Stable magnetic monopole in two Higgs doublet models

Based on arXiv:1904.09269

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SI2019@ Lakai Sandpine (18-23 Aug. 2019)

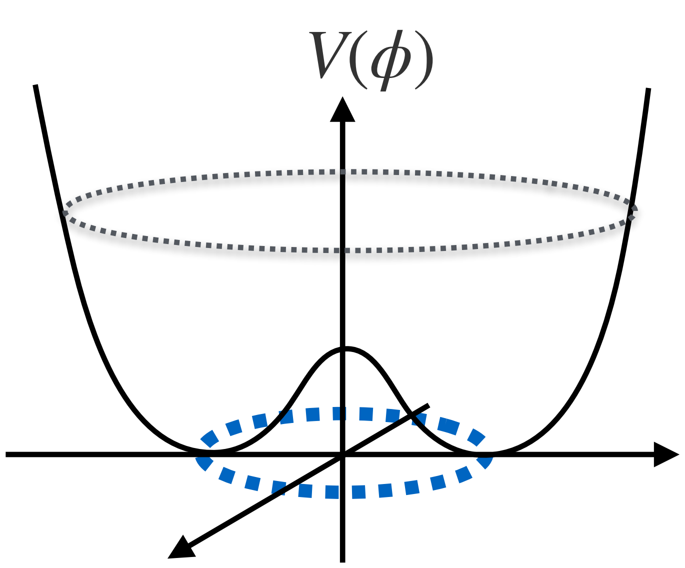
Introduction

Topological Soliton

- Topological solitons are topologically stable excitations in field theories.

- Topological solitons can exist if vacuum is topologically nontrivial.

homotopy group		topological soliton
$\pi_0(\mathscr{M}) \neq 0$	$\longrightarrow$	Domain wall (kink)
$\pi_1(\mathscr{M}) \neq 0$	$\longrightarrow$	Vortex (cosmic string)
$\pi_2(\mathscr{M}) \neq 0$	$\longrightarrow$	Monopole



$\pi_1(S^1) = \mathbb{Z} \neq 0$

$\mathscr{M}$: Vacuum manifold

Topology of SM

- **In SM,** $SU(2)_W \times U(1)_Y \rightarrow U(1)_{EM}$

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad \Rightarrow \quad |\Phi_{\text{vac.}}|^2 = v^2$$

Vacuum manifold : $\mathscr{M} \simeq S^3$

- Homotopy groups in SM :

$\pi_0(S^3) = 0$	**No domain wall**
$\pi_1(S^3) = 0$	**No vortex**
$\pi_2(S^3) = 0$	**No monopole**

Topology of SM is trivial !

How about Beyond the SM?

- Some BSM predict topological solitons

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- **Some BSM predict topological solitons**
  - If they are found ➧ **strong evidence of New Physics**
  - If they are not found ➧ **constraints on the BSM**

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Today:

BSM = Two Higgs doublet model (2HDM)

topological soliton = Magnetic monopole

Plan of talk

- Introduction (5p.) ←Done
- Vortex in 2HDM (Review) (8p.)
- Magnetic Monopole in 2HDM (7p.)
- Summary

Vortex in 2HDM

[Dvali, Senjanovic ’93]

[Eto, Kurachi, Nitta ’18]

Two Higgs doublet model (2HDM)

- Higgs potential

$$V(\Phi_1,\Phi_2) = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right) + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$

$$+\beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \mathrm{h.c.}\right\}$$

- VEVs $\langle\Phi_1\rangle = \begin{pmatrix}0\\ v_1\end{pmatrix}$ $\langle\Phi_2\rangle = \begin{pmatrix}0\\ v_2\end{pmatrix}$ $v_{EW}^2 = 2(v_1^2+v_2^2) \simeq (246\ \mathrm{GeV})^2$

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- 2HDM is well motivated by **simpleness / EW baryogenesis / SUSY.**

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(broken in vacuum)

$\longrightarrow \pi_1(\mathscr{M}) = \mathbb{Z}$

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$\boxed{\bullet\ (\mathbb{Z}_2)_C \text{ sym. :}}$

(not broken in vacuum)

$$\begin{cases} \Phi_1 \to (i\sigma^2)\Phi_2^* \\ \Phi_2 \to (i\sigma^2)\Phi_1^* \\ W_i \to (i\sigma^1)\ W_i\ (i\sigma^1)^\dagger \\ B_i \to -B_i \end{cases}$$

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$\longrightarrow$ $m_{11} = m_{22} \quad \beta_1 = \beta_2$

$\longrightarrow \tan\beta \equiv v_2/v_1 = 1$

(not broken in vacuum)

Topological Z-strings in 2HDM

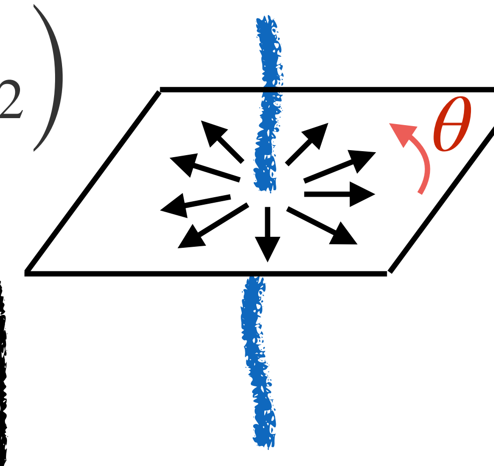
[Dvali, Senjanovic '93]

[Eto, Kurachi, Nitta '18]

- There are two topological Z-strings (Z-flux tubes).

2 x 2 matrix notation: $H \equiv \left(i\sigma_2 \Phi_1^*,\ \Phi_2 \right)$

- **(0,1)-string**



$$H^{(0,1)} \sim v \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \qquad Z_i^{(0,1)} \sim \frac{\cos\theta_W}{g} \frac{\epsilon_{3ij}\ x^j}{r^2}$$

Φ_2 has a winding # confined **Z-flux :** $\Phi_Z = \dfrac{2\pi}{g_Z}$

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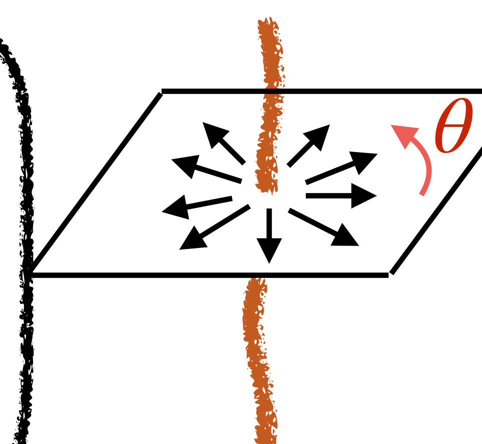

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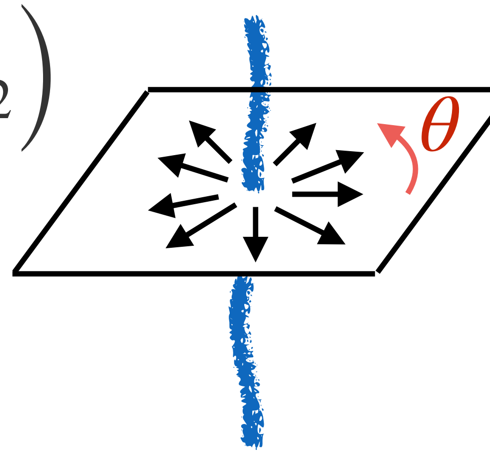

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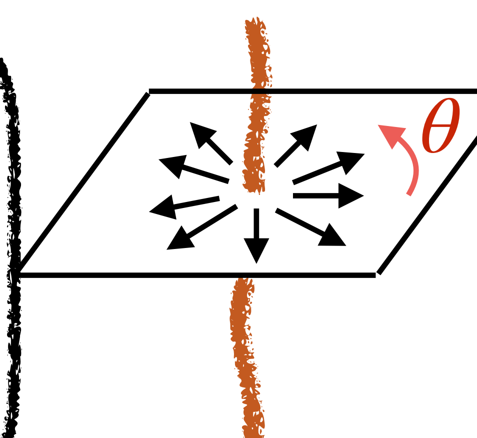

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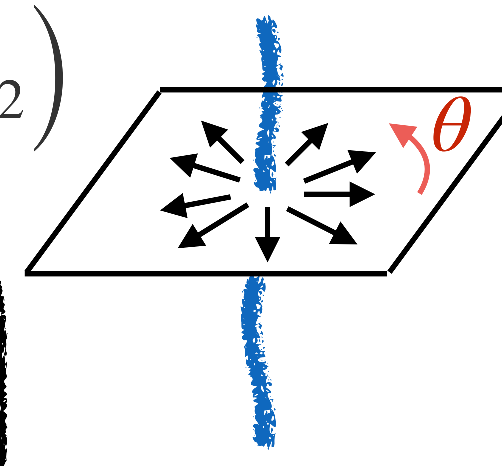

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The tensions are exactly degenerate by $(\mathbb{Z}_2)_C$ sym.

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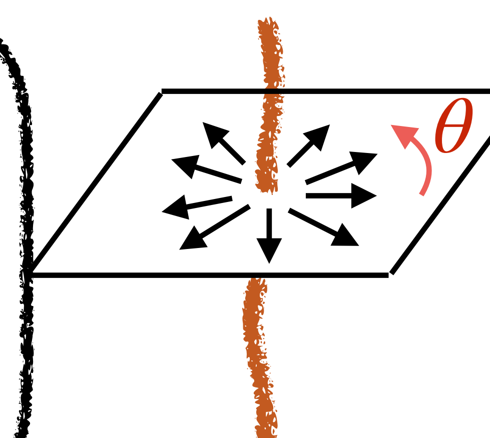

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Magnetic Monopole in 2HDM

[Eto, **Hamada**, Kurachi, Nitta ’19]

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- Symmetry breaking is $SU(2)_W \times U(1)_Y \times U(1)_a \to U(1)_{EM}$
  - $\pi_2(\mathscr{M}) \simeq \pi_2(U(2)) = 0$ **No stable magnetic monopole?**

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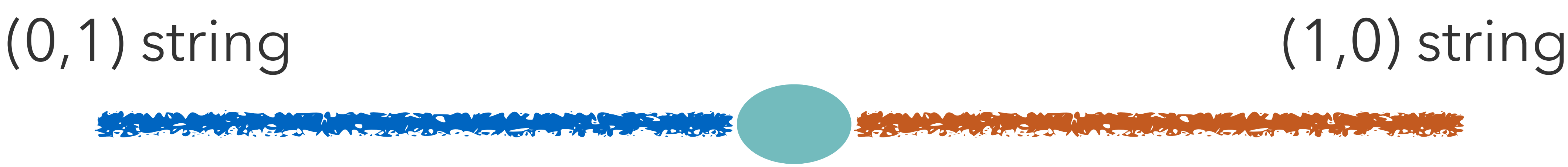

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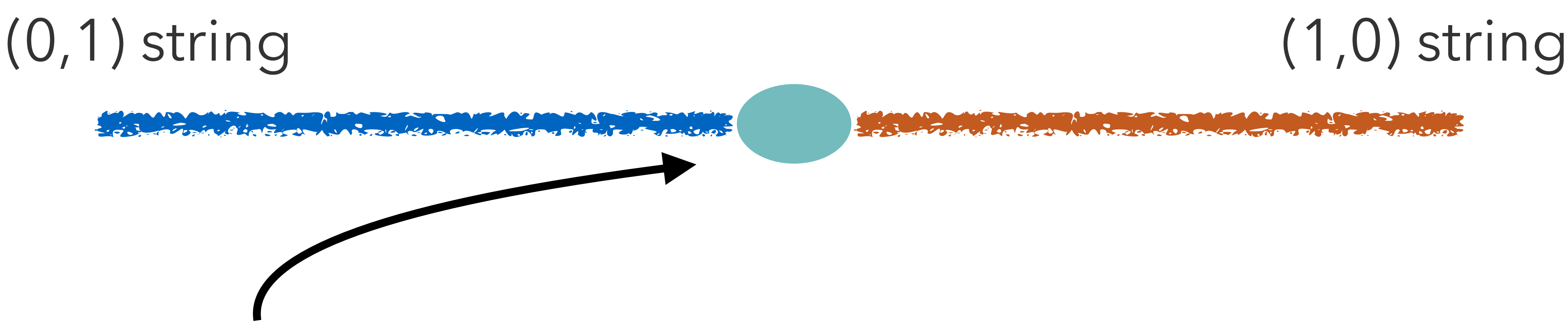

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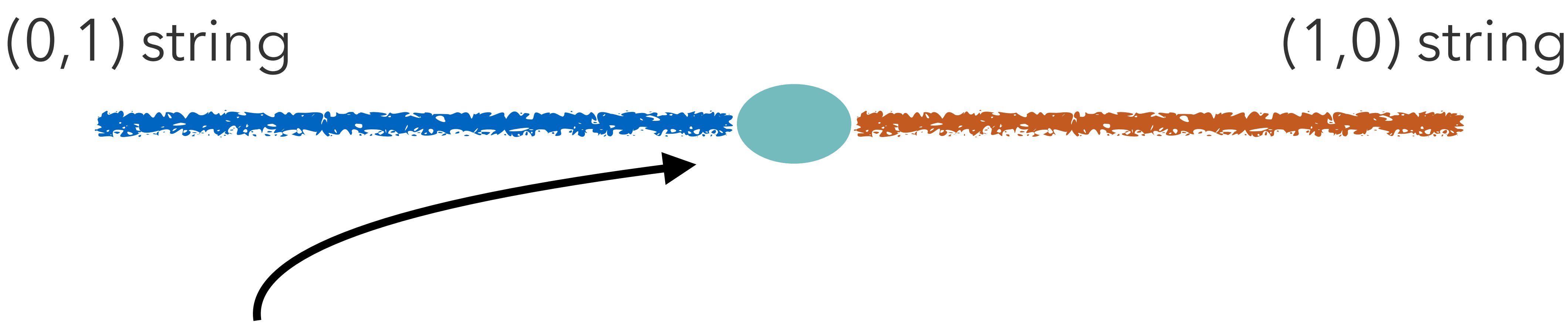

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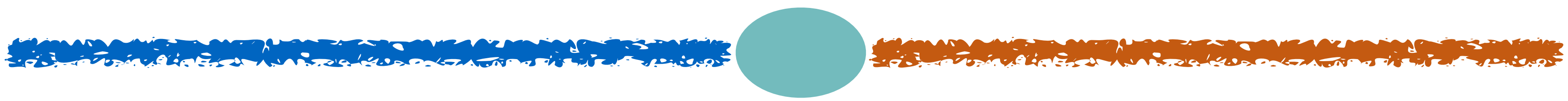
This behaves as a magnetic monopole.

Magnetic Flux

- This configuration can be regarded as embedding 't Hooft-Polyakov monopole into $SU(2)_W$ doublets.



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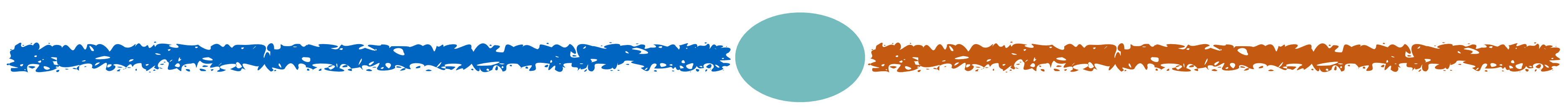

$$n^a \equiv \sum_{i=1,2} \frac{\Phi_i^\dagger \ \sigma^a \ \Phi_i}{\Phi_i^\dagger \Phi_i}$$

Hedgehog Form : $n^a \sim \dfrac{x^a}{r}$

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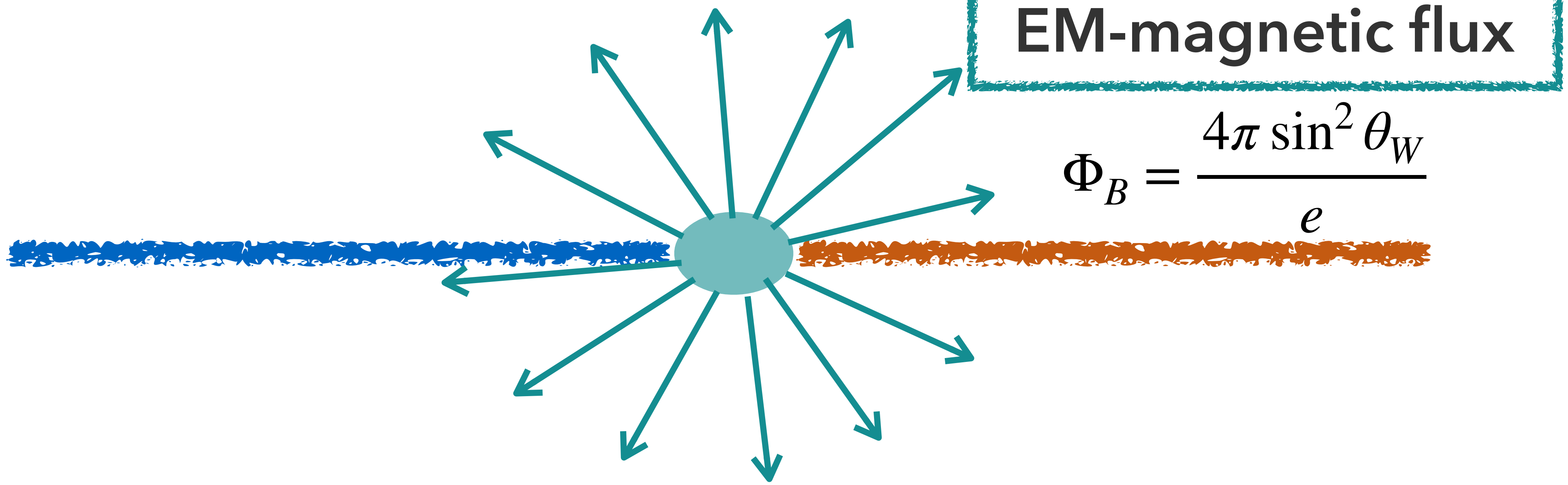

$$\Phi_B = \frac{4\pi \sin^2\theta_W}{e}$$

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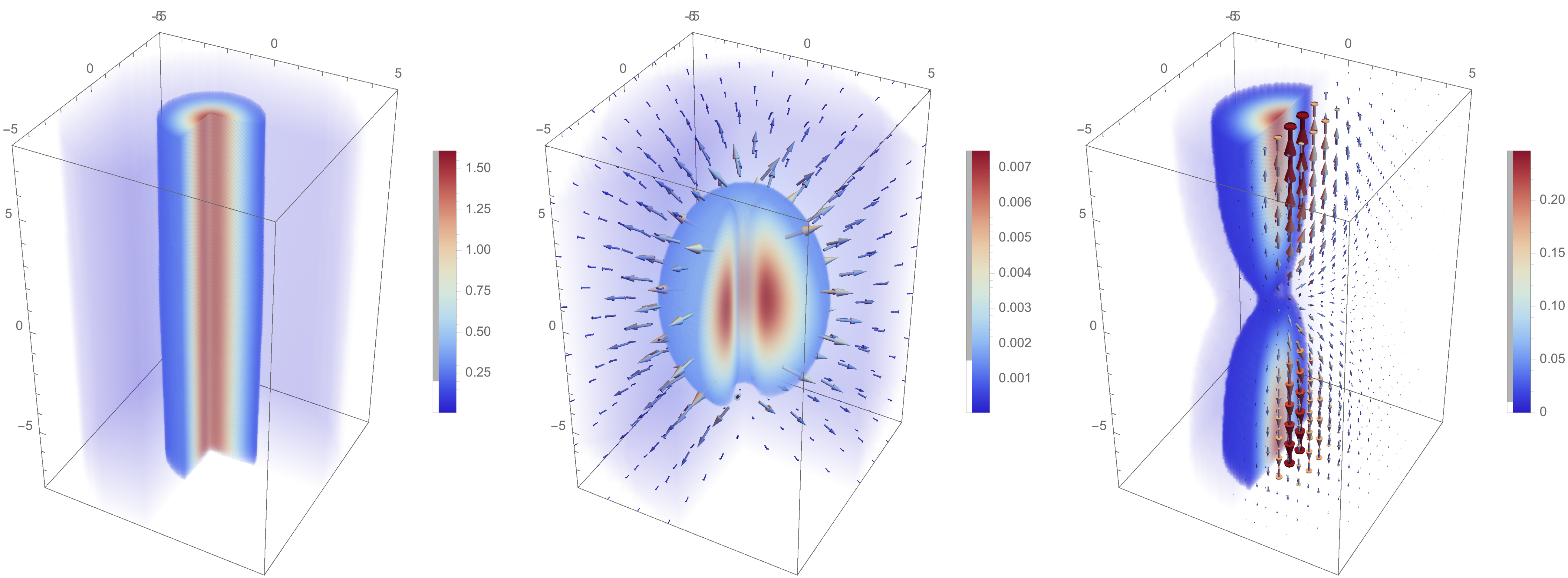
- **Magnetic flux spreads spherically like tHP monopole**
- Clearly **stable** (topological $(\mathbb{Z}_2)_C$ kink)

  (In other words, the two string tensions are balanced.)

Numerical Result

- Numerical solution to EOMs based on relaxation method



(a) energy density (b) magnetic flux (c) Z flux

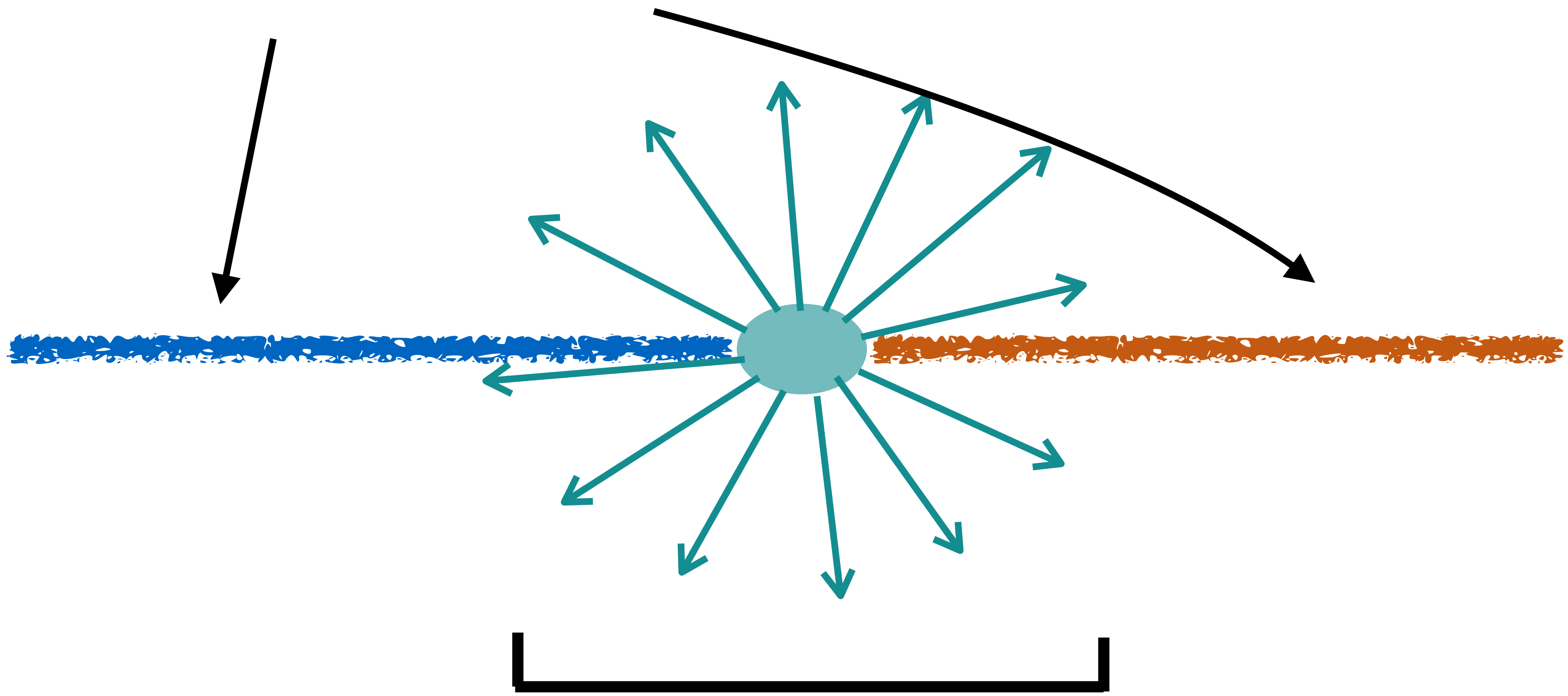
with $\sin^2\theta_W = 0.23,\ m_W = 80\ \text{GeV},\ v_{\text{EW}} = 246\ \text{GeV},$

$$m_h = 125\ \text{GeV},\ m_H = m_{H^\pm} = 400\ \text{GeV}$$

Monopole Energy

- We can numerically calculate the energy of the monopole.

string tension $\simeq (\mathcal{O}(100)\ \mathrm{GeV})^2$



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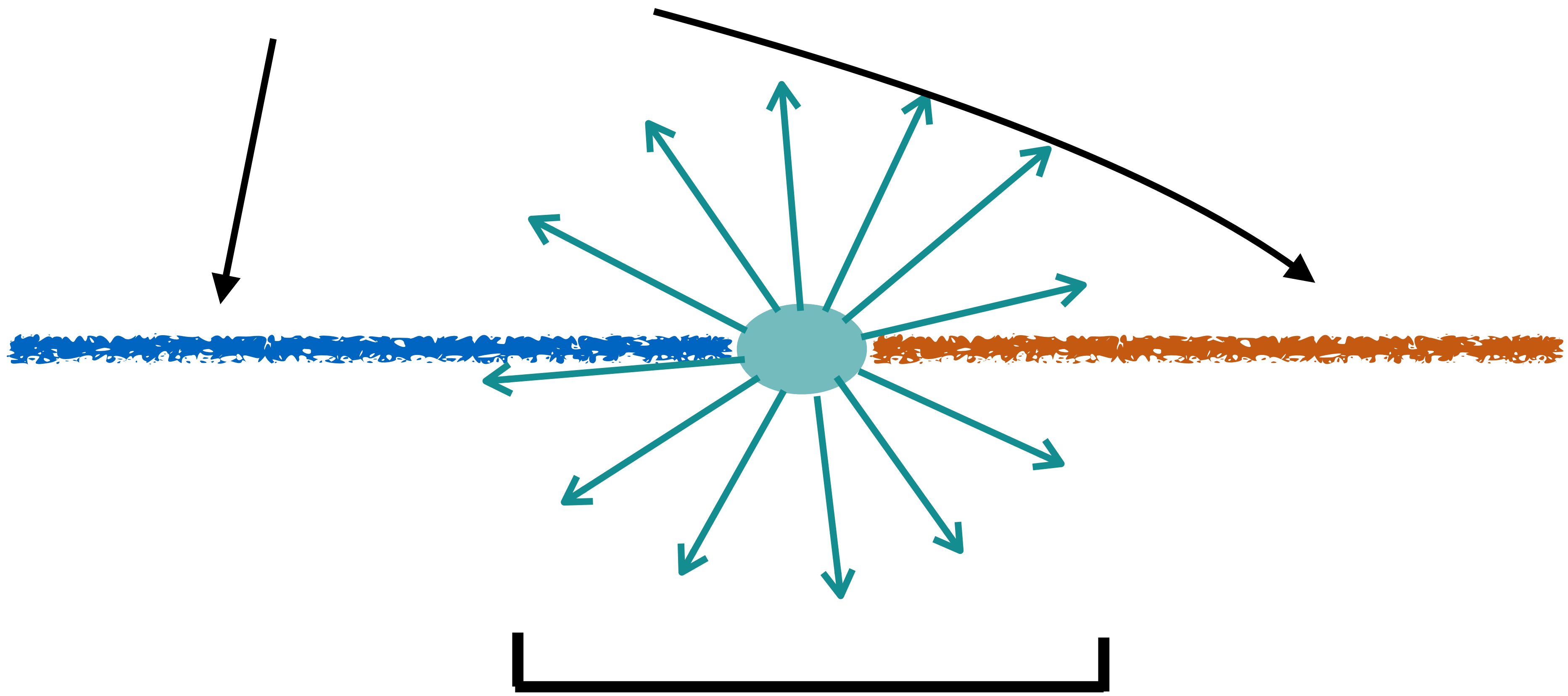

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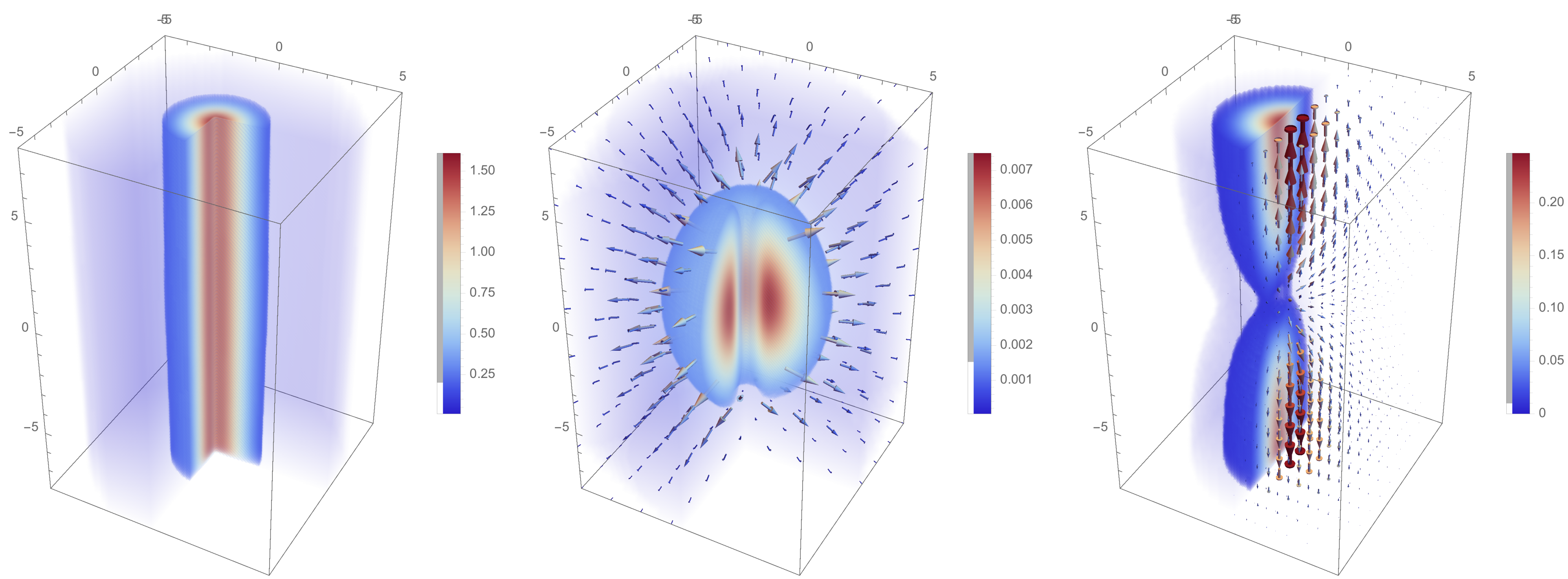
TeV scale phenomenon ! ➡ **can be produced at LHC !?**

Summary

- **Stable magnetic monopole exists in 2HDM.**
- Key symmetries:
  - $U(1)_a \;\Rightarrow$ **topological vortices**
  - $(\mathbb{Z}_2)_C \;\Rightarrow$ **monopole as topological kink**



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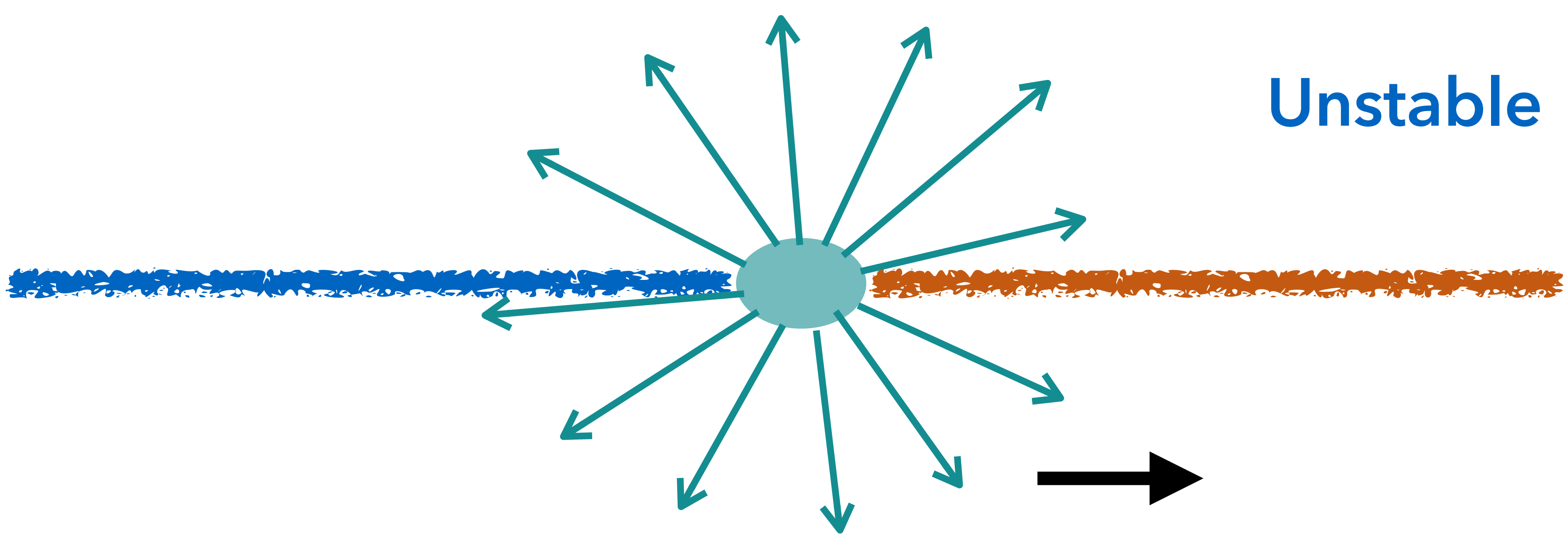
Future works:
- $\cancel{U(1)_a}\,,\quad \cancel{(\mathbb{Z}_2)_C} \;\Rightarrow$ How unstable?
- How is it produced in accelerators?

Backup Slides

Comments: $\cancel{(\mathbb{Z}_2)_C}$

- $(\mathbb{Z}_2)_C$ symmetry is **not exact** because of Yukawa couplings.
  - Quantum corrections break $(\mathbb{Z}_2)_C$ symmetry in the Higgs potential.
  - The tensions of Z-strings are not degenerate.
  - The monopole is pulled to the heavier string.



Comments: $\cancel{U(1)}_a$

[Eto, Kurachi, Nitta '18]

- When $U(1)_a$ symmetry is exact, NG boson appears
  (massless CP-odd Higgs)

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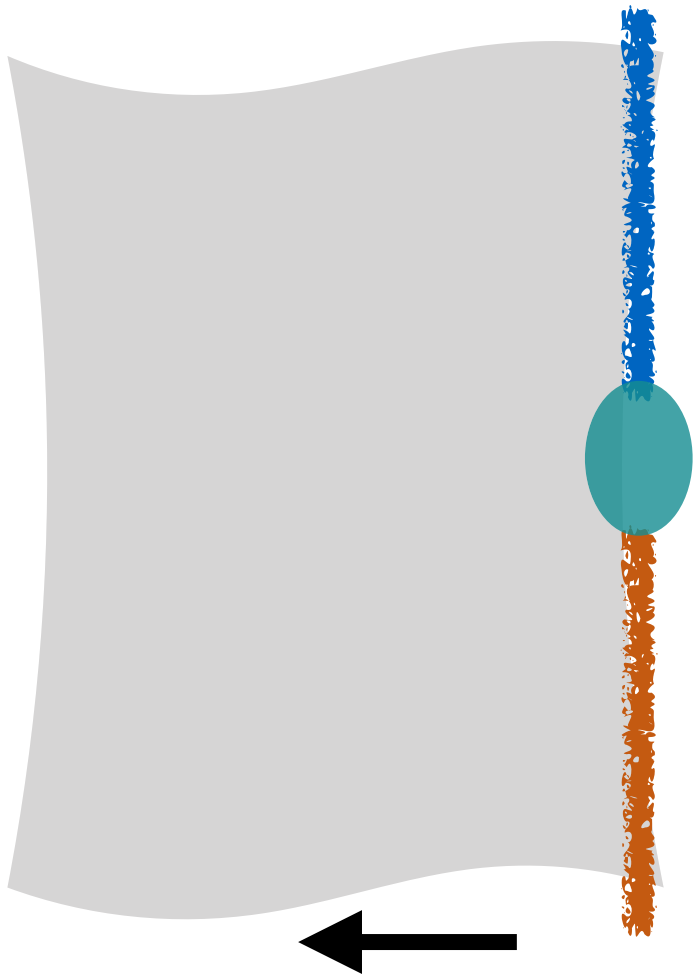
  $\longrightarrow$ wall (membrane) attaches



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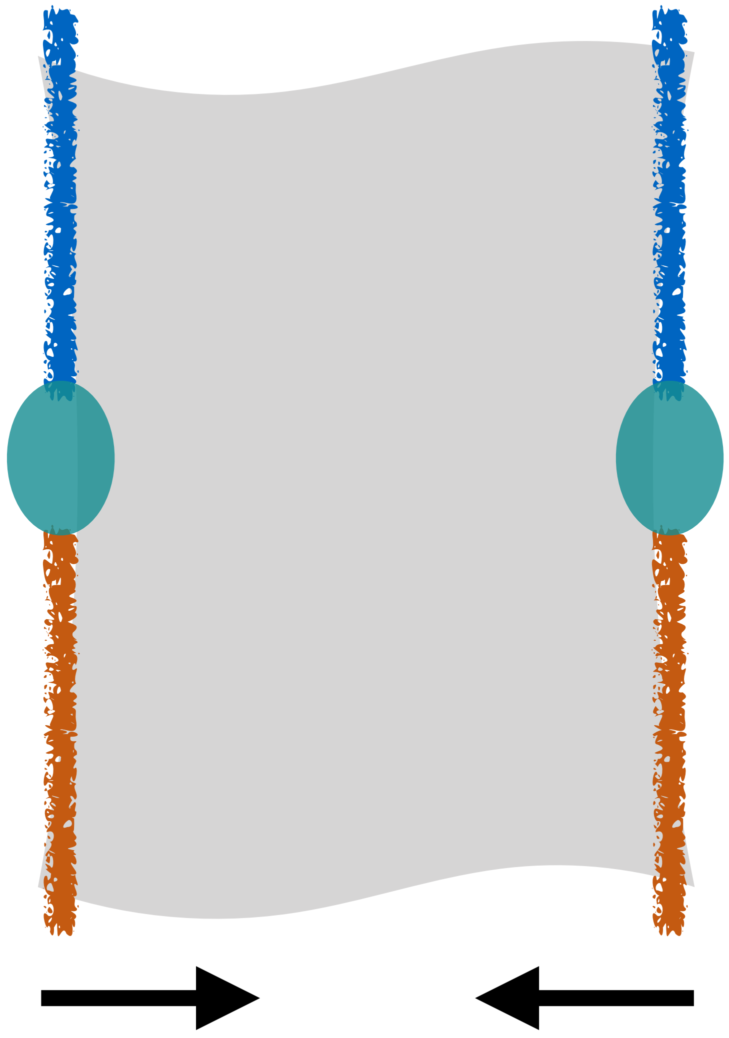

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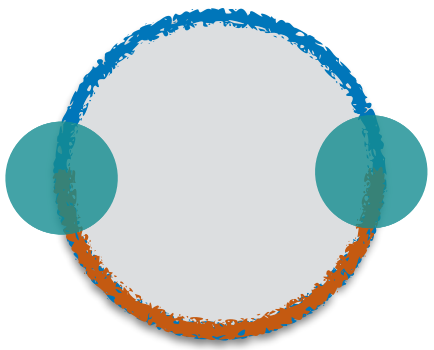
Disappear ?

or



``vorton'' ?

(superconducting string loop)

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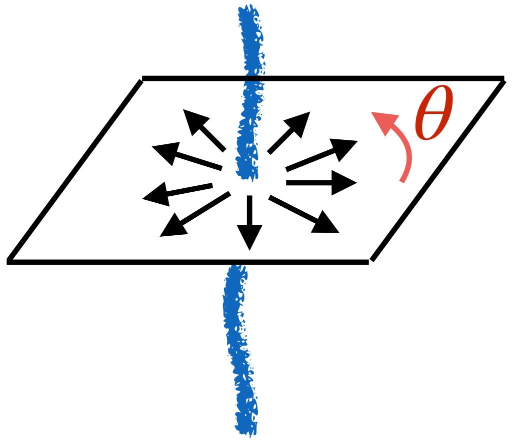
$\Rightarrow$ $\pi_1(\mathscr{M}) \simeq \pi_1(U(2)) = \mathbb{Z}$ **Topological vortex exist !**

- **Topological Z-string ((0,1)-string)**



2 x 2 matrix notation: $H \equiv \left(i\sigma_2\Phi_1^*,\, \Phi_2\right)$

$$H^{(0,1)} \sim v \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = v\, \boxed{e^{i\frac{\theta}{2}}}\, \boxed{e^{-i\frac{\theta}{2}\sigma_3}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

contribute to Z flux
(local symmetry)

$$Z_i^{(0,1)} \sim \frac{\cos\theta_W}{g} \frac{\epsilon_{3ij}\, x^j}{r^2}$$

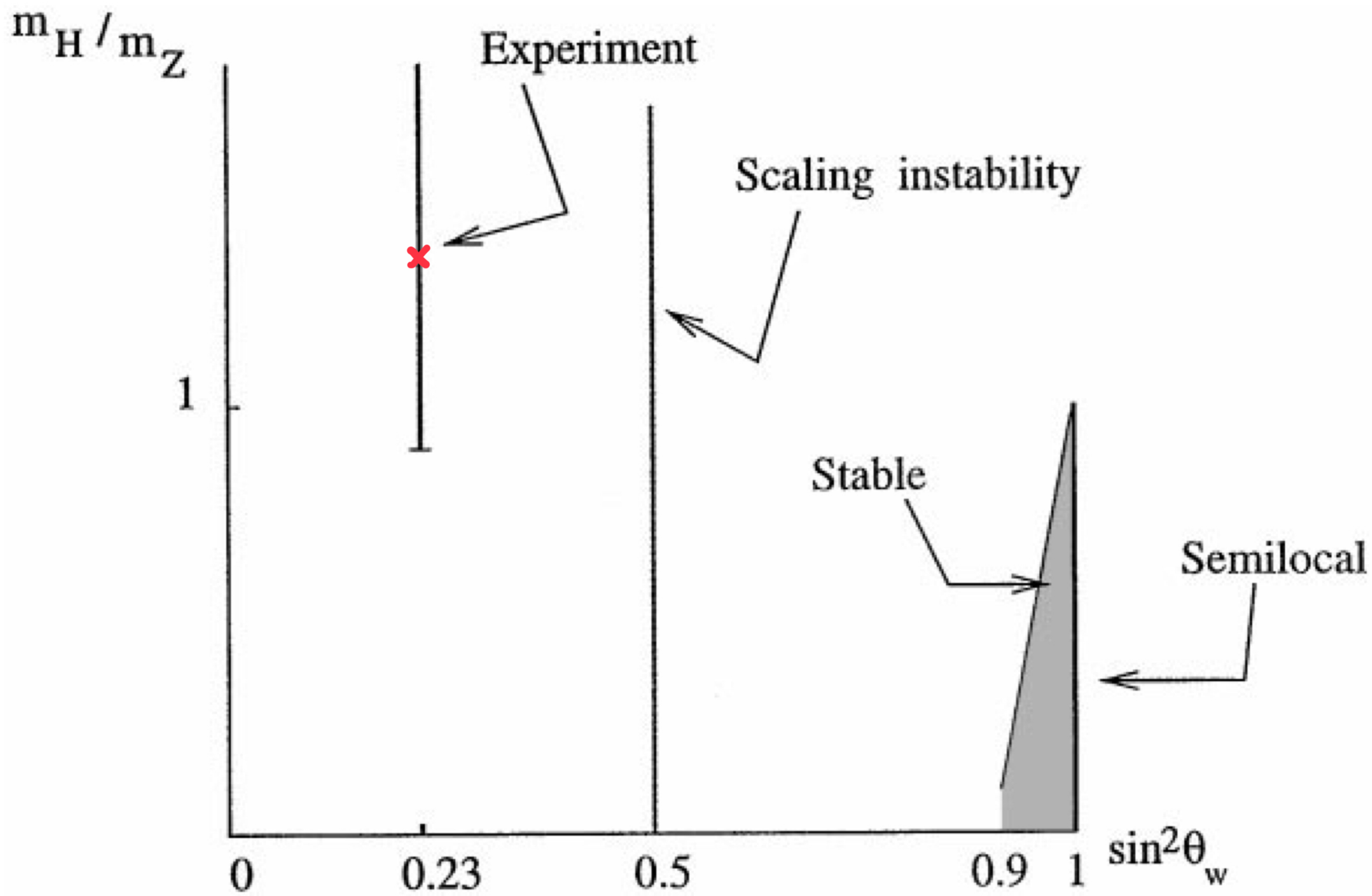
$U(1)_a$ **phase :** $-\pi/2 \sim \pi/2$
(global symmetry)

- confined **Z-flux** $\Phi_Z = \dfrac{2\pi}{g_Z}$

semi-local string

- global vortex $\longrightarrow$ **tension** $\sim \pi v^2 \log \Lambda_{IR}$

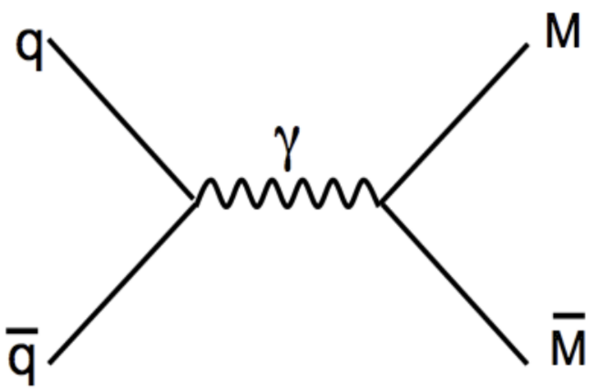
Stability of Z-string in SM



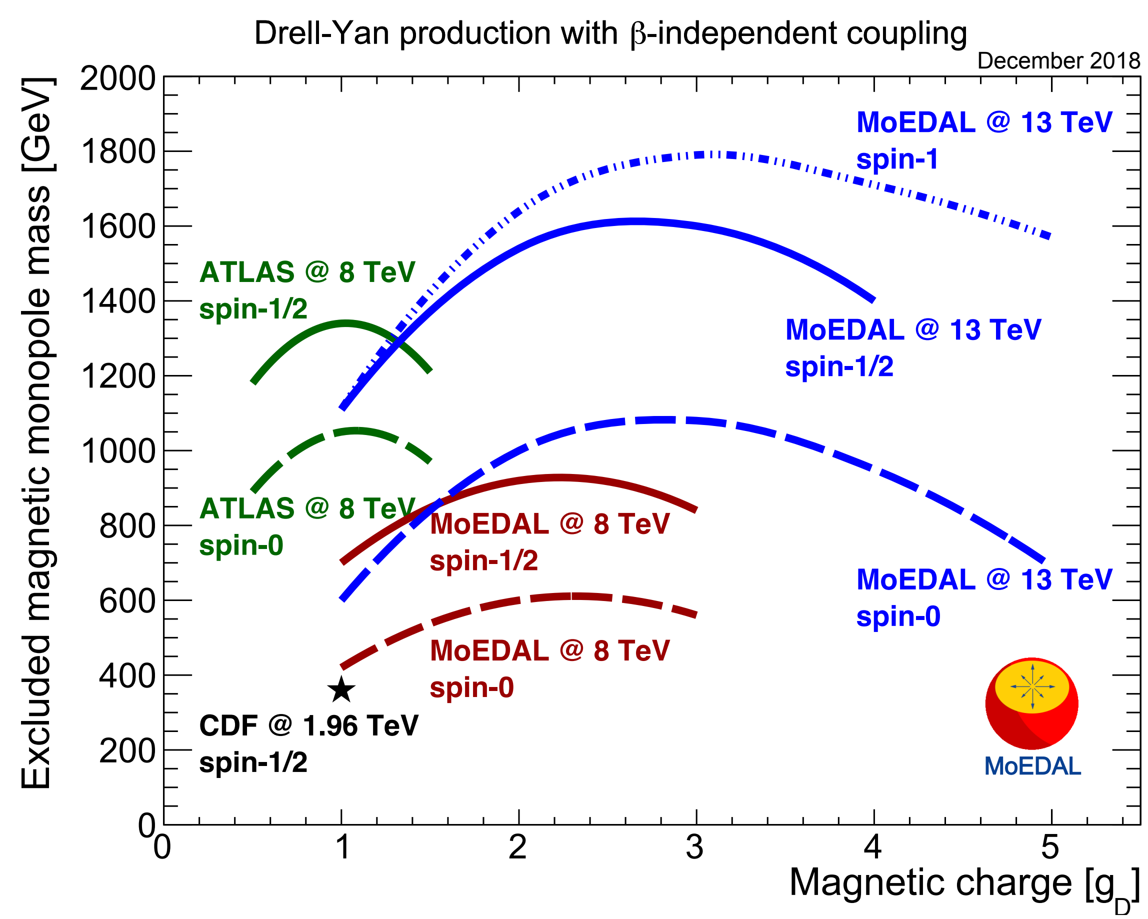
[Achucarro, Vachaspati, hep-ph/9904229]

Monopole production at colliders

- Conventional process

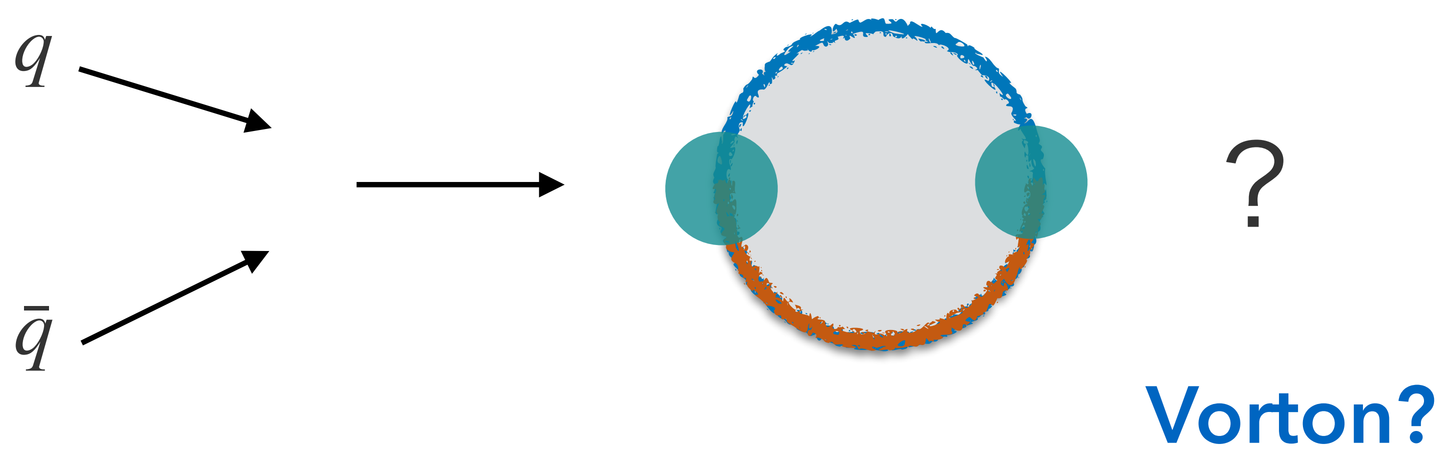


Drell Yan mechanism



From A. Santra's slide

- For our monopole,



If it decays, we can see it as a resonance ?

2HDM in Matrix Notation

$$V(\Phi_1,\Phi_2) = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right) + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$

$$+\beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \mathrm{h.c.}\right\}$$

$$m_{11}^2 = -m_1^2 - m_2^2, \quad m_{22}^2 = -m_1^2 + m_2^2, \quad m_{12} = m_3,$$

$$\beta_1 = 2(\alpha_1+\alpha_2+\alpha_3+\alpha_4), \quad \beta_2 = 2(\alpha_1+\alpha_2+\alpha_3-\alpha_4),$$

$$\beta_3 = 2(\alpha_1+\alpha_2-\alpha_3), \quad \beta_4 = 2(\alpha_3-\alpha_1), \quad \beta_5 = 2\alpha_5$$

$$V(\Phi_1,\Phi_2) = -m_1^2\,\mathrm{Tr}|H|^2 - m_2^2\,\mathrm{Tr}\left(|H|^2\sigma_3\right) - \left(m_3^2\det H + \mathrm{h.c.}\right)$$

$$+\alpha_1\,\mathrm{Tr}|H|^4 + \alpha_2\left(\mathrm{Tr}|H|^2\right)^2 + \alpha_3\,\mathrm{Tr}\left(|H|^2\sigma_3|H|^2\sigma_3\right)$$

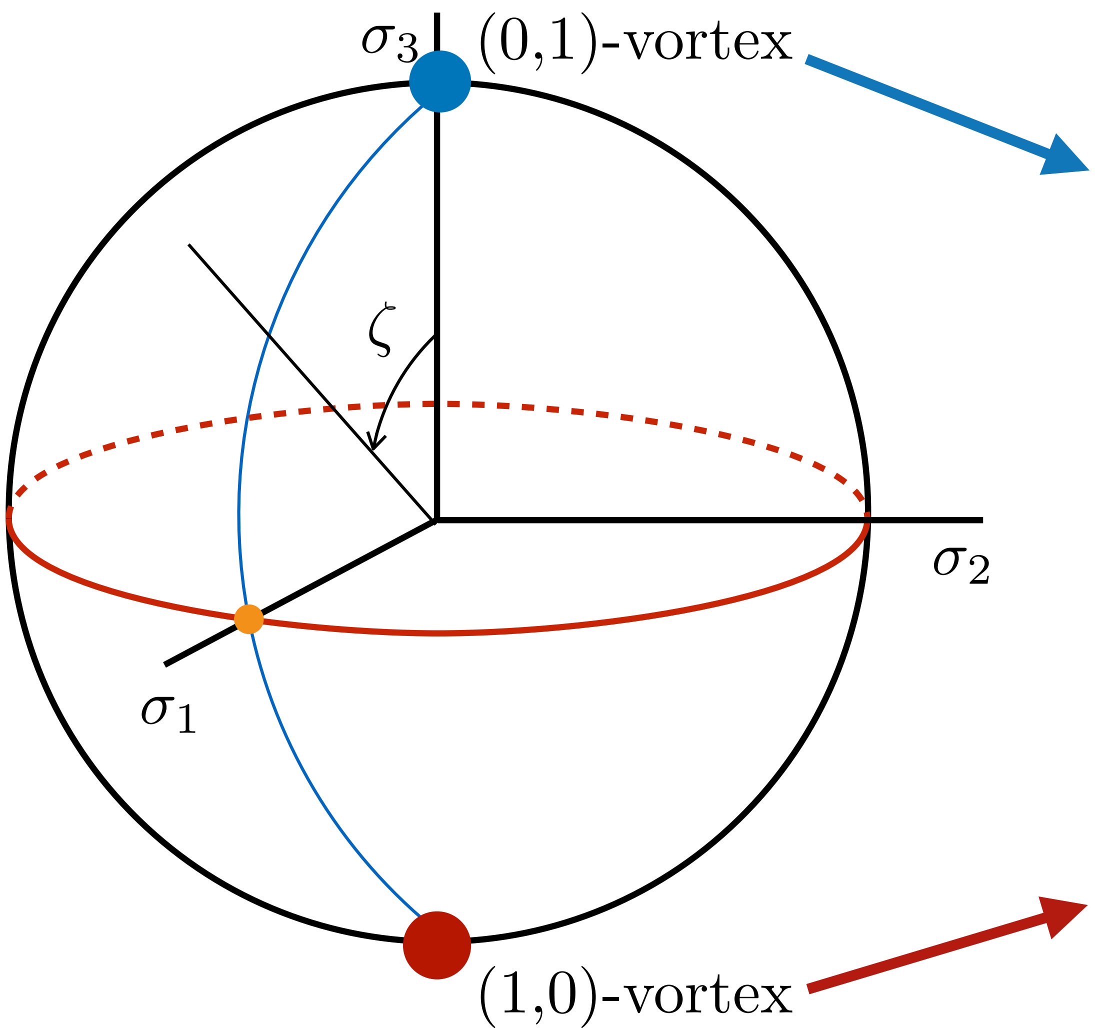
$$+\alpha_4\,\mathrm{Tr}\left(|H|^2\sigma_3|H|^2\right) + \left(\alpha_5\det H^2 + \mathrm{h.c.}\right)$$

$$|H|^2 \equiv H^\dagger H$$

Moduli space of vortices

- There are more topological vortices.
- Space of topological vortices = moduli space S^2

⟺ Each point on the moduli space S^2 corresponds to a vortex.



Two Z-strings :

$$H^{(0,1)} \sim v \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

Z flux: $\Phi_Z^{(0,1)} = \dfrac{2\pi}{g_Z}$

$$H^{(1,0)} \sim v \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix}$$

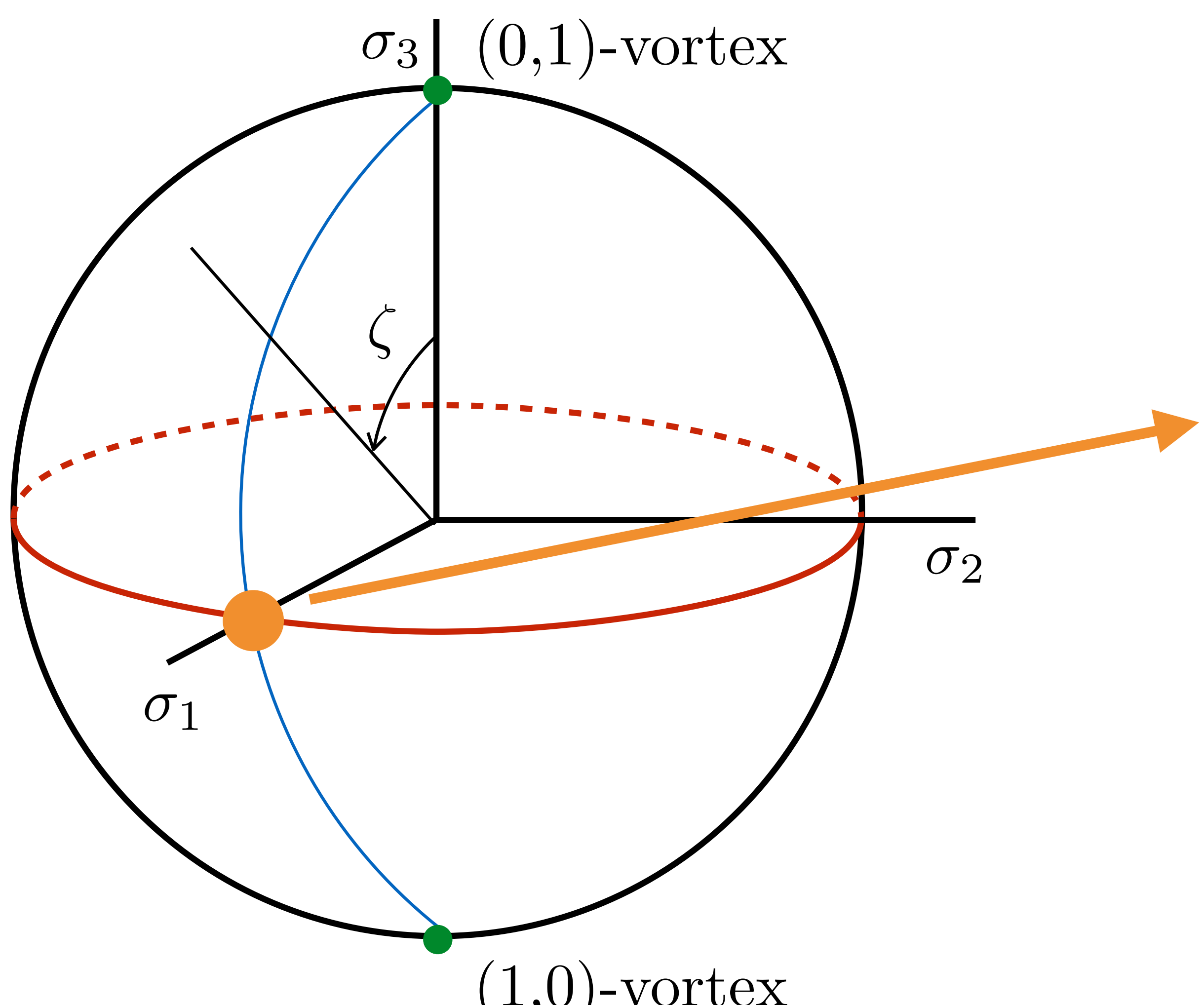
Z flux: $\Phi_Z^{(1,0)} = -\dfrac{2\pi}{g_Z}$

Moduli space of vortices

- There are more topological vortices.

- Space of topological vortices = moduli space S^2

⇔ Each point on the moduli space S^2 corresponds to a vortex.



W-string

$$H \sim v\, e^{\frac{i\theta}{2}}\, e^{\frac{i\theta}{2}\sigma_1}$$

W flux: $\Phi_{W^1} = \dfrac{2\pi}{g}$

Tensions of topological vortices

- Because of $U(1)_Y$, W string and Z strings have different tensions. (lifted moduli space)