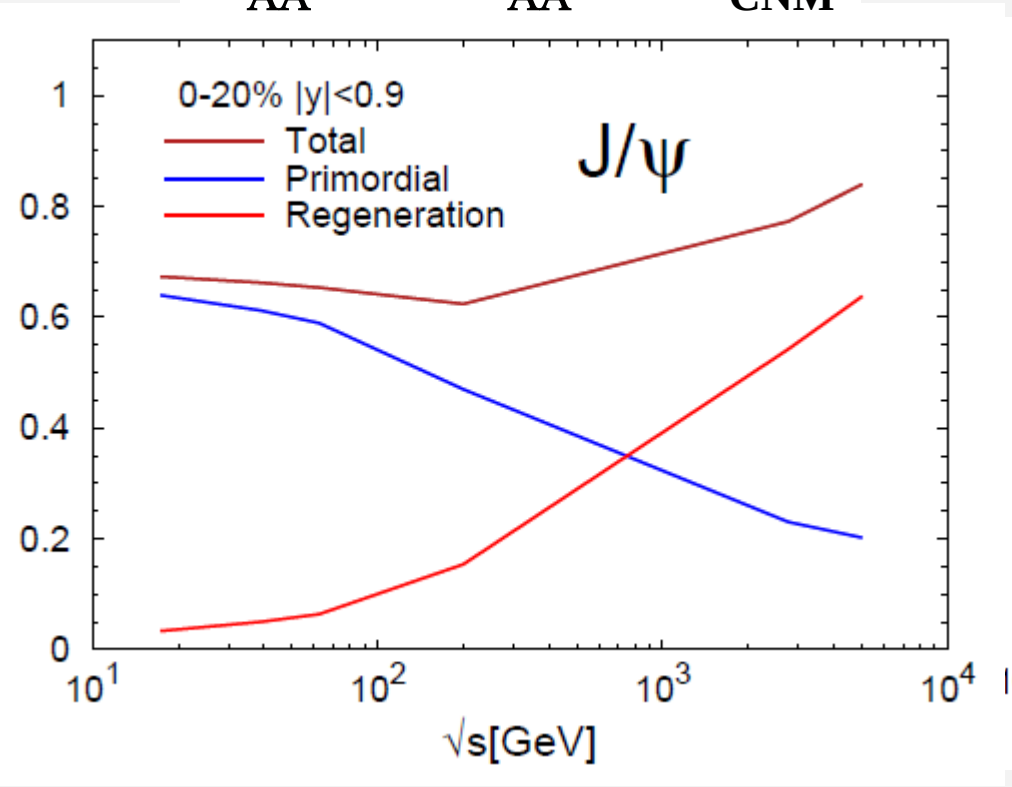
$$T_0^{SPS}(\sim 240) < T_{melt}(J/\psi, \Upsilon') \leq T_0^{RHIC}(\sim 350) < T_{melt}(\Upsilon) \leq T_0^{LHC}(\sim 550)$$

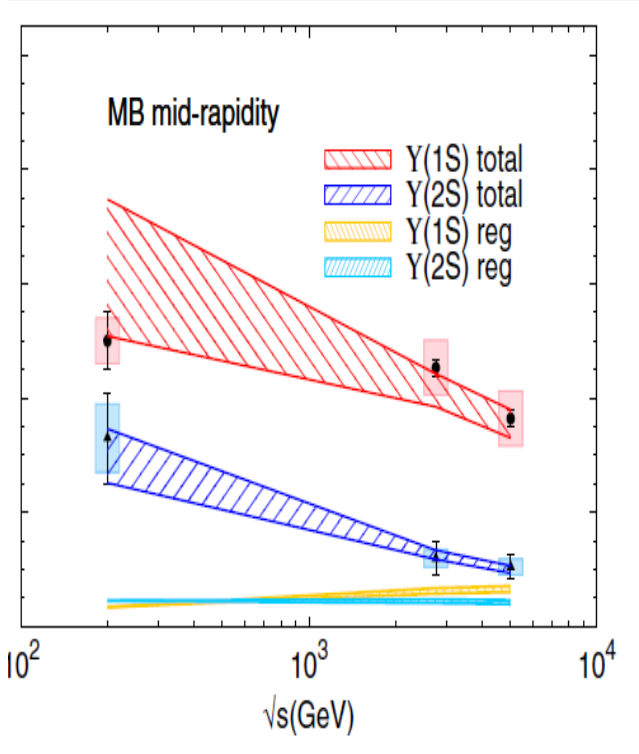
- **Remnants of confining force survive at SPS** [hold **J/ψ** together]
- **Confining force screened at RHIC+LHC** [“melts” **J/ψ + ϒ(2S)**]
- **Color-Coulomb screening at LHC** [**ϒ(1S)** suppression]
- **Thermalizing charm quarks recombine at LHC** [large **J/ψ** yield]



4.1 Divide out Cold-Nuclear-Matter Effects

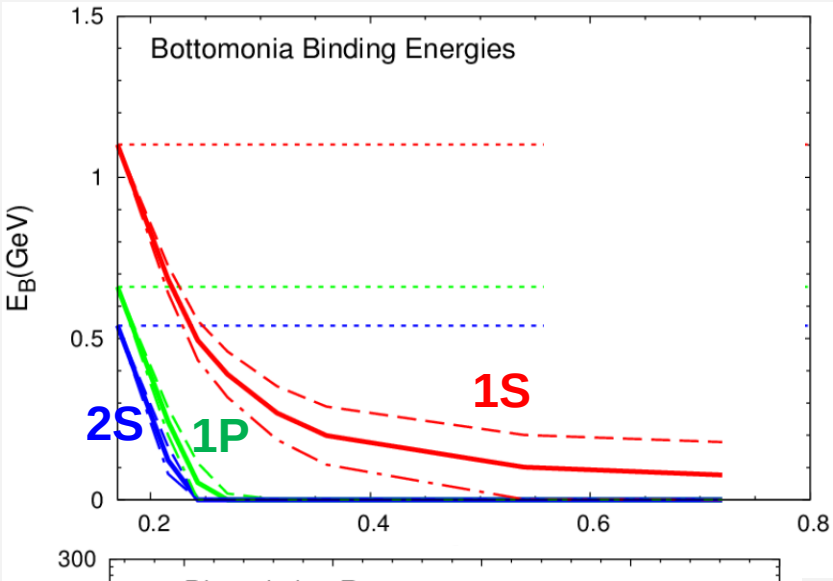
$$R_{AA}^{hot} \equiv R_{AA}^{tot} / S_{CNM}$$

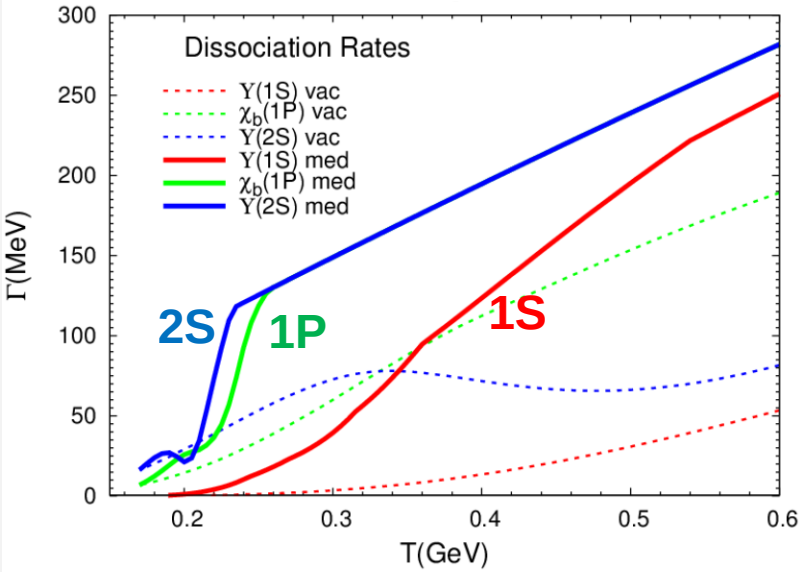


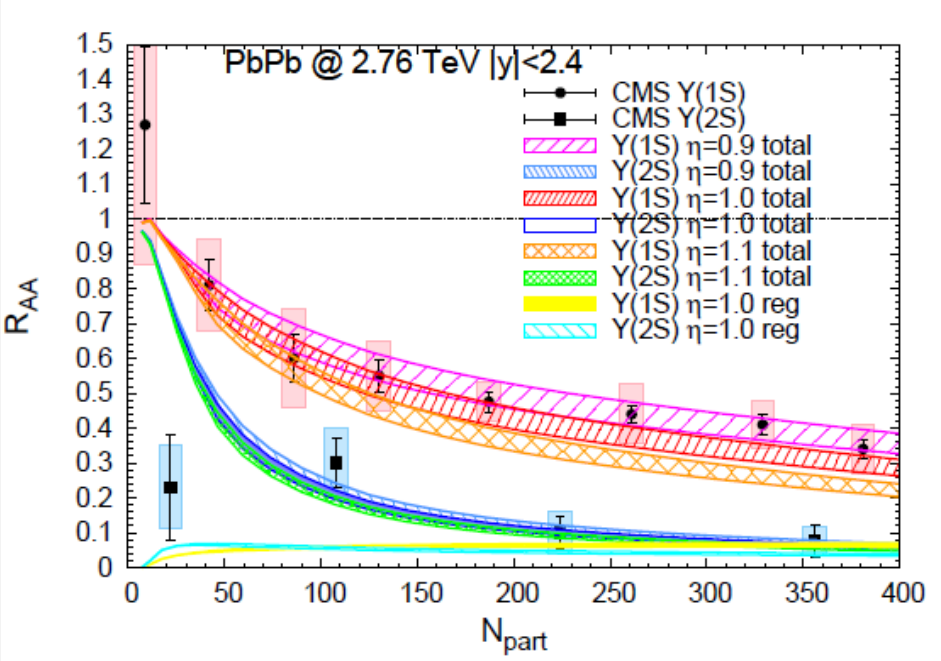


- **J/ψ** suppression at **SPS** mostly from feeddown (**$\sigma_{\psi N}$ ~7.5mb**), melts in the **RHIC → LHC** regime (not unlike **ϒ(2S)**)

2.2 In-Medium ϒ Suppression

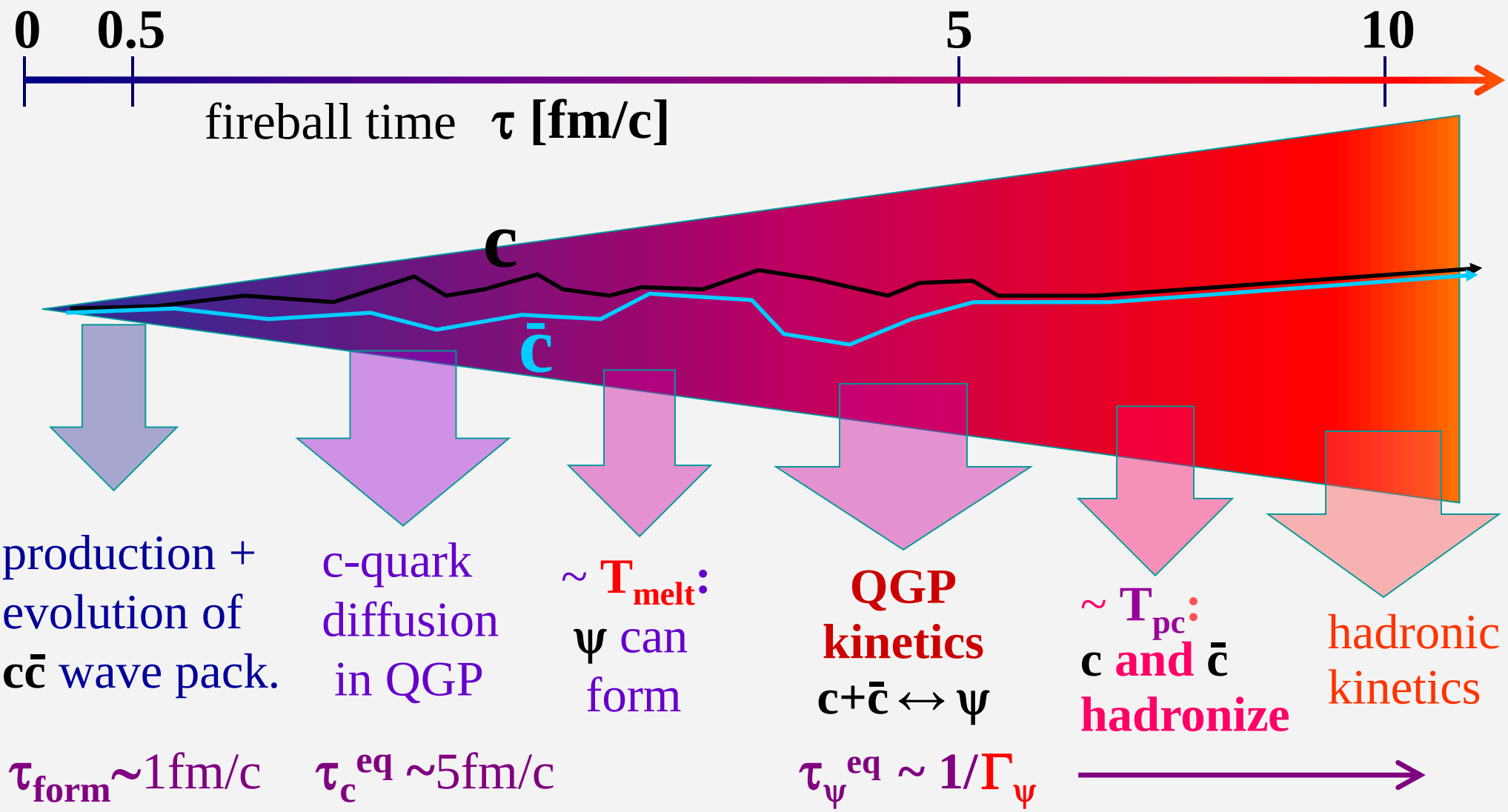






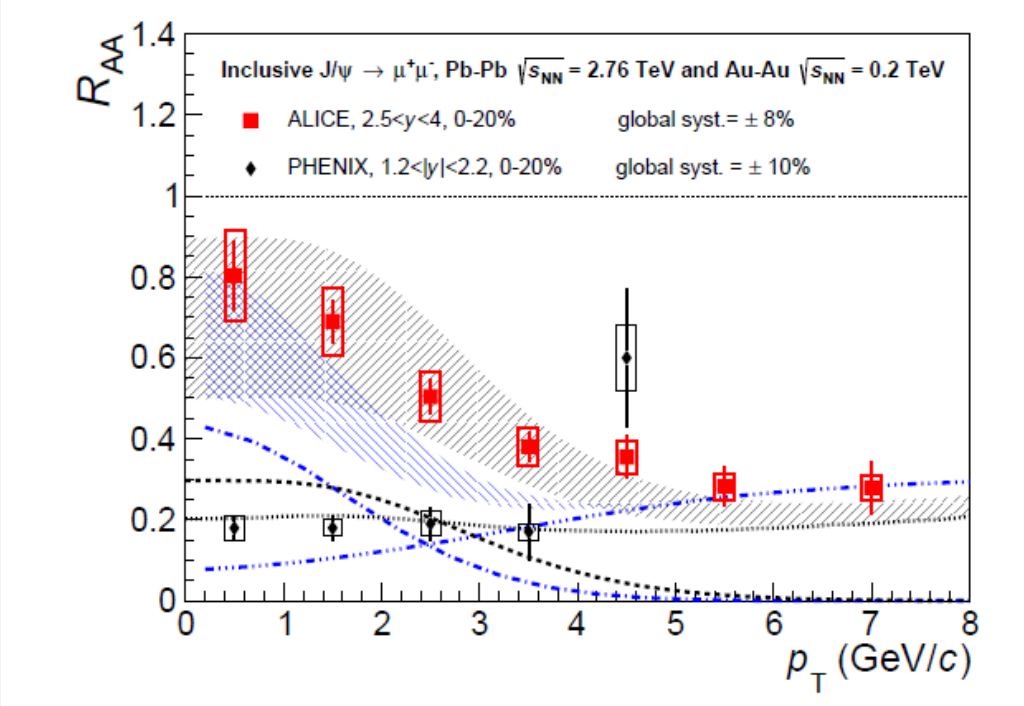
- Binding energies control reaction rates
- **ϒ(1S)** suppression **sensitive to $E_B(T)$**
- significant **regeneration** for **ϒ(2S)**

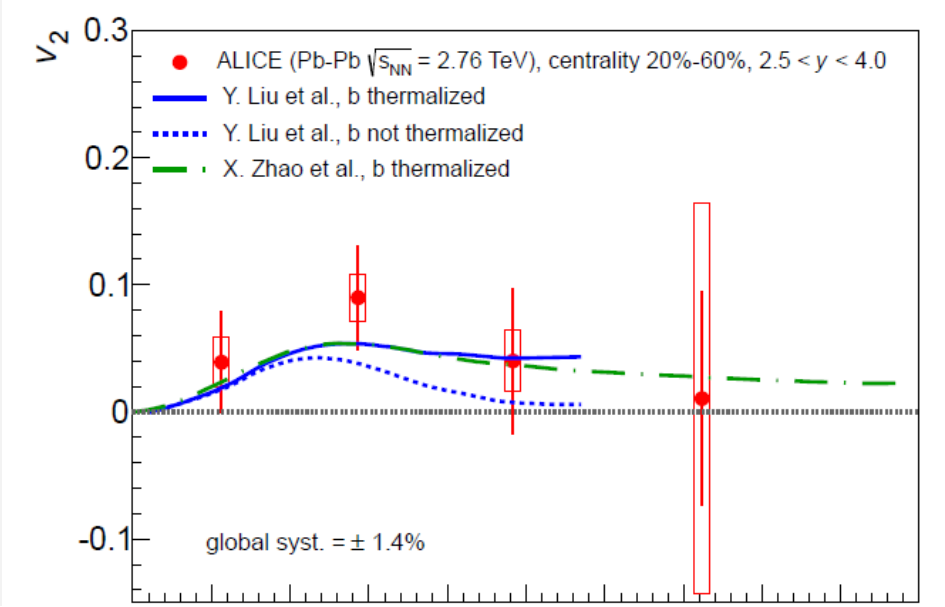
2.1 Quarkonium Transport in URHICs



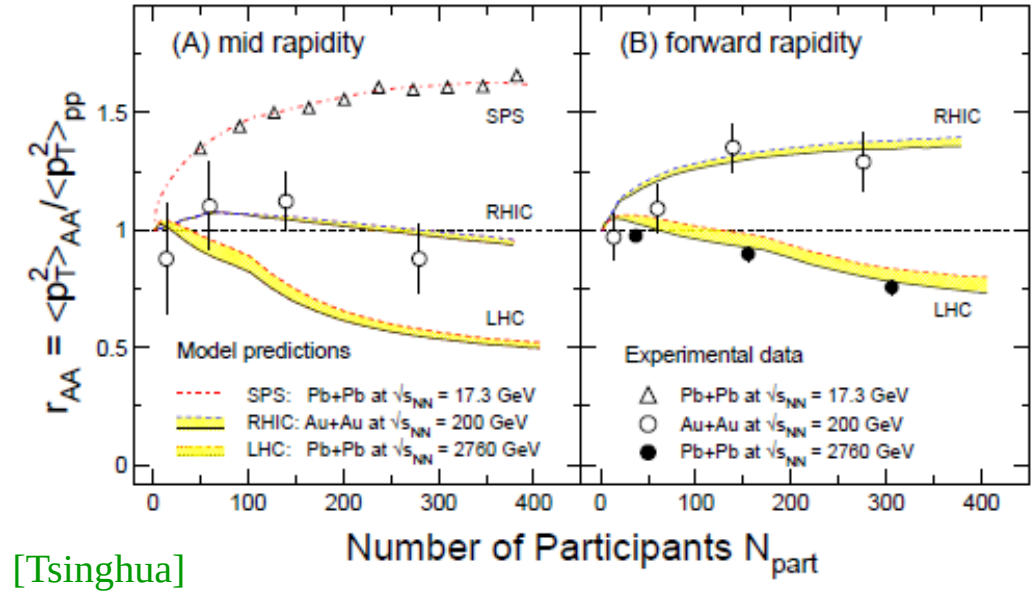
[Satz et al, Capella et al, Spieles et al, PBM et al, Thews et al, Grandchamp et al, Ko et al, Zhuang et al, Zhao et al, Chaudhuri, Gossiaux et al, Young et al, Ferreiro et al, Strickland et al, Brambilla et al, …]

3.3 Properties of Charmonium Excess





- excess concentrated at low p_T
- systematic softening of J/ψ p_T-spectra with increasing √s → nature of source changes

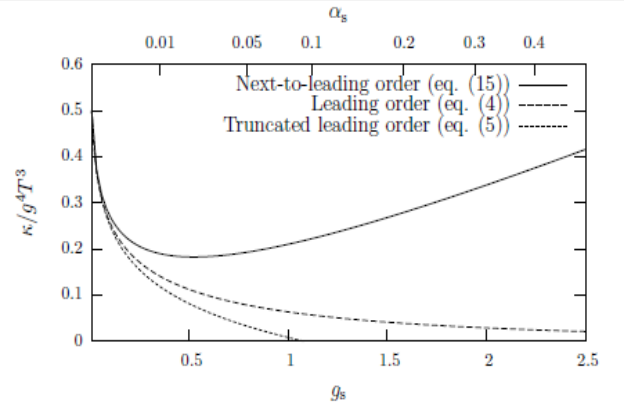


[Tsinghua]

3.3 Heavy-Quark Interactions in QGP

Minimal / Desirable Ingredients and Features

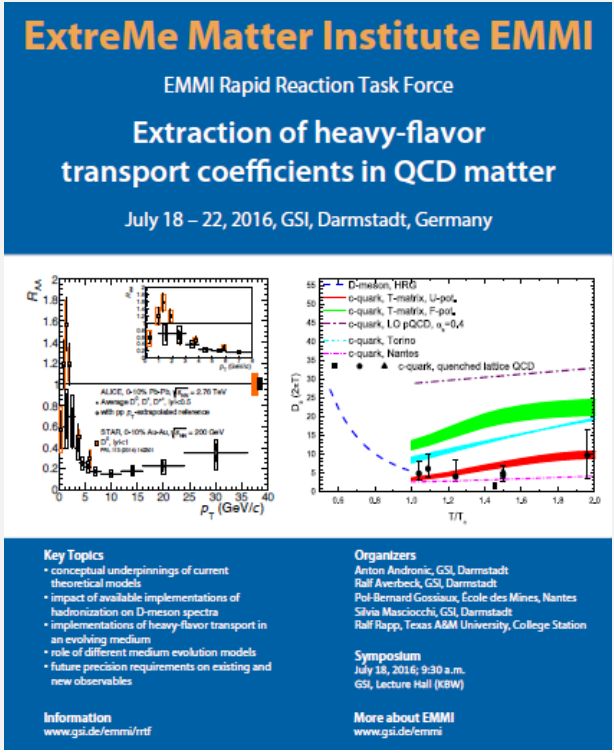
- **Microscopic** description of scattering **amplitude**
- Realistic in-medium **interaction kernel** (screening)
- **Nonperturbative** interactions
  (color-Coulomb / pQCD not enough)
- **Resummation** (strong coupling)
- **Hadronization** approaching T_c from above
  (bound states)
- Elastic + radiative processes (low / high p_T)
- Realistic **medium partons**:
  quasiparticles, widths, parton spectral functions, …? Equation of state?
- Quantitatively rooted in **constraints from lattice QCD**



[Caron-Huot+Moore '08]

3.4 Broader Theory Efforts

- **EMMI Rapid Reaction Task Force** **[NPA '18 in press]**
- HQ-jet working group
- Scrutinize components of open HF phenomenology
  - initial spectra, CNM + pre-equil. effects
  - bulk evolution, implementation of transport
  - **p**-, **T**-dep. transport coeffs. (QGP+hadronic)
  - hadronization mechanisms



- Example:

use common transport coefficient (**5 x** pQCD) in different bulk medium evolution and hadronization models

$$\Rightarrow \mathcal{D}_s(2\pi T) < 6$$