# Stringy Gravity: $\mathbf{O}(D, D)$ completion of General Relativity

**Einstein Double Field Equations**

$$G_{AB} = 8\pi G T_{AB}$$

**Hereafter $A, B$ are $\mathbf{O}(D, D)$ indices**


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***25th International Summer Institute on Phenomenology of Elementary Particle Physics and Cosmology***

***19th August 2019***

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  **while modified gravities involve arbitrariness and are thus often ugly.**

- **Today I am going to talk about a certain modified gravity which**

  - **has no arbitrariness;**
  - **completes GR with Symmetry Principle, O($D$, $D$);**
  - **and is perhaps twice (doubly) more beautiful than GR.**[1]

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# Introduction

– General Relativity is based on Riemannian geometry, where the only geometric and gravitational field is the Riemannian metric, $g_{\mu\nu}$. Other fields are meant to be extra matter.

– On the other hand, string theory suggests to put a two-form gauge potential, $B_{\mu\nu}$, and a scalar dilaton, $\phi$, on an equal footing along with the metric:

- They form the closed string massless (NS-NS) sector, being ubiquitous in all string theories,

$$\int \mathrm{d}^D x \sqrt{-g} e^{-2\phi} \left( R_g + 4\partial_\mu \phi \partial^\mu \phi - \tfrac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} \right) \qquad \text{where} \qquad H = \mathrm{d}B \,.$$

This action hides $\mathbf{O}(D,D)$ symmetry of T-duality which transforms $g, B, \phi$ into one another. Buscher 1987

– T-duality hints at a natural extension of GR, or the $\mathbf{O}(D,D)$ completion of GR, in which the above closed string massless sector constitutes the fundamental gravitational multiplet.

Double Field Theory (DFT), initiated by Siegel 1993 & Hull, Zwiebach 2009-2010, turns out to provide a concrete realization for this idea of Stringy Gravity by manifesting $\mathbf{O}(D,D)$ T-duality.

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# Philosophy

– **To view DFT as Stringy Gravity, or the O$(D, D)$ completion of GR, where the whole closed-string massless (NS-NS) sector is to be the fundamental gravitational multiplet.**

The previous Lagrangian itself should be identified as a generalized scalar curvature,

$$R_g + 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} \quad \equiv S_{(0)} \quad : \quad \text{Pure Gravity}$$

– **To employ novel stringy differential geometry, beyond Riemann.**

– **To formulate the theory in terms of O$(D, D)$ covariant fields only,** rather than conventional ones in SUGRAs such as $g_{\mu\nu}$, $B_{\mu\nu}$, $\phi$, $p$-forms,...

– Consequently, DFT not only reformulates SUGRA but also unifies non-Riemannian or chiral gravities (Newton-Cartan, Carroll, Gomis-Ooguri,...).

Further, it implies, rather inevitably, **modifications to General Relativity**.

# Plan

I. Review of covariant derivatives, $\nabla_A$, and curvatures, $S_{AB}$, $S_{(0)}$ in DFT.

: *classification of the most general DFT backgrounds.*

II. Derivation of the Einstein Double Field Equations, $G_{AB} = 8\pi G T_{AB}$, as the unifying single expression for the closed-string massless NS-NS sector:

$$G_{AB} = 4P_{[A}{}^{C}\bar{P}_{B]}{}^{D}S_{CD} - \tfrac{1}{2}\mathcal{J}_{AB}S_{(0)}\,, \qquad\qquad \nabla_A G^{AB} = 0\,,$$

$$T_{AB} = e^{2d}\left(8\bar{P}_{C[A}P_{B]D}\frac{\delta\mathcal{L}_{\rm matter}}{\delta\mathcal{H}_{CD}} - \tfrac{1}{2}\mathcal{J}^{AB}\frac{\delta\mathcal{L}_{\rm matter}}{\delta d}\right), \qquad\qquad \nabla_A T^{AB} = 0\,.$$

III. *i)* Applicatioin to Cosmology, $\mathbf{O}(D,D)$ *completion of the Friedmann equations*;

*ii)* Spherical (regular) solution to $G_{AB} = 8\pi G T_{AB}$ for a ‘stringy star’

**Collaborators:**

Stephen Angus, Kyungho Cho, Guilherme Franzmann, Shinji Mukohyama, Kevin Morand (recent)

as well as Imtak Jeon and Kanghoon Lee (earlier)

# Notation

| Index | Representation | Metric (raising/lowering indices) |
|---|---|---|
| $A, B, \cdots, M, N, \cdots$ | $\mathbf{O}(D, D)$ vector | $\mathcal{J}_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ |
| $p, q, \cdots$ | $\mathbf{Spin}(1, D{-}1)_L$ vector | $\eta_{pq} = \mathrm{diag}(- + + \cdots +)$ |
| $\alpha, \beta, \cdots$ | $\mathbf{Spin}(1, D{-}1)_L$ spinor | $C_{\alpha\beta}, \qquad (\gamma^p)^T = C\gamma^p C^{-1}$ |
| $\bar{p}, \bar{q}, \cdots$ | $\mathbf{Spin}(D{-}1, 1)_R$ vector | $\bar{\eta}_{\bar{p}\bar{q}} = \mathrm{diag}(+ - - \cdots -)$ |
| $\bar{\alpha}, \bar{\beta}, \cdots$ | $\mathbf{Spin}(D{-}1, 1)_R$ spinor | $\bar{C}_{\bar{\alpha}\bar{\beta}}, \qquad (\bar{\gamma}^{\bar{p}})^T = \bar{C}\bar{\gamma}^{\bar{p}}\bar{C}^{-1}$ |

– The twofold local Lorentz symmetries indicate two distinct locally inertial frames for the left-moving and the right-moving closed string sectors separately $\Rightarrow$ Unification of IIA and IIB.

## Closed-string massless sector as 'Stringy Graviton Fields'

The stringy graviton fields consist of the DFT dilaton, $d$, and DFT metric, $\mathcal{H}_{MN}$ :

$$\mathcal{H}_{MN} = \mathcal{H}_{NM}\,, \qquad\qquad \mathcal{H}_{K}{}^{L}\mathcal{H}_{M}{}^{N}\mathcal{J}_{LN} = \mathcal{J}_{KM}\,.$$

Combining $\mathcal{J}_{MN}$ and $\mathcal{H}_{MN}$, we get a pair of symmetric projection matrices,

$$P_{MN} = P_{NM} = \tfrac{1}{2}(\mathcal{J}_{MN} + \mathcal{H}_{MN})\,, \qquad\qquad P_{L}{}^{M}P_{M}{}^{N} = P_{L}{}^{N}\,,$$

$$\bar{P}_{MN} = \bar{P}_{NM} = \tfrac{1}{2}(\mathcal{J}_{MN} - \mathcal{H}_{MN})\,, \qquad\qquad \bar{P}_{L}{}^{M}\bar{P}_{M}{}^{N} = \bar{P}_{L}{}^{N}\,,$$

which are orthogonal and complete,

$$P_{L}{}^{M}\bar{P}_{M}{}^{N} = 0\,, \qquad\qquad P_{M}{}^{N} + \bar{P}_{M}{}^{N} = \delta_{M}{}^{N}\,.$$

Further, taking the "square roots" of the projectors,

$$P_{MN} = V_{M}{}^{p}V_{N}{}^{q}\eta_{pq}\,, \qquad\qquad \bar{P}_{MN} = \bar{V}_{M}{}^{\bar{p}}\bar{V}_{N}{}^{\bar{q}}\bar{\eta}_{\bar{p}\bar{q}}\,,$$

we get a pair of DFT vielbeins satisfying their own defining properties,

$$V_{Mp}V^{M}{}_{q} = \eta_{pq}\,, \qquad\qquad \bar{V}_{M\bar{p}}\bar{V}^{M}{}_{\bar{q}} = \bar{\eta}_{\bar{p}\bar{q}}\,, \qquad\qquad V_{Mp}\bar{V}^{M}{}_{\bar{q}} = 0\,,$$

or equivalently

$$V_{M}{}^{p}V_{Np} + \bar{V}_{M}{}^{\bar{p}}\bar{V}_{N\bar{p}} = \mathcal{J}_{MN}\,.$$

Solution to the defining relation, $\mathcal{H}_{MN}=\mathcal{H}_{NM},\ \mathcal{H}_K{}^L\mathcal{H}_M{}^N\mathcal{J}_{LN}=\mathcal{J}_{KM}$ ?

$$\mathcal{H}_{MN}=\left(\begin{array}{cc} g^{-1} & -g^{-1}B \\ Bg^{-1} & g-Bg^{-1}B \end{array}\right) \qquad \text{or} \qquad \mathcal{H}_{MN}=\mathcal{J}_{MN}=\left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right)$$

The left one is well-known: it contains a Riemannian metric and reduces DFT to SUGRA.

The right one admits no Riemannian nor SUGRA interpretation.

Thus, DFT describes not only Riemannian SUGRA but also non-Riemannian novel geometries.

The most general form of the DFT metric, $\mathcal{H}_{MN}=\mathcal{H}_{NM}$, $\mathcal{H}_{K}{}^{L}\mathcal{H}_{M}{}^{N}\mathcal{J}_{LN}=\mathcal{J}_{KM}$, is characterized by two non-negative integers, $(n,\bar{n})$, $0\leq n+\bar{n}\leq D$:

$$\mathcal{H}_{AB} = \left(\begin{array}{cc} H^{\mu\nu} & -H^{\mu\sigma}B_{\sigma\lambda}+Y_{i}^{\mu}X_{\lambda}^{i}-\bar{Y}_{\bar{\imath}}^{\mu}\bar{X}_{\lambda}^{\bar{\imath}} \\ B_{\kappa\rho}H^{\rho\nu}+X_{\kappa}^{i}Y_{i}^{\nu}-\bar{X}_{\kappa}^{\bar{\imath}}\bar{Y}_{\bar{\imath}}^{\nu} & K_{\kappa\lambda}-B_{\kappa\rho}H^{\rho\sigma}B_{\sigma\lambda}+2X_{(\kappa}^{i}B_{\lambda)\rho}Y_{i}^{\rho}-2\bar{X}_{(\kappa}^{\bar{\imath}}B_{\lambda)\rho}\bar{Y}_{\bar{\imath}}^{\rho} \end{array}\right)$$

$$= \left(\begin{array}{cc} 1 & 0 \\ B & 1 \end{array}\right)\left(\begin{array}{cc} H & Y_{i}(X^{i})^{T}-\bar{Y}_{\bar{\imath}}(\bar{X}^{\bar{\imath}})^{T} \\ X^{i}(Y_{i})^{T}-\bar{X}^{\bar{\imath}}(\bar{Y}_{\bar{\imath}})^{T} & K \end{array}\right)\left(\begin{array}{cc} 1 & -B \\ 0 & 1 \end{array}\right)$$

*i)* Symmetric and skew-symmetric fields : $\quad H^{\mu\nu}=H^{\nu\mu}, \quad K_{\mu\nu}=K_{\nu\mu}, \quad B_{\mu\nu}=-B_{\nu\mu}\,;$

*ii)* Two kinds of zero eigenvectors: with $i,j=1,2,\cdots,n$ & $\bar{\imath},\bar{\jmath}=1,2,\cdots,\bar{n}$,

$$H^{\mu\nu}X_{\nu}^{i}=0\,,\qquad H^{\mu\nu}\bar{X}_{\nu}^{\bar{\imath}}=0\,,\qquad K_{\mu\nu}Y_{j}^{\nu}=0\,,\qquad K_{\mu\nu}\bar{Y}_{\bar{\jmath}}^{\nu}=0\,;$$

*iii)* Completeness relation: $\quad H^{\mu\rho}K_{\rho\nu}+Y_{i}^{\mu}X_{\nu}^{i}+\bar{Y}_{\bar{\imath}}^{\mu}\bar{X}_{\nu}^{\bar{\imath}}=\delta^{\mu}{}_{\nu}.$

- Note $\mathcal{H}_{A}{}^{A}=2(n-\bar{n})$ and $\frac{\mathbf{O}(D,D)}{\mathbf{O}(t+n,s+n)\times\mathbf{O}(s+\bar{n},t+\bar{n})}$ having dimension $D^{2}-(n-\bar{n})^{2}$ .

The most general form of the DFT metric, $\mathcal{H}_{MN}=\mathcal{H}_{NM}$, $\mathcal{H}_K{}^L\mathcal{H}_M{}^N\mathcal{J}_{LN}=\mathcal{J}_{KM}$, is characterized by two non-negative integers, $(n,\bar{n})$, $0\leq n+\bar{n}\leq D$:

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# Classification

Kevin Morand & JHP 1707.03713

**I.** $(n, \bar{n}) = (0, 0)$ corresponds to the Riemannian case or Generalized Geometry à la Hitchin :

$$\mathcal{H}_{MN} \equiv \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}, \qquad e^{-2d} \equiv \sqrt{|g|}e^{-2\phi}$$

Giveon, Rabinovici, Veneziano '89, Duff '90

**II.** Generically, on worldsheet, string becomes chiral and anti-chiral over the $n$ and $\bar{n}$ dimensions:

$$X^i_\mu\, \partial_+ x^\mu(\tau,\sigma) \equiv 0\,, \qquad \bar{X}^{\bar{\imath}}_\mu\, \partial_- x^\mu(\tau,\sigma) \equiv 0\,,$$

as can be shown using 'doubled-yet-gauged' string action.

– Non-Riemannian examples include

- $(1, 0)$ Newton-Cartan gravity ($\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2$, $\lim_{c\to\infty} g^{-1}$ is finite & degenerate)
- $(1, 1)$ Gomis-Ooguri non-relativistic string — Melby-Thompson, Meyer, Ko, JHP 2015
- $(D{-}1, 0)$ ultra-relativistic Carroll gravity
- $(D, 0)$ is uniquely given by $\mathcal{H} = \mathcal{J}$ : maximally non-Riemannian.
  This is the completely $\mathbf{O}(D, D)$-symmetric vacuum of DFT. It naturally realizes Siegel's chiral string.
  "*Spacetime emerges after SSB of* $\mathbf{O}(D, D)$*, identifying* $\{g_{\mu\nu}, B_{\rho\sigma}\}$ *as Goldstone bosons.*"
  Berman, Blair, and Otsuki 2019

  Further, taken as an internal space, it gives a 'moduli-free' (Scherk-Schwarz twistable) Kaluza-Klein reduction of DFT, in fact, to heterotic supergravity/string. — Cho, Morand, JHP 2018

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# Semi-covariant formalism

Imtak Jeon, Kanghoon Lee, & JHP 2010, 2011

- Semi-covariant derivative :

$$\nabla_C T_{A_1 A_2 \cdots A_n} := \partial_C T_{A_1 A_2 \cdots A_n} - \omega_T \Gamma^B{}_{BC} T_{A_1 A_2 \cdots A_n} + \sum_{i=1}^{n} \Gamma_{CA_i}{}^B T_{A_1 \cdots A_{i-1} B A_{i+1} \cdots A_n} ,$$

for which the 'DFT-Christoffel' connection can be uniquely fixed,

$$\Gamma_{CAB} = 2\left(P\partial_C P\bar{P}\right)_{[AB]} + 2\left(\bar{P}_{[A}{}^D \bar{P}_{B]}{}^E - P_{[A}{}^D P_{B]}{}^E\right)\partial_D P_{EC} - \tfrac{4}{D-1}\left(\bar{P}_{C[A}\bar{P}_{B]}{}^D + P_{C[A}P_{B]}{}^D\right)\left(\partial_D d + (P\partial^E P\bar{P})_{[ED]}\right)$$

by demanding compatibility with $\{\mathcal{J}_{AB}, \mathcal{H}_{AB}, d\}$, torsionless condition, and projection property,

$$\nabla_A P_{BC} = \nabla_A \bar{P}_{BC} = \nabla_A d = 0 , \qquad \hat{\mathcal{L}}_\xi^{\partial} = \hat{\mathcal{L}}_\xi^{\nabla} \quad \Leftrightarrow \quad \Gamma_{[ABC]} = 0 , \qquad (\mathcal{P} + \bar{\mathcal{P}})_{ABC}{}^{DEF}\Gamma_{DEF} = 0 ,$$

where multi-indexed projectors are

$$\mathcal{P}_{ABC}{}^{DEF} := P_A{}^D P_{[B}{}^{[E} P_{C]}{}^{F]} + \tfrac{2}{P_M{}^M - 1} P_{A[B} P_{C]}{}^{[E} P^{F]D} , \qquad \text{same for } \bar{\mathcal{P}}_{ABC}{}^{DEF} \text{ with } P_{AB} \leftrightarrow \bar{P}_{AB} .$$

The "generalized Lie derivative" (DFT-diffeomorphism generator) is

$$\hat{\mathcal{L}}_\xi T_{A_1 \cdots A_n} := \xi^C \partial_C T_{A_1 \cdots A_n} + \omega_T \partial_C \xi^C T_{A_1 \cdots A_n} + \sum_{i=1}^{n} (\partial_{A_i}\xi^C - \partial^C \xi_{A_i}) T_{A_1 \cdots C \cdots A_n} ,$$

and in particular,

$$\hat{\mathcal{L}}_\xi \mathcal{H}_{AB} = 8\bar{P}_{(A}{}^{[C} P_{B)}{}^{D]} \nabla_C \xi_D , \qquad \hat{\mathcal{L}}_\xi d = -\tfrac{1}{2}\nabla_A \xi^A .$$

* **There are no normal coordinates where $\Gamma_{CAB}$ would vanish point-wise: Equivalence Principle is broken for string (*i.e.* extended object), but recoverable coupled to point particle (scalar field).**

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- Semi-covariant Riemann curvature :

$$S_{ABCD}=S_{[AB][CD]}=S_{CDAB}:=\tfrac{1}{2}\left(R_{ABCD}+R_{CDAB}-\Gamma^{E}{}_{AB}\Gamma_{ECD}\right)\,,\qquad S_{[ABC]D}=0\,,$$

where $R_{ABCD}$ denotes the ordinary "field strength", $R_{CDAB}{=}\partial_{A}\Gamma_{BCD}{-}\partial_{B}\Gamma_{ACD}{+}\Gamma_{AC}{}^{E}\Gamma_{BED}{-}\Gamma_{BC}{}^{E}\Gamma_{AED}$ .

By construction, it varies as 'total derivative',

$$\delta S_{ABCD}=\nabla_{[A}\delta\Gamma_{B]CD}+\nabla_{[C}\delta\Gamma_{D]AB}\,,$$

which is useful for Lagrangian variation, *i.e.* action principle.

- Semi-covariant 'Master' derivative :

$$\mathcal{D}_{A}:=\partial_{A}+\Gamma_{A}+\Phi_{A}+\bar{\Phi}_{A}=\nabla_{A}+\Phi_{A}+\bar{\Phi}_{A}\,.$$

The two spin connections are determined in terms of the DFT-Christoffel connection,

$$\Phi_{Apq}=V^{B}{}_{p}\nabla_{A}V_{Bq}\,,\qquad\qquad \bar{\Phi}_{A\bar{p}\bar{q}}=\bar{V}^{B}{}_{\bar{p}}\nabla_{A}\bar{V}_{B\bar{q}}\,,$$

by requiring the compatibility with the vielbeins,

$$\mathcal{D}_{A}V_{Bp}=\nabla_{A}V_{Bp}+\Phi_{Ap}{}^{q}V_{Bq}=0\,,\qquad \mathcal{D}_{A}\bar{V}_{B\bar{p}}=\nabla_{A}\bar{V}_{B\bar{p}}+\bar{\Phi}_{A\bar{p}}{}^{\bar{q}}\bar{V}_{B\bar{q}}=0\,.$$

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$$\mathcal{D}_A V_{Bp}=\nabla_A V_{Bp}+\Phi_{Ap}{}^{q}V_{Bq}=0\,,\qquad \mathcal{D}_A\bar{V}_{B\bar{p}}=\nabla_A\bar{V}_{B\bar{p}}+\bar{\Phi}_{A\bar{p}}{}^{\bar{q}}\bar{V}_{B\bar{q}}=0\,.$$

## Anomaly is under control owing to the six-indexed projectors

- Semi-covariance:

$$\delta_\xi\left(\nabla_C T_{A_1\cdots A_n}\right)=\hat{\mathcal{L}}_\xi\left(\nabla_C T_{A_1\cdots A_n}\right)+\sum_{i=1}^{n} 2(\mathcal{P}+\bar{\mathcal{P}})_{CA_i}{}^{BDEF}\partial_D\partial_E\xi_F\, T_{A_1\cdots A_{i-1}BA_{i+1}\cdots A_n}\,,$$

$$\delta_\xi S_{ABCD}=\hat{\mathcal{L}}_\xi S_{ABCD}+2\nabla_{[A}\Big((\mathcal{P}+\bar{\mathcal{P}})_{B][CD]}{}^{EFG}\partial_E\partial_F\xi_G\Big)+2\nabla_{[C}\Big((\mathcal{P}+\bar{\mathcal{P}})_{D][AB]}{}^{EFG}\partial_E\partial_F\xi_G\Big)\,.$$

- This is due to

$$\delta_\xi\Gamma_{CAB}=\hat{\mathcal{L}}_\xi\Gamma_{CAB}+2\Big[(\mathcal{P}+\bar{\mathcal{P}})_{CAB}{}^{FDE}-\delta_C^F\delta_A^D\delta_B^E\Big]\partial_F\partial_{[D}\xi_{E]}\,.$$

Ideally one might desire to cancel these red-colored anomalies by adding extra terms to $\Gamma_{CAB}$.

But, since

$$\delta\mathcal{H}_{AB}=(P\delta\mathcal{H}\bar{P})_{AB}+(\bar{P}\delta\mathcal{H}P)_{AB}\,,\qquad \delta_\xi(\partial_C\mathcal{H}_{AB})=\hat{\mathcal{L}}_\xi(\partial_C\mathcal{H}_{AB})+8\bar{P}_{(A}{}^D P_{B)}{}^E\partial_C\partial_{[D}\xi_{E]}\,,$$

it is impossible to construct such compensating terms out of the derivatives of $\mathcal{H}_{AB}$.

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# Complete covariantization

– Tensors:

$$P_C{}^D \bar{P}_{A_1}{}^{B_1} \cdots \bar{P}_{A_n}{}^{B_n} \nabla_D T_{B_1 \cdots B_n} \quad \Longrightarrow \quad \mathcal{D}_p T_{\bar{q}_1 \bar{q}_2 \cdots \bar{q}_n} \,,$$

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– Yang-Mills:

$$\mathcal{F}_{p\bar{q}} := \mathcal{F}_{AB} V^A{}_p \bar{V}^B{}_{\bar{q}} \qquad \text{where} \qquad \mathcal{F}_{AB} := \nabla_A W_B - \nabla_B W_A - i\,[W_A, W_B] \,.$$

– Spinors, $\rho^\alpha$, $\psi^\alpha_{\bar{p}}$ :

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– RR sector, $\mathcal{C}^\alpha{}_{\bar{\alpha}}$ :

$$\mathcal{D}_\pm \mathcal{C} := \gamma^p \mathcal{D}_p \mathcal{C} \pm \gamma^{(D+1)} \mathcal{D}_{\bar{p}} \mathcal{C} \bar{\gamma}^{\bar{p}} \,, \quad (\mathcal{D}_\pm)^2 = 0 \qquad \Longrightarrow \qquad \mathcal{F} := \mathcal{D}_+ \mathcal{C} \qquad (\text{RR flux}) \,.$$

– Curvatures:

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# $\mathbf{O}(D,D)$ symmetric coupling to matter

- $D=10$ Maximally Supersymmetric DFT — Jeon-Lee-JHP-Suh 2012 [**Full order construction**]

$$\mathcal{L}_{\rm type\,II}=e^{-2d}\Big[\,\tfrac{1}{8}S_{(0)}+\tfrac{1}{2}{\rm Tr}(\mathcal{F}\bar{\mathcal{F}})+i\bar{\rho}\mathcal{F}\rho'+i\bar{\psi}_{\bar{p}}\gamma_{q}\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^{q}+i\tfrac{1}{2}\bar{\rho}\gamma^{p}\mathcal{D}_{p}\rho-i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho'$$
$$-i\bar{\psi}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho-i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{q}\mathcal{D}_{q}\psi_{\bar{p}}+i\bar{\psi}'^{p}\mathcal{D}_{p}\rho'+i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\mathcal{D}_{\bar{q}}\psi'_{p}\,\Big]$$

which unifies IIA & IIB SUGRAs, and Gomis-Ooguri gravity as different solution sectors.

- Minimal coupling to the $D=4$ Standard Model, — Kangsin Choi & JHP 2015 [PRL]

$$\mathcal{L}_{\rm SM}\;=\;e^{-2d}\left[\begin{array}{l}\frac{1}{16\pi G_{N}}S_{(0)}\\[2mm] +\sum_{\mathcal{V}}{\rm Tr}(\mathcal{F}_{p\bar{q}}\mathcal{F}^{p\bar{q}})+\sum_{\psi}\bar{\psi}\gamma^{a}\mathcal{D}_{a}\psi+\sum_{\psi'}\bar{\psi}'\bar{\gamma}^{\bar{a}}\mathcal{D}_{\bar{a}}\psi'\\[2mm] -\mathcal{H}^{AB}(\mathcal{D}_{A}\phi)^{\dagger}\mathcal{D}_{B}\phi\,-\,V(\phi)\,+y_{d}\,\bar{q}\cdot\phi\,d+y_{u}\,\bar{q}\cdot\tilde{\phi}\,u+y_{e}\,\bar{l}'\cdot\phi\,e'\end{array}\right]$$

⋆ Every single term above is completely covariant, w.r.t. $\mathbf{O}(D,D)$, DFT-diffeomorphisms, and twofold local Lorentz symmetries.

- Henceforth, we consider a general action for Stringy Gravity coupled to matter fields, $\Upsilon_a$,

$$\text{Action} = \int_\Sigma e^{-2d}\Big[\,\tfrac{1}{16\pi G}S_{(0)} + L_{\rm matter}\big(\Upsilon_a, \mathcal{D}_A\Upsilon_b\big)\Big]\,,$$

and seek the variation of the action induced by all the fields, $d$, $V_{Ap}$, $\bar{V}_{Ap}$, $\Upsilon_a$.

Note $\delta V_{Ap} = (\bar{P}+P)_A{}^B\delta V_{Bp} = \bar{V}_{A\bar{q}}\bar{V}^{B\bar{q}}\delta V_{Bp} + (\delta V_{B[p}V^B{}_{q]})V_A{}^q$ . The 2nd term is a local Lorentz rotation and can be absorbed into $\delta\Upsilon_a$. Thus, only the projected variation, $\bar{V}^B{}_{\bar{q}}\delta V_{Bp} = -V^B{}_p\delta\bar{V}_{B\bar{q}}$ , appears.

  - Firstly, the 'pure' Stringy Gravity term transforms, up to total derivatives ($\simeq$), as

$$\delta\Big(e^{-2d}S_{(0)}\Big) \simeq 4e^{-2d}\Big(\bar{V}^{B\bar{q}}\delta V_B{}^p S_{p\bar{q}} - \tfrac{1}{2}\delta d\, S_{(0)}\Big)\,.$$

  - Secondly, the matter Lagrangian transforms as

$$\delta\Big(e^{-2d}L_{\rm matter}\Big) \simeq e^{-2d}\left(-2\bar{V}^{A\bar{q}}\delta V_A{}^p K_{p\bar{q}} + \delta d\, T_{(0)} + \delta\Upsilon_a\frac{\delta L_{\rm matter}}{\delta\Upsilon_a}\right)$$

where we have been naturally led to define

$$K_{p\bar{q}} := \frac{1}{2}\left(V_{Ap}\frac{\delta L_{\rm matter}}{\delta\bar{V}_A{}^{\bar{q}}} - \bar{V}_{A\bar{q}}\frac{\delta L_{\rm matter}}{\delta V_A{}^p}\right)\,,\qquad\qquad T_{(0)} := e^{2d}\times\frac{\delta\Big(e^{-2d}L_{\rm matter}\Big)}{\delta d}\,.$$

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# Derivation of EDFE from General Covariance of Stringy Gravity 1/2

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- Combining the two results, the variation of the action reads

$$\delta\text{Action} = \int_\Sigma e^{-2d}\left[\tfrac{1}{4\pi G}\bar{V}^{A\bar{q}}\delta V_A{}^p(S_{p\bar{q}} - 8\pi G K_{p\bar{q}}) - \tfrac{1}{8\pi G}\delta d(S_{(0)} - 8\pi G T_{(0)}) + \delta\Upsilon_a\frac{\delta L_{\mathrm{matter}}}{\delta\Upsilon_a}\right].$$

- Specifically when the variation is generated by diffeomorphisms, we have $\delta_\xi\Upsilon_a = \hat{\mathcal{L}}_\xi\Upsilon_a$ and

$$\bar{V}^{A\bar{q}}\delta_\xi V_A{}^p = \bar{V}^{A\bar{q}}\hat{\mathcal{L}}_\xi V_A{}^p = 2\mathcal{D}_{[A}\xi_{B]}\bar{V}^{A\bar{q}}V^{Bp}\,, \qquad \delta_\xi d = -\tfrac{1}{2}e^{2d}\hat{\mathcal{L}}_\xi\left(e^{-2d}\right) = -\tfrac{1}{2}\mathcal{D}_A\xi^A\,.$$

- The Diffeomorphic General Covariance of the Action then implies

$$0 = \int_\Sigma e^{-2d}\left[\tfrac{1}{8\pi G}\xi^B\mathcal{D}^A\left\{4V_{[A}{}^p\bar{V}_{B]}{}^{\bar{q}}(S_{p\bar{q}} - 8\pi G K_{p\bar{q}}) - \tfrac{1}{2}\mathcal{J}_{AB}(S_{(0)} - 8\pi G T_{(0)})\right\} + \delta_\xi\Upsilon_a\frac{\delta L_{\mathrm{matter}}}{\delta\Upsilon_a}\right].$$

This gives the definitions of the off-shell conserved **stringy Einstein curvature,**

$$G_{AB} := 4V_{[A}{}^p\bar{V}_{B]}{}^{\bar{q}}S_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{AB}S_{(0)}\,, \qquad \mathcal{D}_A G^{AB} = 0 \qquad \text{(off-shell)}\,,$$

JHP-Rey-Rim-Sakatani 2015

and the on-shell conserved **stringy Energy-Momentum tensor,**

$$T_{AB} := 4V_{[A}{}^p\bar{V}_{B]}{}^{\bar{q}}K_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{AB}T_{(0)}\,, \qquad \mathcal{D}_A T^{AB} = 0 \qquad \text{(on-shell)}\,.$$

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$$G_{AB} := 4V_{[A}{}^p\bar{V}_{B]}{}^{\bar{q}}S_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{AB}S_{(0)}, \qquad \mathcal{D}_A G^{AB} = 0 \qquad \text{(off-shell)},$$

JHP-Rey-Rim-Sakatani 2015

and the on-shell conserved **stringy Energy-Momentum tensor,**

$$T_{AB} := 4V_{[A}{}^p\bar{V}_{B]}{}^{\bar{q}}K_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{AB}T_{(0)}, \qquad \mathcal{D}_A T^{AB} = 0 \qquad \text{(on-shell)}.$$

- Combining the two results, the variation of the action reads

$$\delta\text{Action} = \int_\Sigma e^{-2d}\left[\tfrac{1}{4\pi G}\bar{V}^{A\bar{q}}\delta V_A{}^p(S_{p\bar{q}} - 8\pi G K_{p\bar{q}}) - \tfrac{1}{8\pi G}\delta d(S_{(0)} - 8\pi G T_{(0)}) + \delta\Upsilon_a \frac{\delta L_{\mathrm{matter}}}{\delta\Upsilon_a}\right].$$

- Specifically when the variation is generated by diffeomorphisms, we have $\delta_\xi\Upsilon_a = \hat{\mathcal{L}}_\xi\Upsilon_a$ and

$$\bar{V}^{A\bar{q}}\delta_\xi V_A{}^p = \bar{V}^{A\bar{q}}\hat{\mathcal{L}}_\xi V_A{}^p = 2\mathcal{D}_{[A}\xi_{B]}\bar{V}^{A\bar{q}}V^{Bp}\,, \qquad \delta_\xi d = -\tfrac{1}{2}e^{2d}\hat{\mathcal{L}}_\xi\left(e^{-2d}\right) = -\tfrac{1}{2}\mathcal{D}_A\xi^A\,.$$

- The Diffeomorphic General Covariance of the Action then implies

$$0 = \int_\Sigma e^{-2d}\left[\tfrac{1}{8\pi G}\xi^B\mathcal{D}^A\left\{4V_{[A}{}^p\bar{V}_{B]}{}^{\bar{q}}(S_{p\bar{q}} - 8\pi G K_{p\bar{q}}) - \tfrac{1}{2}\mathcal{J}_{AB}(S_{(0)} - 8\pi G T_{(0)})\right\} + \delta_\xi\Upsilon_a\frac{\delta L_{\mathrm{matter}}}{\delta\Upsilon_a}\right].$$

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- All the EOMs of the vielbeins and dilaton can be unified into a single expression,

$$G_{AB} = 8\pi G T_{AB} \quad : \quad \textbf{Einstein Double Field Equations}$$

which is naturally consistent with our central idea that Stringy Gravity treats the entire closed-string massless sector as the geometrical stringy graviton multiplet.

- Restricting to the Riemannian $(0,0)$ parametrization, EDFE reduce to

$$R_{\mu\nu} + 2\nabla_\mu(\partial_\nu\phi) - \tfrac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} = 8\pi G K_{(\mu\nu)}\,,$$

$$e^{2\phi}\nabla^\rho\left(e^{-2\phi}H_{\rho\mu\nu}\right) = 16\pi G K_{[\mu\nu]}\,,$$

$$R + 4\Box\phi - 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} = 8\pi G T_{(0)}\,.$$

- For other non-Riemannian cases, $(n,\bar{n}) \neq (0,0)$, EDFE govern the dynamics of 'chiral' gravities, *e.g.* Newton-Cartan, Carroll, Gomis-Ooguri, *etc.*

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## **Examples:** $T_{AB} := 4V_{[A}{}^{p}\bar{V}_{B]}{}^{\bar{q}}K_{p\bar{q}} - \frac{1}{2}\mathcal{J}_{AB}T_{(0)}$

- Scalar field,

$$L_{\varphi} = -\tfrac{1}{2}\mathcal{H}^{MN}\partial_M\varphi\partial_N\varphi - V(\varphi)\,, \qquad K_{p\bar{q}} = \partial_p\varphi\partial_{\bar{q}}\varphi\,, \qquad T_{(0)} = -2L_{\varphi}\,.$$

- Spinor field,

$$L_{\psi} = \bar{\psi}\gamma^p\mathcal{D}_p\psi + m_{\psi}\bar{\psi}\psi\,, \qquad K_{p\bar{q}} = -\tfrac{1}{4}(\bar{\psi}\gamma_p\mathcal{D}_{\bar{q}}\psi - \mathcal{D}_{\bar{q}}\bar{\psi}\gamma_p\psi)\,, \qquad T_{(0)} \equiv 0\,.$$

- RR sector,

$$L_{\mathrm{RR}} = \tfrac{1}{2}\mathrm{Tr}(\mathcal{F}\bar{\mathcal{F}})\,, \qquad K_{p\bar{q}} = -\tfrac{1}{4}\mathrm{Tr}(\gamma_p\mathcal{F}\bar{\gamma}_{\bar{q}}\bar{\mathcal{F}})\,, \qquad T_{(0)} = 0\,.$$

- Fundamental string: with $D_iy^M = \partial_iy^M - \mathcal{A}_i^M$ (doubled-yet-gauged),

$$e^{-2d}L_{\mathrm{string}} = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\left[-\tfrac{1}{2}\sqrt{-h}h^{ij}D_iy^MD_jy^N\mathcal{H}_{MN}(y) - \epsilon^{ij}D_iy^M\mathcal{A}_{jM}\right]\delta^D(x-y(\sigma))\,,$$

$$K_{p\bar{q}} = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{-h}h^{ij}D_iy^MD_jy^N\,V_{Mp}\bar{V}_{N\bar{q}}\,e^{2d(x)}\delta^D(x-y(\sigma))\,, \qquad T_{(0)} = 0\,.$$

– More examples include Yang-Mills, point particle, Green-Schwarz superstring, *etc.* 1804.00964

# Cosmology

Stephen Angus, Kyungho Cho, G. Franzmann, S. Mukohyama & JHP 1905.03620

- **$\mathbf{O}(D,D)$ completion of the Friendamann equations:**

$$\frac{8\pi G}{3}\rho e^{2\phi}+\frac{h^2}{12a^6} = H^2-2\left(\frac{\phi'}{N}\right)H+\frac{2}{3}\left(\frac{\phi'}{N}\right)^2+\frac{k}{a^2}$$

$$\frac{4\pi G}{3}(\rho+3p)e^{2\phi}+\frac{h^2}{6a^6} = -H^2-\frac{H'}{N}+\left(\frac{\phi'}{N}\right)H-\frac{2}{3}\left(\frac{\phi'}{N}\right)^2+\frac{1}{N}\left(\frac{\phi'}{N}\right)'$$

$$\frac{8\pi G}{3}\left(\rho e^{2\phi}-\frac{1}{2}T_{(0)}\right) = -H^2-\frac{H'}{N}+\frac{2}{3N}\left(\frac{\phi'}{N}\right)'$$

**which imply the conservation equation,**

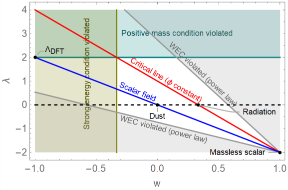
$$\rho'+3NH(\rho+p)+\phi' T_{(0)}e^{-2\phi}=0\,.$$

Here most general homogeneous and isotropic cosmological ansatzes have been adopted:

$$\rho:=\left(-K^t{}_t+\tfrac{1}{2}T_{(0)}\right)e^{-2\phi}\,,\qquad p:=\left(K^r{}_r-\tfrac{1}{2}T_{(0)}\right)e^{-2\phi}\,,\qquad H_{(3)}=\frac{hr^2}{\sqrt{1-kr^2}}\sin\vartheta\,\mathrm{d}r\wedge\mathrm{d}\vartheta\wedge\mathrm{d}\varphi\,.$$

▶ This gives an enriched and novel framework beyond typical string cosmology, enjoying two equation-of-state parameters, $w=p/\rho$ (conventional) and $\lambda=T_{(0)}e^{-2\phi}/\rho$ (new).

Most previous literatures assumed no dilaton coupling, $\lambda=0$.



In remaining time,

I wish to discuss briefly the

**most general spherically symmetric, asymptotically flat, regular solution to $D = 4$ EDFE,**

namely

**stringy 'star' of radius $r_{\mathrm{c}}$ :**

$$G_{AB} = \begin{cases} 8\pi G T_{AB} & \text{for} \quad r \leq r_{\mathrm{c}} \quad \text{(spherical)} \\ 0 & \text{for} \quad r > r_{\mathrm{c}} \end{cases}$$

This will contrast Stringy Gravity with GR as

$D^2+1$ components in $T_{AB}$ *vs.* $\frac{D(D+1)}{2}$ components in $T_{\mu\nu} = T_{\nu\mu}$

**Stringy Gravity = enriched and novel framework beyond GR**

## Spherical Symmetry in DFT

- In GR, isometries are addressed in terms of ordinary Lie derivative,

$$\mathcal{L}_\xi g_{\mu\nu} = 0\,, \qquad \mathcal{L}_\xi H_{\lambda\mu\nu} = 0 \quad \Longleftrightarrow \quad \mathcal{L}_\xi B_{\mu\nu} = \partial_\nu \tilde{\xi}_\mu - \partial_\mu \tilde{\xi}_\nu\,.$$

- In DFT, the spherical symmetry should be analyzed by generalized Lie derivative, with triple Killing vectors, $\xi_a^M = (\tilde{\xi}_{a\mu}, \xi_a^\nu)$, $a = 1, 2, 3$,

$$\hat{\mathcal{L}}_\xi \mathcal{H}_{AB} = 8\bar{P}_{(A}{}^{[C} P_{B)}{}^{D]} \nabla_C \xi_D = 0\,, \qquad \hat{\mathcal{L}}_\xi d = -\tfrac{1}{2}\nabla_A \xi^A = 0\,, \qquad [\xi_a, \xi_b]_{\mathbf{C}} = \textstyle\sum_c \epsilon_{abc}\xi_c\,.$$

  - However, while generalized Lie derivative is diffeomorphism covariant,

$$\begin{aligned}\delta_\xi(\hat{\mathcal{L}}_\zeta T_{M_1\cdots M_n}) &= \hat{\mathcal{L}}_\zeta(\delta_\xi T_{M_1\cdots M_n}) + \hat{\mathcal{L}}_{\delta_\xi \zeta} T_{M_1\cdots M_n} = \hat{\mathcal{L}}_\zeta \hat{\mathcal{L}}_\xi T_{M_1\cdots M_n} + \hat{\mathcal{L}}_{\hat{\mathcal{L}}_\xi \zeta} T_{M_1\cdots M_n} \\ &= \hat{\mathcal{L}}_\xi \hat{\mathcal{L}}_\zeta T_{M_1\cdots M_n} + \hat{\mathcal{L}}_{[\zeta,\xi]_{\mathrm{C}} + \hat{\mathcal{L}}_\xi \zeta} T_{M_1\cdots M_n} = \hat{\mathcal{L}}_\xi(\hat{\mathcal{L}}_\zeta T_{M_1\cdots M_n})\end{aligned}$$

    it is anomalous under local Lorentz rotations. Hence, potentially problematic with vielbeins:

$$\hat{\mathcal{L}}_\xi V_{Ap} = \bar{P}_A{}^B V^C{}_p \left(\hat{\mathcal{L}}_\xi P_{BC}\right) - \left(\xi^B \Phi_{Bpq} + 2\mathcal{D}_{[p}\xi_{q]}\right) V_A{}^q\,,$$

    where $\xi_q = \xi^A V_{Aq}$. This result can be rewritten as

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## Further-generalized Lie derivative, $\widetilde{\mathcal{L}}_\xi$

- We generalize the generalized Lie derivative one step further by including additional local Lorentz rotations:

$$\begin{aligned}\widetilde{\mathcal{L}}_\xi T_{Mp\bar{p}\alpha\bar{\alpha}} \quad := \quad & \xi^N \mathcal{D}_N T_{Mp\bar{p}\alpha\bar{\alpha}} + \omega_T \mathcal{D}_N \xi^N T_{Mp\bar{p}\alpha\bar{\alpha}} + 2\mathcal{D}_{[M}\xi_{N]} T^N{}_{p\bar{p}\alpha\bar{\alpha}} \\ & +2\mathcal{D}_{[p}\xi_{q]} T_M{}^q{}_{\bar{p}\alpha\bar{\alpha}} - \tfrac{1}{2}\mathcal{D}_{[r}\xi_{s]}(\gamma^{rs})^\beta{}_\alpha T_{Mp\bar{p}\beta\bar{\alpha}} \\ & +2\mathcal{D}_{[\bar{p}}\xi_{\bar{q}]} T_{Mp}{}^{\bar{q}}{}_{\alpha\bar{\alpha}} - \tfrac{1}{2}\mathcal{D}_{[\bar{r}}\xi_{\bar{s}]}(\bar{\gamma}^{\bar{r}\bar{s}})^{\bar{\beta}}{}_{\bar{\alpha}} T_{Mp\bar{p}\alpha\bar{\beta}}\,,\end{aligned}$$

$$\left[\widetilde{\mathcal{L}}_\zeta, \widetilde{\mathcal{L}}_\xi\right] = \widetilde{\mathcal{L}}_{[\zeta,\xi]_{\mathrm{C}}} + \omega_{pq}(\zeta,\xi) + \bar{\omega}_{\bar{p}\bar{q}}(\zeta,\xi)\,.$$

- This is covariant under both diffeomorphisms and local Lorentz symmetries, and allows to analyze the spherical **SO**(3) symmetry:

$$\widetilde{\mathcal{L}}_{\xi_a} V_{Ap} = \left(\hat{\mathcal{L}}_{\xi_a} P_{AC}\right) V^C{}_p \equiv 0\,, \qquad \widetilde{\mathcal{L}}_{\xi_a} \bar{V}_{A\bar{p}} = \left(\hat{\mathcal{L}}_{\xi_a} \bar{P}_{AC}\right) \bar{V}^C{}_{\bar{p}} \equiv 0\,, \qquad \widetilde{\mathcal{L}}_{\xi_a} K_{p\bar{q}} \equiv 0\,, \quad \cdots$$

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# Solution: stringy 'star' of radius $r_c$

- Outside the star, $r \geq r_c$, the vacuum geometry is given by [Burgess-Myers-Quevedo '94]

$$e^{2\phi} = \gamma_+ \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{b}{\sqrt{a^2+b^2}}} + \gamma_- \left(\frac{r+\beta}{r-\alpha}\right)^{\frac{b}{\sqrt{a^2+b^2}}} , \qquad H_{(3)} = h \sin\vartheta \, \mathrm{d}t \wedge \mathrm{d}\vartheta \wedge \mathrm{d}\varphi ,$$

$$\mathrm{d}s^2 = e^{2\phi}\left[-\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a}{\sqrt{a^2+b^2}}} \mathrm{d}t^2 + \left(\frac{r+\beta}{r-\alpha}\right)^{\frac{a}{\sqrt{a^2+b^2}}} \{\mathrm{d}r^2 + (r-\alpha)(r+\beta)\mathrm{d}\Omega^2\}\right]$$

which has four parameters, $\{\alpha, \beta, a, h\}$, while

$$b^2 = (\alpha+\beta)^2 - a^2 \geq h^2 , \qquad \gamma_\pm = \tfrac{1}{2}\left(1 \pm \sqrt{1 - h^2/b^2}\right) .$$

If $b = h = 0$, it reduces to Schwarzschild geometry.

- Inside the star, EDFE fix all the constants, $\{\alpha, \beta, a, h\}$, in terms of $T_{AB}$, for example

$$a = \int_0^{r_c} \mathrm{d}r \int_0^{\pi} \mathrm{d}\vartheta \int_0^{2\pi} \mathrm{d}\varphi \; e^{-2d} \left[\tfrac{1}{4\pi} H_{r\vartheta\varphi} H^{r\vartheta\varphi} + 2G\left(K_r{}^r + K_\vartheta{}^\vartheta + K_\varphi{}^\varphi - K_t{}^t - T_{(0)}\right)\right] .$$

Namely, various components of $T_{AB}$ enrich the spherical geometry of Stringy Gravity.

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## Stringy Gravity modifies GR when $\frac{R}{MG}$ is small (e.g. it can be repulsive)

- In terms of Areal Radius, $R$: $\mathrm{d}s^2 = g_{tt}\mathrm{d}t^2 + g_{RR}\mathrm{d}R^2 + R^2\mathrm{d}\Omega^2$, gravitational potential reads

$$\Phi_{\text{Newton}} = -\tfrac{1}{2}(1+g_{tt}) = -\frac{MG}{R} + \left(\frac{2b^2 - h^2 + 2ab\sqrt{1-h^2/b^2}}{a^2+b^2-h^2+2ab\sqrt{1-h^2/b^2}}\right)\left(\frac{MG}{R}\right)^2 + \cdots$$

where the ellipses denote higher order terms in $\frac{MG}{R}$ which is 'dimensionless', and

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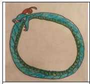
|  | Electron $(R \simeq 0)$ | Proton | Hydrogen Atom | Billiard Ball | Earth | Solar System $(1\mathrm{AU}/M_\odot G)$ | Milky Way (visible) | Galaxy Cluster | Universe $(M \propto R^3)$ |
|---|---|---|---|---|---|---|---|---|---|
| $R/(MG)$ | $0^+$ | $7.1\times10^{38}$ | $2.0\times10^{43}$ | $2.4\times10^{26}$ | $1.4\times10^{9}$ | $1.0\times10^{8}$ | $1.5\times10^{6}$ | $\sim 10^{5}$ | $0^+$ |

The observations of stars/galaxies far away may reveal the short-distance nature of gravity.

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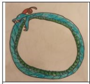
|  | Electron ($R \simeq 0$) | Proton | Hydrogen Atom | Billiard Ball | Earth | Solar System ($1\mathrm{AU}/M_\odot G$) | Milky Way (visible) | Galaxy Cluster | Universe ($M \propto R^3$) |
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**Thank you**