Electroweak baryogenesis and beyond the SM (1)

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Outline

Implication of the baryon number for the Standard Model

Ideas of baryogenesis during EW phase transition
(including basic of thermal field theory)

EWBG models, constraints and possible new directions
Basics of elementary particle for baryogenesis
Standard Model of Elementary Particles

Three generations of matter (fermions):

- **First Generation**
  - **Up Quark (u)**: $\frac{2}{3}$, $\frac{1}{2}$, $\simeq 2.2$ MeV/c$^2$
  - **Down Quark (d)**: $-\frac{1}{3}$, $\frac{1}{2}$, $\simeq 4.7$ MeV/c$^2$
  - **Electron (e)**: $-1$, $\frac{1}{2}$, $<2.2$ eV/c$^2$

- **Second Generation**
  - **Charmed Quark (c)**: $\frac{2}{3}$, $\frac{1}{2}$, $\simeq 1.28$ GeV/c$^2$
  - **Strange Quark (s)**: $-\frac{1}{3}$, $\frac{1}{2}$, $\simeq 96$ MeV/c$^2$
  - **Muon (μ)**: $-1$, $\frac{1}{2}$, $<0.17$ MeV/c$^2$

- **Third Generation**
  - **Top Quark (t)**: $\frac{2}{3}$, $\frac{1}{2}$, $\simeq 173.1$ GeV/c$^2$
  - **Bottom Quark (b)**: $-\frac{1}{3}$, $\frac{1}{2}$, $\simeq 4.18$ GeV/c$^2$
  - **Tau (τ)**: $0$, $\frac{1}{2}$, $<18.2$ MeV/c$^2$

Interactions / Force Carriers (bosons):

- **Gluon (g)**: $0$, $0$, $\simeq 124.97$ GeV/c$^2$
- **Photon (γ)**: $0$, $0$, $\simeq 77.03$ GeV/c$^2$
- **Z boson (Z)**: $0$, $1$, $\simeq 91.19$ GeV/c$^2$
- **W boson (W)**: $\pm 1$, $1$, $\simeq 80.39$ GeV/c$^2$
**Baryon** (βαρύς: heavy) and **Lepton** (λεπτός: thin, small)

**Baryon:** made by (an odd number of) quarks e.g. proton (uud), neutron (udd)

Nucleon is heavy because of quark confinement by strong interaction (+uncertainty principle)

\[ m_N(939.6 \text{ MeV}) \simeq m_P(938.3 \text{ MeV}) \simeq 2000m_e(0.5 \text{ MeV}) \simeq 1000 \text{ MeV} \]

**Lepton:** elementary fermion which does not feel strong interaction e.g. electron, neutrino
Antiparticle

Predicted by theory (unexpectedly)

P.A.M. Dirac provides the relativistic Schrödinger equation for electron: “Dirac equation”

\[i\partial_t \psi = \left( \frac{(\sigma \cdot (i \nabla - e A))^2}{2m} + e\phi \right)\psi \]

Solution to the Dirac equation yields

\[E = \pm \sqrt{p^2 + m^2}\]

The negative energy solution was interpreted as “Dirac sea”.

Excitation of electron from the sea gives the “hole”, the effective particle with the exactly same mass but the opposite charge prediction of positron

After the relativistic Quantum Field Theory (QFT) was developed, the correct interpretation of the hole was made as the anti-particle:

equally elementary particle with the same mass but opposite quantum numbers (charges)

It turns out that all charged particle has its own antiparticle (e.g. antiquark, antineutrino)
Stability of atom?

Classical electrodynamics for the atom

Life time: \( \tau_A \leq 0.016 \text{ns} \)

According to classical physics, an electron in orbit around an atomic nucleus should emit electromagnetic radiation (photons) continuously, because it is continually accelerating in a curved path. The resulting loss of energy implies that the electron should spiral into the nucleus in a very short time (i.e. atoms cannot exist).

Quantum mechanics for the hydrogen atom

\[ E_{\text{binding}} = -\frac{13.6 Z^2}{n^2} \text{ eV} \]

\( Z \) charge of the nucleus, \( n \) principal quantum number.

As the same way, is the positronium stable? No! \( \tau_p = 0.12 \text{ns} - 139 \text{ns} \)

\[ E = 2m_e c^2 - 6.8 \text{ eV}/n^2 \approx \text{MeV} \rightarrow E = 2(3)E_\gamma = 2(3)hv \]

Then why is the atom stable?
An atom is made by baryons and leptons

If baryon number and lepton number are conserved, annihilation between the proton and electron is forbidden. Each particle is also stable: proton is the lightest baryon, electron is the lighted charged lepton.

Atom is stable!

Baryon/Lepton number conservation is essential for life.
List of baryon/lepton number

Notation: for a particle, $\phi$, its antiparticle is denoted as $\bar{\phi}$  ($q$: quark, $l$: lepton)

<table>
<thead>
<tr>
<th></th>
<th>$q$</th>
<th>$\bar{q}$</th>
<th>$l$</th>
<th>$\bar{l}$</th>
<th>$g$</th>
<th>$W^\pm$</th>
<th>$Z^0$</th>
<th>$\gamma$</th>
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<tbody>
<tr>
<td>$B$</td>
<td>1/3</td>
<td>−1/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>$L$</td>
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<td>0</td>
<td>1</td>
<td>−1</td>
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<tr>
<td>$Q_{em}$</td>
<td>2/3, $-1/3$</td>
<td>−2/3, $1/3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1, −1</td>
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<th>$p, n$</th>
<th>$\bar{p}, \bar{n}$</th>
<th>$e, \nu$</th>
<th>$\bar{e}, \bar{\nu}$</th>
<th>$\pi^0$</th>
<th>$\pi^\pm$</th>
<th>$\Lambda^0$</th>
<th>$\bar{\Lambda}^0$</th>
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<tbody>
<tr>
<td>$B$</td>
<td>1</td>
<td>−1</td>
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There is NO observed event that violates baryon and lepton number conservation
Is there other symmetry of the theory?
fermions have the spin angular momentum \( s_z = \pm \hbar/2 \)

C : Charge conjugation ( \( Q \rightarrow -Q \) ) : \( Q \rightarrow -Q, \hat{p} \rightarrow \hat{p}, \hat{s} \rightarrow \hat{s} \)
P : Parity ( \( x \rightarrow -x \) ) : \( Q \rightarrow Q, \hat{p} \rightarrow -\hat{p}, \hat{s} \rightarrow \hat{s} \)
T : Time reversal ( \( t \rightarrow -t \) ) : \( Q \rightarrow Q, \hat{p} \rightarrow -\hat{p}, \hat{s} \rightarrow -\hat{s} \)

\[
\text{helicity} = \frac{\hat{s} \cdot \hat{p}}{|p|}
\]

\( ^E \) \( \leftrightarrow \) \( ^b \) \( \leftrightarrow \) \( ^c \)
\( \hat{s} \cdot \hat{p} \)

Table 1. Transformation properties of electric field \( E \), magnetic field \( B \), charge \( q \), velocity \( \mathbf{v} \) and force \( \mathbf{F} \) under charge conjugation (C), parity (P) and time reversal (T).

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>P</th>
<th>T</th>
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<tr>
<td>( E )</td>
<td>( -E )</td>
<td>( -E )</td>
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<td>( B )</td>
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<td>( q )</td>
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<td>( \mathbf{F} )</td>
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<td>( -\mathbf{F} )</td>
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e.g. Einstein, Maxwell, Dirac equation (so theory) is invariant under the C, P, and T.

However the Standard Model is not invariant under the C, P, and T transformation.
Evidence of matter-antimatter asymmetry
At the present Universe

mater + antimatter $\rightarrow$ fast annihilation into energetic photons

\[ \bar{p}, \bar{n}, \bar{e} \ldots \rightarrow e^+ \]  
\[ p, n, e \ldots \rightarrow e^- \]

Gamma rays

Annihilation

Earth is made by matters (not antiatom, antimolecule)

Sun is made by matters. No observation of antimatter comet in the solar system

No evidence of gamma ray burst between two galaxy induced by intergalactic plasma

Solar wind: energetic electrons, protons
The early Universe

The Universe was extremely hot \((kT \gg m_* c^2)\) in the beginning. All particles and antiparticles form thermal plasma. As the temperature decreases (cool down) and becomes \(kT \ll m_* c^2\), most of particles and antiparticles with the mass \((m_*)\) are annihilated away.

\[
\text{HOT} \quad \text{Cold}
\]

- For \(T \gg m_B\)
  \[
  N_B = 10662827465 \\
  N_{\bar{B}} = 10662827463 \\
  N_\gamma = 14224600201
  \]

- For \(T \ll m_p\)
  \[
  N_B = 2 + O\left(N_\gamma e^{-\frac{m_p}{T}}\right) \\
  N_{\bar{B}} = 0 + O\left(N_\gamma e^{-\frac{m_p}{T}}\right) \\
  N_\gamma = 35550255127
  \]

\[
\frac{N_B}{N_\gamma} = O(1) \rightarrow O\left(10^{-10}\right)
\]
Big Bang Nucleosynthesis (BBN)

Light nuclei (□□□□□□□□□□□□) in the Universe were made between 1 sec and $10^3$ sec.

Input $N_B/N_\gamma$

$Y_p \equiv \rho(^4\text{He})/\rho_B$

$E_{\text{bind}}(D) = 2.2$ MeV

$p + n \rightarrow ^2\text{H} + \gamma$
$p + ^2\text{H} \rightarrow ^3\text{He} + \gamma$
$^2\text{H} + ^2\text{H} \rightarrow ^3\text{He} + n$
$^2\text{H} + ^2\text{H} \rightarrow ^3\text{H} + p$
$^3\text{He} + ^2\text{H} \rightarrow ^4\text{He} + p$
$^3\text{H} + ^2\text{H} \rightarrow ^4\text{He} + n$

Annual Review of Nuclear and Particle Science, 60:539-568, 2010
Big Bang Nucleosynthesis (BBN)

Light nuclei in the Universe were made between 1 sec and $10^3$ sec: primordial abundance

The final abundances of them are sensitive to the initial baryon abundance. Observation of primordial light nuclei can predict the value of baryon asymmetry around 1 sec!

The larger the nucleon density, the higher the temperature at which nucleosynthesis began, and so the less time there was for neutron decay before nucleosynthesis, leading to a higher final $^4$He abundance.

The higher the baryon density, the more complete will be the incorporation of neutrons into $^4$He, and hence the smaller the resulting abundance of deuterium.

[Weinberg, Cosmology]
Cosmic Microwave Background (CMB)

As the temperature becomes lower than the atomic energy ($T < 0\text{(eV)}$), nucleons and electrons are bound and form atoms. Photons are decoupled and freely propagate: CMB

$$\delta_k + c_s^2 k^2 \delta_k + k^2 \phi_k \approx 0$$

gravitational potential from Dark Matter

$$c_s = \frac{1}{\sqrt{3(1 + 3\rho_B/4\rho_\gamma)}}$$

$$T = 2.7\text{ K}$$

$$\frac{\Delta T}{T} = O(10^{-5})$$
Cosmic Microwave Background (CMB)

Baryon abundance is mostly determined by the heights of odd ($c_s k t_{dec} = (2n + 1)\pi$), and even ($c_s k t_{dec} = 2n\pi$) peaks

$$\delta_k(t_{dec}) = \phi_k \left( \frac{1}{c_s^2} - 2 \right) \cos(c_s k t_{dec}) - \frac{\phi_k}{c_s^2}$$

---

Cosmic Microwave Background (CMB)

Consistent with the value from the BBN!

(a) Curvature

(b) Dark Energy

(c) Baryons

\[ \Omega_B h^2 \equiv \frac{\rho_B}{\rho_{\text{tot}}} h^2 \]

(d) Matter

\[ \Omega_{\text{tot}} h^2 \]

\[ \Omega_B h^2 \]

\[ \Omega_{\text{tot}} h^2 \]

Baryo-genesis

If the baryon asymmetry exists at the early Universe, $T \approx \text{MeV} (1\text{sec})$, as $n_B = 6 \times 10^{-10}n_\gamma$, the abundance of light nuclei and the shape of the CMB angular power spectrum, and non-observation of anti star, anti galaxy are all explained consistently.

$$\frac{N_B - N_{\bar{B}}}{N_\gamma} \approx 6 \times 10^{-10}$$

What is the origin of this asymmetry? Is this just an initial condition of the Universe?

Is there any dynamical mechanism for $N_{\Delta B} = 0 \rightarrow N_{\Delta B} > 0$?

Baryon number is the good quantum number for $\tau_U > 1\text{sec}$ as we observed. BUT it should be strongly broken at the early Universe $\tau_U < 1\text{sec}$
Possibility of baryogenesis in the SM
Just an initial condition?

It is commonly accepted that there is super accelerating period of the Universe.

All previous abundance of any particles, asymmetries are diluted away. A new way to generate the asymmetry after inflation is necessary.
The Sakharov’s conditions JETP Lett. 5, 24 (1967)

In order to generate the asymmetry \( N_B \equiv \sum_{\phi} B_{\phi} N_\phi \) of the Universe from \( N_B = 0 \) to \( B_U > 0 \)

1) Baryon number violation:
For some initial, final states with different baryon numbers, \( B_i \neq B_f, \Gamma(i \rightarrow f) \neq 0 \).
If \( \Gamma(i \rightarrow f) = 0 \), any process gives \( \Delta B = 0 \), so that \( N_B = 0 \). No generation

2) C and CP violation: \( \Gamma(i \rightarrow q_L + \cdots) \neq \Gamma(\bar{q}_L (or \bar{q}_R) + \cdots) \) for \( N_i = N_{\bar{t}} \)
If the baryon number violating process preserves Charge conjugation,
\( N_{q_L} = N_{\bar{q}_L}, N_{q_R} = N_{\bar{q}_R} \) Therefore \( N_B = B_{q_L} + B_{q_R} + B_{\bar{q}_L} + B_{\bar{q}_R} = B_{q_L} + B_{q_R} - B_{q_L} - B_{q_R} = 0 \).
Even if C is violated, if CP is conserved,
\( N_{q_L} = N_{\bar{q}_R}, N_{q_R} = N_{\bar{q}_L} \) Therefore \( N_B = B_{q_L} + B_{q_R} + B_{\bar{q}_L} + B_{\bar{q}_R} = B_{q_L} + B_{q_R} - B_{q_R} - B_{q_L} = 0 \).

3) B-violating interactions out-of-thermal equilibrium: \( \langle \Gamma(i \rightarrow f) \rangle_T \neq \langle \Gamma(f \rightarrow i) \rangle_T \)
If the baryon number violating process is in thermal equilibrium,
\[
\langle \tilde{N}_B \rangle = \frac{1}{Z_0} \text{Tr} \left( e^{-\tilde{H}/T} \tilde{N}_B \right) = \frac{1}{Z_0} \text{Tr} \left( (\text{CPT})(\text{CPT})^{-1} e^{-\tilde{H}/T} \tilde{N}_B \right) \\
= \frac{1}{Z_0} \text{Tr} \left( e^{-\tilde{H}/T} (\text{CPT})^{-1} \tilde{N}_B (\text{CPT}) \right) = -\frac{1}{Z_0} \text{Tr} \left( e^{-\tilde{H}/T} \tilde{N}_B \right) = 0
\]
We used \( [(\text{CPT}), \tilde{H}] = 0 \). Any generated asymmetry is washed out.
In other words, if CPT is broken as \( \tilde{H} \rightarrow \tilde{H}_0 + \mu_B \tilde{N}_B, \langle \tilde{N}_B \rangle \propto \mu_B \)
How can we check Sakharov’s conditions in the SM? In the framework of QFT, the evolution of the Universe can be described by the Lagrangian density $\mathcal{L}(x^\mu)$

$$
\mathcal{L} = \frac{1}{2} M_P^2 R + \mathcal{L}_{SM} + \Delta \mathcal{L}_{BSM}
$$

$$
\mathcal{L}_{SM} = -\frac{1}{2} \Tr G_{\mu\nu}G^{\mu\nu} - \frac{1}{2} \Tr W_{\mu\nu}W^{\mu\nu} - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + (D_\mu \phi) \ddagger D^\mu \phi + \mu^2 \phi \ddagger \phi - \frac{1}{2} \lambda (\phi \ddagger \phi)^2 + \sum_{f=1}^3 \left( \bar{\ell}^f_L i \slashed{D} \ell^f_L + \bar{\ell}^f_R i \slashed{D} \ell^f_R + \bar{q}^f_L i \slashed{D} q^f_L + \bar{d}^f_R i \slashed{D} d^f_R + \bar{u}^f_R i \slashed{D} u^f_R \right)
$$

$$
- \sum_{f=1}^3 y_f^f \left( \bar{\ell}^f_L \phi \ell^f_R + \bar{\ell}^f_R \phi \ddagger \ell^f_L \right) - \sum_{f,g=1}^3 \left( y_d^{fg} \bar{q}^f_L \phi q^g_R + (y_d^{fg})^* \bar{d}^g_R \phi \ddagger q^f_L + y_u^{fg} \bar{q}^f_L \phi \ddagger u^g_R + (y_u^{fg})^* \bar{u}^g_R \phi \ddagger q^f_L \right)
$$

There is the possibility to satisfy Sakharov’s three conditions.
B violation in the SM

1) Baryon number seems conserved in the SM (stability of the atom), but not exactly.

   Classical level: baryon current is conserved \( \partial_\mu J_B^\mu = \partial_t n_B - \vec{\nabla} \cdot \vec{J}_B = 0 \rightarrow \frac{dN_B}{dt} = 0 \).

   Quantum level: not conserved \( \partial_\mu J_B^\mu \neq 0 \)

   \[
   (\partial_\mu J_B^\mu) = (\partial_\mu J_L^\mu) = \frac{n_f}{32\pi^2} (2g^2 \text{Tr}[W_{\mu\nu} \tilde{W}^{\mu\nu}] - g'^2 B_{\mu\nu} \tilde{B}_{\mu\nu})
   \]

Nontrivial configuration of the gauge fields can yield the process with \( \Delta B \neq 0 \) such as

\[
p^+p^+ \leftrightarrow \bar{p}^- e^+ \bar{e}^+ + (\sum_i \nu\bar{\nu})
\]

\( (B = 2, L = 0) \leftrightarrow (B = -1, L = -3) \)

How?
The RHS of the equation is total derivative

\[
\frac{g^2}{16\pi^2} \text{Tr}[F_{\mu\nu}F^{\mu\nu}] = \partial_\mu J^\mu_{CS} = \partial_\mu \left[ \frac{g^2}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( A_\nu F_{\rho\sigma} + \frac{2i}{3} g A_\nu A_\rho A_\sigma \right) \right]
\]

Change of the Chern-Simons number is the source of baryon number change

\[
N_{CS}(t) = \int d^3 \vec{x} \, J^0_{CS} = \frac{g^2}{16\pi^2} \int d^3 \vec{x} \, \epsilon^{ijk} \text{Tr} \left( A_i F_{jk} + \frac{2i}{3} g A_i A_j A_k \right) = \text{integers for vacua}
\]

Ex) (with \( A_0 = 0 \))

\[
\tilde{A}_i(t_2, \vec{x}) = \frac{i}{g} U_2^{-1} \vec{\nabla}_i U_2 \to E(t_2) = 0
\]

\[
N_{CS} = \frac{1}{24\pi^2} \int \text{Tr} \left( (U^{-1}dU)^3 \right)
\]

Related by large gauge transformation

\[
\tilde{A}_i(t_1, \vec{x}) = \frac{i}{g} U_1^{-1} \vec{\nabla}_i U_1 \to E(t_1) = 0
\]
B violation in the SM

\[(\partial \mu J_B^\mu) = (\partial \mu J_L^\mu) = \frac{n_f}{32\pi^2} (2g^2 \text{Tr}[W_{\mu\nu} \tilde{W}^{\mu\nu}] - g'^2 B_{\mu\nu} \tilde{B}^{\mu\nu}) \]

The RHS of the equation is total derivative

\[\frac{g^2}{16\pi^2} \text{Tr}[F_{\mu\nu} \tilde{F}^{\mu\nu}] = \partial \mu J_{CS} = \partial \mu \left[ \frac{g^2}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( A_\nu F_{\rho\sigma} + \frac{2i}{3} g A_\nu A_\rho A_\sigma \right) \right] \]

Change of the Chern-Simons number is the source of baryon number change

\[N_{CS}(t) = \int d^3\vec{x} J_{CS}^0 = \frac{g^2}{16\pi^2} \int d^3\vec{x} \epsilon^{ijk} \text{Tr} \left( A_i F_{jk} + \frac{2i}{3} g A_i A_j A_k \right) = \text{intgers for vacua} \]

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\[\vec{A}_i(t_2, \vec{x}) = \frac{i}{g} U_2^{-1} \vec{\partial}_i U_2 \rightarrow E(t_2) = 0 \]

\[E[\{A_i(t, x)\}] \]

Related by large gauge transformation

\[\vec{A}_i(t_1, \vec{x}) = \frac{i}{g} U_1^{-1} \vec{\partial}_i U_1 \rightarrow E(t_1) = 0 \]

\[N_{CS} = \frac{1}{24\pi^2} \int \text{Tr}[(U^{-1}dU)^3] \]

\[E_{saddle} \sim \frac{1}{\alpha W \Delta t} \]
B violation in the SM

At zero temperature, tunneling rate to give ($\Delta N_{CS} = \pm 1$) is exponentially suppressed.

$$\Pr(\text{tunneling}) \propto \left(\exp(-S[A_{\text{inst.}}])\right)^2 \sim e^{-16\pi/g^2} \sim 10^{-170} \ll 1$$

where $S[A]$ is the Euclidean action for the possible tunneling trajectory of the gauge field. The **instanton** action is given by the condition $\delta S = 0$ ($D_\mu F^\mu_{\text{inst.}} = 0$, $F_{\text{inst.}} = \tilde{F}_{\text{inst.}}$).

$$S[A] \geq S[A_{\text{inst.}}] = -\frac{1}{2} \int d^4x E \text{Tr}[F_{\text{inst.}}F_{\text{inst.}}] = \frac{8\pi^2}{g^2}$$

(Because SU(2) is broken and gauge bosons get mass of $m_W$, the dominant contribution of tunneling happens during $\Delta t \leq m^{-1}_W$. For $\Delta t \gg m^{-1}_W$, additional suppression: $\exp(-m^2_W \Delta t^2)$)

Intuitively, $\Pr(\Delta N_{CS} = 1) \sim e^{-2\pi \Delta t E_{\text{saddle}}} \sim e^{-2\pi/\alpha_W}$
At zero temperature, tunneling rate to give \((\Delta N_{CS} = \pm 1)\) is exponentially suppressed.

\[
\text{Pr( tunneling) } \propto (\exp(-S[A_{\text{inst.}]}) )^2 \sim e^{-16\pi/g^2} \sim 10^{-170} \ll 1
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\]

(Because SU(2) is broken and gauge bosons get mass of \(m_W\), the dominant contribution of tunneling happens during \(\Delta t \leq m_W^{-1}\). For \(\Delta t \gg m_W^{-1}\), additional suppression: \(\exp(-m_W^2 \Delta t^2)\))

Such a small probability at zero temperature is good for stability of baryons. What happens in the early universe?
B (and C&P) violation in the SM

\[
(\partial \mu J_B^\mu) = (\partial \mu J_L^\mu) = \frac{n_f}{32\pi^2} \left( 2g^2 \text{Tr}[W_{\mu\nu} \bar{W}^{\mu\nu}] - g'^2 B_{\mu\nu} \bar{B}^{\mu\nu} \right)
\]

\[
\Delta N_B = n_f \Delta N_{CS}
\]

Note that energy barrier is finite. Therefore at the early Universe with finite temperatures, T, there could be enough energy to change the CS number:

\[
\text{Pr}(\Delta N_{CS} = 1) \propto \exp \left( - \frac{E_{\text{sph}}(T)}{T} \right) \sim e^{\frac{8\pi v(T)}{gT}}
\]

Creation and annihilation of sphalerons at the early Universe provide meaningful baryon number violating interactions

Static configuration at this point: sphaleron

(σφαλερός : ready to fall)

\[
E[A_i(t, x)]
\]

\[
E_{\text{sph}} \sim \frac{2m_W(T)}{\alpha_W}
\]
** Sphaleorn transition

from Sam S.C. Won’s slides

Static configuration at this point: sphaleron
(σφαλερός : ready to fall)

\[ E_{\text{sph}} \sim \frac{2m_W(T)}{\alpha_W} \]
B violation in the SM

\[ (\partial_\mu J_B^\mu) = (\partial_\mu J_L^\mu) = \frac{n_f}{32\pi^2} (2g^2 \text{Tr}[W_{\mu\nu} \tilde{W}^{\mu\nu}] - g'^2 B_{\mu\nu} \tilde{B}^{\mu\nu}) \]

\[ \Delta N_B = n_f \Delta N_{CS} \]

Note: since baryons are fermions, creating baryons cost energy as the figure. Therefore even if the baryon number is created, it could be washed out if CPT is preserved after generation \([\text{(CPT)}, \tilde{H}] = 0\) and sphalerons are in equilibrium \(\gamma_{\text{sph}} t \gg 1\):

\[ \dot{N}_B \approx -\gamma_{\text{sph}} N_B \rightarrow N_B \sim e^{-\gamma_{\text{sph}} t} \]

i.e. \(\langle \tilde{N}_B \rangle_t \rightarrow \text{Tr} \left( e^{-\tilde{H}/T} \tilde{N}_B \right) = 0 \)

Need of sudden decoupling for the sphaleron process
2) Electroweak symmetry is spontaneously broken at $T=0$.

At the early Universe with high temperatures, symmetry could be restored. EW phase transition happens as temperature decreases. This is most significant out-of-equil. process in the SM. e.g.

Bubble nucleation/expansion and collision: strong out-of-equilibrium process
Out-of-equilibrium evolution in the SM

**EWPT of the SM** \((m_H = 125 \text{ GeV})\) is not 1\textsuperscript{st}, 2\textsuperscript{nd} order but **cross-over**: similar to the 2\textsuperscript{nd} order PT, but smooth change of Higgs expectation value, i.e. the correlation length does not jump (1\textsuperscript{st}) or diverge (2\textsuperscript{nd}) at \(T = T_c\), just smoothly change \((\xi(T_c)^{-1} = m_H^2(T_c) \approx \lambda(\phi^2)_{T_c} > 0)\).

Out-of-equilibrium process is not so violent.
Out-of-equilibrium evolution in the SM

Sphaleron rate changes smoothly as the temperature decreases. Considering change of $N_{CS}$ as the one-dimensional random walk, the rate per unit volume is given by

$$
\Gamma_{sph}(T) = \lim_{v,t \to \infty} \frac{1}{v t} \text{Tr} \left[ e^{-\beta H} (N_{CS}(t) - N_{CS}(0))^2 \right]
\approx 20 \alpha_w^5 T^4 \\
\approx 0(1) \ 10^{-5} T^4 \left( \frac{\alpha_w}{0.03} \right)^4 \left( \frac{2m_w(T)}{\alpha_w T} \right)^7 \exp \left( - \frac{E_{sph}(T)}{T} \right)
$$

(for $T > E_{sph} \approx 2m_w / \alpha_w$)

$$
\frac{\Gamma_{sph}}{T^3} \frac{1}{H} \approx \gamma_{sph} t \gg 1
$$

$(T) / T$

$\gamma_{sph}$

sphaleron decoupling

\[ \log(\alpha(T)/T) \]

$\gamma_{sph} t \ll 1$ (for $\nu(T)/T > 1$)

D’Onofrio, Rummukainen, Tranberg arXiv:1403.3565

\[ \frac{\nu^2(T)}{T^2} \]

\[ \frac{\ln \Gamma_{sph}(T)}{T^4} \]

multicanonical

standard

perturbative

pure gauge

1

2

0.2

0.4

0.6

0.8

1

130

140

150

160

170

180

130

140

150

160

170

$T / GeV$
C & CP violation in the SM

CP transformation in QFT: The quantum field operators, $\psi_i(t, \vec{x})$, $\chi_i(t, \vec{x})$, $\phi(t, \vec{x})$, transform under the CP as

$$(CP)\psi_i(t, \vec{x})(CP)^{-1} = \bar{\psi}_i(t, -\vec{x})$$
$$(CP)\chi_i(t, \vec{x})(CP)^{-1} = \bar{\chi}_i(t, -\vec{x})$$
$$(CP)\phi(t, \vec{x})(CP)^{-1} = \phi^*(t, -\vec{x})$$

Therefore, the interaction terms $\Delta \mathcal{L}(t, \vec{x})$ given by

$$\Delta \mathcal{L}(t, \vec{x}) = y_{ij} \phi(t, \vec{x})\psi_i(t, \vec{x})\chi_j(t, \vec{x}) + y_{ij}^* \phi^*(t, \vec{x})\bar{\psi}_i(t, \vec{x})\bar{\chi}_j(t, \vec{x})$$

transform as

$$(CP)\Delta \mathcal{L}(t, \vec{x})(CP)^{-1} = y_{ij}^* \phi(t, -\vec{x})\psi_i(t, -\vec{x})\chi_j(t, -\vec{x}) + y_{ij} \phi^*(t, -\vec{x})\bar{\psi}_i(t, -\vec{x})\bar{\chi}_j(t, -\vec{x})$$

CP is conserved i.e. $(CP)\Delta \mathcal{L}(t, \vec{x})(CP)^{-1} = \Delta \mathcal{L}(t, -\vec{x})$, only if the Yukawa couplings, $y_{ij}$, are real: $y_{ij} = y_{ij}^*$.

Complex couplings are necessary to get CP violation

cf) CPT transformation gives $y_{ij} \rightarrow y_{ij}^*$, $O(t, x) \rightarrow O(-t, x)$. Therefore
$$(CPT)\Delta \mathcal{L}(t, \vec{x})(CPT)^{-1} = \Delta \mathcal{L}(-t, -\vec{x})$$ if there is not CPT violating background.
3) C and CP are violated in the SM:

**C violation:** Only left-handed neutrino exists.

Gauge interactions of left-handed and right-handed fermions different

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

**CP violation:** Complex couplings (in mass eigen-basis)

$$\Delta \mathcal{L} = -\frac{g}{\sqrt{2}} (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^\mu W^+_\mu V^+_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$ 

Cabibbo–Kobayashi–Maskawa Matrix (3 × 3)

$$V_{\text{CKM}} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{\text{CKM}}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{\text{CKM}}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{\text{CKM}}} & s_{23} c_{13} \\ s_{12} c_{23} - c_{12} c_{23} s_{13} e^{i\delta_{\text{CKM}}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{\text{CKM}}} & c_{23} c_{13} \end{pmatrix} \neq V^*_{\text{CKM}}$$

$$\theta_{12} \approx 13^0, \theta_{13} \approx 0.2^0, \theta_{23} \approx 2.38^0, \delta_{\text{CKM}} \approx 68^0 \neq 0$$

However, the basis independent CP violation (Jarlskog invariant) gives too small value

$$J = i \text{ Tr } \left( [Y_u^+ Y_u, Y_d^+ Y_d]^3 \right) = \frac{(m^2_u - m^2_c)(m^2_c - m^2_t)(m^2_t - m^2_u)(m^2_d - m^2_s)(m^2_s - m^2_b)(m^2_b - m^2_d)}{v^{12}}$$

$$\times c_{12} c_{13} s_{12} s_{13} s_{23} \sin \delta_{\text{CKM}} \sim 10^{-22}$$
The devil is in the detail

Electroweak phase transition in the SM is not strong out-of-equilibrium process

CP violation of the SM is too small

There are all ingredients but NOT enough! New physics is necessary