HEAVY QUARKONIUM SUPPRESSION BEYOND THE ADIABATIC LIMIT
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Abstract
In Ref [1], we relax the assumption of the adiabatic approximation by solving the 3D Schrödinger equation in real-time in order to compute quarkonium suppression dynamically. We compare results obtained using the adiabatic approximation with real-time calculations for the realistic complex heavy quark potentials.

1. Heavy Quarkonium States
We know that the strong coupling constant decreases with increasing energy. Therefore, at a high energy, the interaction among quarks becomes negligible and quarks behave as free particles. This phenomenon is known as asymptotic freedom.

At temperatures around \(T \approx 170 \text{ MeV}\), the deconfinement phase transition occurs and quarks and gluons are liberated from light hadronic states and form a state consisting of quasi-free quarks and gluons called the Quark-Gluon Plasma (QGP).

At temperatures below roughly 600 MeV, there still remain some quarkonium states which are bound states of a heavy quark and its antiquark. As the QGP temperature increases, the effective color charge is reduced due to the Debye screening, and quark and \(q\) become unbound and the quarkonium state melts in the QGP.

So, now if we can define a term “suppression” indicating the probability of melting of a particular state, one expects that the \(\chi(1P)\) is most suppressed and \(\Upsilon(1S)\) is least suppressed in the QGP.

The suppression factor can be defined as

\[ R_{\text{KA}} = \frac{\text{number produced in } \Lambda}{(N_{\text{ev}})(N_{\text{coll}})} \times \frac{\text{number of nucleon collisions}}{N_{\text{ev}}}, \]

where

- \(N_{\text{ev}}\) is the number of events
- \(N_{\text{coll}}\) is the number of collisions
- \(R_{\text{KA}}\) is the suppression factor
- \(\text{number of nucleon collisions}\)

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2. Adiabatic approximation (KSUs)
Previous works have used the adiabatic approximation to obtain the suppression factor or survival probability. In this approach, they first solved the SWE with a complex-valued potential numerically in order to obtain the real and imaginary parts of the binding energy and then they folded this together with the QGP's hydrodynamic evolution.

In the adiabatic approximation, one ignores the mixing of various excited states which emerge in a time-dependent potential. The result is that in this approximation the time-evolution of each state is governed solely by the time-dependent imaginary part of the states energy, i.e.

\[ \Psi(t) = N_i(t) \exp\left(2 \int_{t_0}^t dt \Im(E_n(t))\right), \]

where

\[ \Psi(t) = |c_n(t)|^2, \]

is the modulus-squared overlap states of type \(n\) at time \(t\). Finally, they used equation (1) to obtain the survival probability i.e. \(|c_n(t)|^2\).

3. Real-time evolution (KSUs)
In the improved method, we use the real-time quantum evolution instead of the adiabatic approximation. At first, we solve the real-time 3D SWE with a complex-valued potential. For this purpose, we make use of a complex Karsch-Mehr-Satz (CKMS) type potential based on the internal energy

\[ V(r, \dot{r}) = V_0[r(1 + m_D r)] \quad \text{exp}(\sigma m_D r), \]

where

\[ \sigma = \frac{2}{m_D^2}, \quad \tau = \frac{2}{m_D^2} \]

\(\tau\) is the QGP Debye mass \(m_D = 0.230 \text{ GeV}\) is the string tension, and \(m_D\) is the QGP Debye mass \(m_D = \sqrt{4N_f(1 + (1 + \epsilon) / 6) \alpha_s T^3 / 3} \), where \(N_f = 3\) and \(N_c = 2\), \(\alpha_s = 4 \alpha_s / 3 = 0.385\), and \(T \equiv m_D / \epsilon\).

We then expand the wave function, i.e., \(\psi(t, x) = \sum_n c_n(t) \psi_n(x, t)\), and obtain the overlap coefficients \(c_n(t)\). Finally, we compute the survival probability i.e., \(|c_n(t)|^2\).

We made several case studies using the CKMS potential (3). In all cases, we assumed that the system was undergoing boost-invariant and transversally homogenous one-dimensional Bjorken flow such that \(T(r) = T_0 \eta(r)^3 / 3\), where \(T_0\) is the initial temperature at the proper time \(\eta\).

We kept the initial time fixed to be \(\tau_0 = 1 \text{ GeV}^{-1}\) and varied the initial temperature to mimic what is seen in a typical Glueball temperature profile in a LHC 2.76 TeV Pb-Pb collision. In all cases shown below, we terminated the evolution when the temperature fell below the freeze-out temperature \(T_{\text{fo}} = 150\text{ MeV}\).

4. Results: Comparison between KSUs and KSUs
In this example, we consider \(T_0 = 0.225 \text{ GeV}\) at \(\tau_0 = 1 \text{ GeV}^{-1}\) for \(T\) and \(T_0 = 0.4 \text{ GeV}\) at \(\tau_0 = 1 \text{ GeV}^{-1}\) for \(J/\psi\).

5. Conclusions
- The adiabatic approximation was accurate to the percent level for the \(\Upsilon\) (1S).
- At low temperatures, we found large corrections to the adiabatic approximation for the \(\Upsilon\) (2S) survival probability.
- In the charmonium sector, we found large corrections to the adiabatic approximation, with the real-time evolution predicting stronger suppression in all cases.

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