Local thermalization of gluons and quarks in a nonlinear model

Georg Wolschin*

Institute for Theoretical Physics, Heidelberg University, Germany, EU

*g.wolschin@thphys.uni-heidelberg.de

Introduction

The aim is to exactly solve a nonlinear kinetic equation for the time evolution of the occupation-number distribution in a finite system of **bosons** including the singularity at $\epsilon = \mu < 0$ with suitable nonequilibrium intial conditions plus boundary conditions at the singularity. The solutions are compared to the ones for **fermions**, and applied to the local equilibration of gluons and quarks in the early stages of relativistic heavy-ion collisions.

Boson equilibration model

The basic nonlinear boson diffusion equation (NBDE) for the occupation-number distribution $n \equiv n(\epsilon, t)$ has been derived in [1] from the Boltzmann collision term in a finite gluon system. Dissipative effects are expressed through the drift term $v(\epsilon, t)$, diffusive effects through the diffusion term $D(\epsilon, t)$. In the limit of constant transport coefficients, it becomes

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial \epsilon}\left[v\, n\,(1+n)\right] + \frac{\partial^2}{\partial \epsilon^2}\left[D\, n\right]. \qquad (1)$$

This kinetic equation can be solved exactly. The thermal equilibrium distribution is a stationary solution

$$n_{\text{eq}}(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/T} - 1}$$

with the chemical potential $\mu < 0$ in a finite Bose system. The equilibrium temperature is $T = -D/v$, with $v < 0$ since the drift is towards smaller energies.

Exact solution

The analytical solution of Eq. (1) is derived through the nonlinear transformation [1]

$$n(\epsilon, t) = -\frac{D}{vP(\epsilon, t)}\frac{\partial P(\epsilon, t)}{\partial \epsilon}.$$

It reduces the NBDE (1) to a linear diffusion equation for $P(\epsilon, t)$ that is solvable and can be retransformed to $n(\epsilon, t)$ as [1]

$$n(\epsilon, t) = \frac{1}{2v}\frac{\int_{-\infty}^{+\infty} \frac{\epsilon - x}{t}\, G(\epsilon, x, t) F(x)\, dx}{\int_{-\infty}^{+\infty} G(\epsilon, x, t) F(x)\, dx} - \frac{1}{2}$$

with the bounded Green's function $G(\epsilon, x, t)$ and $F(x)$ containing an integral over the initial conditions. For schematic initial conditions [2, 3] with $Q_s \approx 1\,\text{GeV}$ and a singularity at $\epsilon = \mu$

$$n_i(\epsilon) = \theta(1 - \epsilon/Q_s)\,\theta(\epsilon - \mu) + \delta(\epsilon - \mu) \qquad (2)$$

that are appropriate for the gluon system in a relativistic heavy-ion collision such as Au-Au or Pb-Pb at RHIC or LHC energies, exact solutions can be obtained [4].

Gluon equilibration

The solutions of the bosonic local equilibration problem are evaluated [4] in closed form for the simplified initial condition Eq. (2). Results at five time steps towards the Bose-Einstein equilibrium $n_{\text{eq}}(\epsilon)$ (solid):

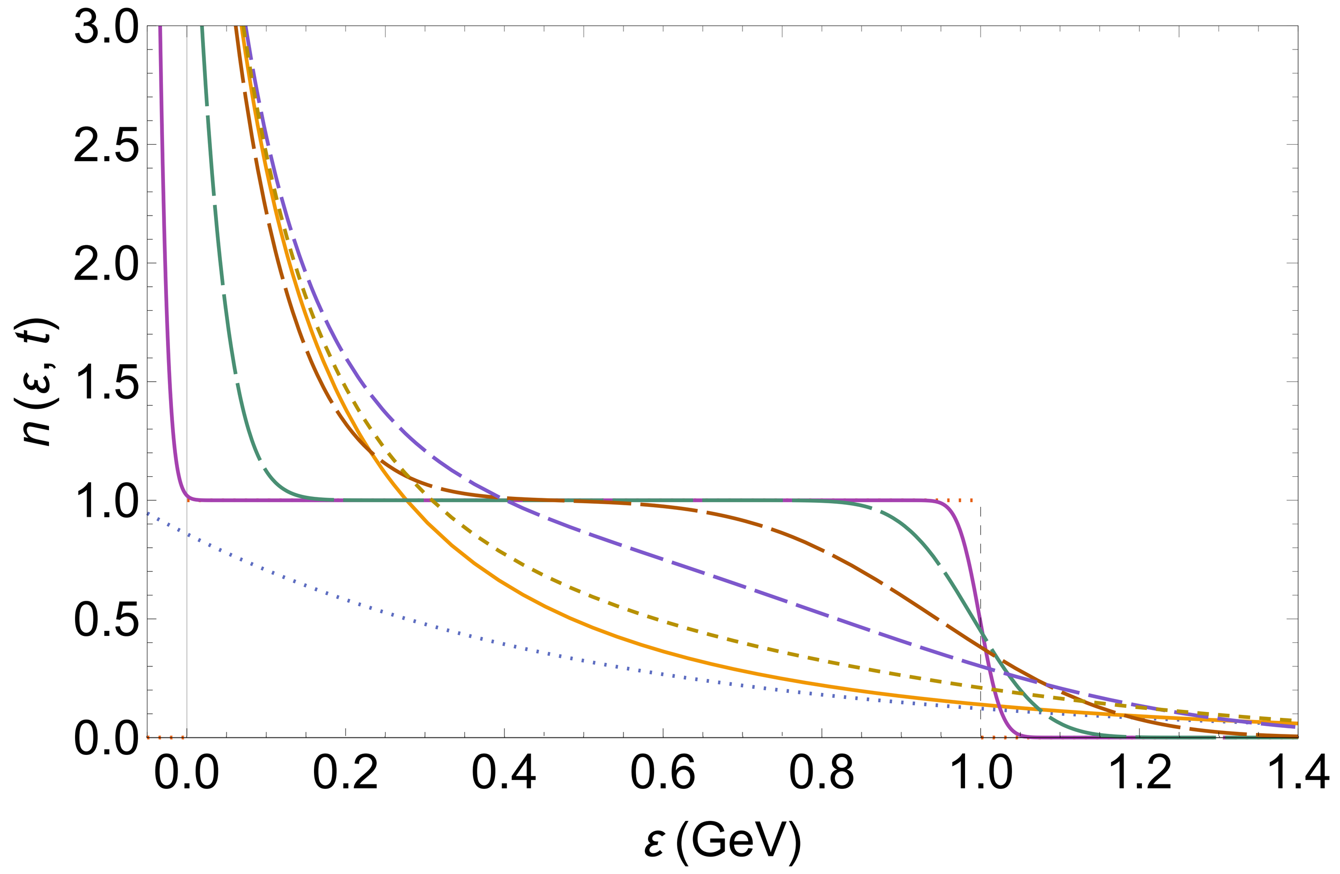

Figure 1: Local equilibration of massless gluons.

Local equilibration time

An explicit expression for the equilibration time follows from an asymptotic expansion of the error functions occurring in the solution

$$\tau_{\text{eq}}^{\text{Bose}} = 4D/(9\,v^2)\,.$$

The Bose equilibration time is almost an order of magnitude shorter than the corresponding equilibration time in a fermion system, which was found to be $\tau_{\text{eq}}^{\text{Fermi}} = 4D/v^2$ [1] for equivalent initial conditions: $\tau_{\text{eq}}^{\text{Fermi}}/\tau_{\text{eq}}^{\text{Bose}} = 9$. This is due to the different statistical properties of bosons and fermions.

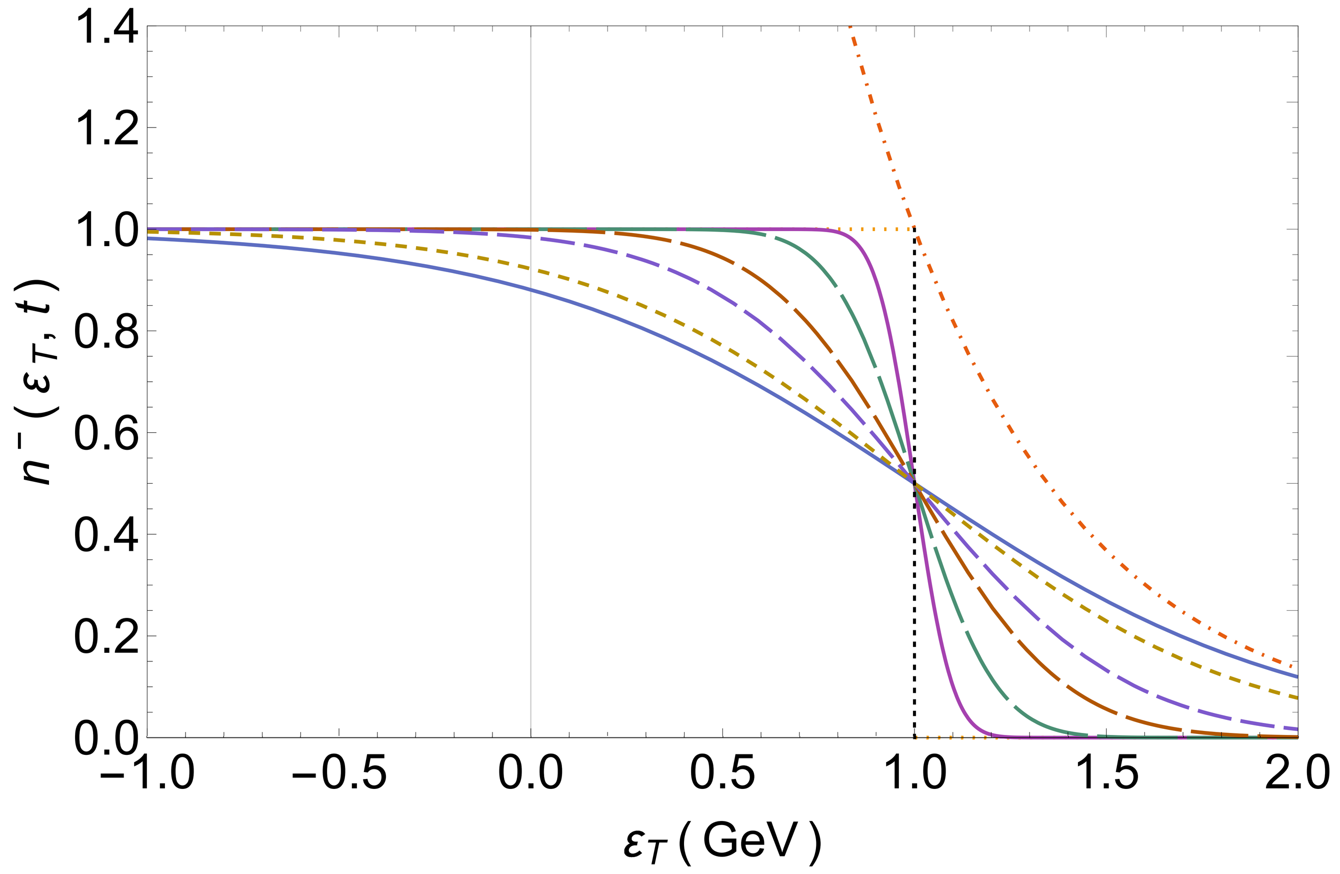

Figure 2: Local equilibration of valence quarks.

Quark-Antiquark equilibration

The corresponding nonlinear fermion diffusion equation for the occupation of fermionic states $n_F(\epsilon, t)$ is easier to solve analytically [1] because no singularity occurs, the distribution functions are limited to $n \leq 1$ due to Pauli's principle. The solutions conserve the total particle number $N_{\text{th}}^{\text{F}}(t)$ (here for constant density of states $g(\epsilon)$) provided antiparticle production from the filled Dirac sea at $\epsilon < 0$ is taken into account,

$$N_{\text{th}}^{\text{F}}(t) = \int_0^{\infty} n_F(\epsilon, t)\, g(\epsilon)\, d\epsilon - \int_{-\infty}^{0} [1 - n_F(\epsilon, t)]\, g(\epsilon)\, d\epsilon = \text{const}.$$

Particle number conservation for fermions including antiparticle creation differs from the bosonic case, where particles can in principle disappear from the (nonequilibrium or equilibrium) statistical part of the distribution and move into the condensed state. The latter process is relevant for cold quantum gases [6] where BEC formation is actually taking place, whereas it appears to be hindered in relativistic collisions due to inelastic processes that violate particle-number conservation [7].

Conclusion

As a consequence of the nonlinearity of the basic bosonic diffusion equation (1), local equilibrium of the gluon distribution with a thermal Bose-Einstein distribution function is rapidly attained within $\tau_{\text{eq}}^{\text{Bose}} \approx 0.1\,\text{fm/c}$, solid curve in Fig. 1. The fast gluon thermalization during the initial stages of the collision is a sufficient condition for the subsequent applicability of relativistic hydrodynamics to the collective expansion and cooling, although it has been proposed that it may not be a necessary condition.

In the infrared, gluons can in principle move into the condensed BEC ground state with $|\mathbf{p}| = p = \epsilon = 0$, but since inelastic collisions do not preserve particle number, this process is strongly hindered: Whereas condensate formation takes place in ultracold atoms at very low energy where particle-number conservation is strictly fulfilled, it is unlikely to occur in relativistic heavy-ion collisions.

Quarks equilibrate about an order of magnitude more slowly towards the Fermi-Dirac equilibrium distribution. The local equilibration time in the UV region is on statistical grounds (Pauli's principle) a factor of nine smaller than the one for gluons, and this difference is increased due to the smaller color factor.

References

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