EFFECTIVE FIELD THEORY OF WIMP DARK MATTER

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MB and V. Vaidya: 1510.02470, JHEP 1603 (2016) 213;
WHY DARK MATTER?

Anomalies on 3 different astrophysical scales!

**Galactic Rotation curves:**
Stars move faster than expected

**Colliding Clusters:**
Gravitational wells nowhere near visible peaks

“Not modified gravity”

Vera Rubin 1928-2016
Established Rotation Curve anomaly
DARK MATTER ABUNDANCE

Cosmic Microwave Background:
Fluctuations measure Dark Matter as 27% of Universe's energy (Planck)

Animations from W. Hu
We know some “dark” particles! Neutrinos!
But they aren’t dark matter

The Great Ruler of Particle Physics

- $10^{-43}$ GeV Compton Wavelength
  Visible Universe
- $10^{-22}$ GeV Compton Wavelength to fit in Dwarf Galaxy
- $10^{-20}$ - $10^{-11}$ GeV QCD axion Dark Matter
- $10^2$ - $10^4$ GeV Weakly-Interacting Massive Particle (WIMP) Dark Matter
- $10^0$ eV Tremaine-Gunn Bound
  Stack fermions in galaxy
- $10^{19}$ GeV Planck Mass
WHY WIMPS?

Cosmic Expansion

DM density decreases: Annihilation & expansion

\[ \langle \sigma v \rangle_{\text{annihilation}} \sim C \frac{\alpha^2}{M_X^2} \]
WIMP MIRACLE

\[ \Omega_{DM} \sim \frac{1}{10^3 \langle \sigma v \rangle} \frac{1}{T_{CMB} M_{Planck}} \sim \frac{1}{10^3 \langle \sigma v \rangle} \frac{1}{\text{TeV}^2} \]

\[ M_X \sim \text{TeV} \left( 10 \sqrt{C \alpha} \right) \sqrt{\frac{\Omega_{DM}}{0.27}} \]

WIMP can be simple addition to known particles & forces. **WHY?**

DM density decreases:
- \( \Omega \): Annihilation & expansion
- \( Y \): Annihilation

See Dimopoulos PLB 246(1990):347-52
STARTING SIMPLE W/ WIMPS

Maybe we already know everything here except $\chi$?

$X$: Z-boson, Higgs?

$\psi$: Elementary Fermion, Higgs?

$\alpha$: $\alpha_{\text{weak}}$?

$<\sigma v>_{\text{annihilation}} \sim C \alpha^2/M_X^2$
**RELIC DENSITY**

$\Omega_{DM} = 0.27$

- **Simple Candidates!**
  - Dark Matter $\leftrightarrow$ Weak Scale:
    - Weak Triplet: “Wino”
    - Weak Doublet: “Higgsino”
    - Weak Quintuplet

- **Correct Dark Matter Density fixes $M_\chi$:**
  - Wino: 3 TeV
  - Higgsino: 1 TeV
  - Quintuplet: 9 TeV

- **Tying WIMP to Weak Scale**
e.g. SUSY & explaining elementary fermion masses
1403.6118: MB, D. Stolarski, T. Zorawski
**Indirect Detection:**
Photons from Dark Matter Annihilation

**HESS/VERITAS** can probe Dark Matter Masses up to 70 TeV

Successor **CTA** will start in 2023, will test up to 100 TeV
WINO SHOT DEAD BY HESS?

From 1307.4082:

**Blue line:** Wino annihilation rate

**Blue shade:** Exclusion from HESS

**Yellow shade:** All DM is thermal wino

*1307.4082: Cohen, Lisanti, Pierce, and Slatyer; see also “In Wino Veritas”, 1307.4400: Fan & Reece

The Loophole

Navarro-Frank-White (cusped) halo profile assumed

Cusp vs. Core

We are here ~ 8.5 kpc

Ruled out by 16x?
WHAT COULD SAVE THE WINO?

## Halo Model
- Flatten distribution in galactic center (core)

## Quantum Corrections
- Claim in literature of **75% reduction** from first quantum corrections

### Core needed:
- **1.5 kpc**
- **0.5 kpc**

Simulations with baryons show **cusped profiles down to 1 kpc** (1208.4844, 1305.5360, 1306.0898)

**O(1-10) Factor at stake,**
need state of the art calculation to determine

From Cohen et al. 1307.4082
• 3 separate threats to perturbation theory!
  
• $M_X/m_w >> 1 \rightarrow$ Long range force

• $M_X/m_w >> 1 \rightarrow$ Electroweak shower

• $\log(1 - z_{cut}) \rightarrow$ Phase space restriction

• Proliferation of scales $\rightarrow$ Effective Field Theory
Effective Field Theory: Systematically decouple High-Energy Physics

Dropping high-frequency modes can suffice

Quantum relativist terminology: Energy=Mass=Momentum=Frequency
MODERN EFFECTIVE FIELD THEORY

Eliminate modes

MP3: Psychoacoustics
Effective Field Theory: Kinematics

Shower of radiation:
Less Low-energy and Collinear enhancement

1007.0758: MB, Marcantonini, C., Stewart, I.
LONG-RANGE FORCES

• **Sommerfeld Enhancement**, generic for WIMP Dark Matter ($v \sim 10^{-3-6}$)

• **Quantum-Mechanically Potential** drags wavefunction to peak at origin

$$\left\langle \sigma v \right\rangle \equiv |\psi(0)|^2 \Gamma_{\text{pert}}.$$  

Wavefunction at the origin

In absence of gravitation, capture radius is geometric, $R$

Turning on gravity, cross section grows for slower projectile:

$$b_{\text{capture}} = R \sqrt{1 + \frac{2GM}{v^2 R}}$$

$$b_{\text{capture}} \sim \frac{1}{v}$$
\[ A_n \simeq \alpha \left( \frac{\alpha_W M_X}{m_W} \right)^n \]

\[ |\psi(0)|^2 \propto \min \left[ \frac{\alpha_W}{v}, \frac{\alpha_W M_X}{m_W} \right] \]

\[ \text{Parametrically enhanced wavefunction} \]

Reduce Relativistic WIMP QFT to Quantum Mechanics + Potential

[J. Hisano, S. Matsumoto, M. Nojiri, O. Saito, hep-ph/0412403
MB, Rothstein, I., Vaidya, V.: 1412.8698]
**NR“QCD” FOR WIMPS**

### Multimodal Effective Field Theory

<table>
<thead>
<tr>
<th><strong>WIMP</strong> $\chi$</th>
<th>$(E,p) \sim (M_\chi v^2, M_\chi v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Potential</em> $A_\mu \rightarrow V(r)$</td>
<td>$(E,p) \sim (M_\chi v^2, M_\chi v)$</td>
</tr>
<tr>
<td><em>Ultrasoft</em> $S \sim \text{Exp}[i \int v \cdot A_{us}]$</td>
<td>$(E,p) \sim (M_\chi v^2, M_\chi v^2)$</td>
</tr>
<tr>
<td><em>Soft</em> $A_{s\mu}$</td>
<td>$(E,p) \sim (M_\chi v, M_\chi v)$</td>
</tr>
</tbody>
</table>

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*Classic treatment of NRQCD in Bodwin, Braaten, LePage*  
*hep-ph/9407339*

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*Hierarchy of scales for nonrelativistic particles*

```
$M_X$  
$M_\chi v$  
$M_\chi v^2$  
```

*“Soft” Radiation appears as a Wilson Line*
• 3 separate threats to perturbation theory!

• $\frac{M_X}{m_w} \gg 1 \rightarrow$ Long range force

• $\frac{M_X}{m_w} \gg 1 \rightarrow$ Electroweak shower

• $\log(1 - z_{\text{cut}}) \rightarrow$ Phase space restriction

• Proliferation of scales $\rightarrow$ Effective Field Theory
HUGE ACCELERATION $\rightarrow$ CLASSICAL RADIATION

Charged particles in annihilation process radiate ($\gamma, W, Z$) from acceleration

\[
\sigma v = \sigma v_0 \left| \exp \left[ -\frac{\alpha}{2\pi} \log\left( \frac{E_{\text{high}}}{E_{\text{low}}} \right) \log\left( \frac{E_{\text{high}}}{E_{\text{collinear}}} \right) \right]\right|^2
\]

Above rate produces classical spectrum, but hard to see in quantum perturbation theory

\[
\frac{\alpha W}{\pi} \log\left( \frac{M_{\text{wino}}^2}{m_W^2} \right)^2 \approx 0.6
\]

Perturbative factor picks up kinematic enhancements “Sudakov double log”

Double log Large correction!
Electroweak physics has **infrared divergences**, even in **fully inclusive observables**

\[ \sigma_{e^+e^-} \neq \sigma_{e^+\nu_e} \]

Virtual corrections only cancel emission upon color averaging.

B-N VIOLATION IN EFT

\[ S_{12}^{a'b'ab} = \left(0 \left| (Y_n^{3k} Y_{\bar{n}}^{dk})^\dagger (x) \delta(\mathcal{M} - \hat{\mathcal{M}}_s) \left( Y_n^{3g} Y_{\bar{n}}^{df} Y_v^{ag} Y_v^{bf} \right) (0) \right| 0 \right) \delta^{a'b'} , \]

\[ \rightarrow \delta^{a'b'} \delta(m_X^2) \langle 0 \left| e^3 Y_v^{ae} Y_v^{bf} Y_{\bar{n}}^{3f} \right| 0 \rangle \]

Wilson lines collapse from measurement inclusivity, but identifying photon in final state precludes sum over degenerate states.
SOFT/COLLINEAR ENHANCEMENT

Soft radiation: Time-scales much longer than annihilation

Collinear Radiation: Narrow splitting of one particle into 2

SCET for Dark Matter annihilation

[MB, Rothstein, I., Vaidya, V.: 1409.4415]

Keep modes with kinematic enhancement (soft, collinear)

SOFT-COLLINEAR EFFECTIVE THEORY

- Large scale-hierarchies can arise within one field

\[ p_{\perp} \ll Q \]

\[ M_{\text{jet}}^2 \sim p_{\perp} Q \]

- We can use Renormalization Group to resum kinematic logs

Integrate out hard modes, keep those collinear to null directions and soft fields
SCET OBSERVABLES

Factorized Hilbert Space:

\[ |X\rangle = |X_{\text{collinear}}\rangle |X_{\text{soft}}\rangle \]

Squared Wilson coefficient

\[ J_n = \langle 0| B_{n \perp} \delta[f(Q, z_{\text{collinear}})] |X_n\rangle \langle X_n| B_{n \perp} |0\rangle \]

\[ d\sigma = H(Q) J(Q, z_{\text{collinear}}) \otimes S(z_{\text{soft}}) \]

\[ S = \langle 0| (YY)^\dagger \delta[f(z_{\text{soft}})] (YY) |0\rangle \]
**SEMI-INCLUSIVE ANNIHILATION**

- HESS/VERITAS observes photons colliding with the atmosphere

- Therefore, we compute \( \chi \chi \rightarrow \gamma^+ (\text{Whatever else}) \)

- But we have introduced a new scale, \( M_\chi (1 - z_{\text{cut}}) \approx \text{Bin size} \)

*From HESS collaboration 1301.1173*

- at 3 TeV, energy resolution is \( \sim 400 \text{ GeV} \)
- \( m_W = 80 \text{ GeV} \)

\[
E_\gamma \text{ (soft } W) = M_\chi - m_W / 2 \\
E_\gamma \text{ (collinear } Ws) = M_\chi - m_W^2 / M_\chi
\]
SCET WITH 2 EXPANSIONS

Center of Mass Energy

Measured Jet Mass

Soft radiation scale

Electroweak scale
• 3 separate threats to perturbation theory!

- $\frac{M_X}{m_w} \gg 1 \rightarrow \text{Long range force}$
- $\frac{M_X}{m_w} \gg 1 \rightarrow \text{Electroweak shower}$
- $\log(1 - z_{\text{cut}}) \rightarrow \text{Phase space restriction}$

If $J(M_X, (1 - z_{\text{coll}}), m_w)$

$M_X \sqrt{(1 - z_{\text{cut}})} \sim M_X (1 - z_{\text{cut}}) \sim m_W$

Jet $\sim$ Soft $\sim$ Electroweak
SOFT REFACTORIZATION

S: Perform matching
@ \( M_\chi \sqrt{(1 - z_{\text{cut}})} \)

\[
S \rightarrow H_S(M_\chi \sqrt{(1 - z_{\text{cut}})))S(m_\text{W})
\]

Remaining soft:
\((p_+, p_-, p_\perp) \sim M(\lambda, \lambda, \lambda)\)

\[
\lambda = m_\text{W}/M_\chi
\]

**BUT...**

what about measurement function?

\[
(1 - z) = \frac{1}{4 M_\chi^2} m_\chi^2 = \frac{1}{4 M_\chi^2} \left( \sum_{i \in X_s} p_i^\mu + \sum_{i \in X_c} p_i^\mu \right)^2
\]

\[
\equiv (1 - z_s) + (1 - z_c) + O(\lambda^2)
\]
Collinear soft modes account for radiation along photon direction, but contribute to recoil jet mass.

\[
\frac{d\sigma}{dz} = H(m_\chi, \mu) \cdot H_{Jn}(m_\chi, (1 - z), \mu) \cdot H_S(m_\chi, (1 - z), \mu) \\
\quad \cdot J_\gamma(m_W, \mu, \nu) \cdot S(m_W, \mu, \nu) \cdot C_S(m_\chi, (1 - z), m_W, \mu, \nu) \cdot J_n(m_W, \mu, \nu)
\]

Alternate collinear-soft scaling:

\[(p_+, p_-, p_\perp) \sim M(1 - z_{cut})(\lambda^2, 1, \lambda)\]

\[\lambda = \frac{m_W}{M_\chi}(1 - z_{cut})\]

Factorization holds to NLL!

MB et al.: 1808.08956
• 3 separate threats to perturbation theory!

- \( \frac{M_X}{m_W} \gg 1 \rightarrow \text{Long range force} \)
- \( \frac{M_X}{m_W} \gg 1 \rightarrow \text{Electroweak shower} \)
- \( \log(1 - z_{\text{cut}}) \rightarrow \text{Phase space restriction} \)

- Proliferation of scales \( \rightarrow \) Effective Field Theory
\[
\frac{d\sigma}{dz} = \frac{\pi \alpha_W^2 \sin^2 \theta_W}{2M^2}\left(-2C_2(W)\frac{\alpha_W}{\pi} \log^2 \left(\frac{2M}{M}\right)\right) (F_0 + F_1)\delta(1 - z)
\]

\[
+ \left(C_2(W)\frac{\alpha_W}{\pi} \log \left(\frac{4M^2(1 - z)}{M^2}\right) + 3C_2(W)\frac{\alpha_W}{\pi} \log \left(\frac{M}{2M^2(1 - z)}\right)\right) F_0
\]

\[
+ \left(\left(-\frac{3}{2}C_2(W)\frac{\alpha_W}{\pi} \log^2 \left(\frac{M^2}{2M^2(1 - z)}\right) + C_2(W)\frac{\alpha_W}{2\pi} \log^2 \left(\frac{M^2}{4M^2(1 - z)}\right)\right)\right) \frac{1}{1 - z} + \ldots
\]

*Squared Wilson Coefficient for wino annihilation*

*Linear combination of Sommerfeld factors*

MB et al.: 1712.07656
CUMULATIVE RESUMMED ANNIHILATION RATE

Thermal Wino Cross Section

- LL
- NLL

Thermal relic wino rate vs. Energy fraction
Update to HESS 2013 analysis projected to rule out by 30x, halo loophole 1-1.5 kpc

More aggressive analysis with better galactic center understanding, halo loophole closes, $r_c > 2.5$ kpc

Hooper: 1608.00003 limit of 2 kpc
CONCLUSION

• Despite certain lore, **WIMPs remain motivated, viable, simple dark matter candidate.**

• But... **we have the wino on the run.**

• Data already in the can **will kill (or discover) it!**

• Analysis **easily extendable** to other SU(2) reps, general SUSY LSP, Higgs Portal, Bound State effects

• **Multi-TeV regime requires electroweak resummation even at qualitative level.**
**DISCOVERY AT THE LHC? NO**

Find Winos by their charged partner’s disappearing track

\[ M_\chi > 270 \text{ GeV} \]

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**From Cirelli et al. 1407.7058**

500 GeV, LHC Wino reach

Higgsino reach may not improve over LEP: 110 GeV

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**Han et al.: 1401.1235**
MINING FOR WINOS? NO

Example: Coupling Dark Matter to Nucleons through $Z$

Ruled Out

$\sigma_{\text{wino}} \sim 10^{-47} \text{ cm}^2$

Wino: Simple Model, but not Simple Calculation

$10^{-35} \text{ cm}^2$

From Rick Gaitskell (LUX) talk

J. Hisano et al.: 1104.0228
R. Hill & M. Solon: 1309.4092

Higgsino limits even weaker
Above high-frequency cutoff, \( \Lambda \), remove but change coupling

\[
\frac{d\lambda}{d \log(\Lambda)} = \frac{3\lambda^2}{16\pi^2}
\]

\[
\lambda(\Lambda) = \frac{\lambda(\Lambda^0)}{1 - \frac{3}{16\pi^2} \lambda(\Lambda^0) \log(\Lambda/\Lambda^0)}
\]

\[V(\phi) = \frac{\lambda}{4!} \phi^4\]

Included \( \lambda \ \text{Log}(\Lambda/\Lambda^0) \) to all orders!
SOMMERFELD RESONANCES

\[ r_{\text{Bohr}} \sim \frac{1}{\alpha M_X} \]

\[ r_{\text{Range}} \sim \frac{1}{m_W} \]

\[ r_{\text{Bohr}} \gg r_{\text{Range}} \]

No bound state

Bound state forms

Transition from short to long-range force leads to resonance

For wino

\[ m_W = \alpha_w M_X @ M_X = 2.4 \text{ TeV} \]
Zero-energy bound states → Peaks

\[ \alpha_W M_\chi = n^2 m_W \]

\[ \langle 0 | \chi_v^3 T i\sigma_2 \chi_v^3 \mid (\chi^0 \chi^0)_S \rangle = 4\sqrt{2} M_\chi s_{00} ; \]

\[ \langle 0 | \chi_v^{+T} i\sigma_2 \chi_v^- \mid (\chi^0 \chi^0)_S \rangle = 4 M_\chi s_{0\pm} \]
ELECTROWEAK RESUMMATION ERA

- Electroweak parton shower for colliders
- Interplay with QCD shower
- Factorization violation at LHC from Glauber modes*
- High-energy neutrinos (Eₜ > PeV) demanding resummation

\[
\begin{align*}
\text{Log}(\text{PeV}/m_\nu)^2 &\sim 100
\end{align*}
\]

[MB, Erdogan, O., Rothstein, L., Vaidya, V.: 1811.04120]
HIGGSINO RATE AT LL’

I TeV higgsino needs proper treatment of endpoint region

Minimally split Higgsino.*

Purest viable doublet

*Cheung et al: hep-ph:0512192

\[ \Delta M_0 = 200 \text{ keV}; \]
\[ \Delta M_+ = 350 \text{ MeV} \]

\[ \Delta M_0 = 2 \text{ GeV}; \]
\[ \Delta M_+ = 480 \text{ MeV} \]

Thermal Higgsino unconstrained by current/future tests

MB, Vaidya, V.: 1510.02470
Initial motivation for Wino stressed its simplicity, but perhaps its role in Dark Matter is more involved, including a non-thermal history, multi-component DM, mixing with other electroweak states.

Possibility for the wino to make up some fraction of DM with NFW profile flattened to a constant core at some radius.
CAPTURE TO BOUND STATE

Sommerfeld Resonances reveal bound states in spectrum

Blue: $e^+e^- \rightarrow \text{Positronium} + \gamma$
Yellow: $e^+e^- \rightarrow \gamma\gamma$ (Sommerfeld)
Green: $e^+e^- \rightarrow \gamma\gamma$ (No Sommerfeld)

For postitronium
Capture $\approx 3\times$ Annihilation

$$\sigma v|_{\text{capture}} = \frac{2^{10}\pi^2}{3} e^{-4} \alpha^3 \frac{1}{m_e^2} \frac{1}{v_{\text{rel}}}$$
$$\sigma v|_{\text{ann.}} = \frac{2\pi^2\alpha^3}{m_e^2} \frac{1}{v_{\text{rel}}}$$
What is the fate of a $\chi^0\chi^0$ (wino) Initial state?

Capture loses to annihilation, but new signature (capture photons)

Importance depends on identity of WIMP
GOLDILOCKS & THE 3 RESEARCH GROUPS

**Exclusive**

$2 \rightarrow 2$ annihilation: \( \gamma \gamma + \gamma Z \)

Ovanesyan, Slatyer, and Stewart: 1409.8294
Cohen et al.: 1409.7392

**Semi-inclusive**

Integrate out recoil state with OPE: \( \gamma + X \)

MB, I.Z. Rothstein, V. Vaidya: 1409.4415,
MB, I.Z. Rothstein, V. Vaidya: 1412.8698

**Endpoint Region**

Measurement forces recoil into jet: \( \gamma + X \)

MB, Cohen, Moult et al.: 1712.07656

**Semi-inclusive with cutoff**

MB and V. Vaidya: 1510.02470
TRADITIONAL FACTORIZATION

• SCET modes’ lightcone power counting
  • Collinear \( (p_+, p_-, p_\perp) \sim Q(1, \lambda^2, \lambda) \)
  • Soft \( (p_+, p_-, p_\perp) \sim Q(\lambda, \lambda, \lambda) \)
  • Ultrasoft \( (p_+, p_-, p_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2) \)

Pull ultrasoft Wilson line out of collinear field → Collinear/Soft Factorization!

hep-ph/0109045: Bauer, Pirjol, & Stewart
COLLINEAR REFACTORIZATION

Original SCET doesn’t distinguish soft scales: \((1 - z_{\text{coll}}), mw\)

\(J_n: \) Perform matching 
@ \(M_X \sqrt{(1 - z_{\text{cut}})}\)

\(J_n \to H_{J_n}(M_X \sqrt{(1 - z_{\text{cut}})}) J_n(m_W)\)

Remaining collinear:

\((p_+, p_-, p_\perp) \sim M(1, \lambda^2, \lambda)\)

\(\lambda = m_W/M_X\)
WHITHER SOFT DIVERGENCE?

\[ S_{11}^{a'b'ab}(x) = \left\langle 0 \left\vert (Y_n^{3k}Y_{\bar{n}}^{dk})^\dagger \delta \left[ M^2_\chi(1 - z_{\text{soft}}) - M^2_\chi(1 - \hat{z}_{\text{soft}}) \right] \right\rangle \times \left\langle X_s \right\vert X_s \right\rangle \left( Y_n^{3j}Y_{\bar{n}}^{dj} \right)^\dagger \delta^{a'b'} \delta^{ab} , \]

\[ \rightarrow S_{11}^{a'b'ab} \sim \delta^{a'b'} \delta^{ab} \langle 0 \left\vert \left[ Y^{ce'}_n Y^{3e'}_{\bar{n}} \right]^\dagger \delta \left[ M^2_\chi(1 - z_{\text{soft}}) \right] \left[ Y^{ce}_n Y^{3e}_{\bar{n}} \right] \right\rangle \]

\[ = \delta^{a'b'} \delta^{ab} \delta \left[ (M^2_\chi(1 - z_{\text{soft}})) \right] \]

One-loop soft anomalous dimension

\[ \Rightarrow \text{???)} \]

Can’t live in \( H_S \), because \( H_S \) doesn’t know about scale \( m_W \)
COLLINEAR SOFT MODE

Alternate soft scaling:

\[(p^+, p^-, p_\perp) \sim M(1-z_{\text{cut}})(\lambda^2, 1, \lambda)\]

\[\lambda = m_W/M_X(1-z_{\text{cut}})\]

\[\tilde{S}_{11}^{aba'b'} = \left\langle 0 \left| \left( Y_n^{3f'} Y_{\bar{n}}^{dg'} \right)^\dagger (0) \delta(M - \tilde{M}_{C_s}) \right. \left. \right| X_{C_s} \right\rangle \left\langle X_{C_s} \right| \left( Y_n^{3f} Y_{\bar{n}}^{dg} \right)(0) |0\rangle \delta^{f'g'} \delta^{a'b'} \delta^{fg} \delta^{ab}.\]

Get back pre-refactorized divergence!
RAPIDITY RG

- SCET is a “modal” theory

\[ A_\mu = A_{c;n}^\mu + A_{c;\bar{n}}^\mu + A_{soft}^\mu + \ldots \]

- We can get divergences when integrals invade other sectors. Soft-collinear overlap requires boost-violating regulator

- Regulating sets up RG for resumming these rapidity logs

\[ W_n = \sum_{\text{perms}} \exp \left[ -\frac{g}{\bar{n} \cdot P} \nu^\eta \bar{n} \cdot P \eta \bar{n} \cdot A_n \right] \]

Above from Chiu et al. 1202.0814: In SCETII, soft and collinear modes have same virtuality \( \nu \)-running lets us minimize log between soft & collinear scales