



# **Baryogenesis and late-time lepton asymmetry via right-handed neutrino oscillations**

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Based on: S.E. and M. Shaposhnikov, PLB 771 (2017) 288-296

S.E., M. Shaposhnikov and I. Timiryasov, JHEP 1907 (2019) 077

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# Introduction

Observed phenomena beyond the SM;

- Neutrino oscillations
- Baryon Asymmetry of the Universe (BAU)
- Dark Matter (DM)

Seesaw mechanism  
Leptogenesis  
Sterile neutrino

$\nu$ MSM (neutrino Minimal Standard Model)  
[Asaka, Shaposhnikov ('05)]  
[Asaka, Blanchet, Shaposhnikov ('05)]

SM + three right-handed neutrinos  
with masses below electroweak  
scale

Three Generations of Matter (Fermions) spin 1/2

|         | I  | II   | III  |  |
|---------|--|--|--|--|
| mass    | 2.4 MeV  | 1.27 GeV   | 171.2 GeV  | 0  |
| charge  | 2/3  | 2/3  | 2/3  | 0  |
| name    | Left <b>u</b> Right<br>up  | Left <b>c</b> Right<br>charm   | Left <b>t</b> Right<br>top   | 0 <b>g</b><br>gluon                        |
| Quarks  | 4.8 MeV<br><b>~keV</b>   | 104 MeV<br>-1/3 <b>S</b> <b>~GeV</b>   | 1.2 GeV  | 0 <b><math>\gamma</math></b><br>photon     |
|         | Left <b><math>\nu_e</math></b> Right <b><math>N_1</math></b><br>electron neutrino sterile neutrino | Left <b><math>\nu_\mu</math></b> Right <b><math>N_2</math></b><br>muon neutrino sterile neutrino | Left <b><math>\nu_\tau</math></b> Right <b><math>N_3</math></b><br>tau neutrino sterile neutrino | 91.2 GeV <b>Z</b><br>weak force            |
| Leptons | 0.511 MeV<br>-1 <b>e</b>   | 105.7 MeV<br>-1 <b><math>\mu</math></b>  | 1.777 GeV<br>-1 <b><math>\tau</math></b>   | 80.4 GeV <b>W</b><br>weak force            |
|         |  |  |  | >114 GeV <b>H</b><br>Higgs boson<br>spin 0 |

Bosons (Forces) spin 1

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i\bar{\nu}_{R_I}\gamma^\mu\partial_\mu\nu_{R_I} - F_{\alpha I}\bar{L}_\alpha\tilde{\Phi}\nu_{R_I} - \frac{M_{IJ}}{2}\bar{\nu}_{R_I}^c\nu_{R_J} + \text{h.c.}$$

# Motivation

## 1. Parameter space for Baryogenesis

GeV-scale  $\nu_R$  are testable in ground experiments!

Which parameter space is suggested from the successful Baryogenesis?

$$n_B \equiv \frac{n_b - n_{\bar{b}}}{s} \quad \& \quad n_B^{\text{obs}} \simeq 8 \times 10^{-11}$$

Can be tested in near-future experiments?

## 2. Late-time Leptogenesis

Resonant production of sterile neutrino dark matter requires a large lepton asymmetry,  $10^5$  larger than BAU, at  $T \sim 100$  MeV.

Can GeV-scale  $\nu_R$  provide the asymmetry?

**Necessary to investigate the mechanism generating asymmetry via right-handed neutrino oscillations**

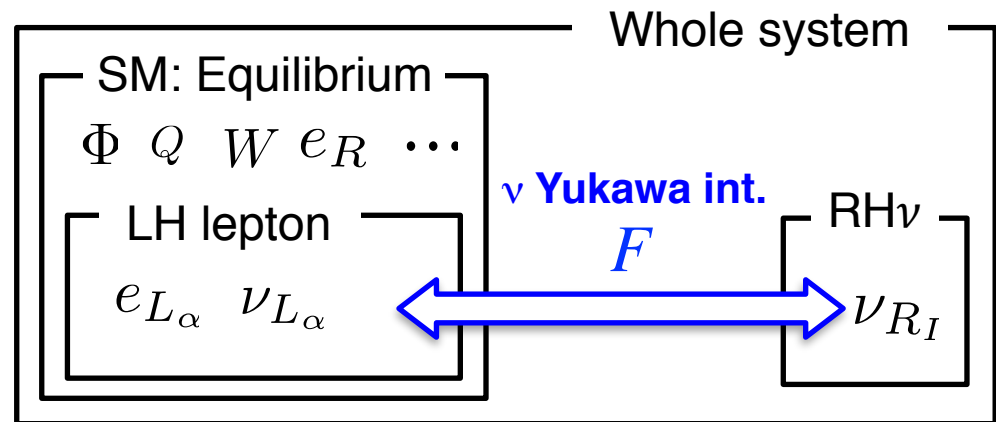
# Baryogenesis via $\nu_R$ oscillation [1]

## Mechanism

Yukawa coupling of  $\nu_{R_{2,3}}$

$$|F_{2,3}| \sim 10^{-7,8}$$

- **Out-of-equilibrium process**
- **CP violation**



$$\nu_{RI} \dots \dots \nu_{RJ}$$

**Coherent oscillations of  $\nu_R$**

$$e^{-i \left( \frac{M_I^2 - M_J^2}{2p} \right) dt}$$



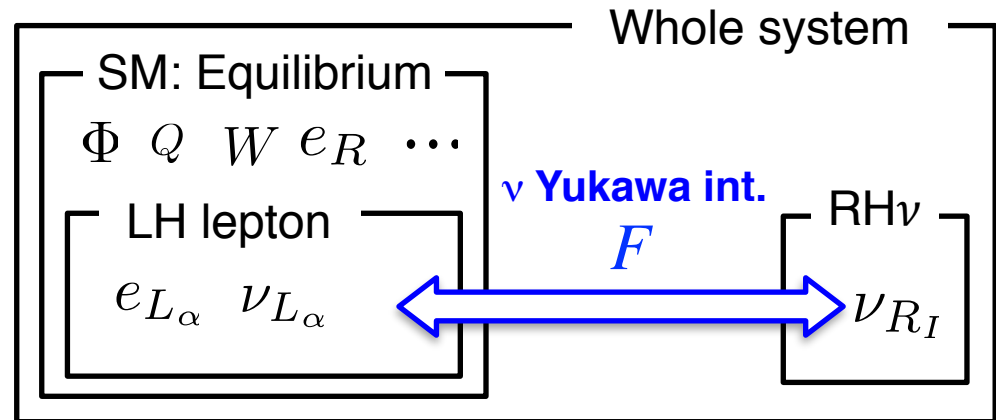
# Baryogenesis via $\nu_R$ oscillation [1]

## Mechanism

Yukawa coupling of  $\nu_{R_{2,3}}$

$$|F_{2,3}| \sim 10^{-7,8}$$

- **Out-of-equilibrium process**
- **CP violation**



$$L_\alpha \xrightarrow{F_{\alpha I}} \nu_{RI} \dots \nu_{RJ} \xrightarrow{F_{\beta J}} L_\beta$$

Coherent oscillations of  $\nu_R$

$$\Phi \quad e^{-i \left( \frac{M_I^2 - M_J^2}{2p} \right) dt} \quad \Phi$$

Evolution

$$L_\beta \xrightarrow{F_{\beta I}} \nu_{RI}$$

$\Phi$

$$\Gamma(L_\alpha \rightarrow L_\beta) \neq \Gamma(\bar{L}_\alpha \rightarrow \bar{L}_\beta)$$

$$\rightarrow n_{L_e} \neq 0, n_{L_\mu} \neq 0, n_{L_\tau} \neq 0$$

$$n_{L_e} + n_{L_\mu} + n_{L_\tau} = 0$$

$$n_L = \sum_\alpha n_{L_\alpha} \neq 0$$

$$n_B(T_{\text{SF}}) = -\frac{28}{79} n_L(T_{\text{SF}}) \quad \text{at } T_{\text{SF}} \simeq 130 \text{ GeV}$$

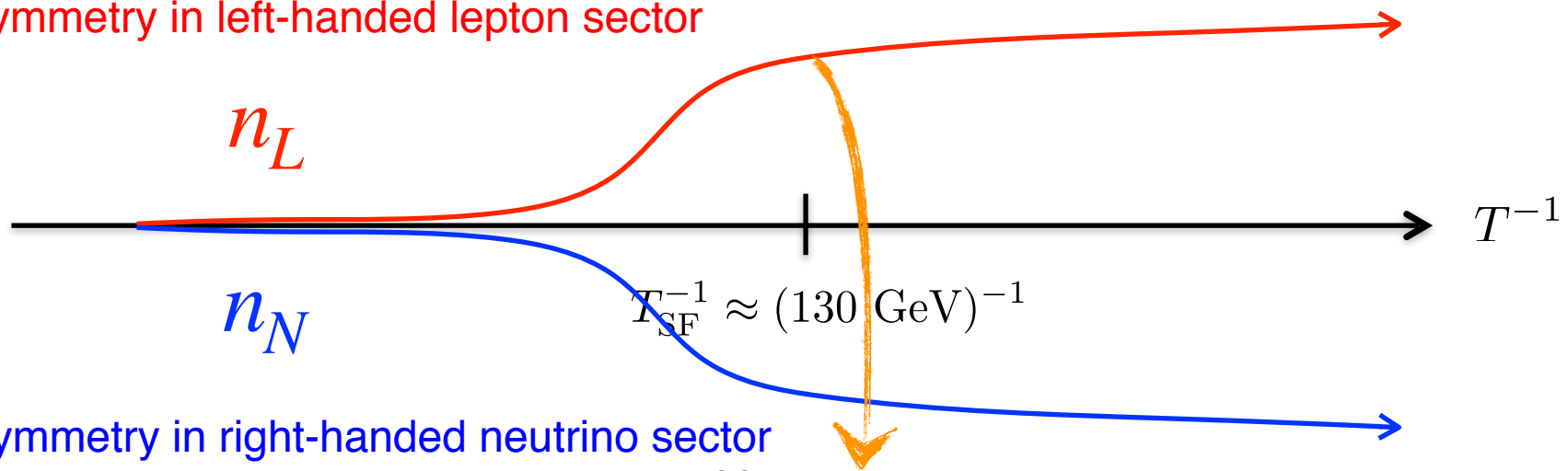
# Baryogenesis via $\nu_R$ oscillation [2]

## Key features

1. Effective lepton number conservation due to  $M \ll T$

$$n_L \approx -n_N$$

Asymmetry in left-handed lepton sector



Asymmetry in right-handed neutrino sector

$$n_B(T_{\text{SF}}) = -\frac{28}{79}n_L(T_{\text{SF}}) \text{ \& } n_B^{\text{obs}} \simeq 8 \times 10^{-11}$$

$n_L$  generation continues even after  $T_{\text{SF}}$  until  $\nu_R$  are thermalized

# Baryogenesis via $\nu_R$ oscillation [3]

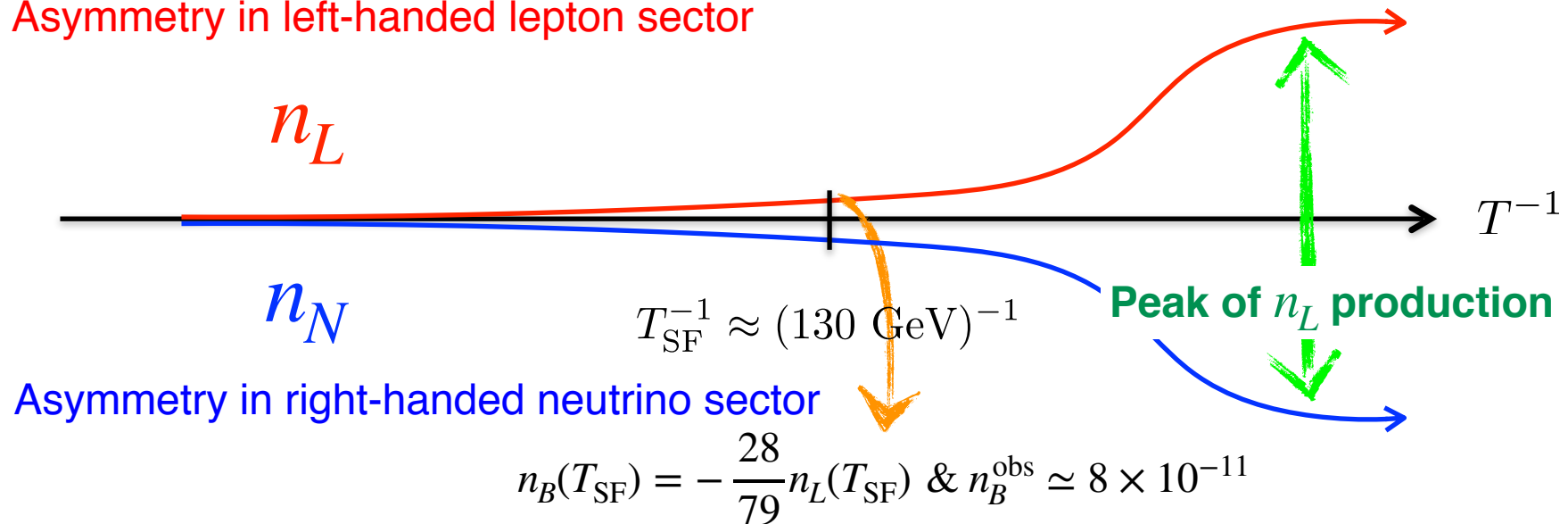
## Key features

### 2. Scale of asymmetry generation can be controlled

$$\int_{t_i}^t \frac{M_i^2 - M_j^2}{2p} dt' \approx 1 \quad \rightarrow \quad \text{Oscillation temperature: } T_{\text{osc}} \simeq (\Delta M M M_0)^{\frac{1}{3}}$$

$$M = \frac{M_3 + M_2}{2}, \quad \Delta M = \frac{M_3 - M_2}{2}, \quad M_0 \simeq 7 \times 10^{17} \text{ GeV}$$

Asymmetry in left-handed lepton sector



Possibility to explain BAU and late-time asymmetry simultaneously



# Kinetic equations for Baryogenesis

To treat coherent and incoherent processes at the same time

Matrix of density :  $\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$  [Diagonal elements : occupation numbers  
Off-diagonal elements : correlations]

For  $\nu_R$  and anti- $\nu_R$  (2x2 matrix)

$$\frac{d\rho_N}{dt} = -i [H_N, \rho_N] - \frac{1}{2} \{\Gamma_N, \rho_N - \rho_N^{\text{eq}}\} - \frac{1}{2} \sum_{\alpha} [\tilde{\Gamma}_N^{\alpha} \Delta\rho_{\nu_{\alpha}}]$$

Oscillation

Production and Destruction

Communication (back-reaction)

$$\frac{d\rho_{\bar{N}}}{dt} = -i [H_N^*, \rho_{\bar{N}}] - \frac{1}{2} \{\Gamma_N^*, \rho_{\bar{N}} - \rho_N^{\text{eq}}\} + \frac{1}{2} \sum_{\alpha} [(\tilde{\Gamma}_N^{\alpha})^* \Delta\rho_{\nu_{\alpha}}]$$

For lepton asymmetries  $\Delta\rho_{\nu_{\alpha}}$  ( $\alpha = e, \nu, \tau$ )

$$\frac{d\Delta\rho_{\nu_{\alpha}}}{dt} = -\Gamma_{\nu_{\alpha}} \Delta\rho_{\nu_{\alpha}} + \text{Tr}[\tilde{\Gamma}_{\nu_{\alpha}} \rho_{\bar{N}}] - \text{Tr}[\tilde{\Gamma}_{\nu_{\alpha}}^* \rho_N]$$

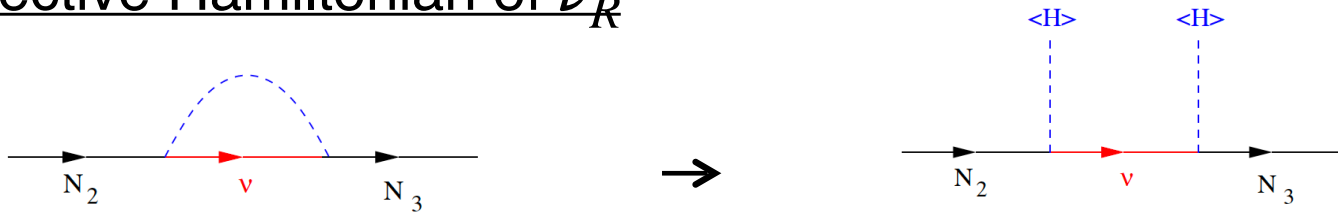
$$H_N = H_0 + H_I$$

$$H_I, \Gamma \propto F^2$$

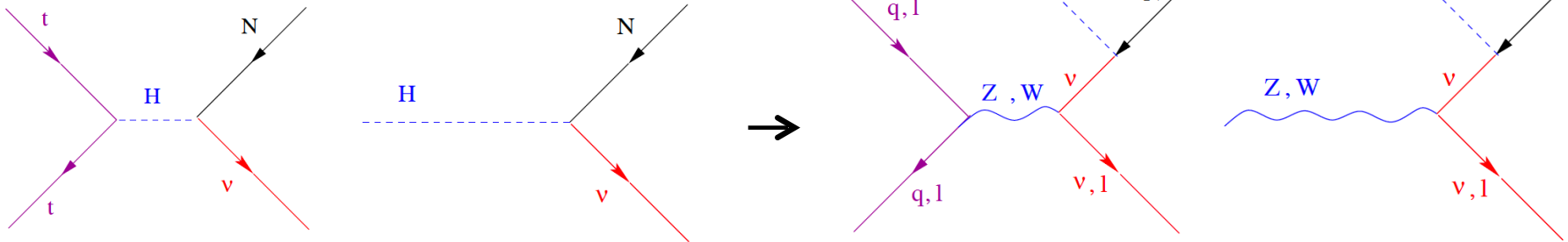
# Our work [1]

Derive kinetic equations in the Higgs phase ( $T < T_{EW}$ )

For effective Hamiltonian of  $\nu_R$



For interactions of  $\nu_R$



Lepton number conserving and violating contributions

$$\Gamma = \Gamma_+ + \Gamma_-$$

$$\Gamma_+, H_+ \propto T \quad ; \text{Lepton \# conserving}$$

$$H = H_+ + H_-$$

$$\Gamma_-, H_- \propto (M/T)^2 T \quad ; \text{Lepton \# violating}$$

# Our work [2]

Impacts on our purposes

## 1. Parameter space for Baryogenesis

Baryogenesis needs information in the Higgs phase

$$T_{\text{SF}} \simeq 130 \text{ GeV} < T_{\text{EW}} \simeq 160 \text{ GeV} \quad \begin{array}{l} \text{[D'Onofrio, Rummukainen, Tranberg ('14)]} \\ \text{[D'Onofrio, Rummukainen ('16)]} \end{array}$$

Perform comprehensive numerical analysis of Baryogenesis taking LNV corrections, plasma neutrality and exact sphaleron treatment into account additionally

[Ghiglieri, Laine ('16, '17)] [Drewes, Garbrecht, Gueter, Klaric ('16)] [SE, Shaposhnikov, Timiryasov ('17)]  
 [Hernandez, Kekic, Lopez-Pavon, Racker, Salvado ('16)] [Hambye, Teresi ('17)]  
 [Antusch, Cazzato, Drewes, Fischer, Garbrecht, Gueter, Klaric ('17)] ...

## 2. Late-time Leptogenesis

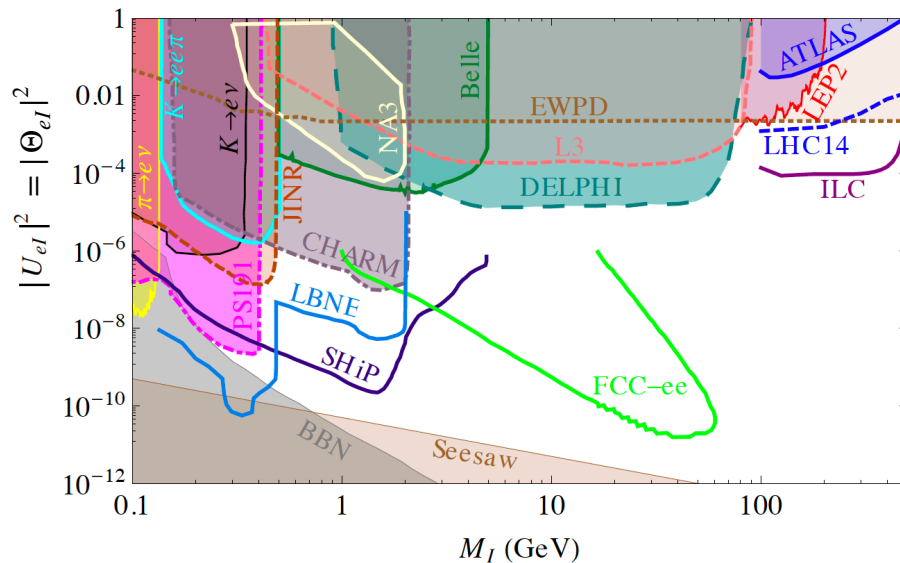
Study the asymmetry production at lower temperatures properly

# Parameter space of Baryogenesis

## Constraints

$$W^+ \rightarrow e^+ \Theta_{eI} N_I \approx \nu_{R,I}$$

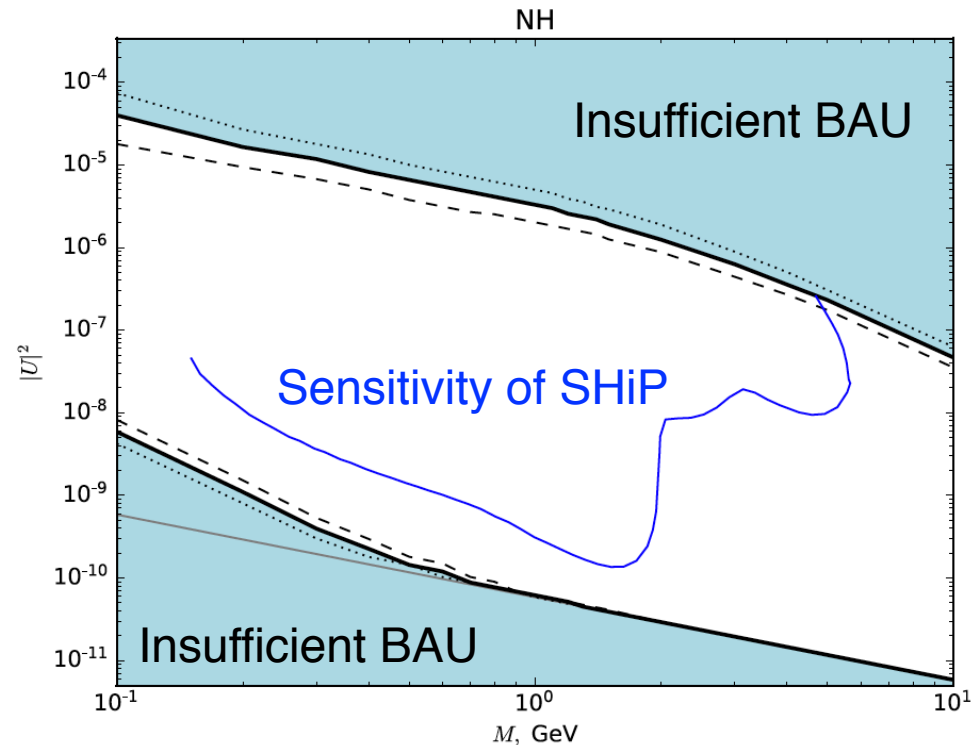
$$\Theta_{\alpha I} \equiv \frac{v F_{\alpha I}}{M_I}$$



[The SHiP collaboration ('15)]

## Numerical result

$$\text{Typical mixing of } \nu_R: U^2 \equiv \sum_{\alpha I} |\Theta_{\alpha I}|^2$$



[SE, Shaposhnikov, Timiryasov ('18)]

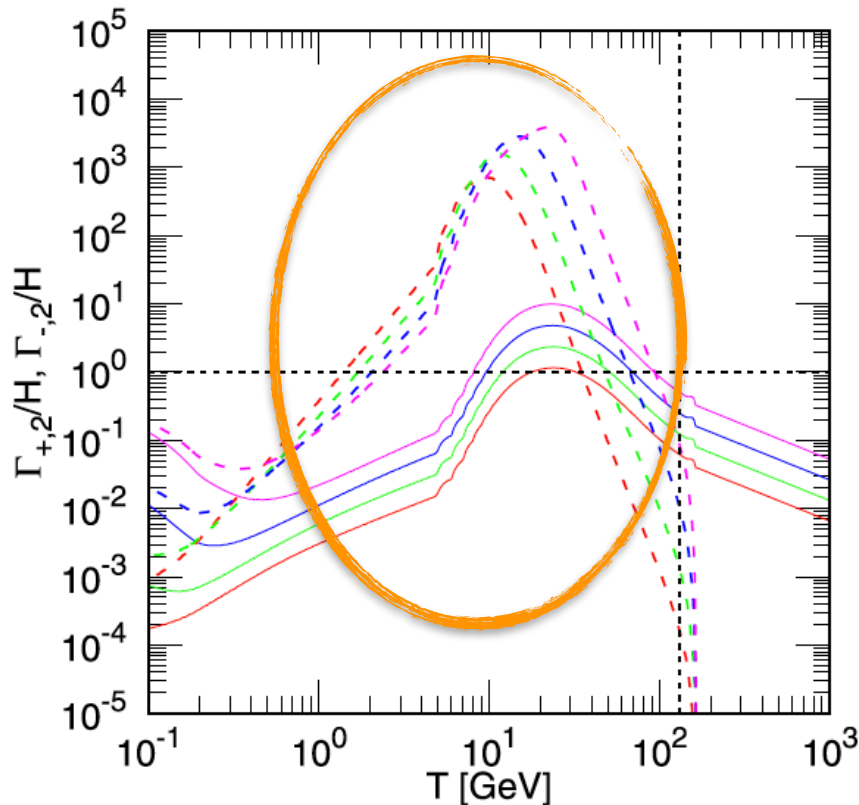
For GeV-scale mass the near-future experiment will probe the wide region in the suggested parameter space!

# Issue of late-time leptogenesis

## Thermalization

Interaction rate:  $\Gamma_N = \Gamma_+ + \Gamma_-$       $\Gamma_+$  ; Lepton number conserving rate  
 $\Gamma_-$  ; Lepton number violating rate

Largest eigenvalues of  $\Gamma_+$  (solid lines) and  $\Gamma_-$  (dashed lines)



—  $M = 0.5$  GeV  
 —  $M = 1.0$  GeV  
 —  $M = 2.0$  GeV  
 —  $M = 4.0$  GeV

$$F_{2,3} \simeq F_{2,3}^{\min}$$

$\nu_R$  are thermalized

Any asymmetries generated before are washed out...

# Protection by conservation law

Combinations of leptonic asymmetries for  $\nu_R$  and active flavors

[Shaposhnikov ('08)]

$$L_{\pm} \equiv n_N \mp n_L$$

When total lepton number is conserved  $n_L \approx -n_N$

$$\rightarrow L_+ \approx -2n_L, L_- \approx 0$$

From the derived kinetic equations

$$\begin{array}{l} \frac{dL_+}{dt} \propto \gamma_+ \quad ; \text{ Lepton \# conserving} \\ \frac{dL_-}{dt} \propto \gamma_- \quad ; \text{ Lepton \# violating} \end{array} \quad \left[ \begin{array}{l} \Gamma_N = \Gamma_+ + \Gamma_- \\ \gamma_+ \sim \Gamma_+/F^2 \\ \gamma_- \sim \Gamma_-/F^2 \end{array} \right]$$

If  $\Gamma_+$  ( $\Gamma_-$ ) doesn't come into equilibrium,  $L_+$  ( $L_-$ ) is a **conserved number**

**Generated asymmetries associated with  $L_+$  ( $L_-$ ) can potentially be protected from the wash-out down to lower temperatures!**

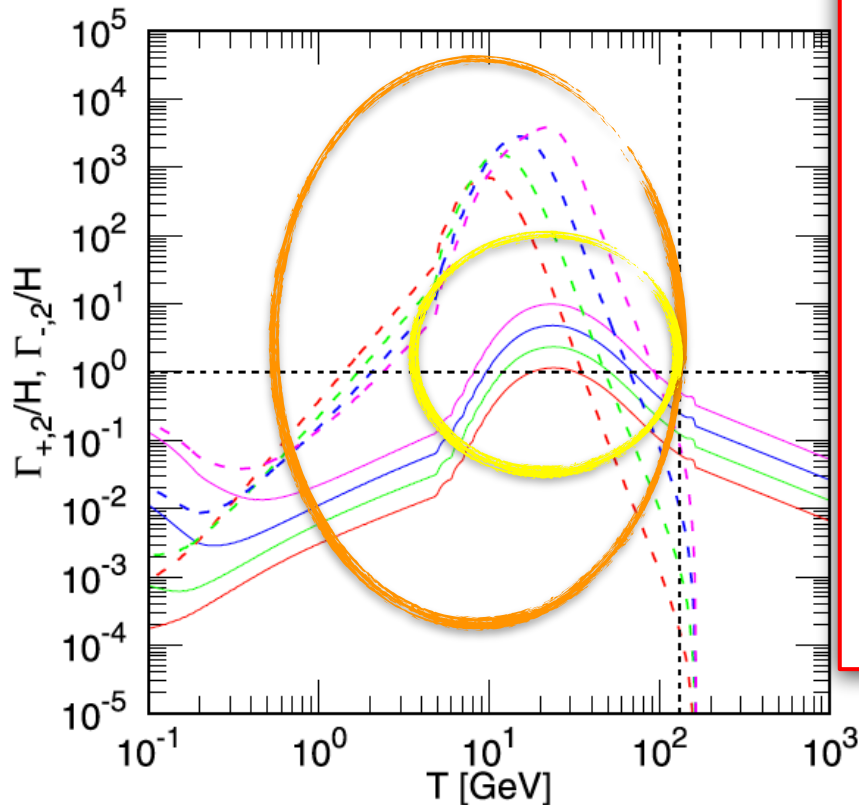
# Survival from wash-out

$$\Gamma_-/H \gg 1$$

$\nu_R$  must be thermalized

No conservation of  $L_-$

Largest eigenvalues of  $\Gamma_+$  (solid)



$$\Gamma_+/H \lesssim \mathcal{O}(1) \text{ for } M \lesssim 2 \text{ GeV}$$

**Effective conservation of  $L_+$**

Just before thermal-in

$$L_+ = n_N - n_L \approx -2n_L$$

**This conservation law can potentially solve the issue in the resonant DM production!**

before are washed out...

# Summary

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Taking into account recent improvements in the Baryogenesis we carried out the comprehensive numerical analysis. **The results show that near future experiments would probe origins of neutrino masses and baryon asymmetry of the universe.**

To extend the kinetic equations into the Higgs phase enables to study the late-time Leptogenesis in details. **The asymmetry protection due to the conservation law could potentially encourage the resonant production of sterile neutrino dark matter.** The numerical computation is now in progress.

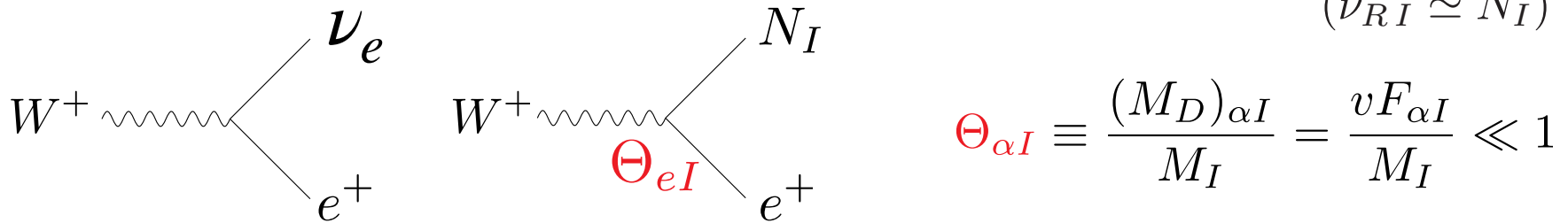


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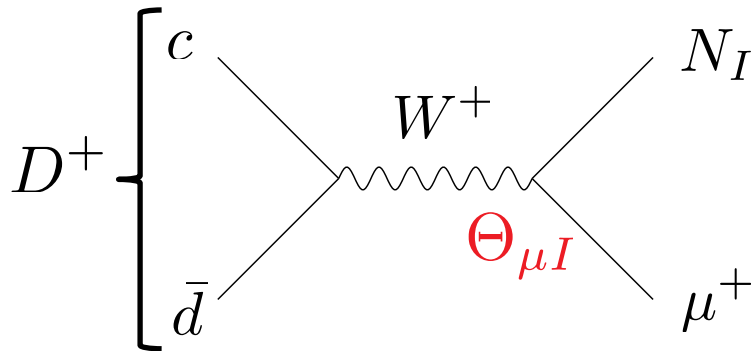
**Backup slides**

# Search for GeV-scale RH $\nu$

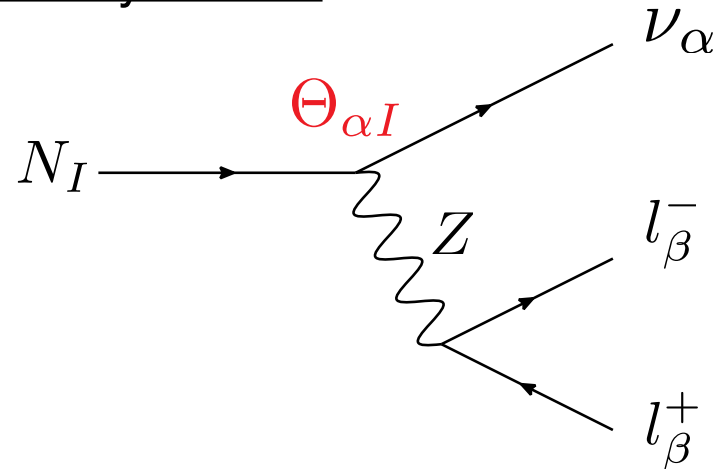
**Weak interaction through neutrino mixing;**  $\nu_{L\alpha} = U_{\alpha i}\nu_i + \Theta_{\alpha I}N_I^c$   
 ( $\nu_{RI} \simeq N_I$ )



## Production from meson decays



## Decay of RH $\nu$



$$N \rightarrow \nu\nu\bar{\nu}, N \rightarrow \nu l\bar{l}, N \rightarrow l^+ M^- \dots$$

# Yukawa coupling constants

In the minimal (two RHvs) case  $F = (i/\langle\Phi\rangle)U D_\nu^{\frac{1}{2}} \Omega D_N^{\frac{1}{2}}$  (3x2 matrix)

[Casas, Ibarra ('01)]

$U$ : PMNS matrix;  $\theta_{12}, \theta_{13}, \theta_{23}$ , CP phases

$$D_\nu^{\frac{1}{2}} = \text{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$$

$$D_N^{\frac{1}{2}} = \text{diag}(\sqrt{M_1}, \sqrt{M_2}) = \text{diag}(\sqrt{\bar{M} - \Delta M}, \sqrt{\bar{M} + \Delta M})$$

$\Omega$ : A orthogonal matrix; A complex mixing angle  $\omega \rightarrow \text{Re}\omega + i\text{Im}\omega$

$$\Omega = \begin{pmatrix} 0 & 0 \\ \cos \omega & \sin \omega \\ -\xi \sin \omega & \xi \cos \omega \end{pmatrix}_{\text{(NH)}}$$

$$\cos(i\text{Im}\omega) = \cosh(\text{Im}\omega) = (e^{\text{Im}\omega} + e^{-\text{Im}\omega})/2$$

$$\sin(i\text{Im}\omega) = i \sinh(\text{Im}\omega) = i(e^{\text{Im}\omega} - e^{-\text{Im}\omega})/2$$

For large  $|\text{Im}\omega|$   $F \propto \exp(|\text{Im}\omega|)$ , BUT neutrino oscillation data can be always explained!

$$\text{cf.) } \hat{M}_\nu = -v^2 F M_M^{-1} F^T$$

# H and $\Gamma$

## Effective Hamiltonian

$$H_N = H_0 + H_I,$$

$$H_0 = -\frac{\Delta MM}{E_N} \sigma_1$$

$$H_I = h_+ \sum_{\alpha} Y_{+,\alpha}^N + h_- \sum_{\alpha} Y_{-,\alpha}^N,$$

## Production and back-reaction rates

$$\Gamma_N = \gamma_+ \sum_{\alpha} Y_{+,\alpha}^N + \gamma_- \sum_{\alpha} Y_{-,\alpha}^N$$

$$\tilde{\Gamma}_N^{\alpha} = -\gamma_+ Y_{+,\alpha}^N + \gamma_- Y_{-,\alpha}^N$$

$$\Gamma_{\nu\alpha} = (\gamma_+ + \gamma_-) \sum_I h_{\alpha I} h_{\alpha I}^*$$

$$\tilde{\Gamma}_{\nu\alpha} = -\gamma_+ Y_{+,\alpha}^{\nu} + \gamma_- Y_{-,\alpha}^{\nu}$$

## Coefficients

$$h_+ = \frac{2\langle\Phi\rangle^2 E_{\nu} (E_N + k)(E_N + E_{\nu})}{k E_N (4(E_N + E_{\nu})^2 + \gamma_{\nu}^2)},$$

$$h_- = \frac{2\langle\Phi\rangle^2 E_{\nu} (E_N - k)(E_N - E_{\nu})}{k E_N (4(E_N - E_{\nu})^2 + \gamma_{\nu}^2)},$$

$$\gamma_+ = \frac{2\langle\Phi\rangle^2 E_{\nu} (E_N + k) \gamma_{\nu}}{k E_N (4(E_N + E_{\nu})^2 + \gamma_{\nu}^2)},$$

$$\gamma_- = \frac{2\langle\Phi\rangle^2 E_{\nu} (E_N - k) \gamma_{\nu}}{k E_N (4(E_N - E_{\nu})^2 + \gamma_{\nu}^2)},$$

$$E_N = \sqrt{k^2 + M^2} \quad E_{\nu} = k - b_L$$

## Matrices of yukawa couplings

$$Y_{+,\alpha}^N = \begin{pmatrix} h_{\alpha 3} h_{\alpha 3}^* & -h_{\alpha 3} h_{\alpha 2}^* \\ -h_{\alpha 2} h_{\alpha 3}^* & h_{\alpha 2} h_{\alpha 2}^* \end{pmatrix},$$

$$Y_{-,\alpha}^N = \begin{pmatrix} h_{\alpha 2} h_{\alpha 2}^* & -h_{\alpha 3} h_{\alpha 2}^* \\ -h_{\alpha 2} h_{\alpha 3}^* & h_{\alpha 3} h_{\alpha 3}^* \end{pmatrix},$$

$$Y_{+,\alpha}^{\nu} = \begin{pmatrix} h_{\alpha 3} h_{\alpha 3}^* & -h_{\alpha 2} h_{\alpha 3}^* \\ -h_{\alpha 3} h_{\alpha 2}^* & h_{\alpha 2} h_{\alpha 2}^* \end{pmatrix},$$

$$Y_{-,\alpha}^{\nu} = \begin{pmatrix} h_{\alpha 2} h_{\alpha 2}^* & -h_{\alpha 2} h_{\alpha 3}^* \\ -h_{\alpha 3} h_{\alpha 2}^* & h_{\alpha 3} h_{\alpha 3}^* \end{pmatrix},$$

# H and $\Gamma$

## Effective Hamiltonian

$$H_N = H_0 + H_I,$$

$$H_0 = -\frac{\Delta MM}{E_N} \sigma_1$$

$$H_I = h_+ \sum_{\alpha} Y_{+,\alpha}^N + h_- \sum_{\alpha} Y_{-,\alpha}^N,$$

## Production and b

$$\Gamma_N = \gamma_+ \sum_{\alpha} Y_{+,\alpha}^N + \gamma_- \sum_{\alpha} Y_{-,\alpha}^N$$

$$\tilde{\Gamma}_N^{\alpha} = -\gamma_+ Y_{+,\alpha}^N + \gamma_- Y_{-,\alpha}^N$$

## Coefficients

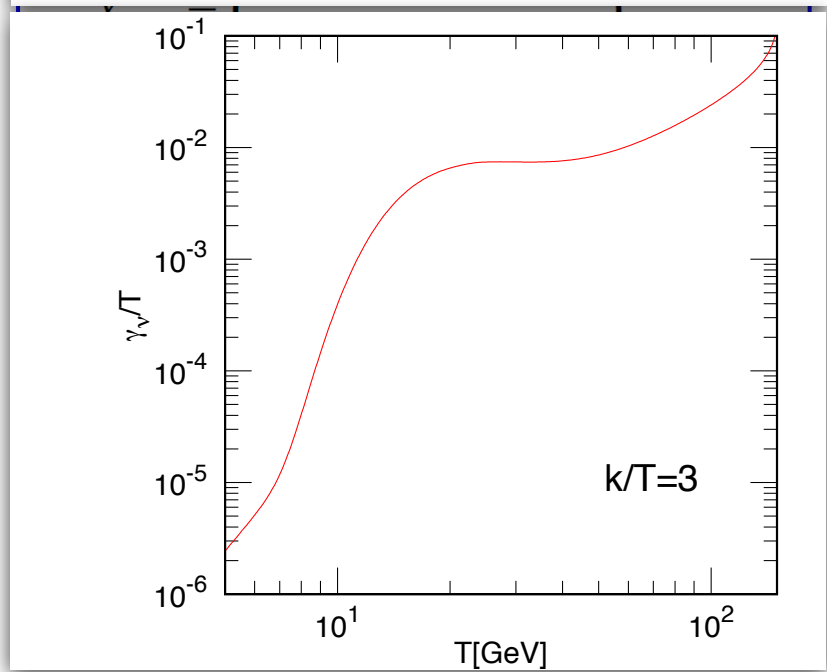
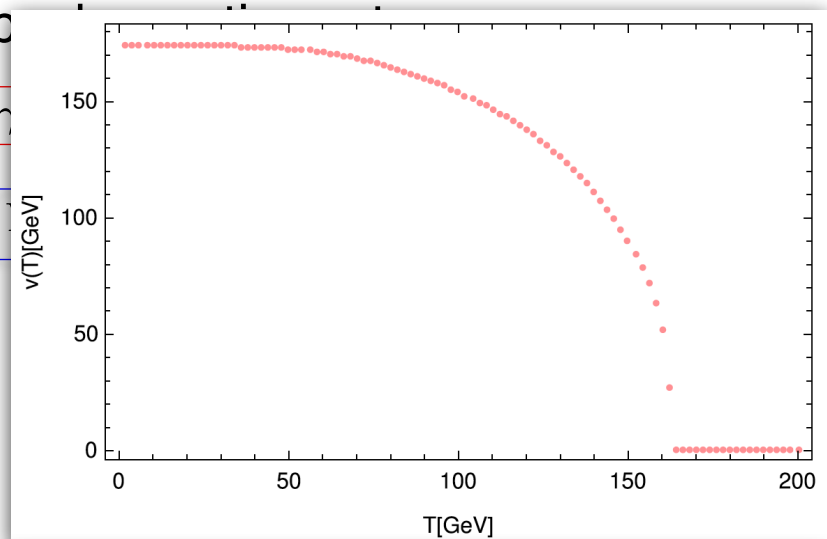
$$h_+ = \frac{2\langle\Phi\rangle^2 E_{\nu}(E_N + k)(E_N + E_{\nu})}{kE_N(4(E_N + E_{\nu})^2 + \gamma_{\nu}^2)},$$

$$h_- = \frac{2\langle\Phi\rangle^2 E_{\nu}(E_N - k)(E_N - E_{\nu})}{kE_N(4(E_N - E_{\nu})^2 + \gamma_{\nu}^2)},$$

$$\gamma_+ = \frac{2\langle\Phi\rangle^2 E_{\nu}(E_N + k)\gamma_{\nu}}{kE_N(4(E_N + E_{\nu})^2 + \gamma_{\nu}^2)},$$

$$\gamma_- = \frac{2\langle\Phi\rangle^2 E_{\nu}(E_N - k)\gamma_{\nu}}{kE_N(4(E_N - E_{\nu})^2 + \gamma_{\nu}^2)},$$

$$E_N = \sqrt{k^2 + M^2} \quad E_{\nu} = k - b_L$$



# H and $\Gamma$

## Higgs phase

$$h_+ = \mathcal{K}(m_h) \frac{T^2}{8k} + \frac{2\langle\Phi\rangle^2 E_\nu (E_N + k)(E_N + E_\nu)}{kE_N(4(E_N + E_\nu)^2 + \gamma_\nu^2)},$$

$$\gamma_+ = \gamma_+^{\text{direct}} + \frac{2\langle\Phi\rangle^2 E_\nu (E_N + k)\gamma_\nu}{kE_N(4(E_N + E_\nu)^2 + \gamma_\nu^2)},$$

$$\gamma_+^{\text{direct}} = \mathcal{K}(m_h) \frac{1}{E_N} \text{Im} \Pi_R + \gamma_{ph},$$

$$\mathcal{K}(m_h) = \frac{3}{\pi^2 T^3} \int_0^\infty dp p^2 f_b(E_h)(1 + f_b(E_h)),$$

$$\gamma_{ph} = \frac{1}{E_N} \frac{m_h^2 T}{32\pi k} \ln \left\{ \frac{1 + e^{-\frac{m_h^2}{4kT}}}{1 - e^{-\frac{1}{T}(k + \frac{m_h^2}{4k})}} \right\}, \text{ [Ghiglieri, Laine ('16)]}$$

$$h_- = \frac{2\langle\Phi\rangle^2 E_\nu (E_N - k)(E_N - E_\nu)}{kE_N(4(E_N - E_\nu)^2 + \gamma_\nu^2)},$$

$$\gamma_- = \frac{2\langle\Phi\rangle^2 E_\nu (E_N - k)\gamma_\nu}{kE_N(4(E_N - E_\nu)^2 + \gamma_\nu^2)},$$

## Symmetric phase

$$h_+ = \frac{T^2}{8k},$$

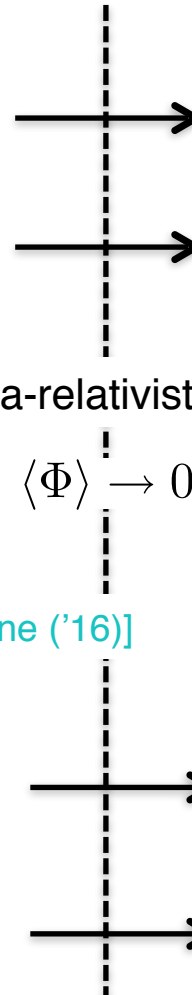
$$\gamma_+ = \frac{1}{E_N} \text{Im} \Pi_R,$$

Ultra-relativistic limit

$$\langle\Phi\rangle \rightarrow 0$$

$$h_- = 0,$$

$$\gamma_- = 0,$$

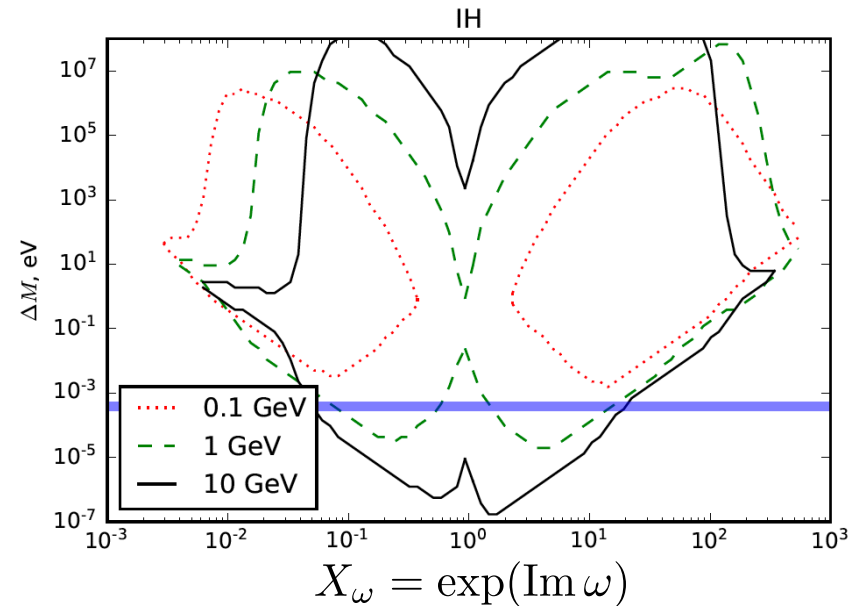
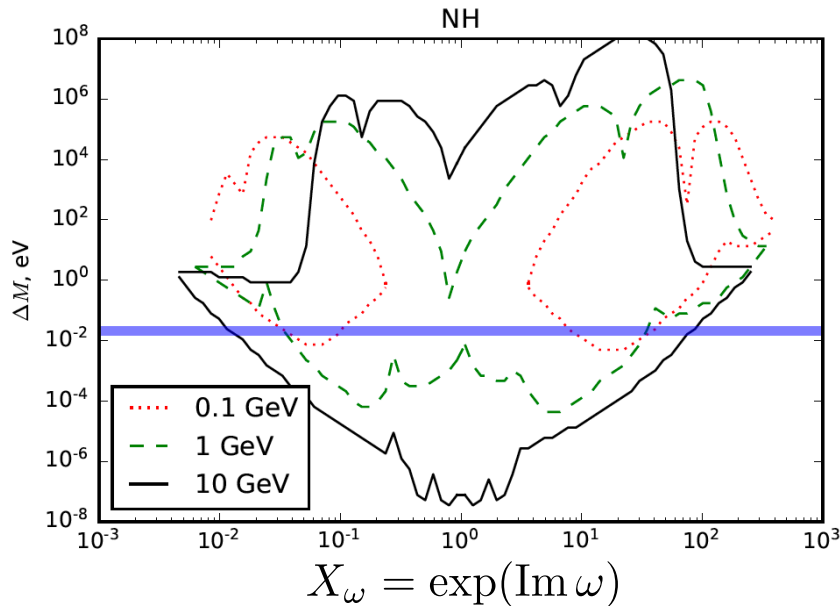


# Parameter dependence of Baryogenesis

## Mass dependence in the baryogenesis

$$\text{NH: } \delta = \pi, \quad \eta = 3\pi/2, \quad \text{Re } \omega = \pi/4,$$

$$\text{IH: } \delta = 0, \quad \eta = \pi/2, \quad \text{Re } \omega = \pi/4.$$

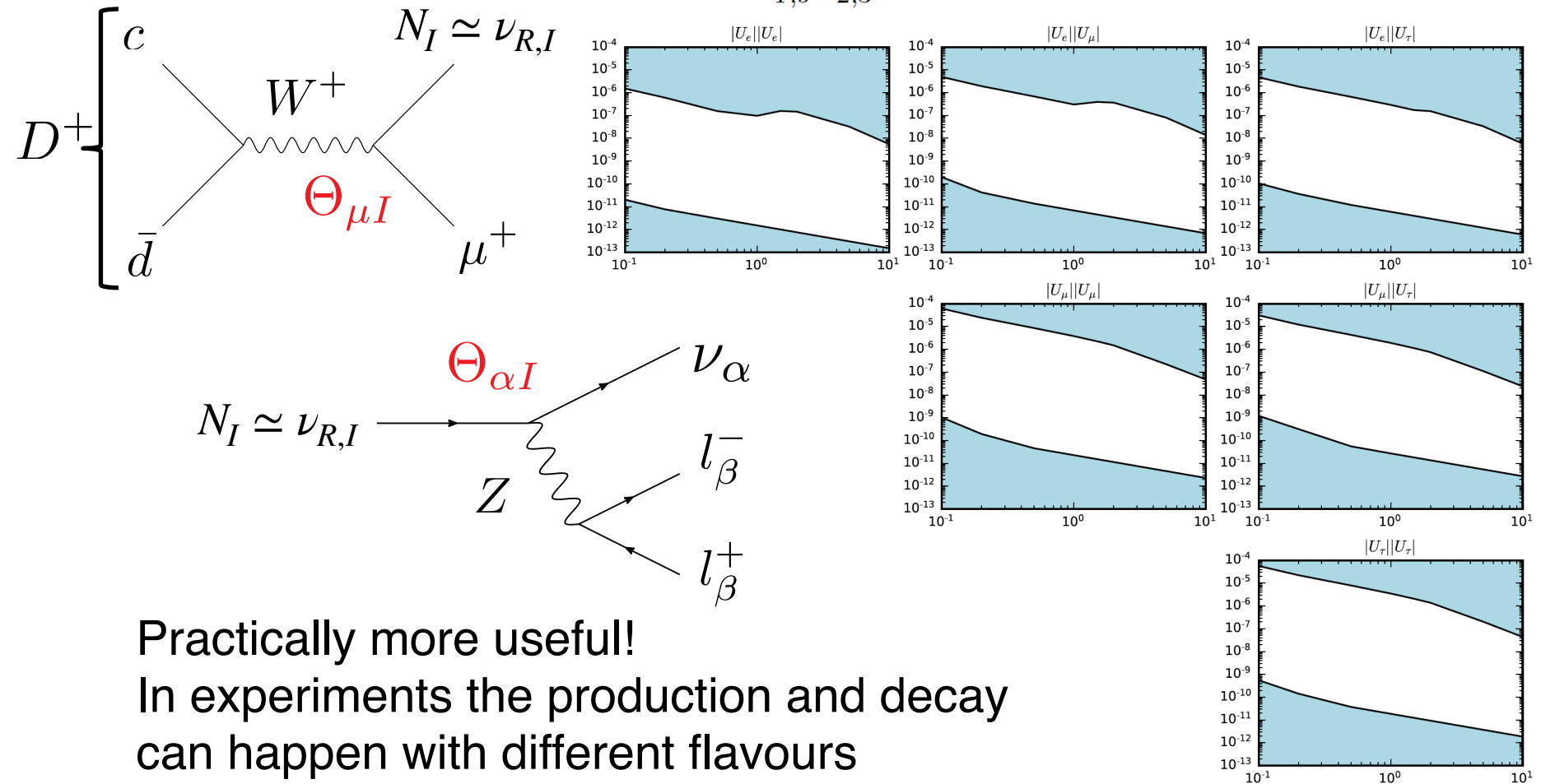


[SE, Shaposhnikov, Timiryasov ('18)]

# Parameter space of Baryogenesis [2]

## Full parameter scanning

Individual mixings of HNLs:  $\sum_{I,J=2,3} |\Theta_{I\alpha}|^2 \cdot |\Theta_{J\beta}|^2 = |U_\alpha|^2 \cdot |U_\beta|^2$



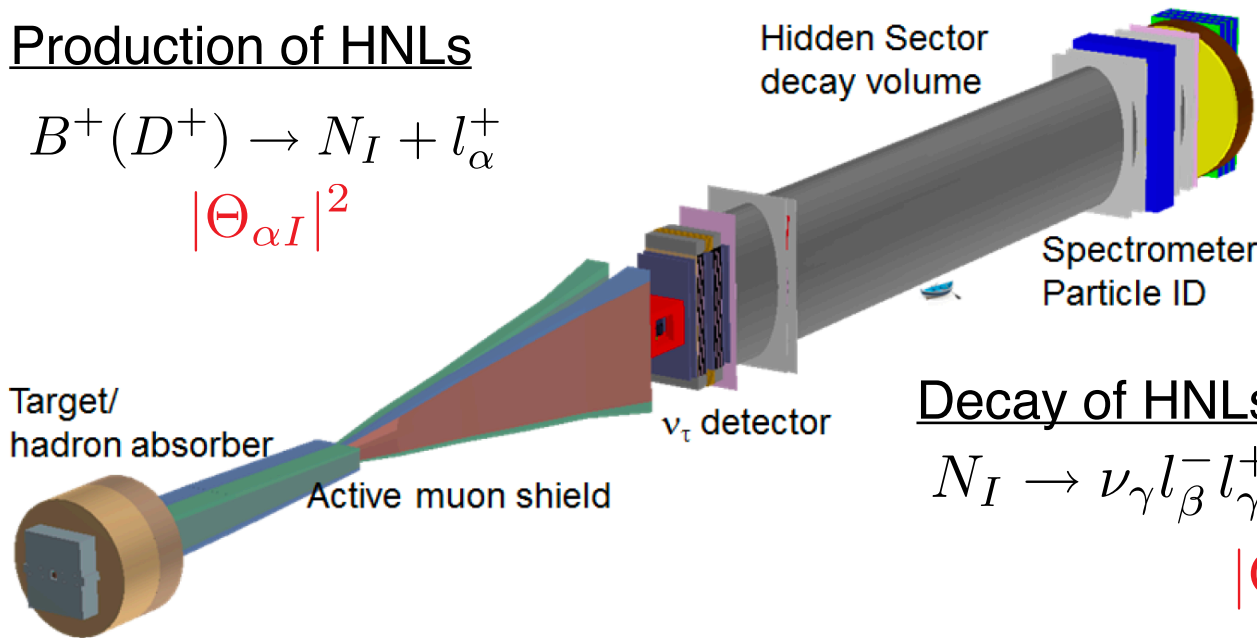
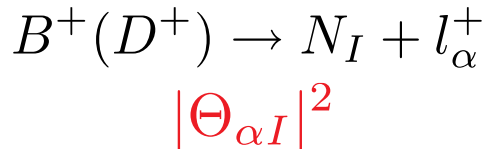


# SHiP experiment

Search for **Hidden Particle (SHiP)** is a proposed beam-dump experiment by using the SPS in CERN.

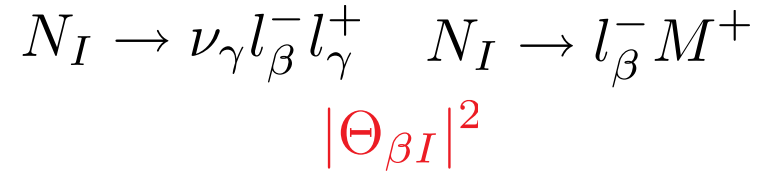


## Production of HNLs



[The SHiP collaboration ('15)]

## Decay of HNLs



Individual flavor mixings are also important information for the search!

In this method relevant mixings are  $\sum_{I,J=1,2} |\Theta_{\alpha I}|^2 |\Theta_{\beta J}|^2 = |U_\alpha|^2 |U_\beta|^2$ .

# Resonant production of sterile $\nu$ DM

Production with lepton asymmetries

$$\Gamma_N \sim \Theta_M^2(T) \Gamma_\nu \sim \Theta_M^2(T) G_F^2 T^5$$

$$\Theta_M^2(T) \simeq \frac{\Theta_1^2}{\left(1 + \frac{2p}{M_1^2} (b(p, T) \pm c(T))\right)^2 + \Theta_1^2}$$

When  $\left[1 + \frac{2p}{M_1^2} (b(p, T) \pm c(T))\right] \simeq 0$   
the mixing is enhanced

$$\begin{cases} b(p, T) = \frac{16G_F^2}{\pi\alpha_W} p(2 + \cos^2 \theta_W) \frac{7\pi^2 T^4}{360} \\ c(T) = 3\sqrt{2}G_F(1 + 4\sin^2 \theta_W)(n_{\nu_e} - n_{\bar{\nu}_e}) \end{cases}$$

$$L_6 = \frac{n_L}{s} \times 10^6$$

$n_{\nu_\alpha}/s \gtrsim 10^{-5}$  is necessary

$\left(\sum_\alpha n_{\nu_\alpha}/s \simeq 10^{-10} \text{ for the BAU}\right)$

