The Improved Opacity Expansion

higher order results and the k_t differential spectrum

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based on arXiv:2004.02323 [hep-ph] and ongoing work

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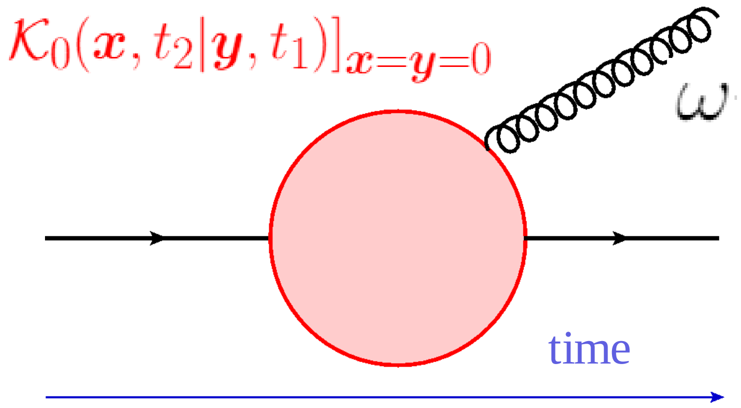
The medium induced gluon spectrum

$$\omega\frac{dI}{d\omega}=\frac{\alpha_s C_R}{\omega^2}\int_0^\infty dt_2\int_0^{t_2}dt_1\,\boldsymbol{\partial_x}\cdot\boldsymbol{\partial_y}\left[\mathcal{K}(\boldsymbol{x},t_2|\boldsymbol{y},t_1)-\mathcal{K}_0(\boldsymbol{x},t_2|\boldsymbol{y},t_1)\right]_{\boldsymbol{x}=\boldsymbol{y}=0}$$

- The propagator obeys

$$\left[i\partial_t+\frac{\boldsymbol{\partial}^2}{2\omega^2}+iv(\boldsymbol{x})\right]\mathcal{K}(\boldsymbol{x},t_2|\boldsymbol{y},t_1)=i\delta(\boldsymbol{x}-\boldsymbol{y})\delta(t_2-t_1)$$



- Analytic treatment only possible in 2 extreme limits

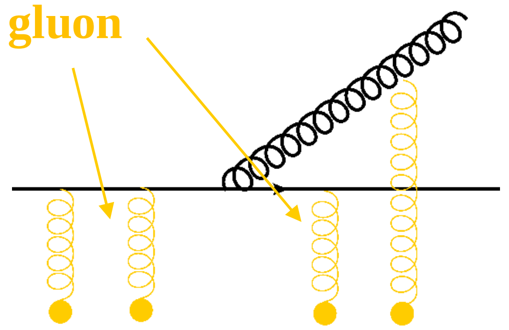
BDMPS-Z (HO) VS GLV (N=1 Opacity)

BDMPS-Z (HO)

✔ Resumation of multiple soft gluon exchanges with the medium

✖ Fails at describing the tail of the real potential

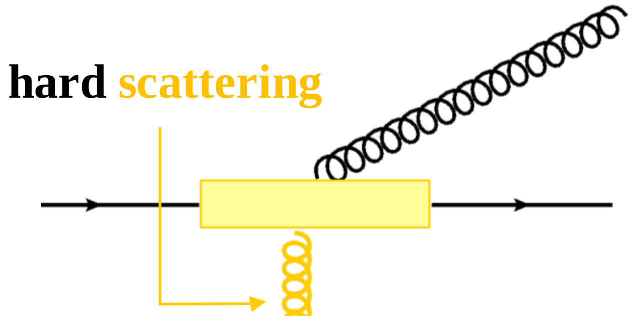
$$v(\boldsymbol{x},t)_{HO}=\frac{1}{4}\boldsymbol{x}^2\hat{q}_0(t)\log\left(\frac{Q^2}{\mu^{\star 2}}\right)$$



GLV (N=1 Opacity)

✔ Full potential $v(\boldsymbol{x},t)_{GLV}=v(\boldsymbol{x},t)_{GW}$

✖ Single hard scattering



In-medium scattering potential models

- We consider 2 models treated at full leading logarithmic (LL) accuracy

⇨ Gyulassy-Wang (Yukawa potential) $\left(\frac{d\sigma}{d^2\boldsymbol{q}}\right)^{GW}=\frac{g^4 n(t)}{(\boldsymbol{q}^2+\mu^2)^2}$ — Scattering centres' density in the medium; GW screening mass

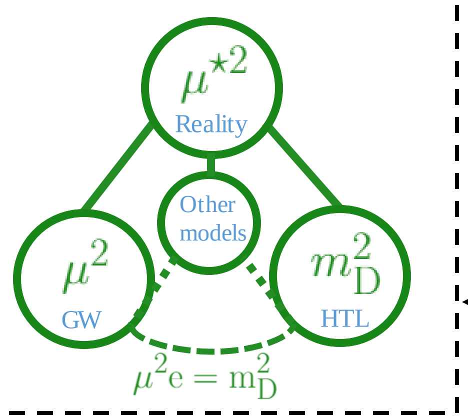
⇨ Thermal model (HTL potential) $\left(\frac{d\sigma}{d^2\boldsymbol{q}}\right)^{HTL}=\frac{g^2 m_D^2 T}{\boldsymbol{q}^2(\boldsymbol{q}^2+m_D^2)}$ — Medium Debye mass; Medium temperature

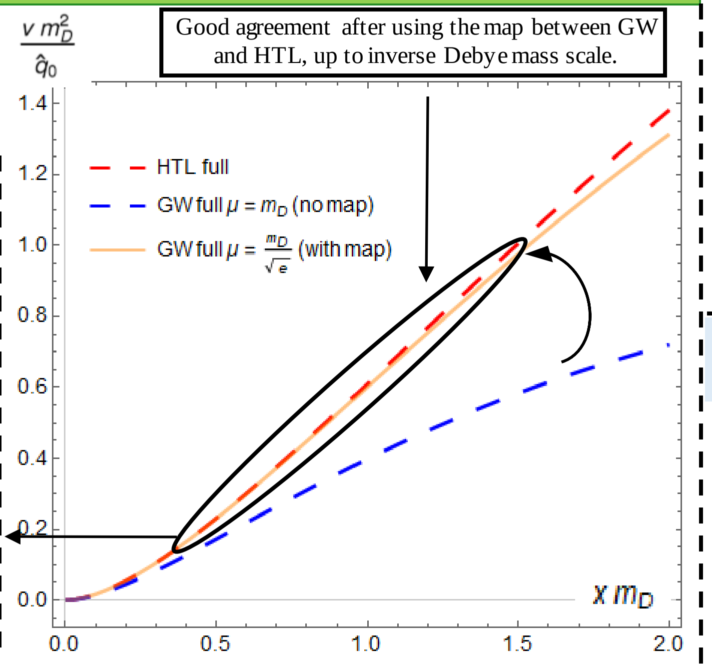
LL accuracy map between the physical screening mass and the models' parameters

The LL potential is given by $v(\boldsymbol{x},t)_{LL}=\frac{1}{4}\boldsymbol{x}^2\hat{q}_0(t)\log\left(\frac{1}{\boldsymbol{x}^2\mu^{\star 2}}\right)$

Medium transport parameter stripped of any logarithm

with $\mu^{\star 2}=\begin{cases}\frac{\mu^2}{4}e^{-1+2\gamma_E}\ \textbf{(GW)}\\ \frac{m_D^2}{4}e^{-2+2\gamma_E}\ \textbf{(HTL)}\end{cases}$ $\quad v(\boldsymbol{x},t)\equiv C_A\int_{\boldsymbol{q}}\left(\frac{d\sigma}{d^2\boldsymbol{q}}\right)^{model}(1-e^{i\boldsymbol{q}\cdot\boldsymbol{x}})$





Beyond GLV and BDMPS-Z: Improved Opacity expansion (IOE)

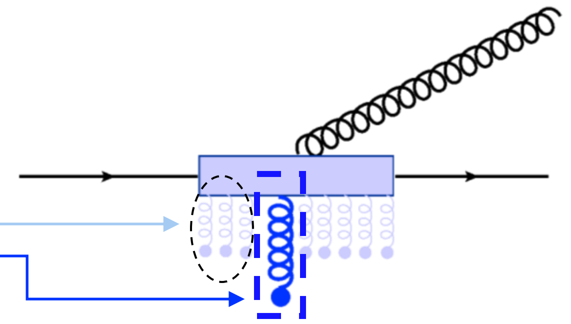
- In the IOE we expand the propagator around the BDMPS-Z solution

$$\mathcal{K}(\boldsymbol{x},t,\boldsymbol{y},s)=-\int_{\boldsymbol{z}}\int_s^t du\,\mathcal{K}_{HO}(\boldsymbol{x},t|\boldsymbol{z},u)\delta v(\boldsymbol{z},u)\mathcal{K}(\boldsymbol{z},u|\boldsymbol{y},s)$$

Hard scattering at time u and postion z

$$v(\boldsymbol{x},t)=\frac{1}{4}\hat{q}_0\boldsymbol{x}^2\log\left(\frac{1}{\mu^{\star 2}\boldsymbol{x}^2}\right)=\frac{1}{4}\hat{q}_0\boldsymbol{x}^2\left(\log\left(\frac{Q^2}{\mu^{\star 2}}\right)+\log\left(\frac{1}{Q^2\boldsymbol{x}^2}\right)\right)\equiv v_{HO}(\boldsymbol{x},t)+\delta v(\boldsymbol{x},t)$$

LL general potential; BDMPS-Z potential; Expansion potential



- We explicitly compute the LO (BDMPS-Z), NLO and NNLO terms in 2 regimes

a) Large gluon frequencies $\omega\gg\omega_c$

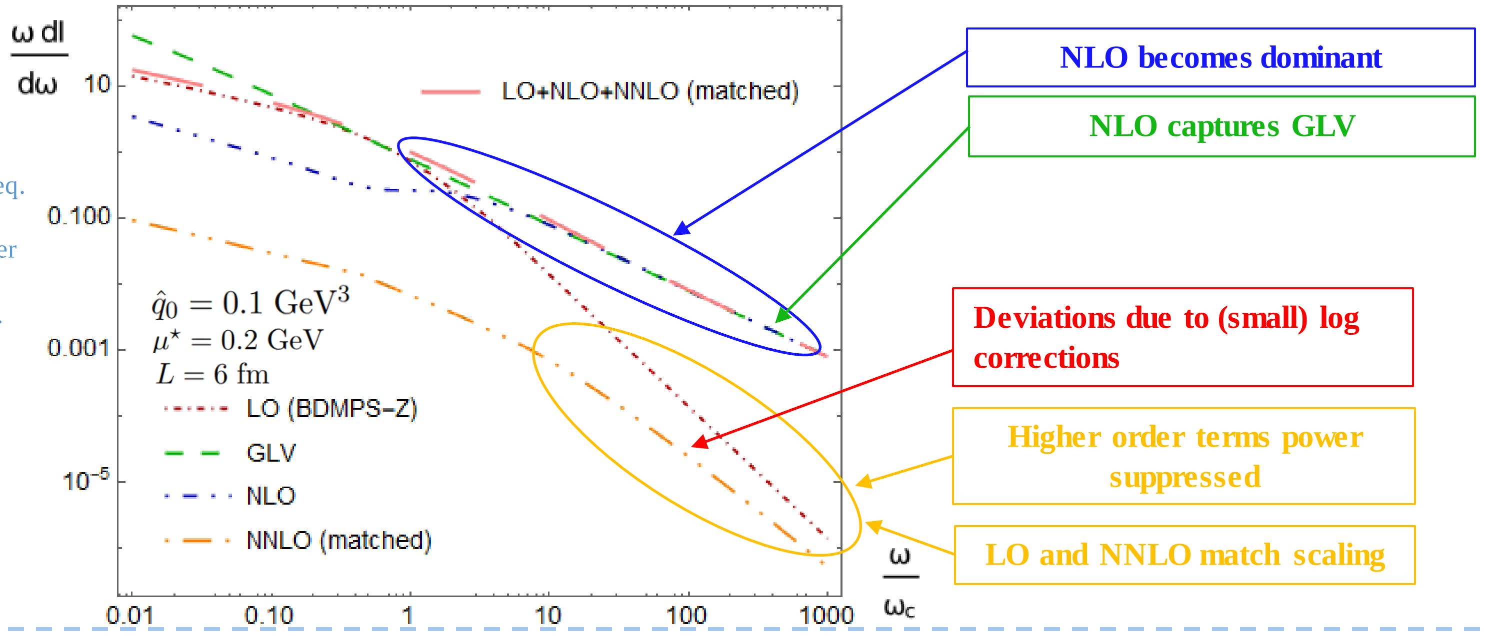
- We find the following scalings

$\omega_c=\hat{q}L^2$ BDMPS cut freq.; $\chi\equiv\frac{\hat{q}_0 L}{\mu^{\star 2}}$ opacity parameter; $\bar{\omega}_c\equiv\frac{\mu^{\star 2}L}{2}$ GLV cut freq.

$$\lim_{\omega\to\infty}\frac{dI^{LO}}{d\omega}=2\bar{\alpha}\frac{1}{12}\boxed{\left(\frac{\omega_c}{\omega}\right)^2}$$

$$\lim_{\omega\to\infty}\omega\frac{dI^{NLO}}{d\omega}\sim\bar{\alpha}\hat{q}_0\frac{\pi}{4}\frac{L^2}{2\omega}=\frac{\bar{\alpha}\hat{q}_0 L}{\mu^{\star 2}}\frac{\pi}{4}\frac{\bar{\omega}_c}{\omega}=\frac{\pi}{4}\chi\bar{\alpha}\boxed{\frac{\bar{\omega}_c}{\omega}}$$

$$\lim_{\omega\to\infty}\omega\frac{dI^{NNLO}}{dLd\omega}\sim\frac{\bar{\alpha}}{L}\chi\boxed{\left(\frac{\bar{\omega}_c}{\omega}\right)^2}\boxed{\log^2\left(\frac{\omega}{Q^2L}\right)}$$

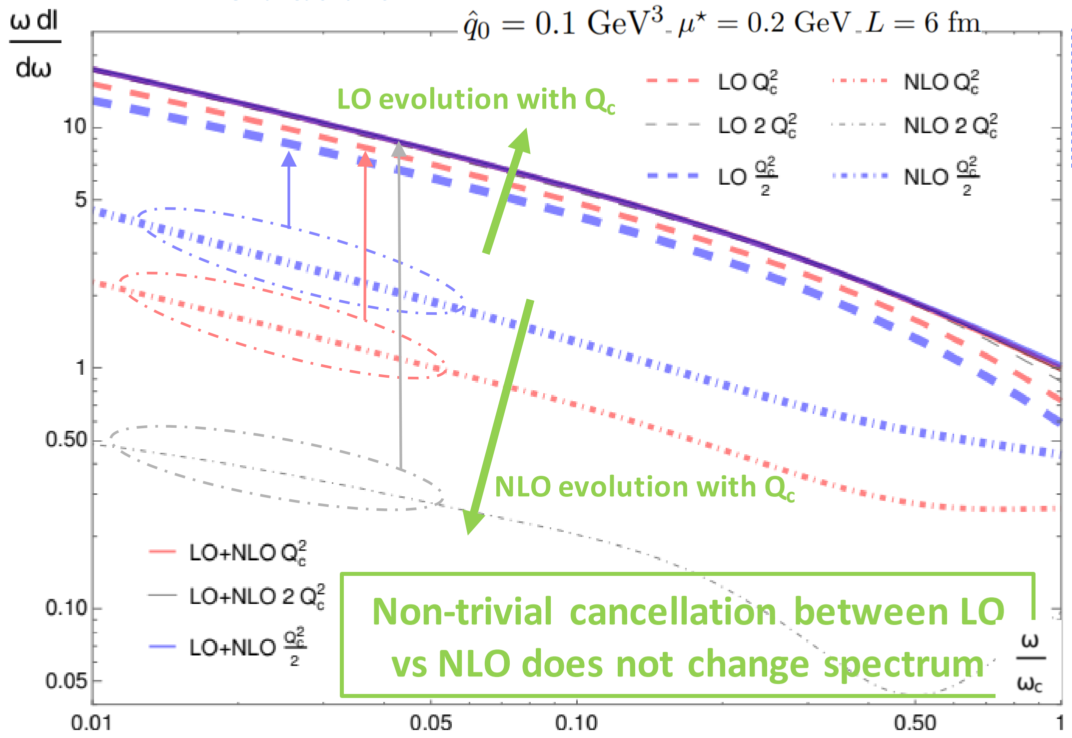


b) Small gluon frequencies $\omega\ll\omega_c$

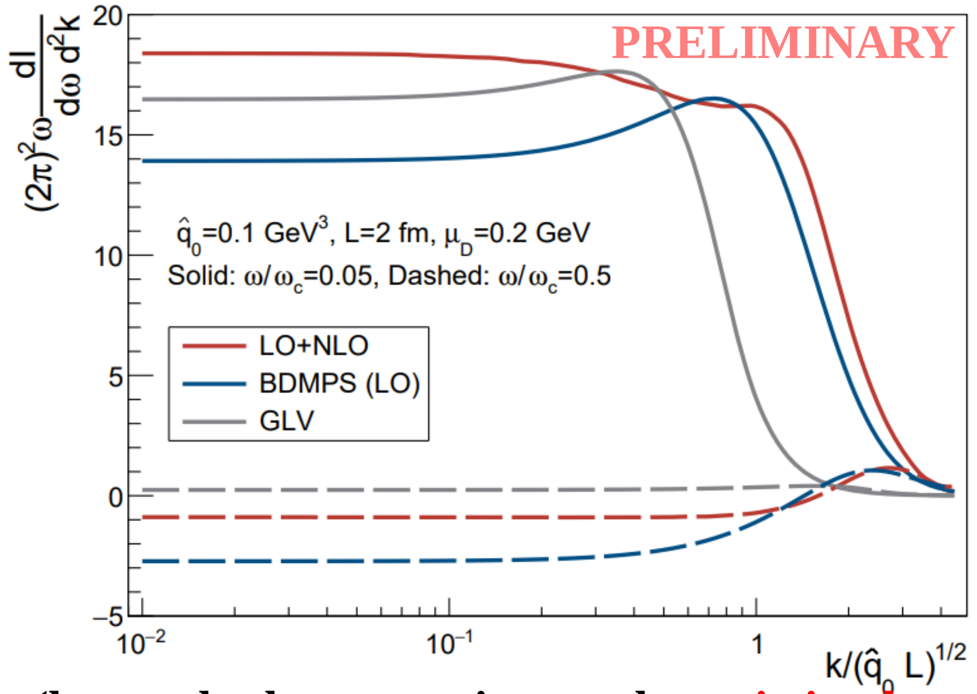
All orders are captured by a renormalization of medium parameter

$$\omega\frac{dI}{d\omega dL}=\bar{\alpha}\sqrt{\frac{\hat{q}_{eff}(Q_c)}{\omega}}\qquad \hat{q}_{eff}(Q_c)=\hat{q}_0\log\left(\frac{Q_c^2}{\mu^{\star 2}}\right)\left[1+\frac{1.016}{\log\left(\frac{Q_c^2}{\mu^{\star 2}}\right)}+\frac{0.316}{\log^2\left(\frac{Q_c^2}{\mu^{\star 2}}\right)}+\mathcal{O}\left(\log^{-3}\left(\frac{Q_c^2}{\mu^{\star 2}}\right)\right)\right]\qquad Q_c^2=\sqrt{\hat{q}_0\,\omega\log\left(\frac{Q_c^2}{\mu^{\star 2}}\right)}$$

LO functional form



BDMPS-Z vs GLV interpolation possible due to non trivial choice for matching scale Q_c



The k_t differential spectrum

- We extend the IOE at NLO to the differential spectrum (here only the corrections to the emission kernel)

$$(2\pi)^2\omega\frac{dI}{d\omega d\boldsymbol{k}}=\frac{\alpha_s C_R}{\omega^2}2\mathrm{Re}\left[\int_0^\infty dt_2\int_0^{t_2}dt_1\int d^2\boldsymbol{x}e^{-i\boldsymbol{k}\boldsymbol{x}}e^{-\int_{t_2}^\infty ds\,v(s,\boldsymbol{x})}\,\boldsymbol{\partial_y}\cdot\boldsymbol{\partial_x}\left(\mathcal{K}(\boldsymbol{x},t_2;\boldsymbol{y},t_1)-\mathcal{K}_0(\boldsymbol{x},t_2;\boldsymbol{y},t_1)\right)_{\boldsymbol{y}=0}\right]$$

Momentum broadening