Symmetry and geometry in generalized Higgs sector

Yoshiki Uchida

Nagoya University

arXiv : 1904.07618

In collaboration with

Ryo Nagai (U. of Tokyo)
Masaharu Tanabashi (Nagoya U.)
Koji Tsumura (Kyoto U.)

Summer institute 2019 @ Sandpine, Gangneung, Korea

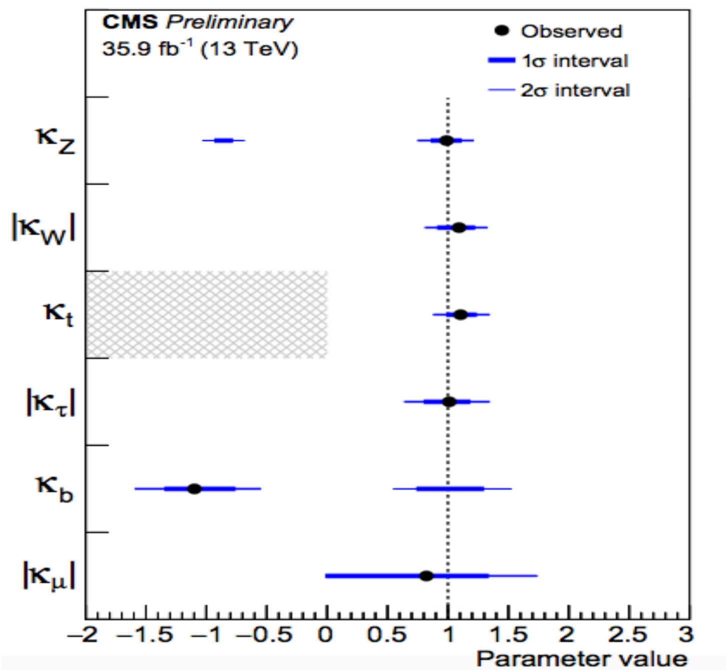
SM is not enough

・Hierarchy problem ・Dark Matter ・・・

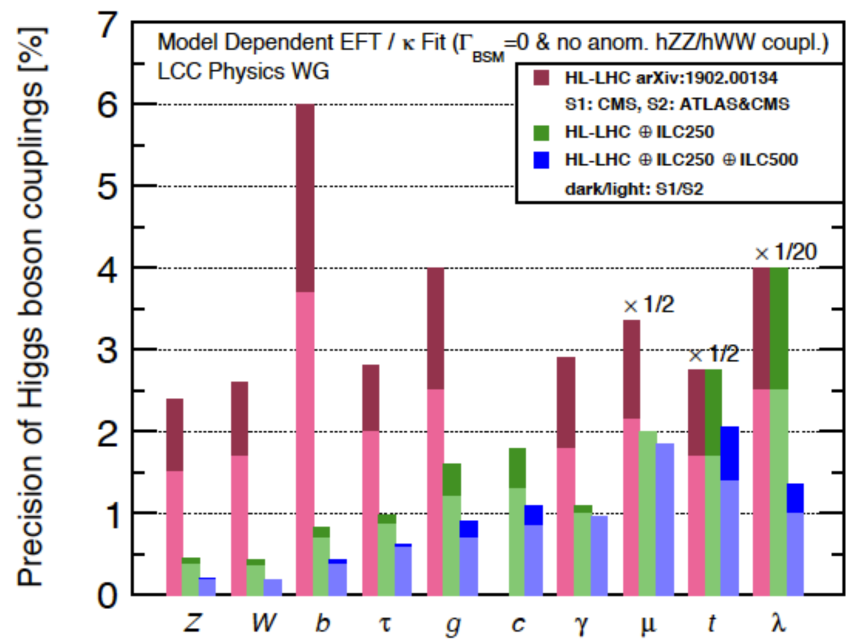
Beyond the standard model (BSM) is needed !

・SUSY ・Composite Higgs ・Extra dim. ・・・

In many BSM, the scalar sector is extended from SM



⬆ CMS-PAS-HIG-17-031



⬆ arXiv:1903.01629

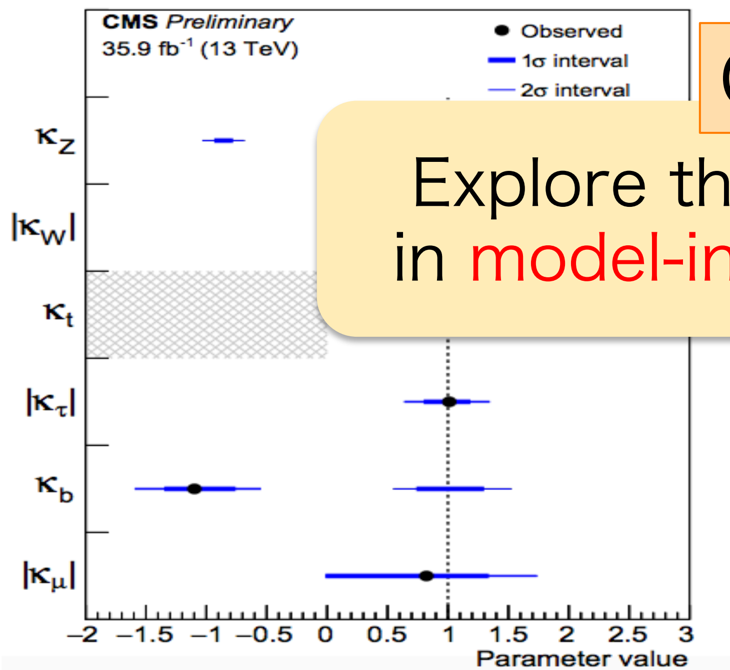
SM is not enough

・Hierarchy problem　・Dark Matter　…

Beyond the standard model (BSM) is needed !

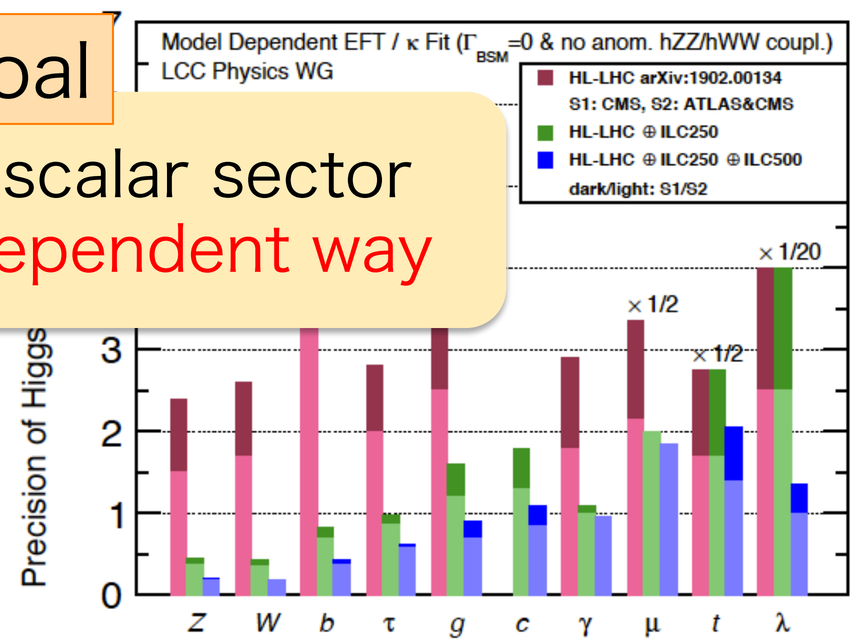
・SUSY　・Composite Higgs　・Extra dim.　…

In many BSM, the scalar sector is extended from SM



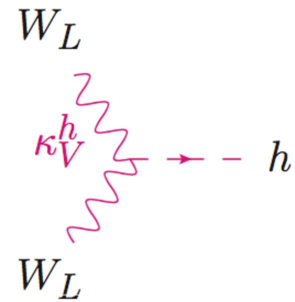
Goal

Explore the scalar sector in model-independent way



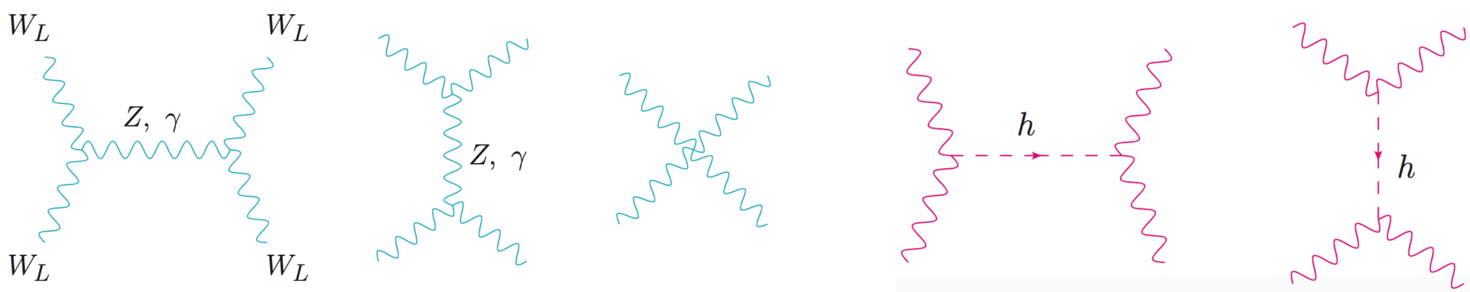
↑ CMS-PAS-HIG-17-031　↑ arXiv:1903.01629

＜Standard Model＞



I. Higgs unitalize W_L scattering amplitude at tree level (tree level unitarity)

$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{E^2}{v^2}\left(1-(\kappa_V^h)^2\right) \quad (\ \kappa_V^h = 1 \ \text{ in SM} \ )$$

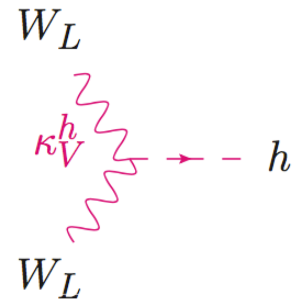


II. Higgs cancels the divergence in oblique corrections

Peskin Takeuchi
Phys. Rev. Lett. 65 (1990) 964

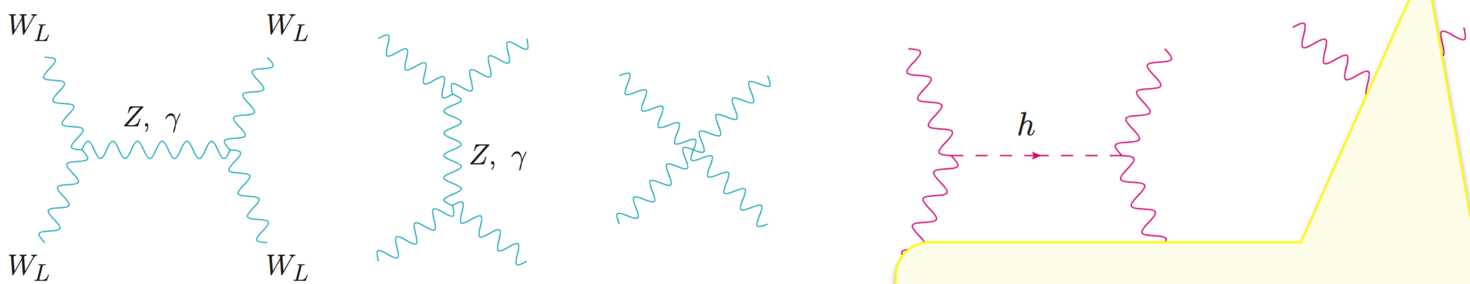
$$S \simeq \frac{1}{12\pi}\left(1-(\kappa_V^h)^2\right)\ln\frac{\Lambda^2}{\mu^2}$$

＜Standard Model＞



I. Higgs unitalize W_L scattering amplitude at tree level (tree level unitarity)

$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{E^2}{v^2}\left(1-(\kappa_V^h)^2\right) \quad (\ \kappa_V^h = 1 \ \text{ in SM })$$



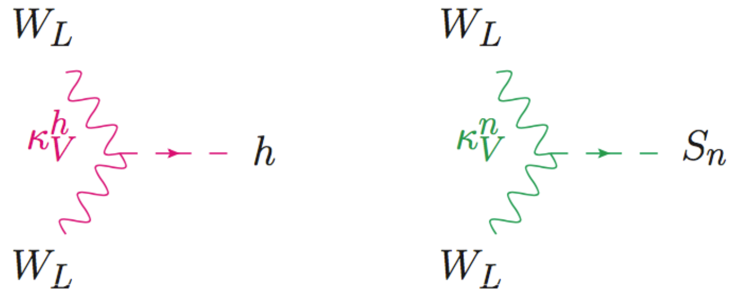
What will happen
if κ_V^h deviate from 1 ?

II. Higgs cancels the divergence in oblique corrections

Peskin Takeuchi
Phys. Rev. Lett. 65 (1990) 964

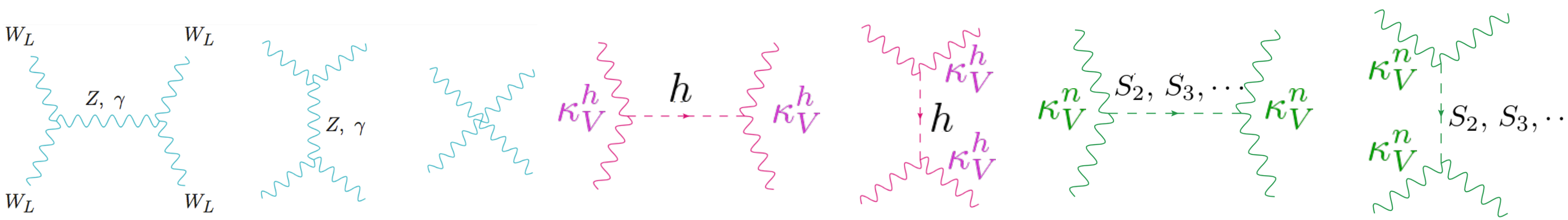
$$S \simeq \frac{1}{12\pi}\left(1-(\kappa_V^h)^2\right)\ln\frac{\Lambda^2}{\mu^2}$$

<Singlet extension>



I. Higgs unitalize W_L scattering amplitude at tree level (tree level unitarity)

$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{E^2}{v^2}\left(1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2\right)$$



unitarity sum rules $\quad 1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2=0$

J F Gunion, H E Haber, J Wudka
Phys. Rev. D 43 904 (1991)

II. Higgs cancels the divergence in oblique corrections

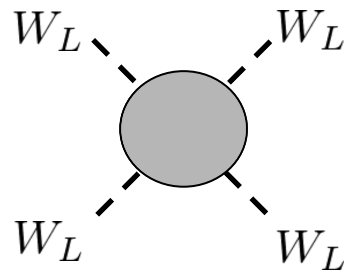
$$S \simeq \frac{1}{12\pi}\left(1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2\right)\ln\frac{\Lambda^2}{\mu^2}$$

finiteness conditions

$$1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2=0$$

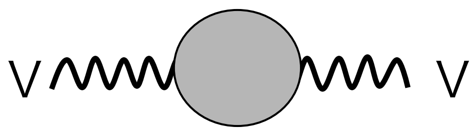
It is reasonable to impose the two conditions on NP scalar sector

I. tree level unitarity



$$\sim \left(\underbrace{1-(\kappa_V^h)^2-\cdots}_{=\,0}\right)E^2$$

II. finiteness of oblique parameter (1-loop)



$$\sim \left(\underbrace{1-(\kappa_V^h)^2-\cdots}_{=\,0}\right)\ln\Lambda^2$$

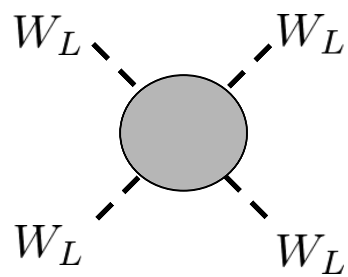
S, T, U

Our Goal

Explore the scalar sector in model-independent way

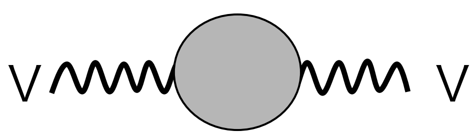
It is reasonable to impose the two conditions on NP scalar sector

I. tree level unitarity



$$\sim \left(\underbrace{1-(\kappa_V^h)^2-\cdots}_{=\,0}\right)E^2$$

II. finiteness of oblique parameter (1-loop)



S, T, U

$$\sim \left(\underbrace{1-(\kappa_V^h)^2-\cdots}_{=\,0}\right)\ln\Lambda^2$$

Our Goal

Explore the scalar sector in model-independent way

How can we achieve this goal ?

Effective field theory approach

SM + Singlet scalar

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + 2c\frac{h}{v} + 2s\frac{H}{v} + c^2\frac{h^2}{v^2} + s^2\frac{H^2}{v^2} + 2cs\frac{hH}{v^2}\right)$$

$$U := e^{i\frac{\pi^a}{v}\tau_a} \qquad + \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu H \partial^\mu H$$

Integrate out H

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + \kappa_1\frac{h}{v} + \kappa_2\frac{h^2}{v^2} + \cdots\right) + \frac{1}{2}\partial_\mu h \partial^\mu h$$

New Physics is encoded in the κ_1 , κ_2 $\cdots$

Alternative : **Approach based on Symmetry and Geometry**

KUNS-2755

Symmetry and geometry in generalized Higgs effective field theory

– Finiteness of oblique corrections v.s. perturbative unitarity –

Ryo Nagai,[1,2,*] Masaharu Tanabashi,[3,4,†] Koji Tsumura,[5,‡] and Yoshiki Uchida[3,§]

[1] *Institute for Cosmic Ray Research (ICRR), The University of Tokyo, Kashiwa, Chiba 277-8582, Japan*

[2] *Department of Physics, Tohoku University, Sendai, Miyagi 980-8578, Japan*

[3] *Department of Physics, Nagoya University, Nagoya 464-8602, Japan*

[4] *Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya 464-8602, Japan*

[5] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

(Dated: April 17, 2019)

Alternative : **Approach based on Symmetry and Geometry**

KUNS-2755

R. Alonso et al.
JHEP08(2016)101

PUBLISHED FOR SISSA BY SPRINGER

RECEIVED: *May 20, 2016*
REVISED: *July 22, 2016*
ACCEPTED: *July 22, 2016*
PUBLISHED: *August 17, 2016*

Geometry of the scalar sector

Rodrigo Alonso,[a] Elizabeth E. Jenkins[a,b] and Aneesh V. Manohar[a,b]

[a] *Department of Physics, University of California at San Diego, La Jolla, CA 92093, U.S.A.*
[b] *CERN TH Division, CH-1211 Geneva 23, Switzerland*

E-mail: ralonsod@ucsd.edu, ejenkins@ucsd.edu, amanohar@ucsd.edu

Effective field theory approach

SM + Singlet scalar

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + 2c\frac{h}{v} + 2s\frac{H}{v} + c^2\frac{h^2}{v^2} + s^2\frac{H^2}{v^2} + 2cs\frac{hH}{v^2}\right) + \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu H \partial^\mu H$$

$$U := e^{i\frac{\pi^a}{v}\tau_a}$$

Integrate out H

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + \kappa_1\frac{h}{v} + \kappa_2\frac{h^2}{v^2} + \cdots\right) + \frac{1}{2}\partial_\mu h \partial^\mu h$$

New Physics is encoded in the κ_1 , κ_2 $\cdots$

Approach based on Symmetry and Geometry

SM + Singlet scalar

$$\mathcal{L}_{\rm SM+S} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + 2c\frac{h}{v} + 2s\frac{H}{v} + c^2\frac{h^2}{v^2} + s^2\frac{H^2}{v^2} + 2cs\frac{hH}{v^2}\right)$$

$$U := e^{i\frac{\pi^a}{v}\tau_a} \qquad + \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{1}{2}\partial_\mu H\partial^\mu H$$

$\Downarrow$ Integrate out H

$$\mathcal{L}_{\rm SM+S} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + \kappa_1\frac{h}{v} + \kappa_2\frac{h^2}{v^2} + \cdots\right) + \frac{1}{2}\partial_\mu h\partial^\mu h$$

Approach based on Symmetry and Geometry

SM + Singlet scalar

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\left(1 + 2c\frac{h}{v} + 2s\frac{H}{v} + c^2\frac{h^2}{v^2} + s^2\frac{H^2}{v^2} + 2cs\frac{hH}{v^2}\right) + \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{1}{2}\partial_\mu H\partial^\mu H$$

$$= \frac{1}{2}\partial_\mu \begin{pmatrix}\pi^a & h & H\end{pmatrix}\begin{pmatrix} F(h,H)(\delta_{ab}+\mathcal{O}(\pi^2)) & & \\ & 1 & \\ & & 1\end{pmatrix}\partial^\mu\begin{pmatrix}\pi^b \\ h \\ H\end{pmatrix}$$

$$= \frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j \qquad \phi^i = \pi^a, h, H$$

Approach based on Symmetry and Geometry

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2} g_{ij}(\phi)(D_\mu \phi)^i (D^\mu \phi)^j - V(\phi)$$

Geometry

$$R_{ijkl}(\phi)$$

(Riemann tensor)

Symmetry

$$w_a^i(\phi), \quad y^i(\phi)$$

(Killing vector)

Geometry

$$R_{ijkl}(\phi)$$

(Riemann tensor)

Symmetry

$$w_a^i(\phi)\,,\quad y^i(\phi)$$

(Killing vector)

Geometry

$$R_{ijkl}(\phi)$$

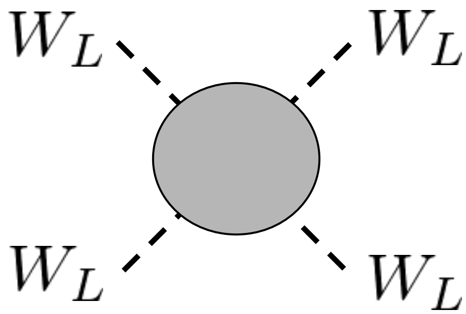
(Riemann tensor)

Symmetry

$$w_a^i(\phi)\,,\ \ y^i(\phi)$$

(Killing vector)

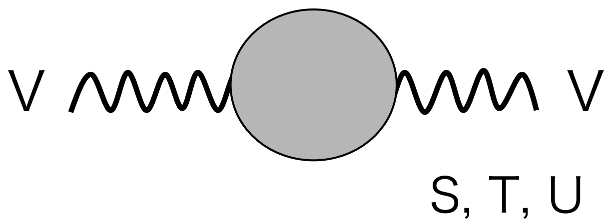
Geometry



$$\bar{R}_{ijkl} = 0$$

pertubative unitarity

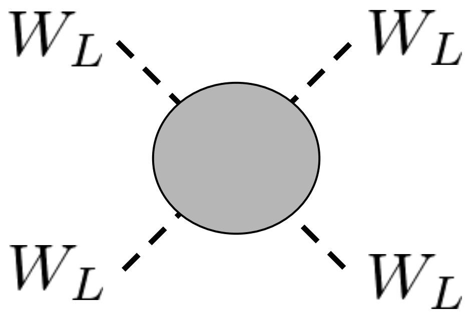
Symmetry



$$(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} = 0$$

1-loop finite S

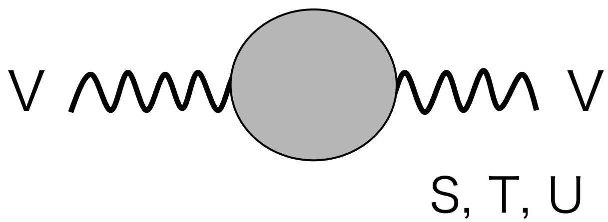
Geometry



$$\bar{R}_{ijkl} = 0$$

pertubative unitarity

Symmetry



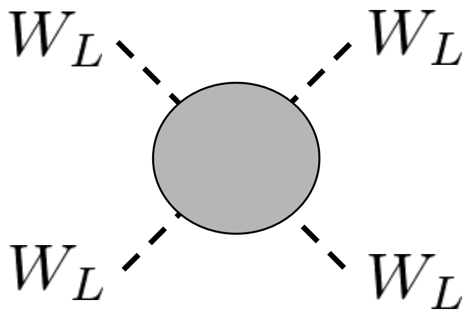
$$(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} = 0$$

1-loop finite S

$$(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i}$$

$$\propto \epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}$$

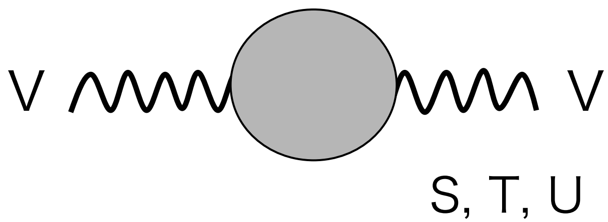
Geometry



$$\boxed{\bar{R}_{ijkl} = 0}$$

pertubative unitarity

Symmetry



$$(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} = 0$$

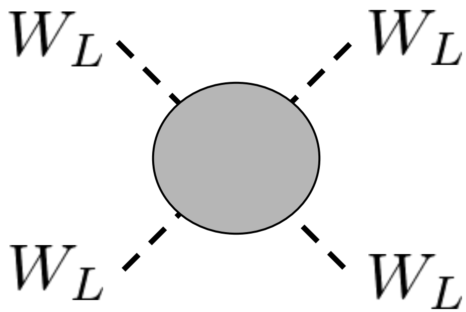
1-loop finite S

$$(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i}$$

If... If...

$$\propto \epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\underbrace{\boxed{\bar{R}^i{}_{jkl}}}_{=0}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\underbrace{\boxed{\bar{R}^i{}_{jkl}}}_{=0}(\bar{y}^j)_{;i}$$

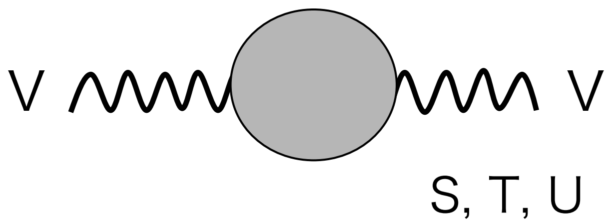
Geometry



$$\bar{R}_{ijkl} = 0$$

pertubative unitarity

Symmetry



$$\boxed{(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} = 0}$$

1-loop finite S

$\boxed{(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i}}$ Then...

$$\propto \epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}$$

take-home message

In arbiterally scalar sector ...

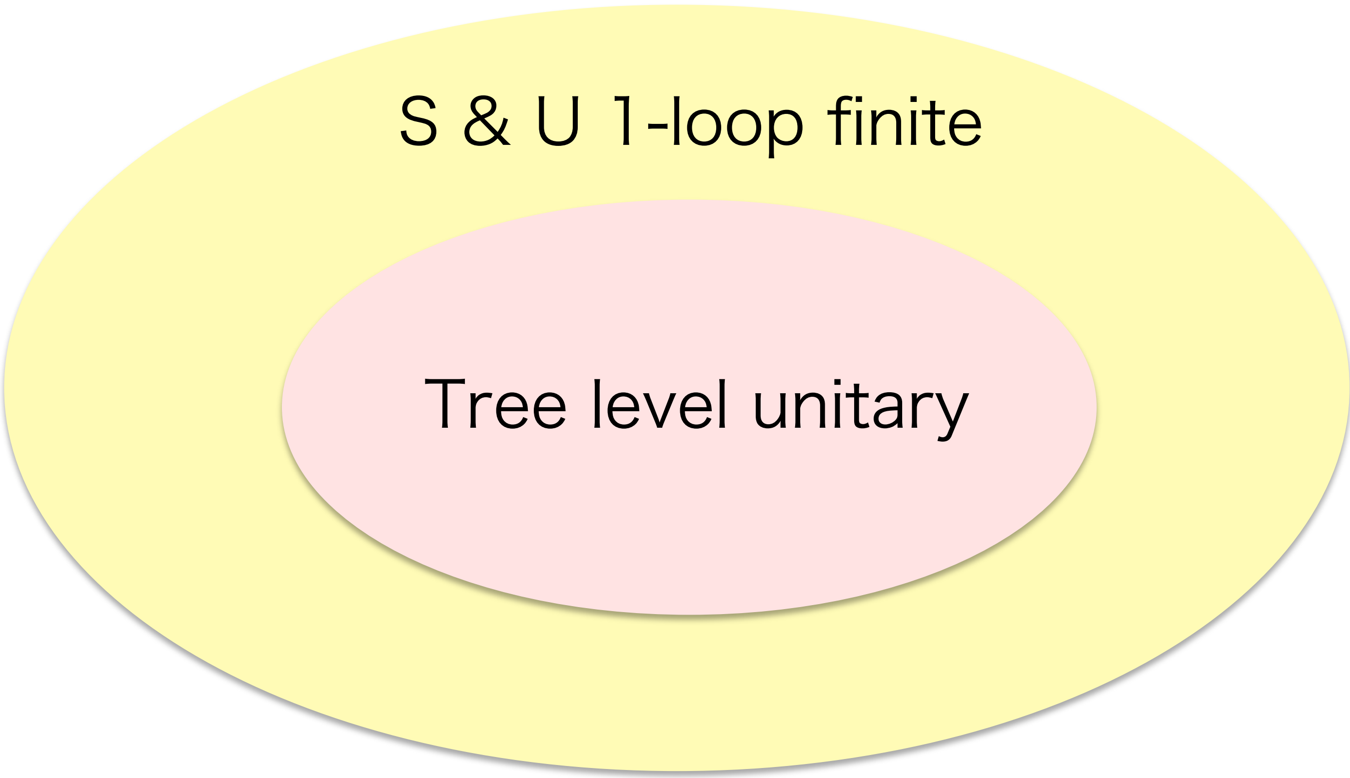
Tree level unitarity $\not\Leftarrow$ $\Rightarrow$ 1-loop finiteness of S & U

take-home message

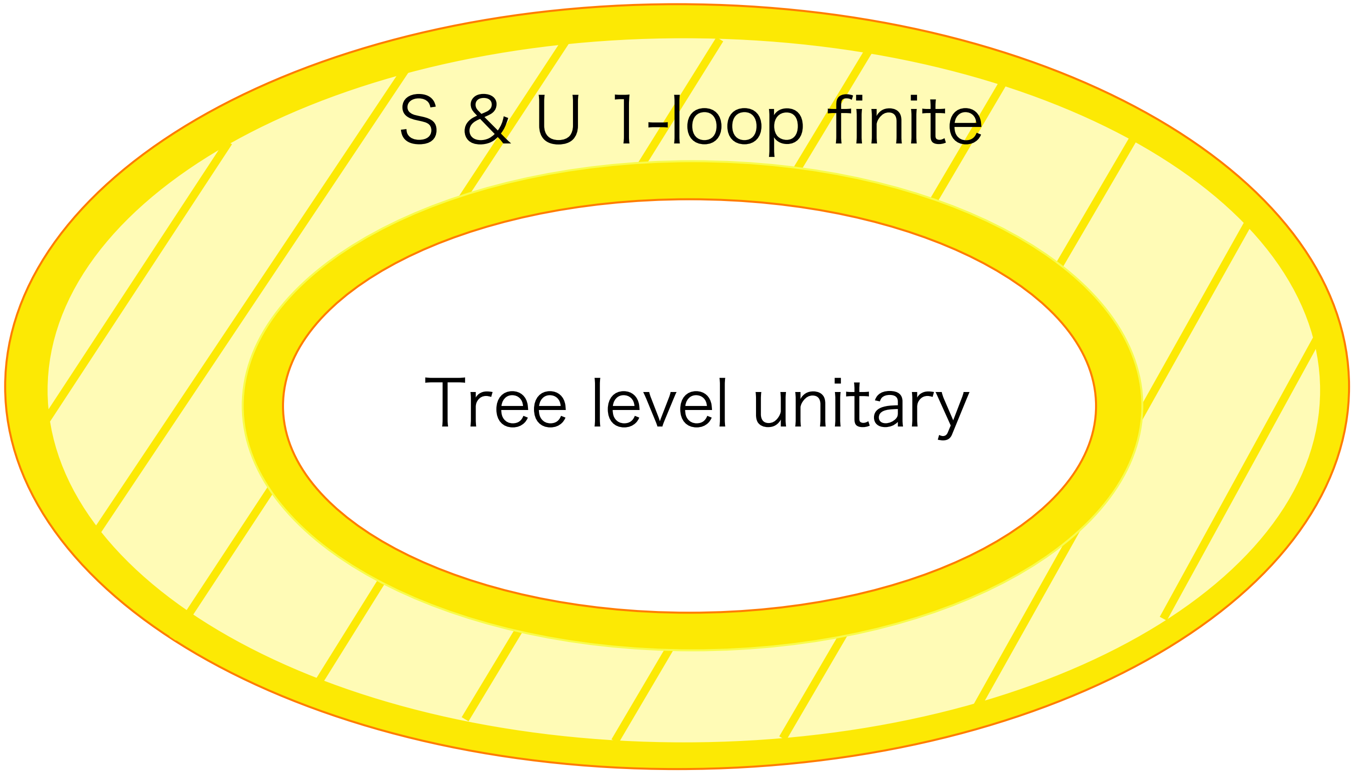
In arbiterally scalar sector ...

Tree level unitarity $\nLeftarrow$ $\Rightarrow$ 1-loop finiteness of S & U

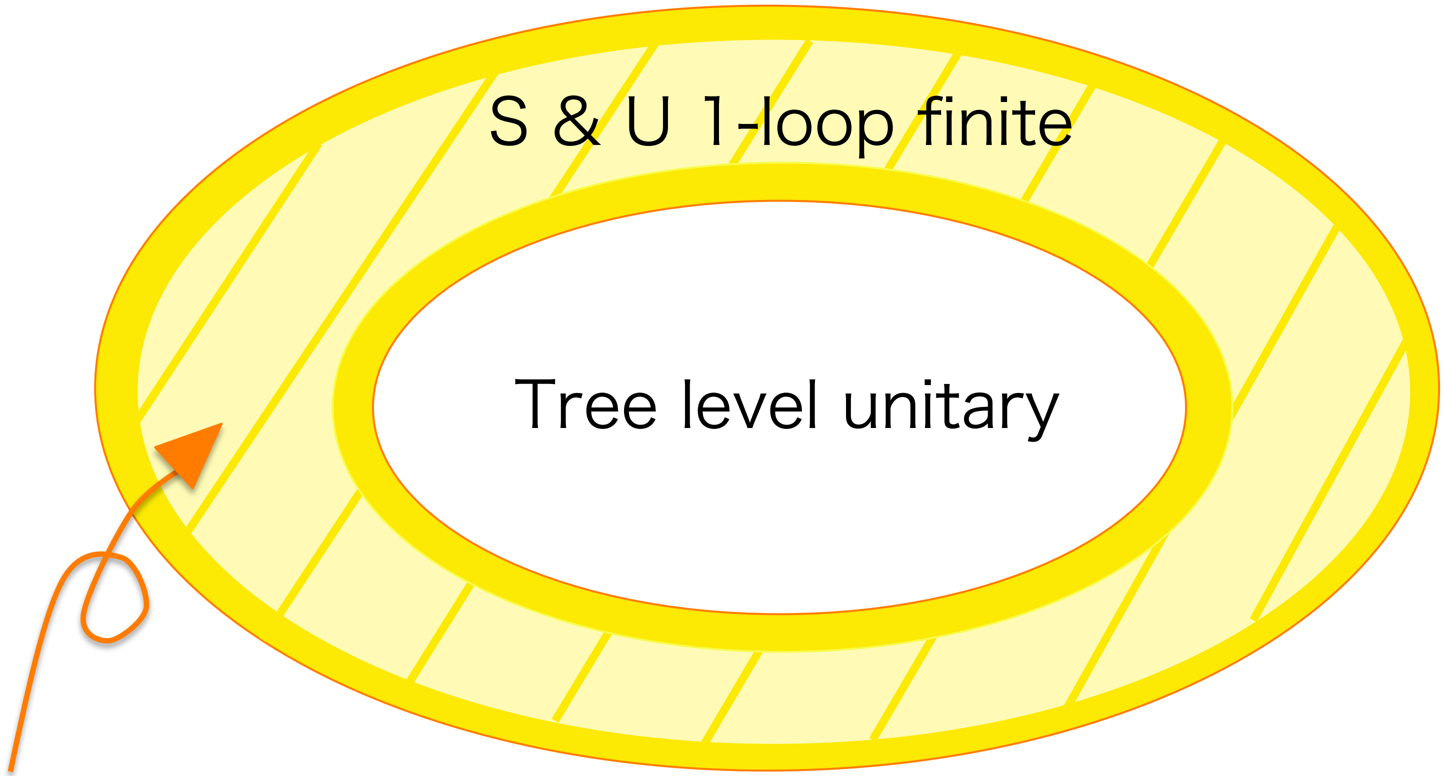
take-home message



take-home message



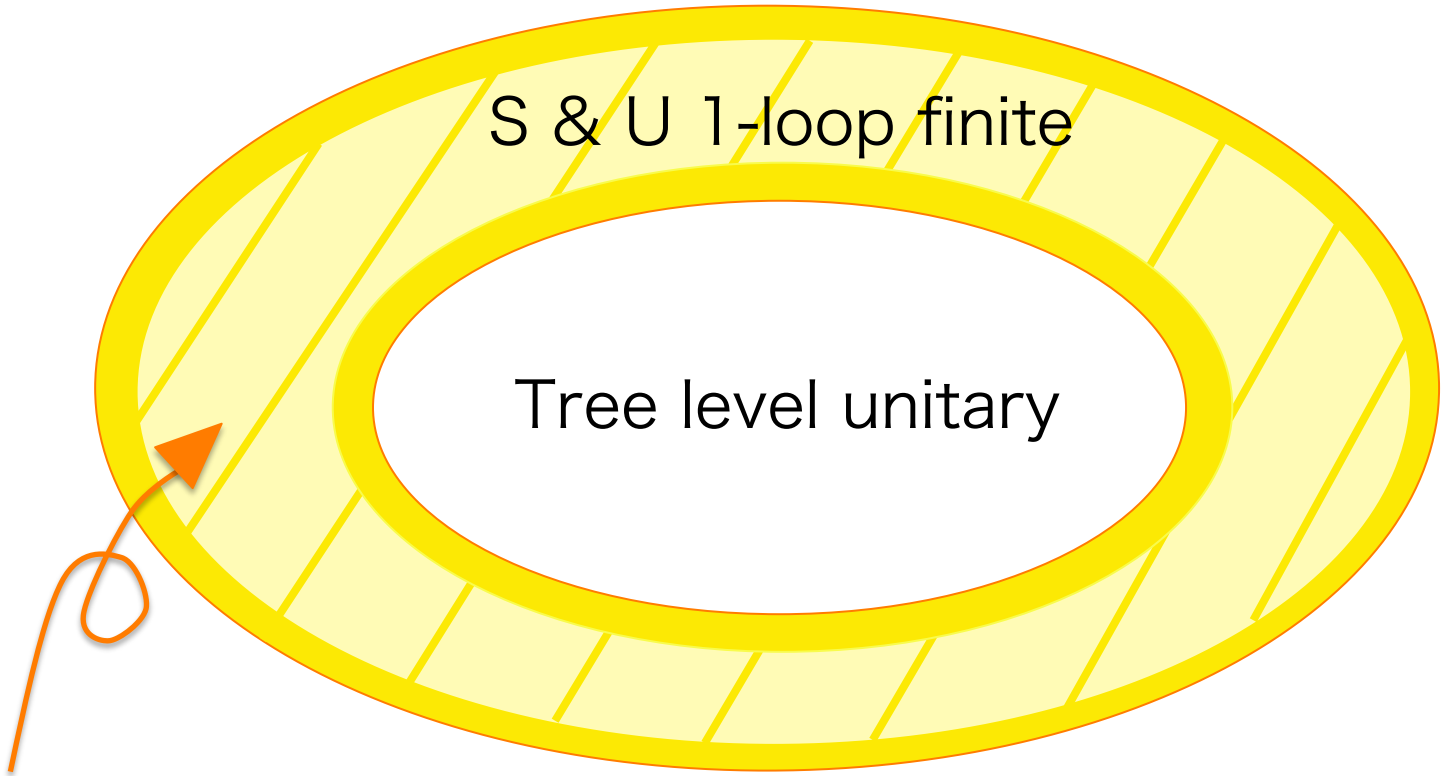
take-home message



Tree level unitary ✕

S & U 1-loop finite ○

take-home message

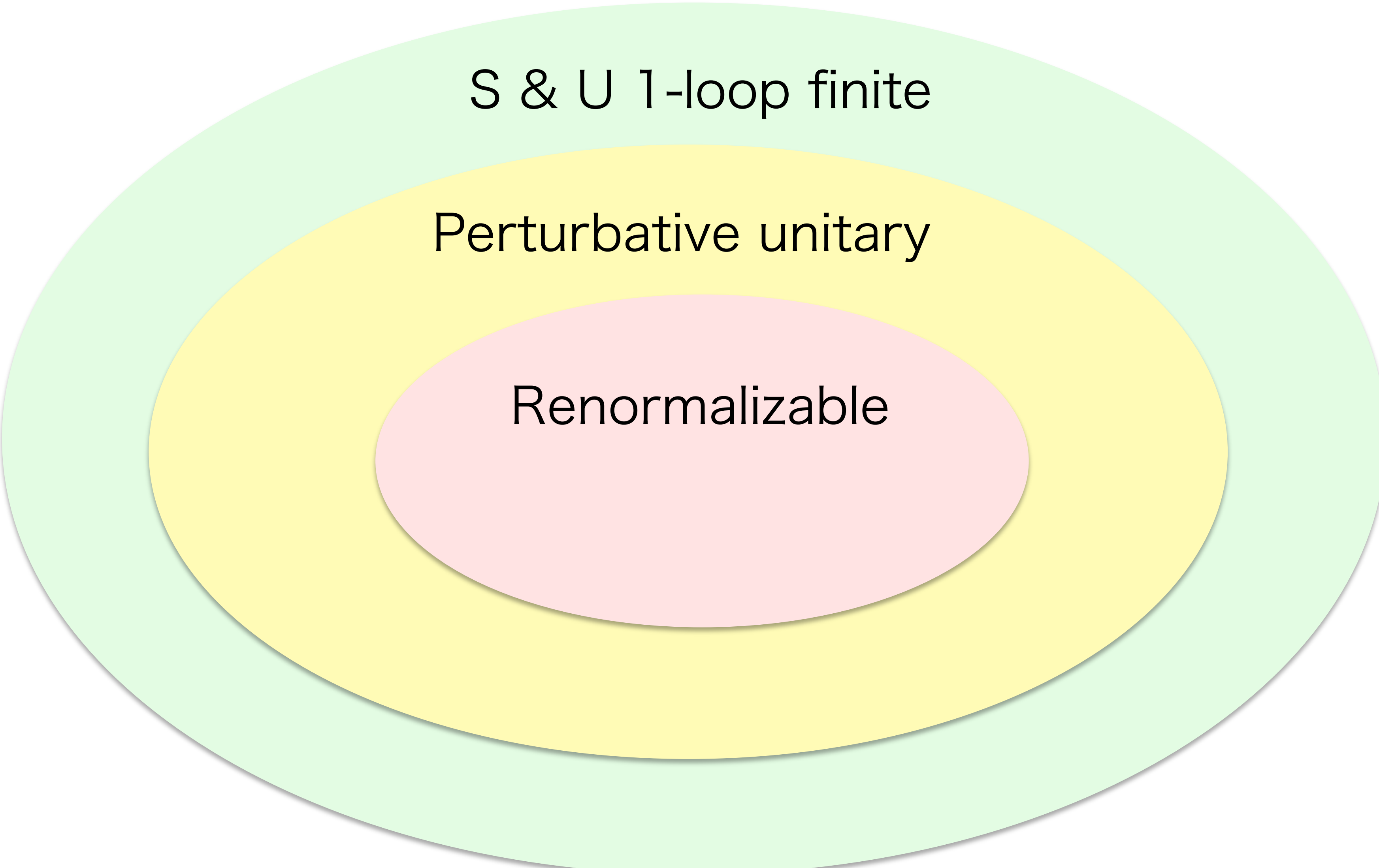


Interesting scenario such that …

Tree level unitarity is broken in certain amplitudes with keeping the consistency with EWPT.

BACK UP

Oblique correction finiteness v.s. Unitarity v.s. Renoralizability



T parameter

$$
\begin{aligned}
T_{\text{div}} \sim & \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right] \\
& \times \Big\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k \bar{w}_a^l + \bar{y}^k \bar{y}^l] \\
& \qquad - 4g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} \Big\} \ln\frac{\Lambda^2}{\mu^2}
\end{aligned}
$$

T parameter

$$T_{\rm div} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right]$$
$$\times \left\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k \bar{w}_a^l + \bar{y}^k \bar{y}^l] \right.$$
$$\left. - 4g_W^2 (\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2 (\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} \right\} \ln \frac{\Lambda^2}{\mu^2}$$

T parameter

$$T_{\rm div} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right]$$

$$\times \Big\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k \bar{w}_a^l + \bar{y}^k \bar{y}^l]$$

$$- 4g_W^2 (\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2 (\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} \Big\} \ln \frac{\Lambda^2}{\mu^2}$$

if $\bar{R}_{ikjl} = 0$

$$T_{\rm div} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right]$$

$$\times \Big\{ - 4g_W^2 (\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2 (\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} \Big\} \ln \frac{\Lambda^2}{\mu^2}$$

= 0 (SM)

= 0 (2HDM)

≠ 0 (Georgi Machacek Model)

SMEFT v.s. HEFT

SMEFT : $\mathcal{L}_{\mathrm{scalar}}$ is written in terms of H

$$\mathcal{L}_{\mathrm{scalar}} = (D_\mu H)^\dagger D^\mu H + \frac{C_1}{\Lambda^2}(H^\dagger D_\mu H)^*(H^\dagger D^\mu H) + \cdots$$

HEFT : $\mathcal{L}_{\mathrm{scalar}}$ is written in terms of π^a (NGBs) and h

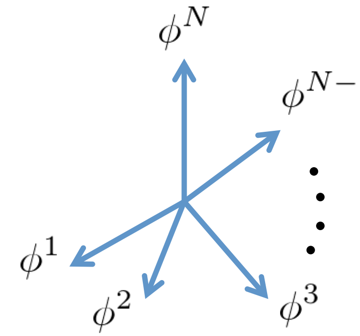
$$\mathcal{L}_{\mathrm{scalar}} = \left(1 + \kappa_1 \frac{h}{v} + \kappa_2 \frac{h^2}{v^2} + \cdots\right) \frac{v^2}{4} \mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right] - V(h)$$

$$U := e^{i \frac{\pi^a}{v} \tau_a}$$

- HEFT is more general than SMEFT
- New Physics effect is encoded in C_i or κ_i

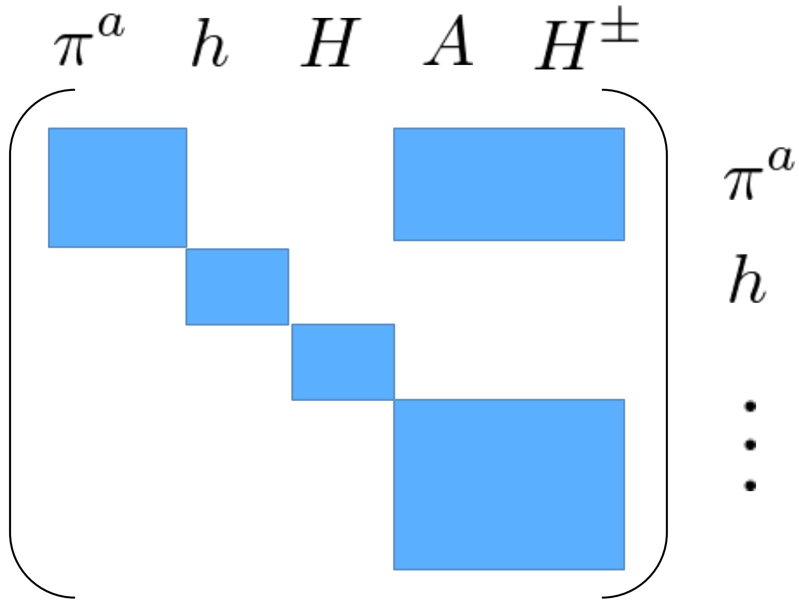
- # of indices = # of scalar fields

- form = expression of the scalars



2HDM

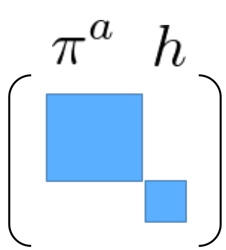
$g_{ij} =$



Nagai-san's
slide at PPP2018

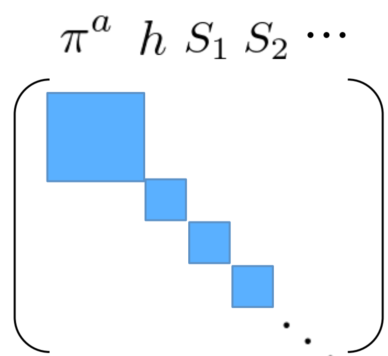
SM

$g_{ij} =$



SM + Singlet

$g_{ij} =$



Effective field theory approach

SMEFT : $\mathcal{L}_{\rm scalar}$ is written in terms of H

$$\mathcal{L}_{\rm scalar} = (D_\mu H)^\dagger D^\mu H + \frac{C_1}{\Lambda^2}(H^\dagger D_\mu H)^*(H^\dagger D^\mu H) + \cdots$$

HEFT : $\mathcal{L}_{\rm scalar}$ is written in terms of π^a (NGBs) and h

$$\mathcal{L}_{\rm scalar} = \left(1 + \kappa_1 \frac{h}{v} + \kappa_2 \frac{h^2}{v^2} + \cdots\right) \frac{v^2}{4} {\rm tr}\left[(D_\mu U)^\dagger D^\mu U\right] - V(h)$$

$$U := e^{i\frac{\pi^a}{v}\tau_a}$$

- HEFT is more general than SMEFT
- New Physics effect is encoded in C_i or κ_i

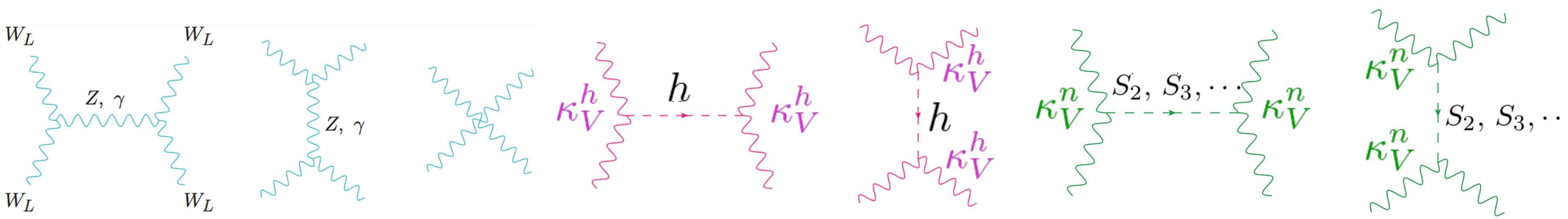
< Singlet extension >

$$\mathcal{L} = \frac{v^2}{4} F(h, S_n) \mathrm{Tr}[(D_\mu U)^\dagger D^\mu U] + \mathcal{L}_{kinetic} - V(h, S_n)$$

$$F(h, S_n) = 1 + 2\kappa_V^h \frac{h}{v} + 2\sum_n \kappa_V^n \frac{S_n}{v} + \cdots \qquad U = \exp\left(i\frac{\pi^a}{v}\frac{\tau^a}{2}\right)$$

I. Higgs unitalize W_L scattering amplitude at tree level (tree level unitarity)

$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{s+t}{v^2}\left(1 - (\kappa_V^h)^2 - \sum_{n=2}^{N} (\kappa_V^n)^2\right)$$



II. Higgs cancels the divergence in oblique corrections

Peskin Takeuchi
Phys. Rev. Lett. 65 (1990) 964

$$S \simeq \frac{1}{12\pi}\left(1 - (\kappa_V^h)^2 - \sum_{n=2}^{N} (\kappa_V^n)^2\right) \ln\frac{\Lambda^2}{\mu^2}$$