Stable magnetic monopole in two Higgs doublet models

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Introduction
Topological solitons are topologically stable excitations in field theories.

Topological solitons can exist if vacuum is topologically nontrivial.

**Homotopy group** | **Topological soliton**
--- | ---
$\pi_0(\mathcal{M}) \neq 0$ & Domain wall (kink)
$\pi_1(\mathcal{M}) \neq 0$ & Vortex (cosmic string)
$\pi_2(\mathcal{M}) \neq 0$ & Monopole

$\mathcal{M}$ : Vacuum manifold

$\pi_1(S^1) = \mathbb{Z} \neq 0$
In SM, \( SU(2)_W \times U(1)_Y \rightarrow U(1)_{EM} \)

\[
\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad |\Phi_{\text{vac.}}|^2 = v^2
\]

Vacuum manifold: \( \mathcal{M} \cong S^3 \)

Homotopy groups in SM:

\[
\begin{align*}
\pi_0(S^3) &= 0 \quad \text{No domain wall} \\
\pi_1(S^3) &= 0 \quad \text{No vortex} \\
\pi_2(S^3) &= 0 \quad \text{No monopole}
\end{align*}
\]

Topology of SM is trivial!
How about Beyond the SM?

- Some BSM predict topological solitons
How about Beyond the SM?

Some BSM predict topological solitons

- If they are found → strong evidence of New Physics
- If they are not found → constraints on the BSM
How about Beyond the SM?

- Some BSM predict topological solitons
  - If they are found: strong evidence of New Physics
  - If they are not found: constraints on the BSM

Today:

BSM = Two Higgs doublet model (2HDM)

topological soliton = Magnetic monopole
Plan of talk

- Introduction (5p.) ← Done
- Vortex in 2HDM (Review) (8p.)
- Magnetic Monopole in 2HDM (7p.)
- Summary
Vortex in 2HDM

[Dvali, Senjanovic ‘93]

[Eto, Kurachi, Nitta ‘18]
Two Higgs doublet model (2HDM)

- Higgs potential

\[ V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left( m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) + \frac{\beta_1}{2} \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{\beta_2}{2} \left( \Phi_2^\dagger \Phi_2 \right)^2 \]

\[ + \beta_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) + \beta_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \left\{ \frac{\beta_5}{2} \left( \Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right\} \]

- VEVs

\[ \langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ \nu_1 \end{pmatrix} \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ \nu_2 \end{pmatrix} \quad \nu_{EW}^2 = 2(\nu_1^2 + \nu_2^2) \simeq (246 \text{ GeV})^2 \]
Two Higgs doublet model (2HDM)

- Higgs potential

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V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left( m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) + \frac{\beta_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\beta_2}{2} (\Phi_2^\dagger \Phi_2)^2 \\
+ \beta_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \beta_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \frac{\beta_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right\}
\]

- VEVs

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\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ \nu_1 \end{pmatrix} \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ \nu_2 \end{pmatrix} \quad \nu_{EW}^2 = 2(\nu_1^2 + \nu_2^2) \simeq (246 \text{ GeV})^2
\]

- 2HDM is well motivated by **simpleness / EW baryogenesis / SUSY.**
Two Higgs doublet model (2HDM)

- Higgs potential

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\[ + \beta_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) + \beta_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \left\{ \frac{\beta_5}{2} \left( \Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right\} \]

- Impose two global symmetries:
Two Higgs doublet model (2HDM)

- Higgs potential

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V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) + \frac{\beta_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\beta_2}{2} (\Phi_2^\dagger \Phi_2)^2 \\
+ \beta_3 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_2) + \beta_4 (\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1) + \left\{ \frac{\beta_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right\}
\]

- Impose two global symmetries:
  - \( U(1)_a \) sym.: \( \Phi_1 \rightarrow e^{-i\alpha} \Phi_1, \ \Phi_2 \rightarrow e^{i\alpha} \Phi_2 \)
  - \( \pi_1(\mathcal{M}) = \mathbb{Z} \) (broken in vacuum)
Two Higgs doublet model (2HDM)

- Higgs potential

\[
V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) + \frac{\beta_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\beta_2}{2} (\Phi_2^\dagger \Phi_2)^2
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+ \beta_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \beta_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \frac{\beta_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right\}
\]

- Impose two global symmetries:

  - \(U(1)\) sym.: \(\Phi_1 \rightarrow e^{-i\alpha} \Phi_1, \ \Phi_2 \rightarrow e^{i\alpha} \Phi_2\)

  \(\pi_1(\mathcal{M}) = \mathbb{Z}\)

  - \((\mathbb{Z}_2)\) sym.: 

    \[
    \begin{align*}
    \Phi_1 &\rightarrow (i\sigma^2) \Phi_2^* \\
    \Phi_2 &\rightarrow (i\sigma^2) \Phi_1^*
    \end{align*}
    \]

    \[
    \begin{align*}
    W_i &\rightarrow (i\sigma^1) \ W_i \ (i\sigma^1)^\dagger \\
    B_i &\rightarrow - B_i
    \end{align*}
    \]

  (not broken in vacuum)
Two Higgs doublet model (2HDM)

- Higgs potential

\[
V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) + \frac{\beta_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\beta_2}{2} (\Phi_2^\dagger \Phi_2)^2
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\[+ \beta_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \beta_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \left\{ \frac{\beta_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right\} \]

- Impose two global symmetries:

  - $U(1)_a$ sym.: \( \Phi_1 \to e^{-i\alpha} \Phi_1, \quad \Phi_2 \to e^{i\alpha} \Phi_2 \)

  \[\pi_1(\mathcal{M}) = \mathbb{Z}\]

  \[m_{11} = m_{22} \quad \beta_1 = \beta_2\]

  \[\tan \beta \equiv \frac{v_2}{v_1} = 1\]

- $(\mathbb{Z}_2)_C$ sym.:

  \[
  \begin{align*}
  \Phi_1 & \to (i\sigma^2) \Phi_2^* \\
  \Phi_2 & \to (i\sigma^2) \Phi_1^* \\
  W_i & \to (i\sigma^1) W_i (i\sigma^1)^\dagger \\
  B_i & \to -B_i
  \end{align*}
  \]
There are two topological Z-strings (Z-flux tubes).

- **(0,1)-string**

  \[ H^{(0,1)} \sim v \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \quad Z_{i}^{(0,1)} \sim \frac{\cos \theta_{W} \epsilon_{3ij} x^{j}}{g} \frac{1}{r^{2}} \]

  \( \Phi_{2} \) has a winding \#  
  confined \textbf{Z-flux}: \( \Phi_{Z} = \frac{2\pi}{g_{Z}} \)

- **(1,0)-string**

  \[ H^{(1,0)} \sim v \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix} \quad Z_{i}^{(1,0)} \sim \frac{-\cos \theta_{W} \epsilon_{3ij} x^{j}}{g} \frac{1}{r^{2}} \]

  \( \Phi_{1} \) has a winding \#  
  confined \textbf{Z-flux}: \( \Phi_{Z} = \frac{-2\pi}{g_{Z}} \)
There are two topological Z-strings (Z-flux tubes).

- **(0,1)-string**
  
  $H^{(0,1)} \sim v \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$
  
  $Z_i^{(0,1)} \sim \frac{\cos \theta_W}{g} \frac{\epsilon_{3ij}}{r^2} x^j$

  Φ₂ has a winding #

  Confined Z-flux: $\Phi_Z = \frac{-2\pi}{g_Z}$

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  Confined Z-flux: $\Phi_Z = \frac{-2\pi}{g_Z}$
There are two topological Z-strings (Z-flux tubes).

- **(0,1)-string**
  
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  \( \Phi_2 \) has a winding \#  

  confined **Z-flux**: \( \Phi_Z = \frac{2\pi}{g_Z} \)

  \( (\mathbb{Z}_2)_C \) transf.

- **(1,0)-string**
  
  \[ H^{(1,0)} \sim v \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix} \quad Z^{(1,0)}_i \sim -\cos \theta_W \frac{\epsilon_{3ij} x^j}{g r^2} \]

  \( \Phi_1 \) has a winding \#  

  confined **Z-flux**: \( \Phi_Z = -\frac{2\pi}{g_Z} \)

  \( (\mathbb{Z}_2)_C \) sym.

The tensions are exactly degenerate by \( (\mathbb{Z}_2)_C \) sym.
Magnetic Monopole in 2HDM

[Eto, Hamada, Kurachi, Nitta ’19]
Symmetry breaking is $SU(2)_W \times U(1)_Y \times U(1)_a \rightarrow U(1)_{EM}$

$\pi_2(\mathcal{M}) \simeq \pi_2(U(2)) = 0$  
No stable magnetic monopole?
Symmetry breaking is \( SU(2)_W \times U(1)_Y \times U(1)_a \rightarrow U(1)_{EM} \)

\[ \pi_2(\mathcal{M}) \simeq \pi_2(U(2)) = 0 \]

No stable magnetic monopole? Does exist!
Magnetic Monopole in 2HDM

- Symmetry breaking is \( SU(2)_W \times U(1)_Y \times U(1)_a \rightarrow U(1)_{EM} \)

\[ \pi_2(\mathcal{M}) \simeq \pi_2(U(2)) = 0 \]

No stable magnetic monopole? **Does exist!**

- Connect the two Z-strings smoothly

(0,1) string \hspace{1cm} (1,0) string
Magnetic Monopole in 2HDM

- Symmetry breaking is $SU(2)_W \times U(1)_Y \times U(1)_a \to U(1)_{EM}$

$$\pi_2(\mathcal{M}) \simeq \pi_2(U(2)) = 0$$

No stable magnetic monopole? Does exist!

- Connect the two Z-strings smoothly

- This object is a topological $(\mathbb{Z}_2)_C$ kink interpolating the two Z-strings.
Magnetic Monopole in 2HDM

- Symmetry breaking is $SU(2)_W \times U(1)_Y \times U(1)_a \to U(1)_{EM}$

$$\pi_2(\mathcal{M}) \simeq \pi_2(U(2)) = 0$$

No stable magnetic monopole? Does exist!

- Connect the two $Z$-strings smoothly

- This object is a topological $(\mathbb{Z}_2)_C$ kink interpolating the two $Z$-strings.

This behaves as a magnetic monopole.
This configuration can be regarded as embedding ‘t Hooft-Polyakov monopole into $SU(2)_W$ doublets.
Magnetic Flux

- This configuration can be regarded as embedding ‘t Hooft-Polyakov monopole into $SU(2)_W$ doublets.

$$n^a \equiv \sum_{i=1,2} \frac{\Phi_i \sigma^a \Phi_i}{\Phi_i \Phi_i} \quad \text{Hedgehog Form : } n^a \sim \frac{x^a}{r}$$
Magnetic Flux

- Magnetic flux spreads spherically like tHP monopole

- Clearly stable (topological \((\mathbb{Z}_2)_C\) kink)

(In other words, the two string tensions are balanced.)

\[ n^a \equiv \sum_{i=1,2} \frac{\Phi_i^\dagger \sigma^a \Phi_i}{\Phi_i^\dagger \Phi_i} \]

\[ \Phi_B = \frac{4\pi \sin^2 \theta_W}{e} \]

Hedgehog Form: \[ n^a \sim \frac{x^a}{r} \]

This configuration can be regarded as embedding ‘t Hooft-Polyakov monopole into \(SU(2)_W\) doublets.
Numerical Result

- Numerical solution to EOMs based on relaxation method

(a) energy density
(b) magnetic flux
(c) Z flux

with \( \sin^2 \theta_W = 0.23, \ m_W = 80 \text{ GeV}, \ \nu_{EW} = 246 \text{ GeV}, \)

\[ m_h = 125 \text{ GeV}, \ m_H = m_{H^\pm} = 400 \text{ GeV} \]
We can numerically calculate the energy of the monopole.

\[ \text{string tension} \simeq (\mathcal{O}(100) \text{ GeV})^2 \]

\[ \text{defect energy + magnetic energy} \simeq \mathcal{O}(1) \text{ TeV} \]
We can numerically calculate the energy of the monopole.

\[
\text{string tension} \approx (\mathcal{O}(100) \text{ GeV})^2
\]

\[
\text{defect energy + magnetic energy} \approx \mathcal{O}(1) \text{ TeV}
\]

TeV scale phenomenon! can be produced at LHC!?
Summary

- Stable magnetic monopole exists in 2HDM.

- Key symmetries:
  - $U(1)_a \Rightarrow$ topological vortices
  - $(\mathbb{Z}_2)_C \Rightarrow$ monopole as topological kink

Future works:
- $U(1)_a$, $(\mathbb{Z}_2)_C \Rightarrow$ How unstable?
- How is it produced in accelerators?
Backup Slides
$(\mathbb{Z}_2)_C$ symmetry is not exact because of Yukawa couplings.

Quantum corrections break $(\mathbb{Z}_2)_C$ symmetry in the Higgs potential.

The tensions of $Z$-strings are not degenerate.

The monopole is pulled to the heavier string.
When $U(1)_a$ symmetry is exact, NG boson appears (massless CP-odd Higgs)

Phenomenologically disfavored
When $U(1)_a$ symmetry is exact, NG boson appears (massless CP-odd Higgs)

- Phenomenologically disfavored
- $U(1)_a$ to give a mass ($m_{12}, \beta_5$ switch on)
- Wall (membrane) attaches

[Eto, Kurachi, Nitta '18]
When $U(1)_a$ symmetry is exact, NG boson appears (massless CP-odd Higgs)

Phenomenologically disfavored

- $U(1)_a$ to give a mass ($m_{12}, \beta_5$ switch on)

Wall (membrane) attaches

String + monopole are pulled by the wall

Monopole abundance is non-trivial
When $U(1)_a$ symmetry is exact, NG boson appears (massless CP-odd Higgs)

- Phenomenologically disfavored

  - $U(1)_a$ to give a mass ($m_{12}, \beta_5$ switch on)

  $\downarrow$ wall (membrane) attaches

```
String + monopole are pulled by the wall
monopole abundance is non-trivial
```

(Eto, Kurachi, Nitta '18)

Disappear ?

or

``vorton'' ?

(or superconducting string loop)
Topological Z-string in 2HDM

- Symmetry breaking is $SU(2)_W \times U(1)_Y \times U(1)_a \rightarrow U(1)_{EM}$

  $\pi_1(\mathcal{M}) \simeq \pi_1(U(2)) = \mathbb{Z}$  \hspace{1cm} \text{Topological vortex exist!}

- Topological Z-string ( (0,1)-string )

  2 x 2 matrix notation: $H \equiv \left( i\sigma_2 \Phi_1^*, \Phi_2 \right)$

  $H^{(0,1)} \sim v \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = v \begin{pmatrix} e^{i\theta/2} & e^{-i\theta/2} \sigma_3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

  $Z^{(0,1)}_i \sim \frac{\cos \theta_W}{g} \frac{e_{3ij} x^j}{r^2}$

  \begin{itemize}
  \item confined Z-flux $\Phi_Z = \frac{2\pi}{g_Z}$
  \item global vortex $\rightarrow \text{tension} \sim \pi v^2 \log \Lambda_{IR}$
  \end{itemize}

$U(1)_a$ phase: $-\pi/2 \sim \pi/2$ (global symmetry)

[Refs: Dvali, Senjanovic ‘93, Eto, Kurachi, Nitta ‘18]
Stability of Z-string in SM

$\frac{m_H}{m_Z}$ vs $\sin^2\theta_w$

- Experiment
- Scaling instability
- Stable
- Semilocal

[Achucarro, Vachaspati, hep-ph/9904229]
Monopole production at colliders

- Conventional process

![Drell-Yan mechanism](image)

- For our monopole,

\[ q \rightarrow q \]

\[ \bar{q} \rightarrow \bar{q} \]

If it decays, we can see it as a resonance?

![Graph showing excluded magnetic monopole mass](image)

From A. Santra’s slide

Magnetic Monopoles via Photon Fusion

Magnetic Monopoles via Photon Fusion
2HDM in Matrix Notation

\[ V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) + \frac{\beta_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\beta_2}{2} (\Phi_2^\dagger \Phi_2)^2 \]

\[ + \beta_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \beta_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \left\{ \frac{\beta_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right\} \]

\[
m_{11}^2 = -m_1^2 - m_2^2, \quad m_{22}^2 = -m_1^2 + m_2^2, \quad m_{12} = m_3, \\
\beta_1 = 2(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4), \quad \beta_2 = 2(\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4), \\
\beta_3 = 2(\alpha_1 + \alpha_2 - \alpha_3), \quad \beta_4 = 2(\alpha_3 - \alpha_1), \quad \beta_5 = 2\alpha_5
\]

\[ V(\Phi_1, \Phi_2) = -m_1^2 \text{Tr} |H|^2 - m_2^2 \text{Tr} \left( |H|^2 \sigma_3 \right) - (m_3^2 \det H + \text{h.c.}) \]

\[ + \alpha_1 \text{Tr} |H|^4 + \alpha_2 \left( \text{Tr} |H|^2 \right)^2 + \alpha_3 \text{Tr} \left( |H|^2 \sigma_3 |H|^2 \sigma_3 \right) \]

\[ + \alpha_4 \text{Tr} \left( |H|^2 \sigma_3 |H|^2 \right) + (\alpha_5 \det H^2 + \text{h.c.}) \]

\[ |H|^2 \equiv H^\dagger H \]
There are more topological vortices.

Space of topological vortices = moduli space $S^2$

Each point on the moduli space $S^2$ corresponds to a vortex.

Two Z-strings:

$H^{(0,1)} \sim \nu \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$

$Z$ flux: $\Phi_{Z}^{(0,1)} = \frac{2\pi}{8Z}$

$H^{(1,0)} \sim \nu \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix}$

$Z$ flux: $\Phi_{Z}^{(1,0)} = -\frac{2\pi}{8Z}$
Moduli space of vortices

- There are more topological vortices.
- Space of topological vortices = moduli space $S^2$

Each point on the moduli space $S^2$ corresponds to a vortex.

$H \sim v e^{\frac{i\theta}{2}} e^{\frac{i\sigma_1}{2}}$

W flux: $\Phi_{W^1} = \frac{2\pi}{g}$
Because of $U(1)_Y$, W string and Z strings have different tensions. (lifted moduli space)