SI 2019
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Inflation & Cosmology III

- primordial black hole and gravitational waves -

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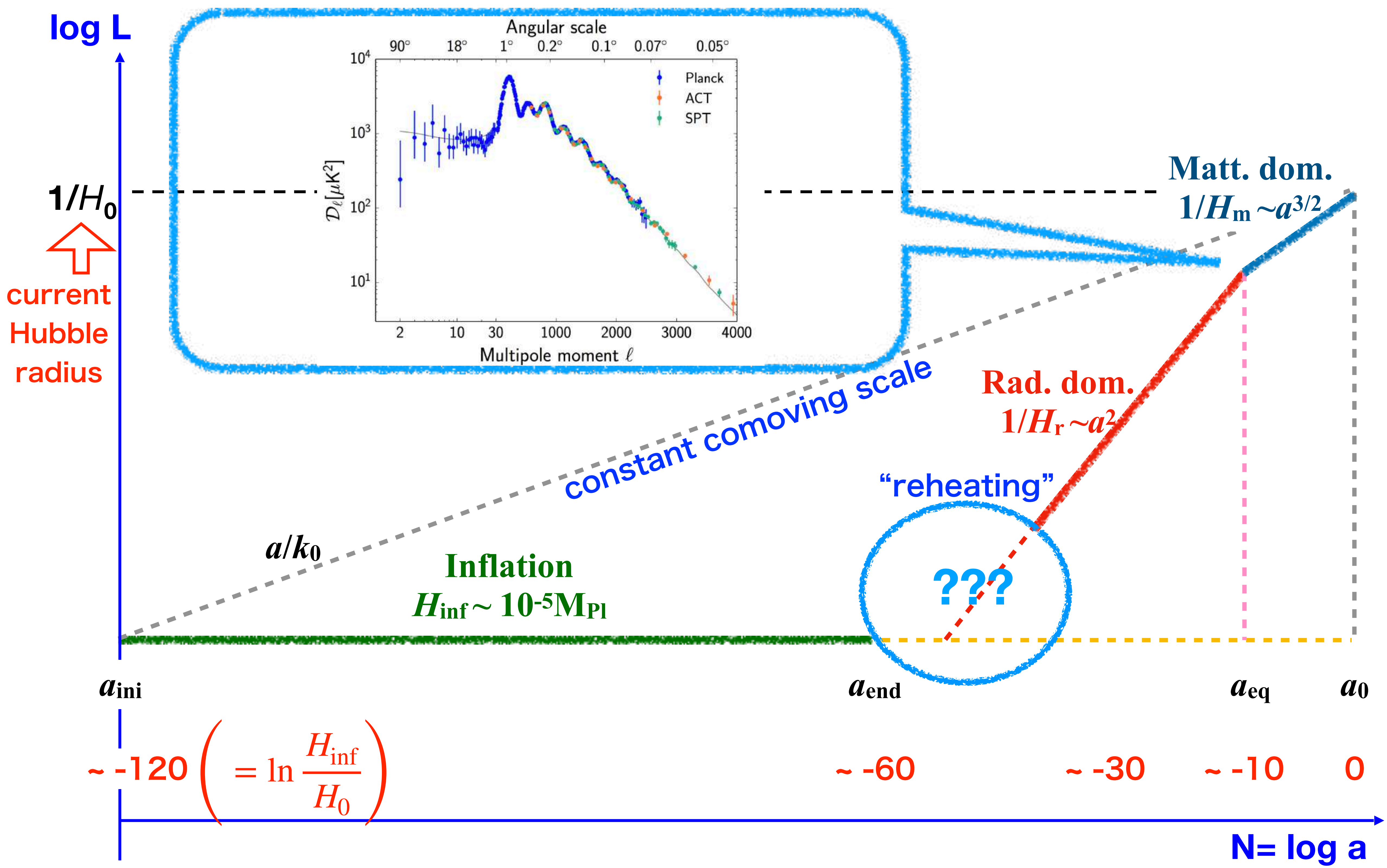
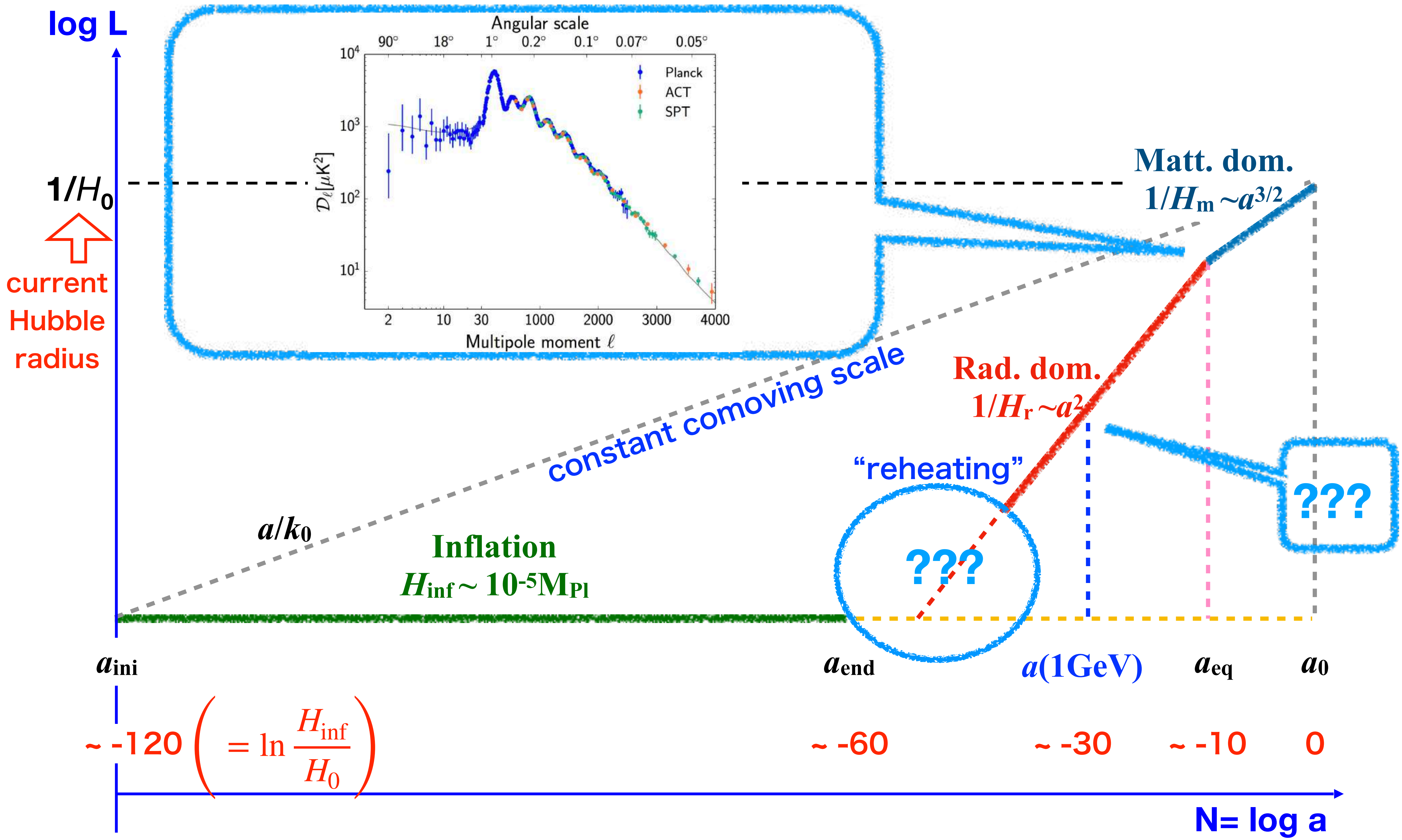
PBH Cosmology!

cosmic spacetime diagram

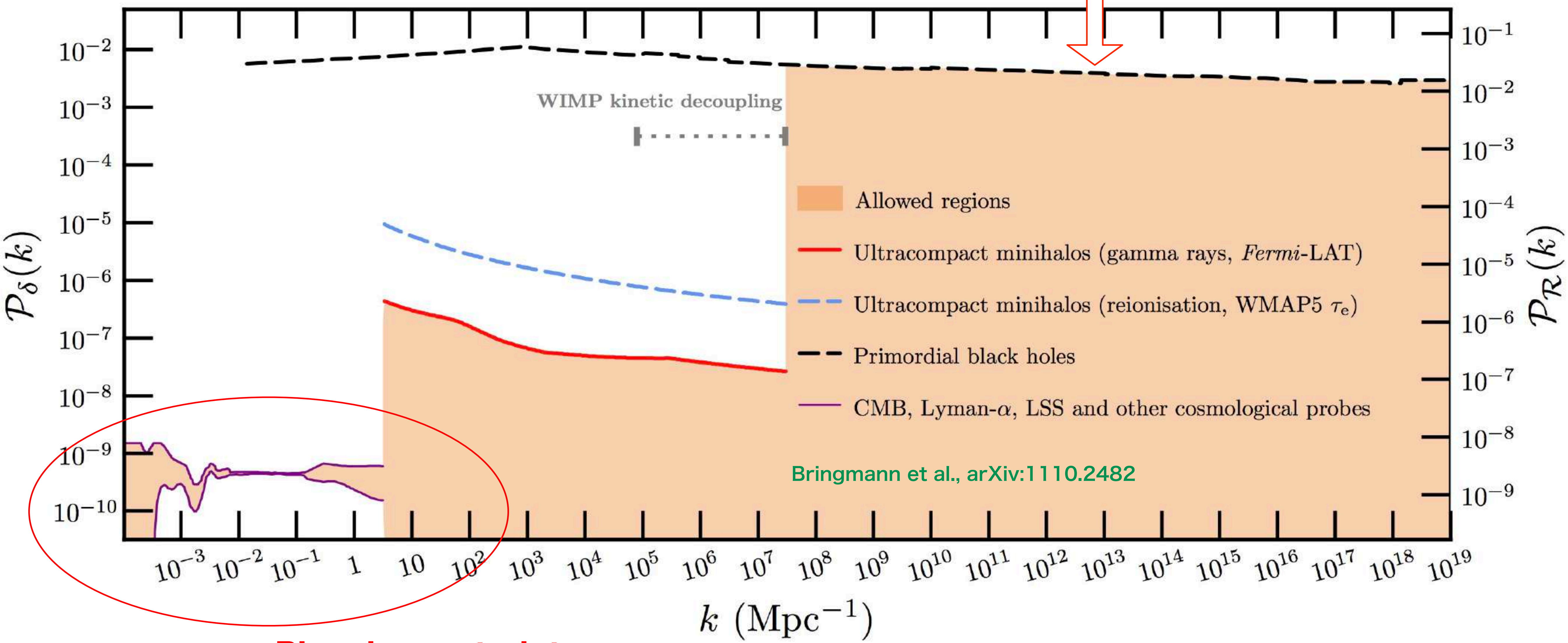


cosmic spacetime diagram



observational constraint on inflation

constraints on small scales are from PBHs

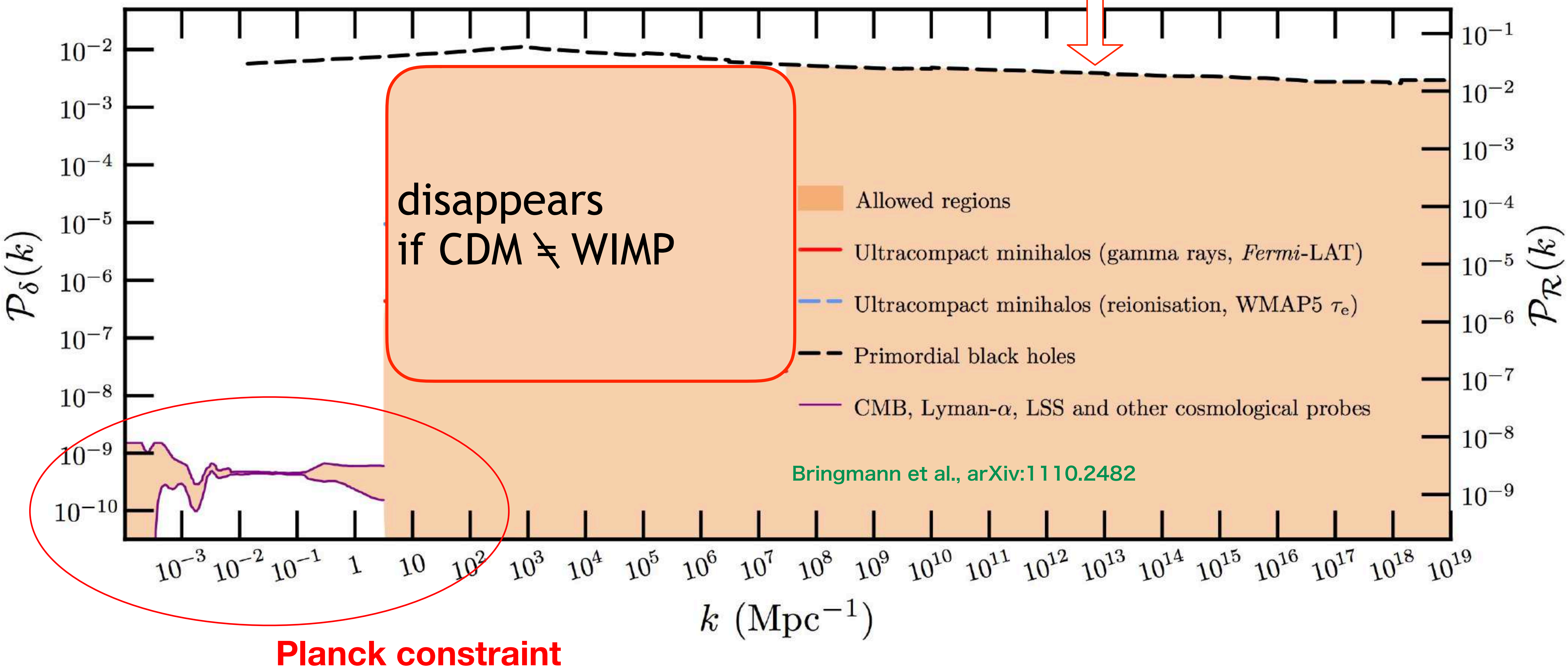


Planck constraint

There are some constraints on small scales, but quite weak.

observational constraint on inflation

constraints on small scales are from PBHs



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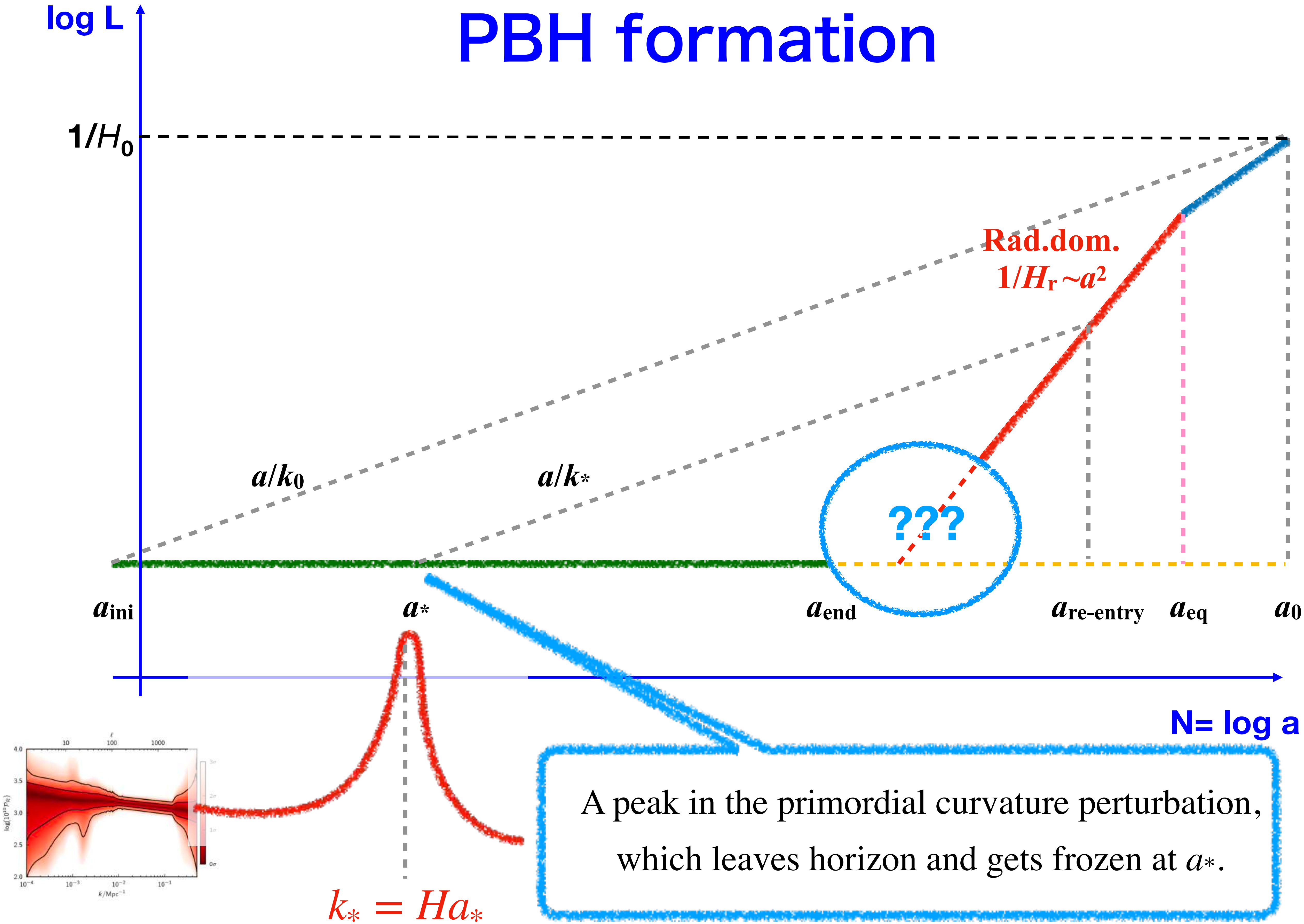
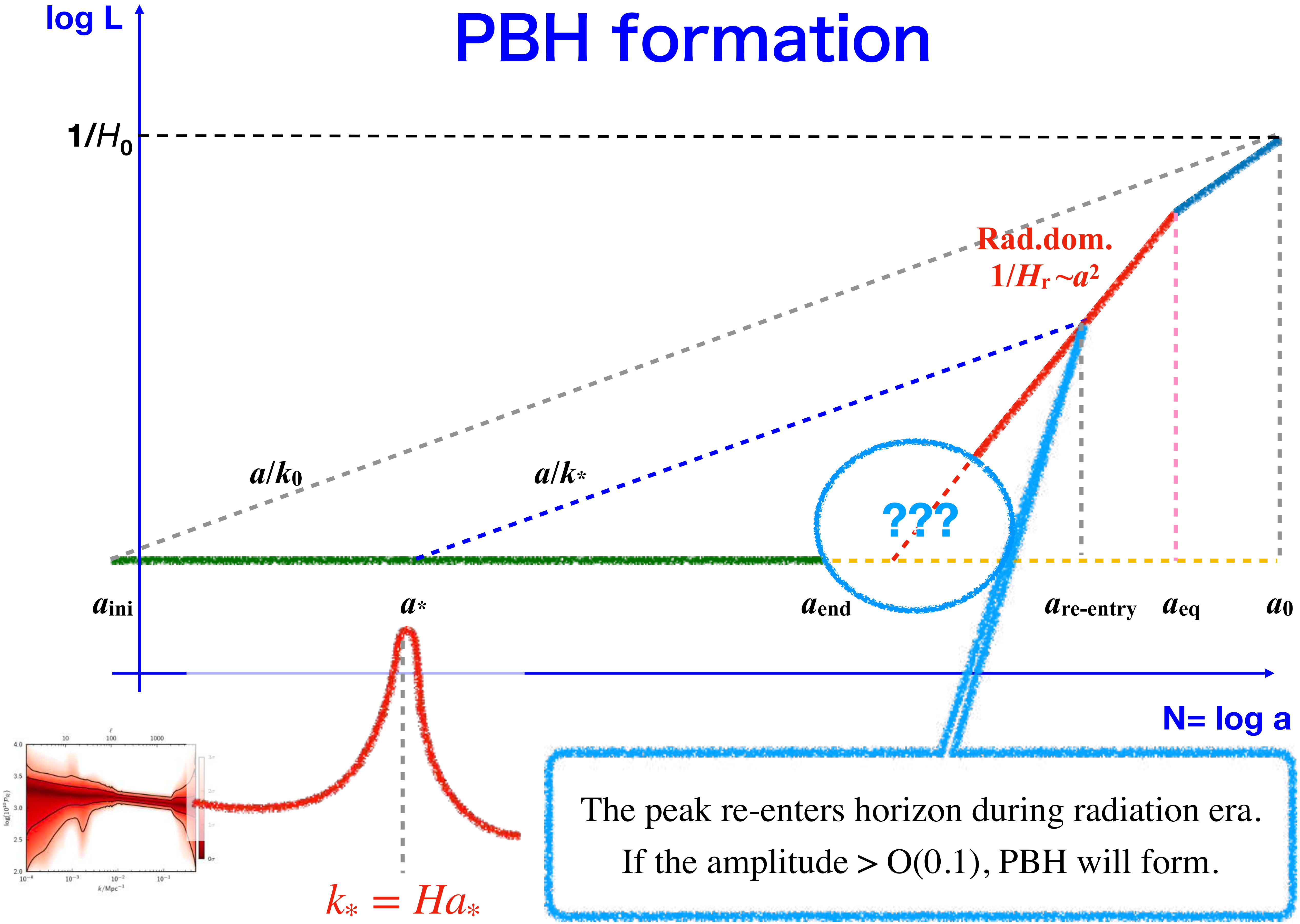
PBH formation

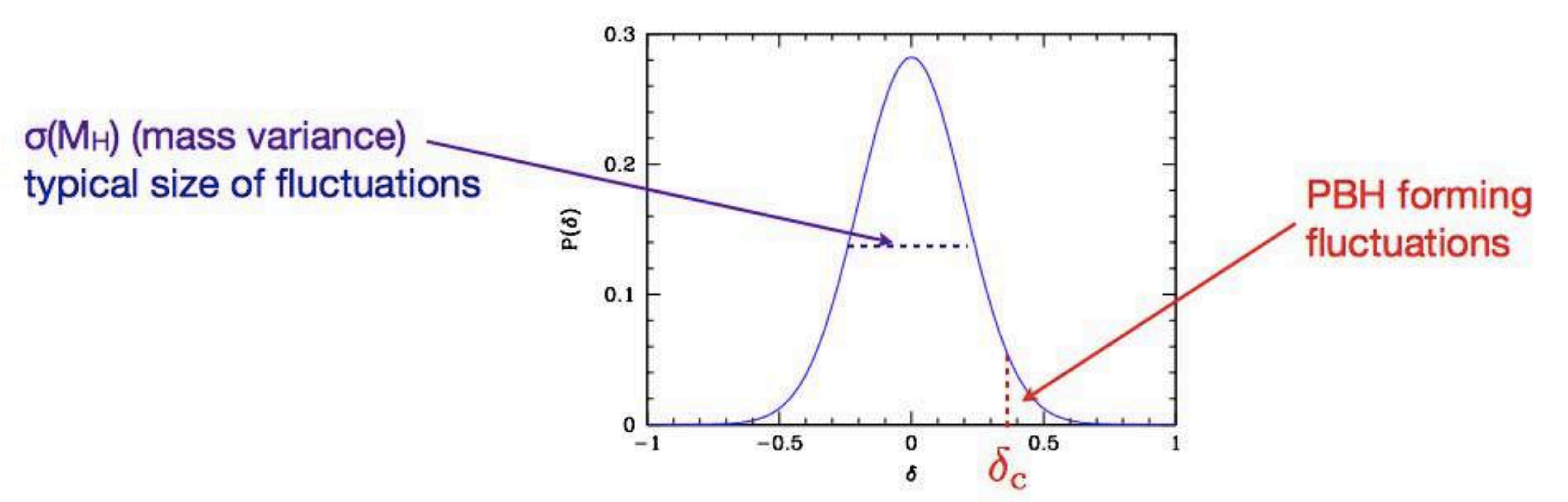


PBH formation



fraction β that turns into PBHs

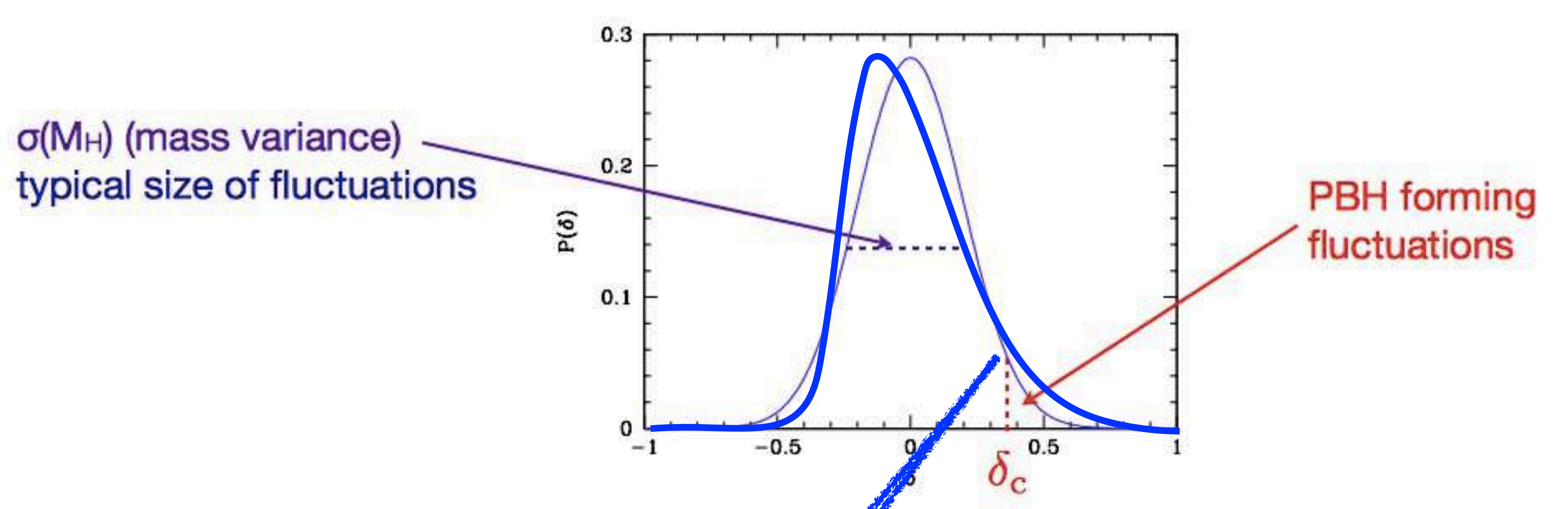
for Gaussian probability distribution



- When $\sigma_M << \delta_c$, β can be approximated by exponential:

$$\beta \approx \sqrt{\frac{2}{\pi}} \frac{\sigma_M}{\delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)$$

effect or non-Gaussianity



Non-Gaussianity can increase (f_{NL}>0) or decrease (f_{NL}<0) the PBH adundances, substantially if $\sigma(M_H) \ll 1$.

Young & Byrnes, 1307.4995

Formation of PBHs

Curvature perturbation to PBH

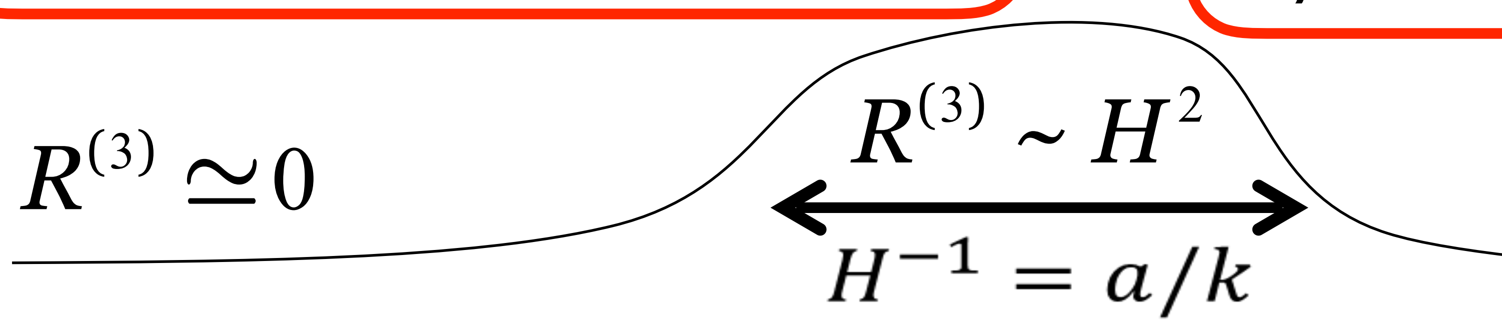
- gradient expansion/separate universe approach

$$6H^2(t,x) + R^{(3)}(t,x) = 16\pi G\rho(t,x) + \cdots$$

Hamiltonian constraint (Friedmann eq.)

$$R^{(3)} \approx -\frac{4}{a^2}\nabla^2\mathcal{R}_c \approx \frac{8\pi G}{3}\delta\rho_c$$

$$\frac{\delta\rho_c}{\rho} \approx \mathcal{R}_c \quad \text{at} \quad \frac{k^2}{a^2} = H^2$$



formation of a closed universe

- If $R^{(3)} \sim H^2$ $(\Leftrightarrow \delta\rho_c/\rho \sim 1)$, it collapses to form BH

Young, Byrnes & MS '14

$$M_{\rm PBH} \sim \rho H^{-3} \sim 10^5 M_\odot \left(\frac{t}{1s}\right) \sim 20 M_\odot \left(\frac{k}{1{\rm pc}^{-1}}\right)^{-2}$$

- Spins of PBHs are expected to be very small

simple 2-field model

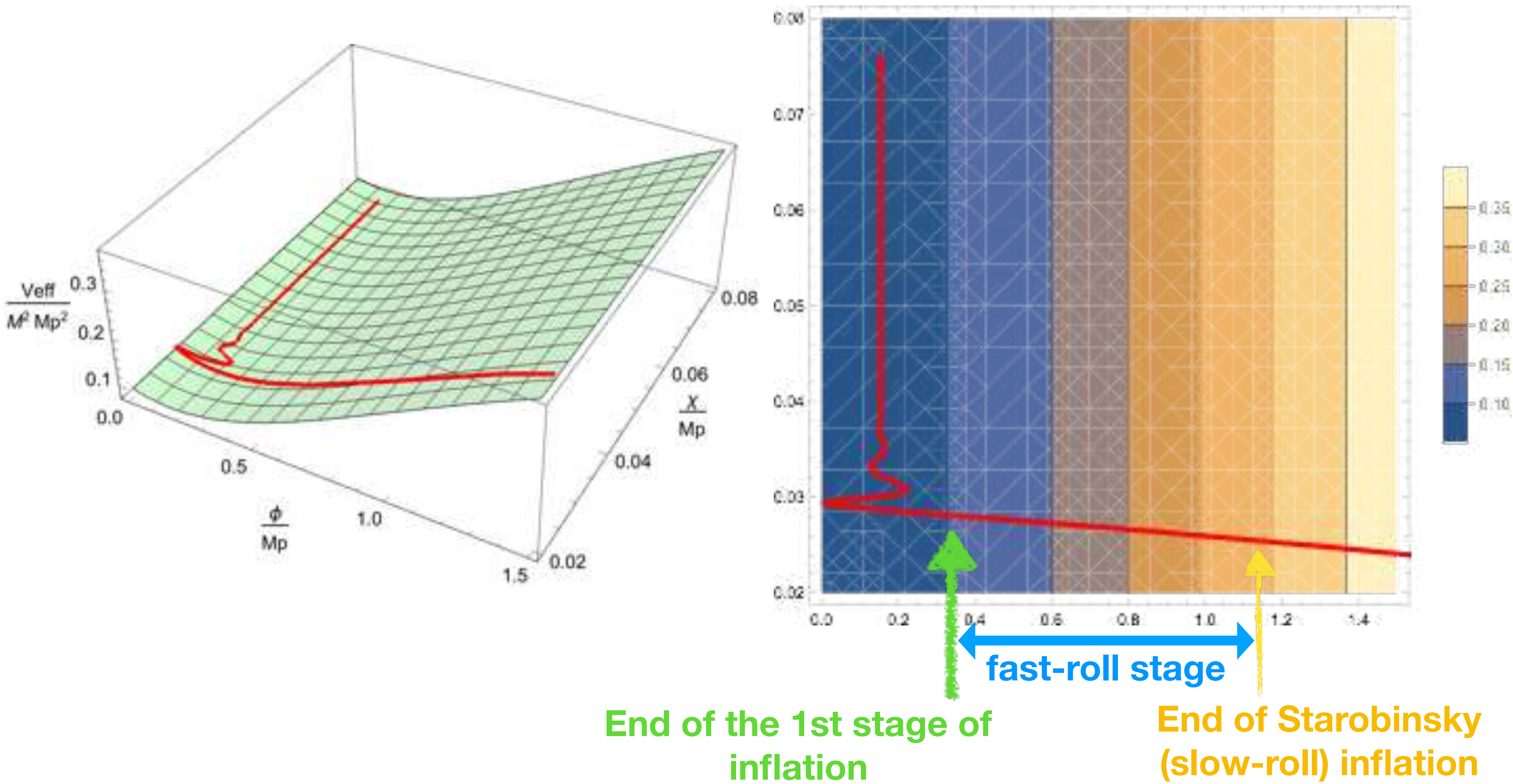
- Starobinsky R^2 gravity plus a scalar field χ, non-minimally coupled to gravity: (→ scalaron ϕ)

$$S_J = \int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}\left(R+\frac{R^2}{6M^2}\right)-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)-\frac{1}{2}\xi R\chi^2\right\}.$$

- for $V(\chi)$ we pick the small-field form:

$$V(\chi)=V_0-\frac{1}{2}m^2\chi^2+\cdots$$

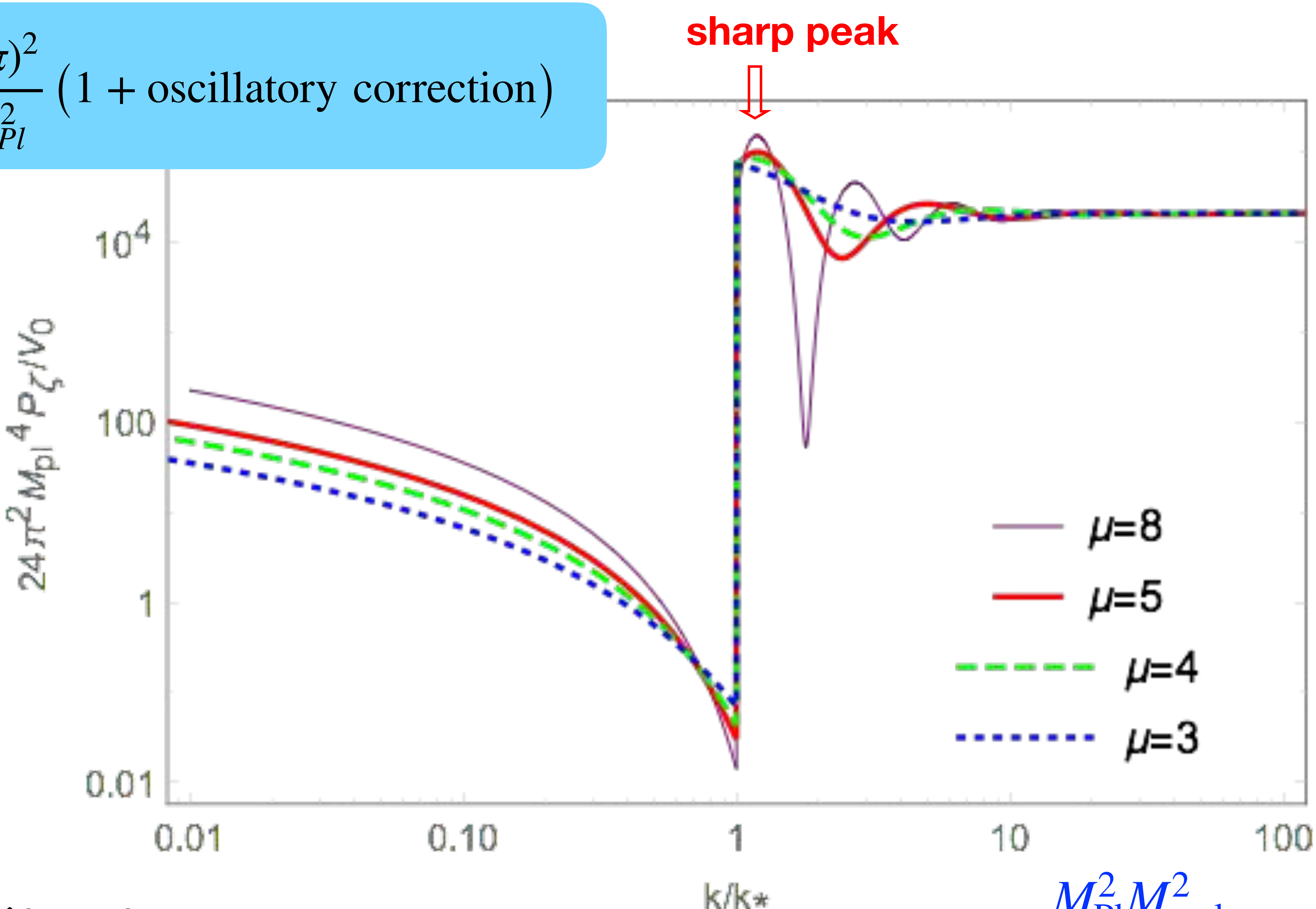
- ξ-term is the non-minimally coupled term to stabilize the initial condition problem. A version of SSB in χ direction.



- Scalaron ϕ becomes massive at the end of the 1st stage.
- Field χ plays the role of inflaton at the 2nd stage.

Curvature Perturbation Power Spectrum

$$P \approx \frac{(H/2\pi)^2}{2\epsilon M_{Pl}^2}\left(1 + \text{oscillatory correction}\right)$$



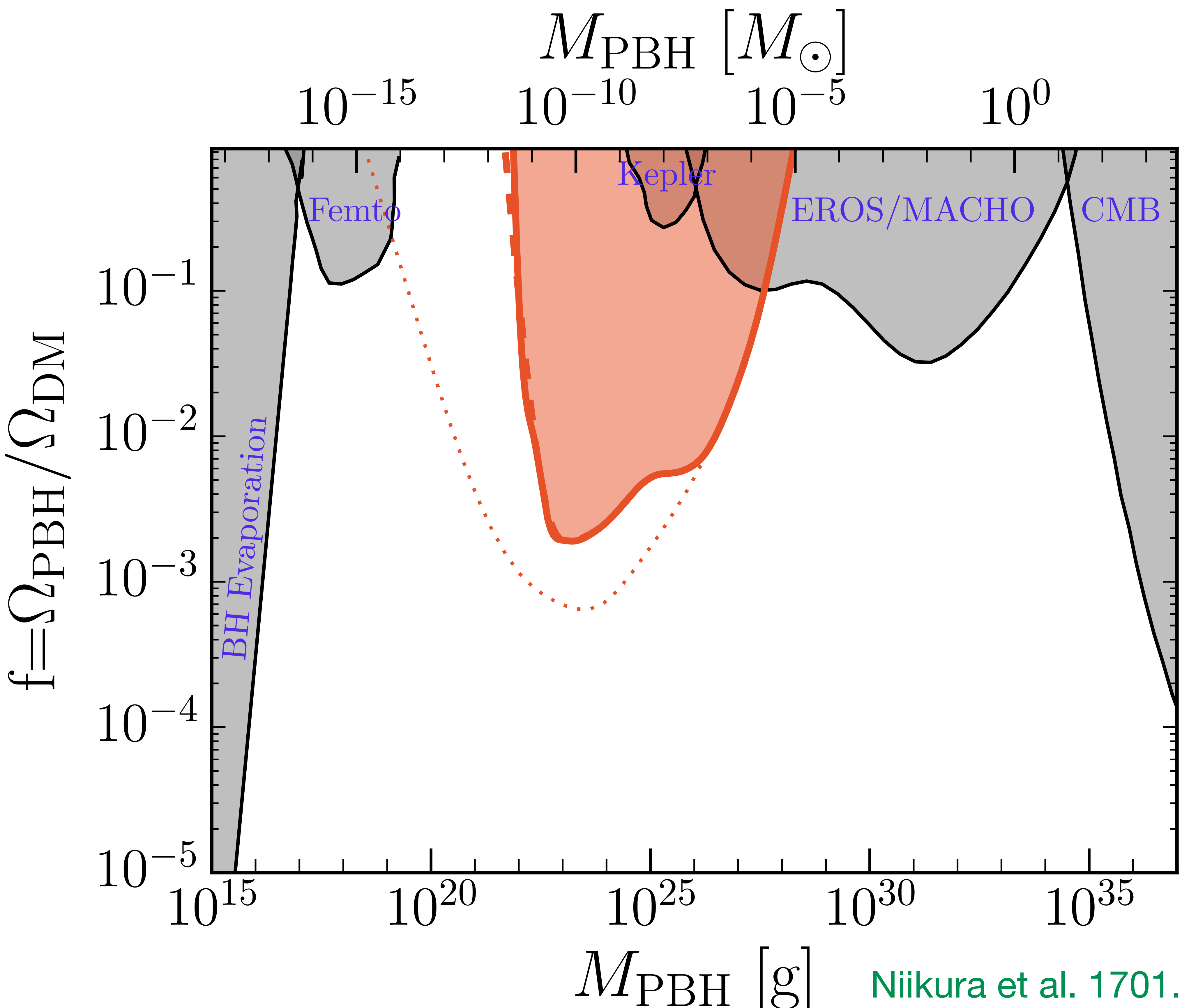
$$\epsilon = \frac{3(\dot\phi^2 + \dot\chi^2)}{2V}$$

$\eta_H = 0.02, \quad \chi_*/M_{\rm Pl} = 0.1,$

$\mu = 2, 3, 5, 8$

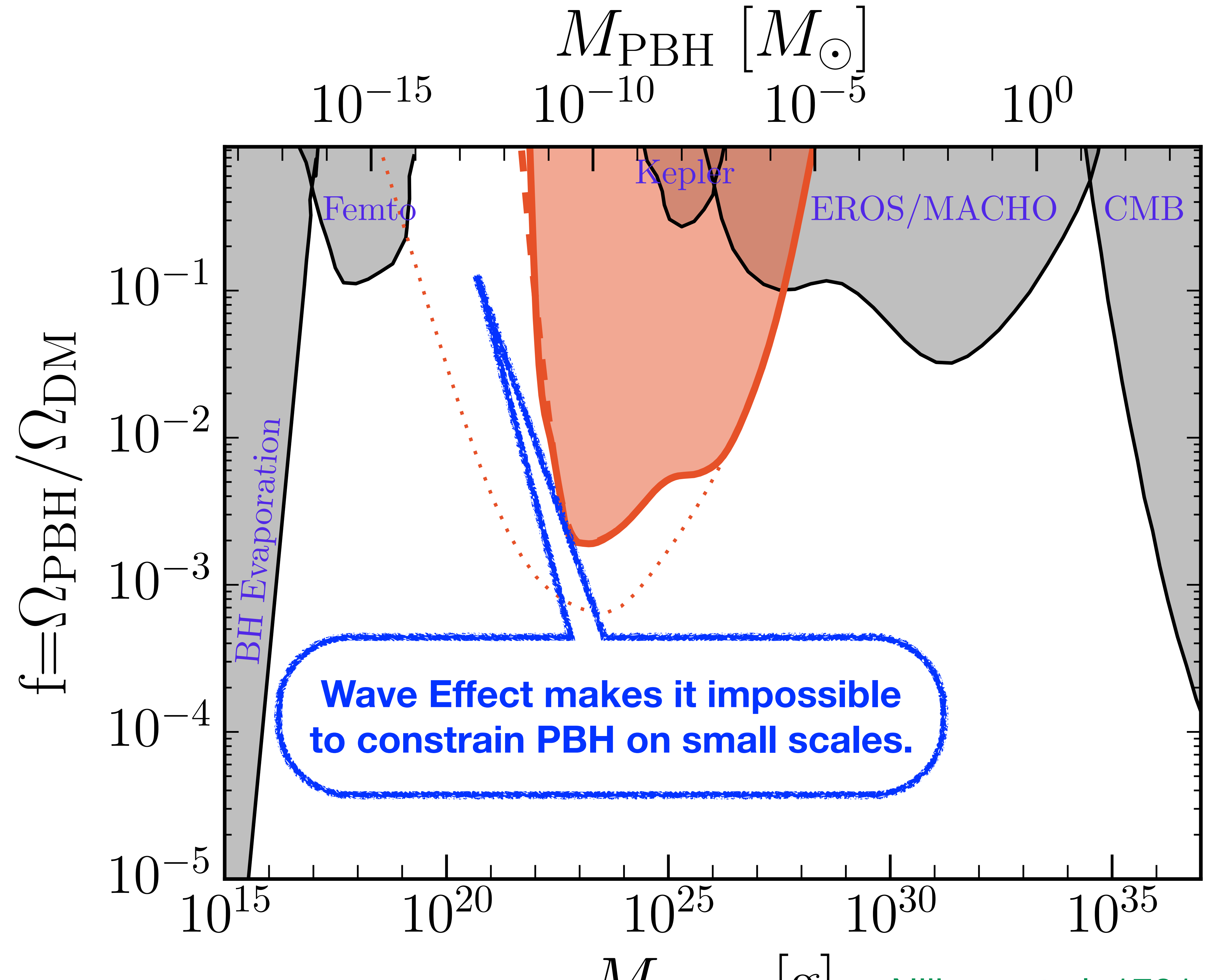
$$\mu^2 \equiv \frac{M_{\rm Pl}^2 M_{\rm scalaron}^2}{V_0}$$

Constraints on PBH mass spectrum



Niikura et al. 1701.02151v3

Constraints on PBH mass spectrum



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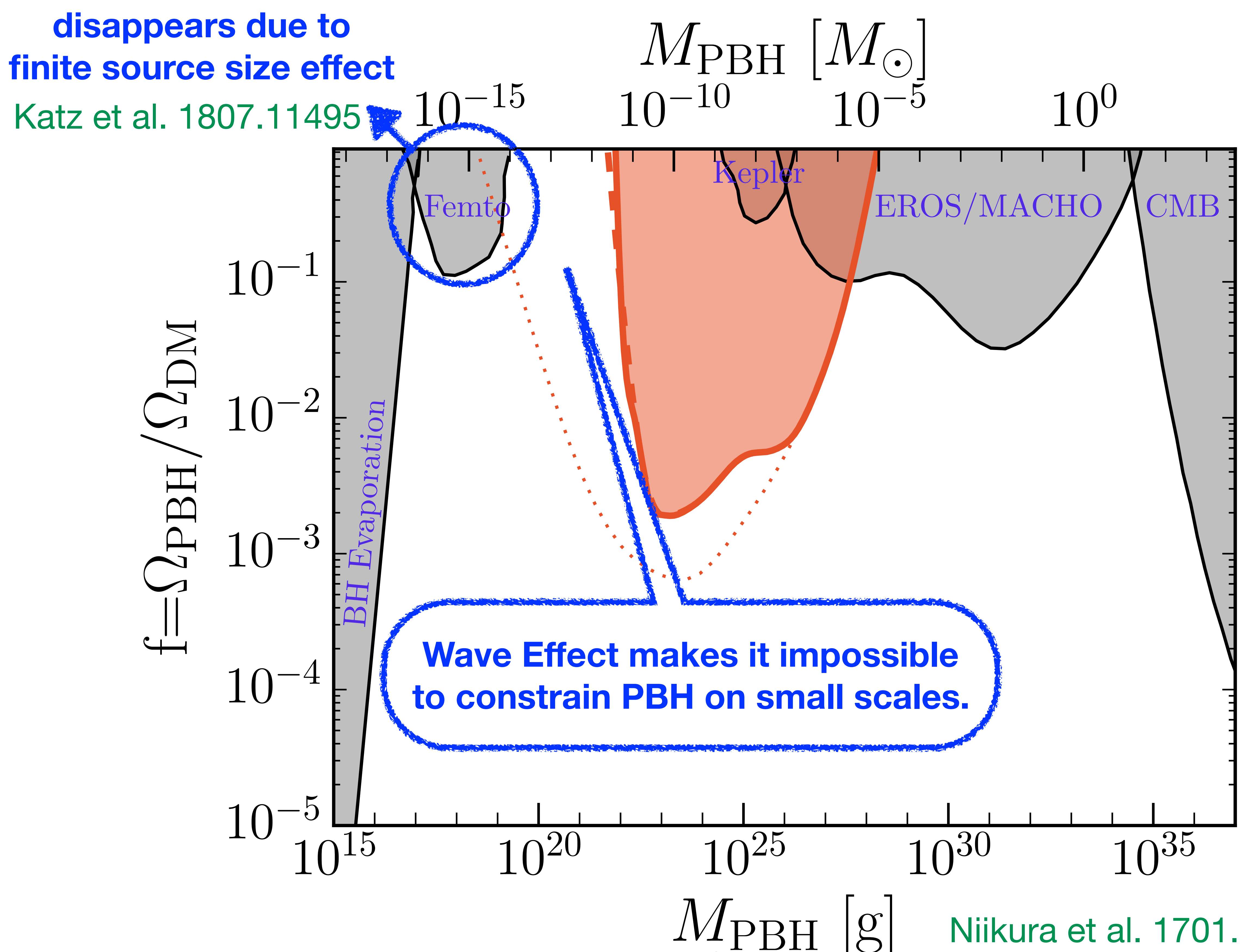

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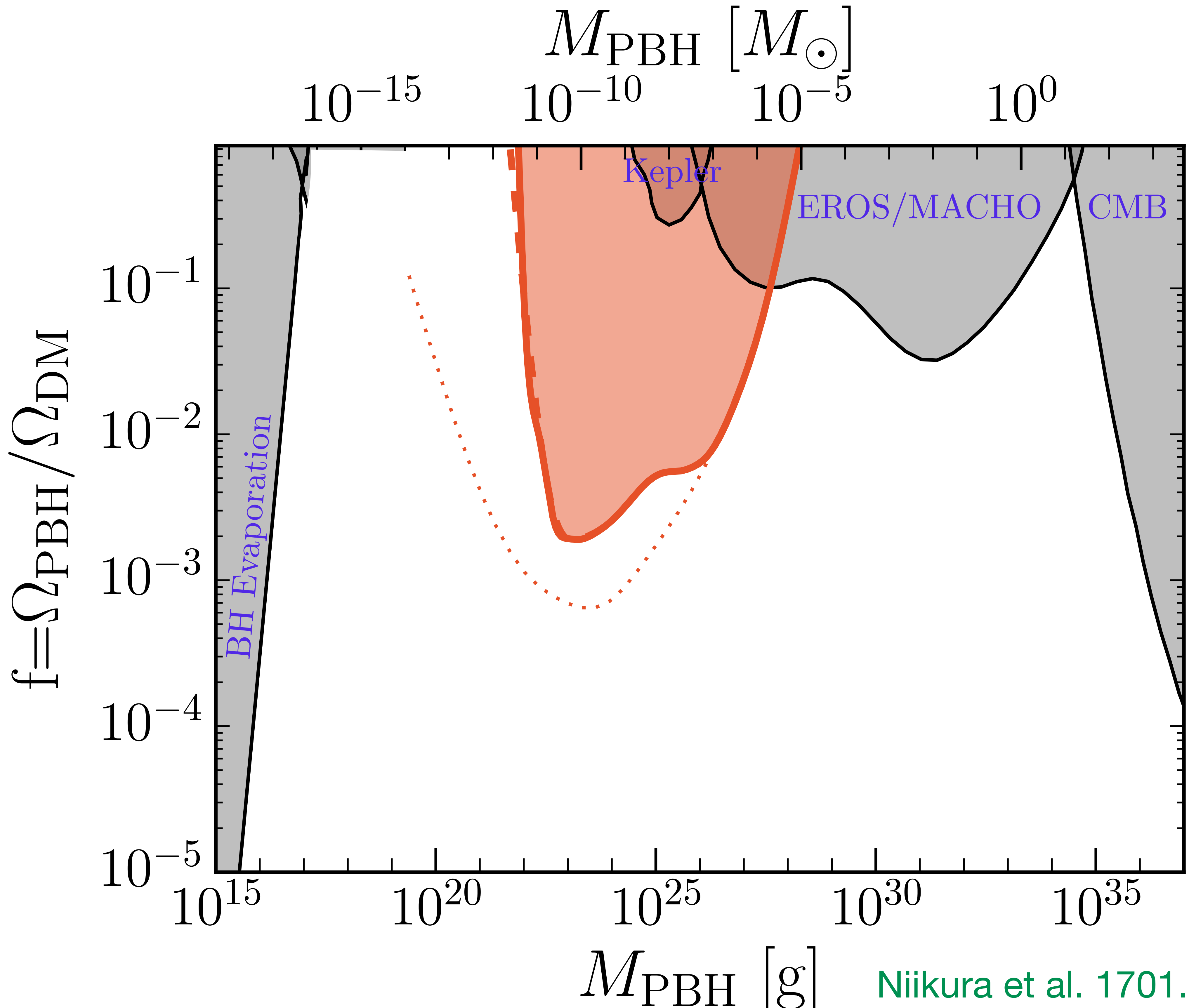

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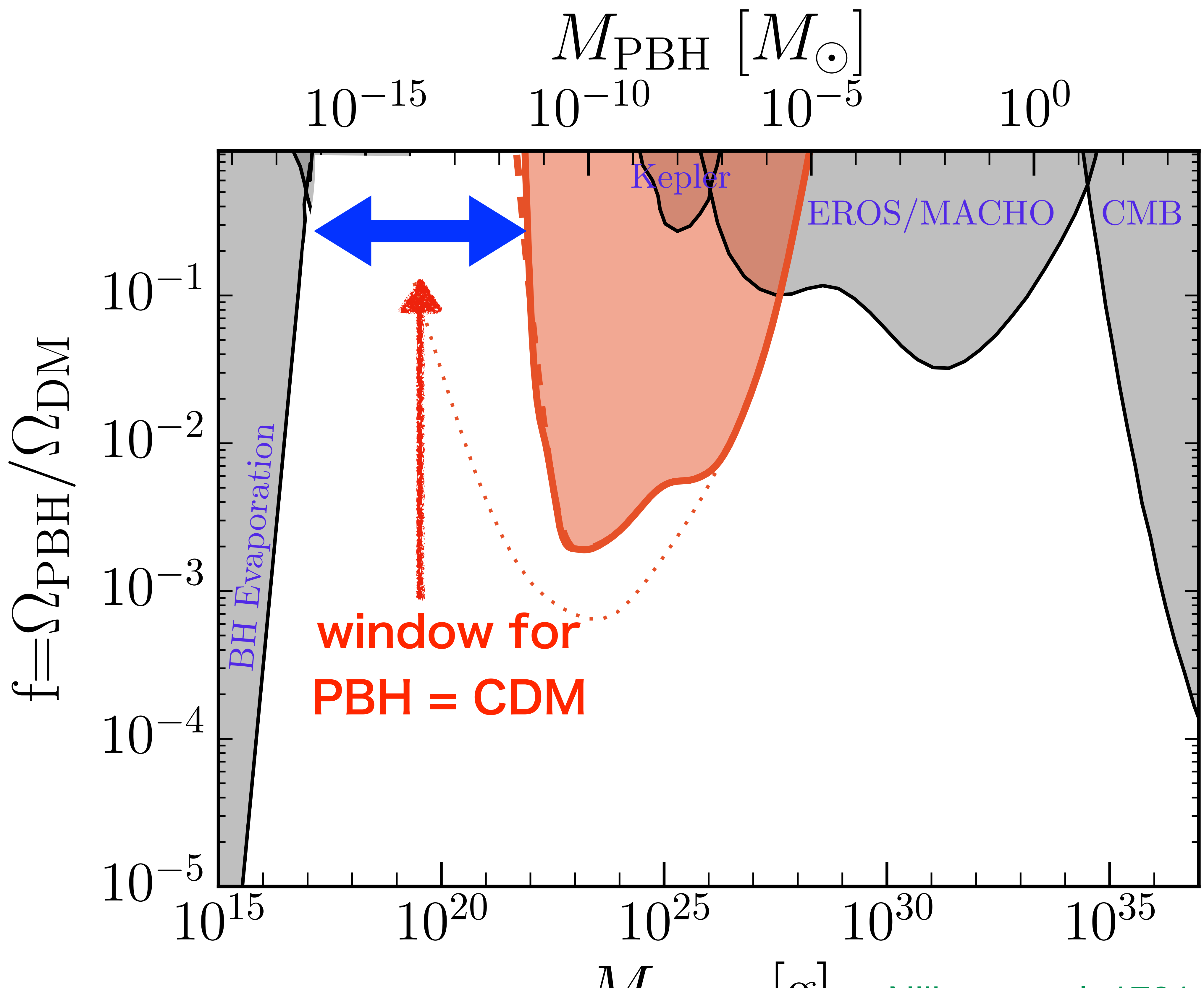

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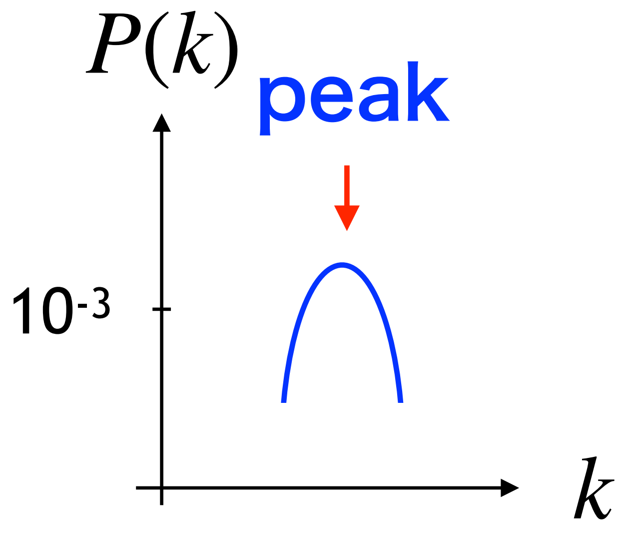
PBHs as CMD

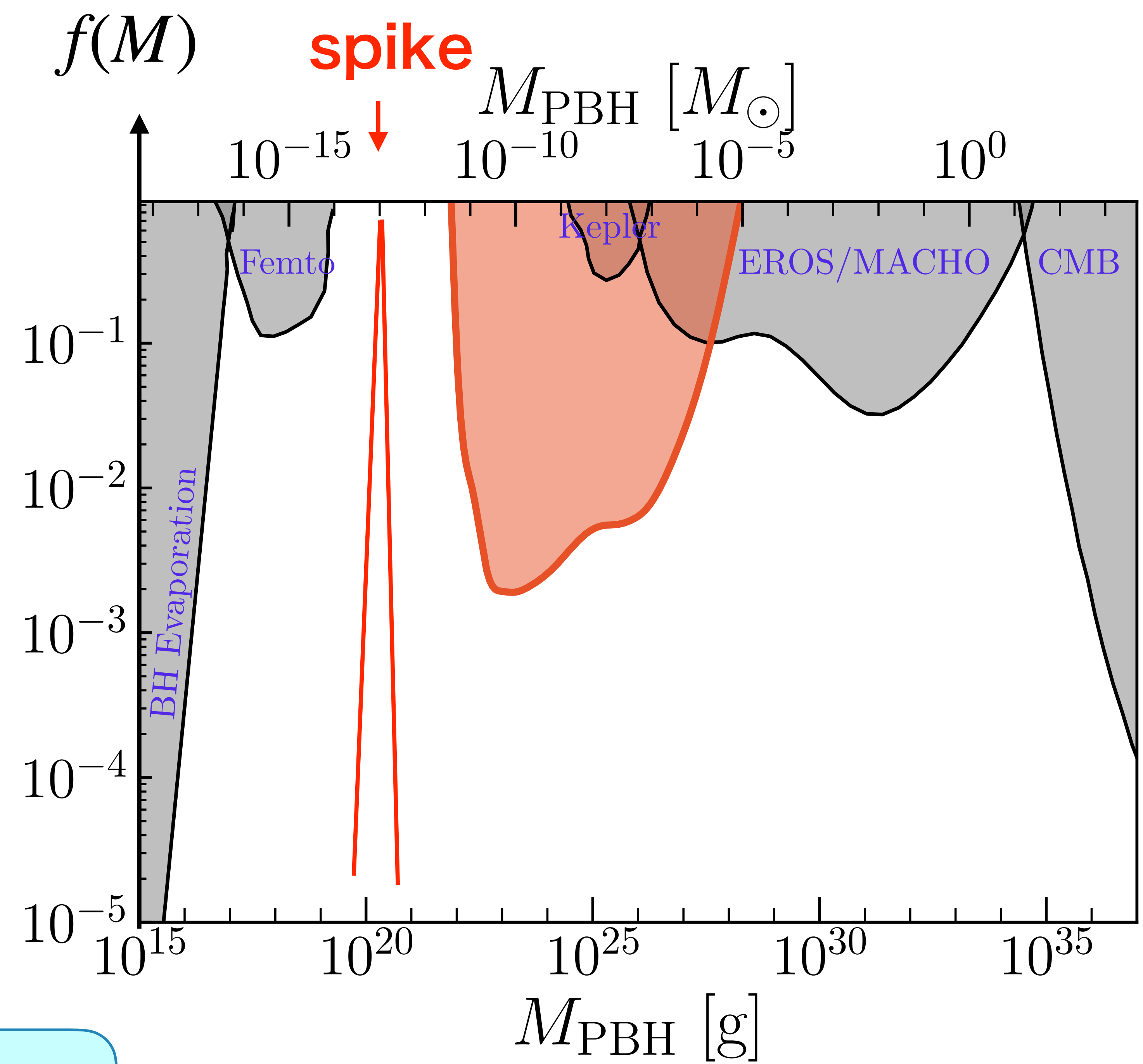


$$f(M) \equiv \frac{\Omega_{PBH}}{\Omega_{DM}} \propto \exp\left[-\frac{O(0.1)}{P(k)}\right]$$

a sharp peak in $P(k)$

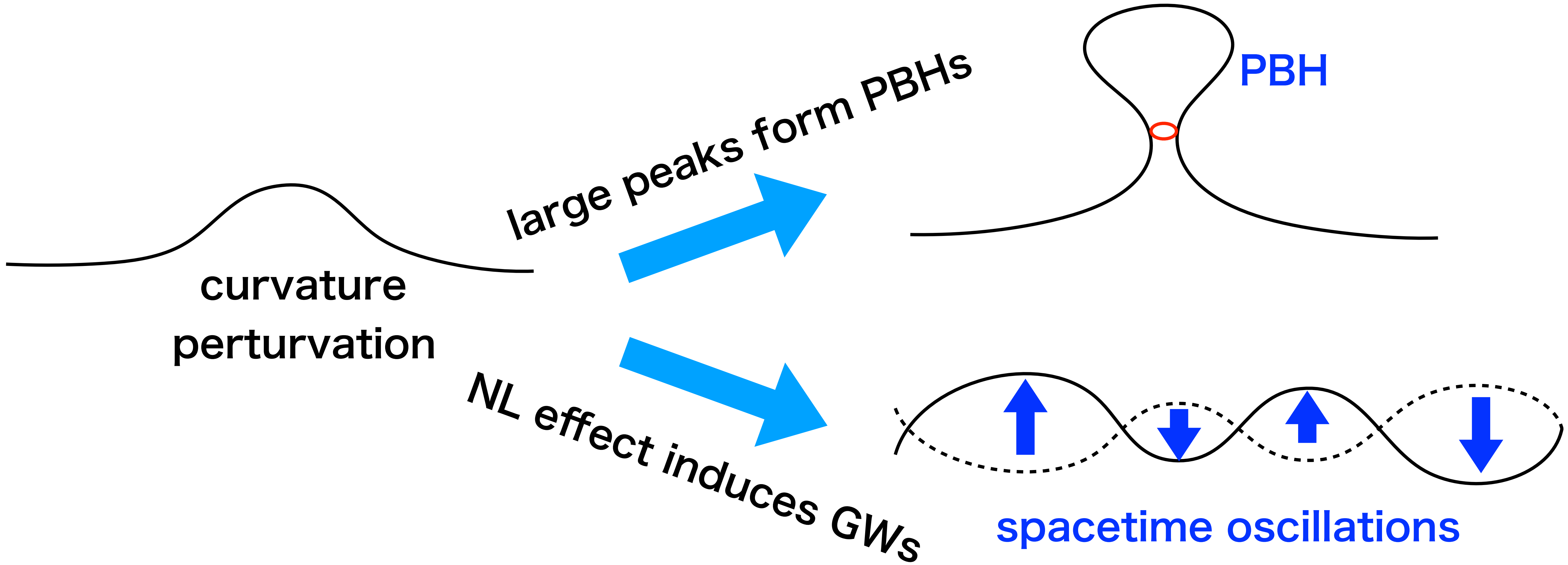
a spike in $f(M)$

monochromatic PBH mass fcn



GWs from Large Scalar Curvature Perturbation

GWs can capture PBHs!



PBHs = CDM with M_{PBH} ~10^{21}g
generates GWs with f~10^{-3} Hz

⇨ Background GWs
at LISA band

⇔ LIGO-Virgo :10 - 1000 Hz

Induced GWs

- The equation of motion for the tensor perturbation with source

$$h''_{\mathbf{k}} + 2\mathscr{H} h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = \mathcal{S}(\mathbf{k}, \eta) \sim \int d^3 l \; l_i l_j \Phi_{\mathbf{l}}(\eta) \Phi_{\mathbf{k}-\mathbf{l}}(\eta)$$

- The quantity we want to calculate is

$$\Omega_{\rm GW}(k) \equiv \frac{1}{12}\left(\frac{k}{Ha}\right)^2 \frac{k^3}{\pi^2} \overline{\langle h_{\mathbf{k}}(\eta) h_{\mathbf{k}}(\eta)\rangle}.$$

$$\Rightarrow \quad \Omega_{\rm GW} \sim \langle hh \rangle \sim \langle \mathcal{S}\mathcal{S} \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathscr{P}_{\Phi}^2$$

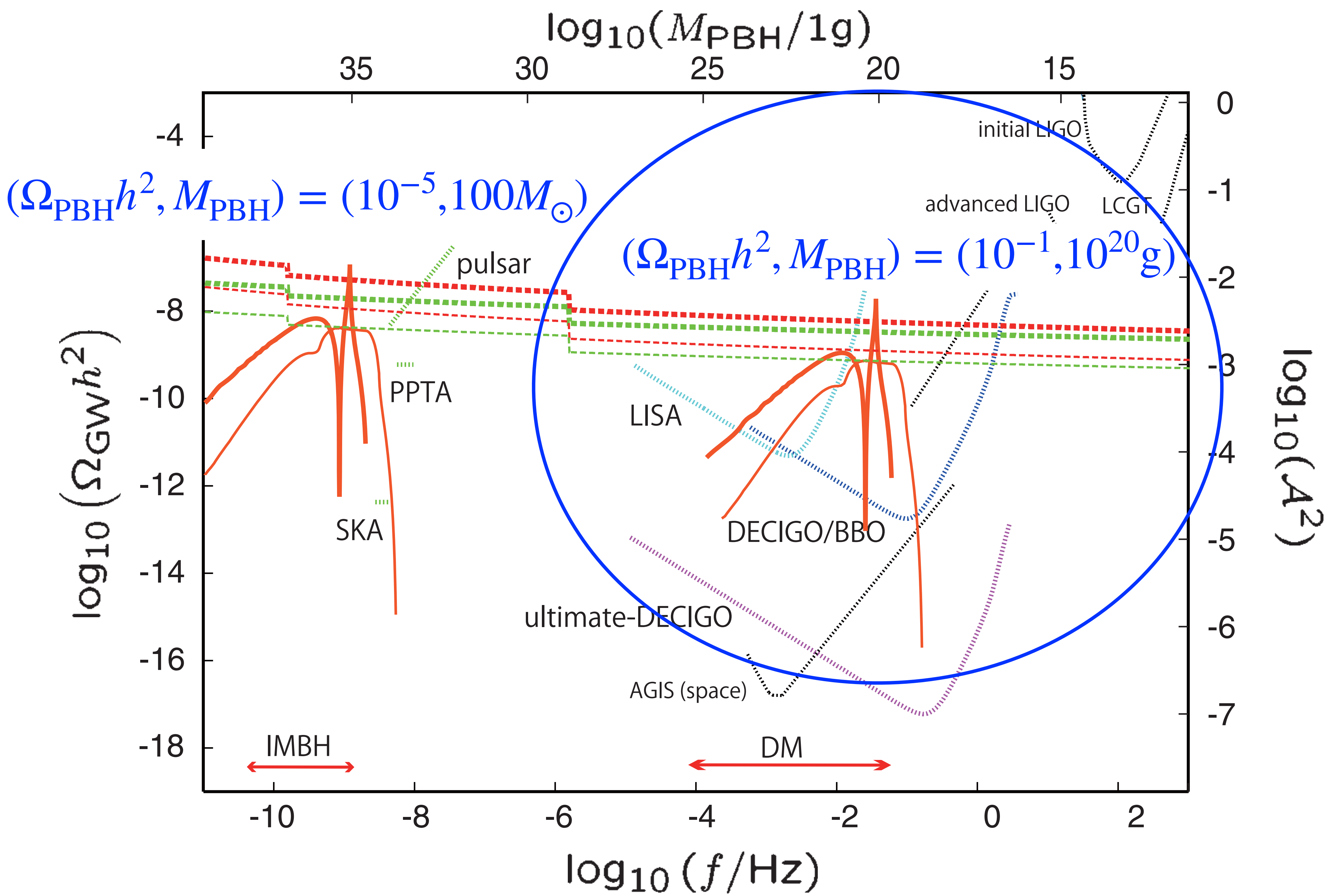
- Φ may not be Gaussian. So consider a non-Gaussianity:

$$\mathscr{R}(\mathbf{x}) = \mathscr{R}_g(\mathbf{x}) + F_{\rm NL}\left[\mathscr{R}_g^2(\mathbf{x}) - \langle \mathscr{R}_g^2(\mathbf{x}) \rangle\right].$$

$$\Phi = \frac{2}{3}\mathscr{R}$$

at radiation-dominated stage

GWs test PBH=DM!

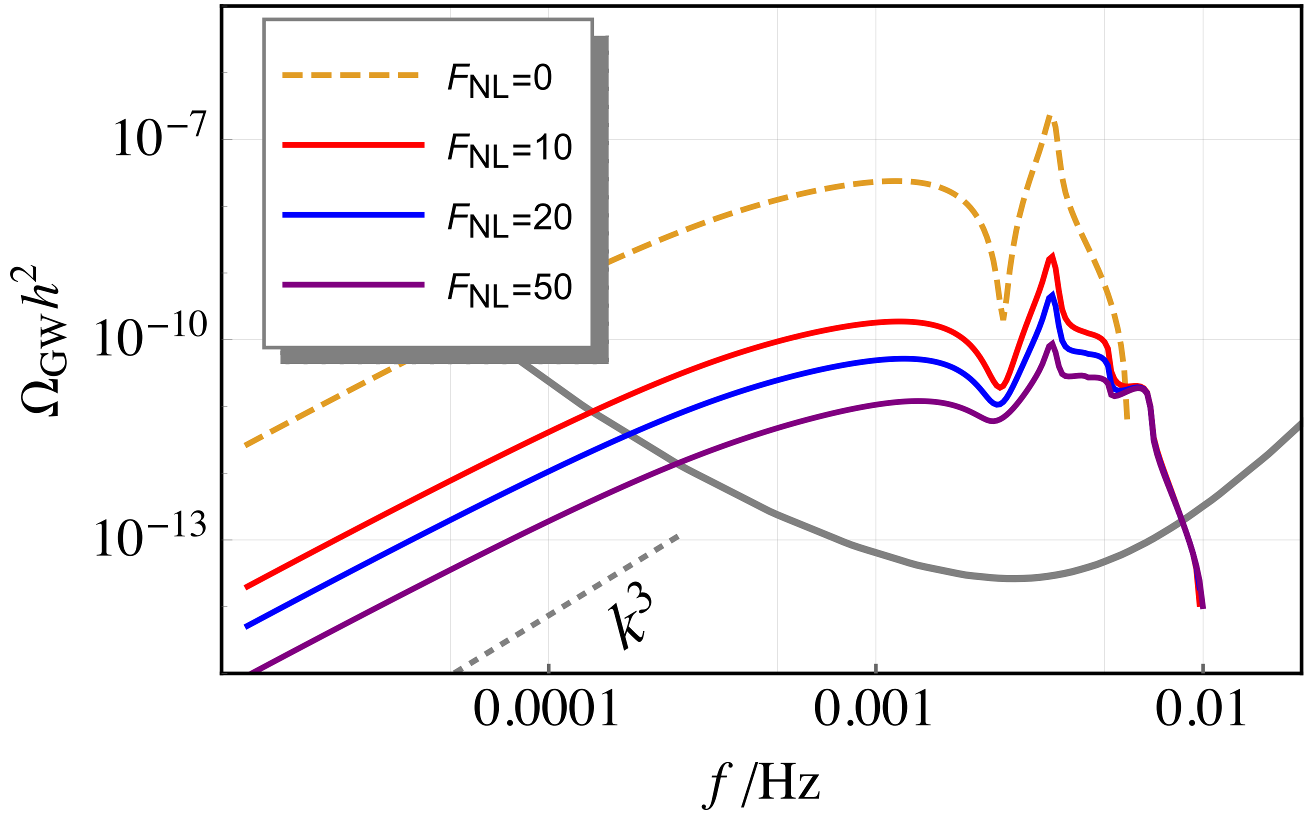


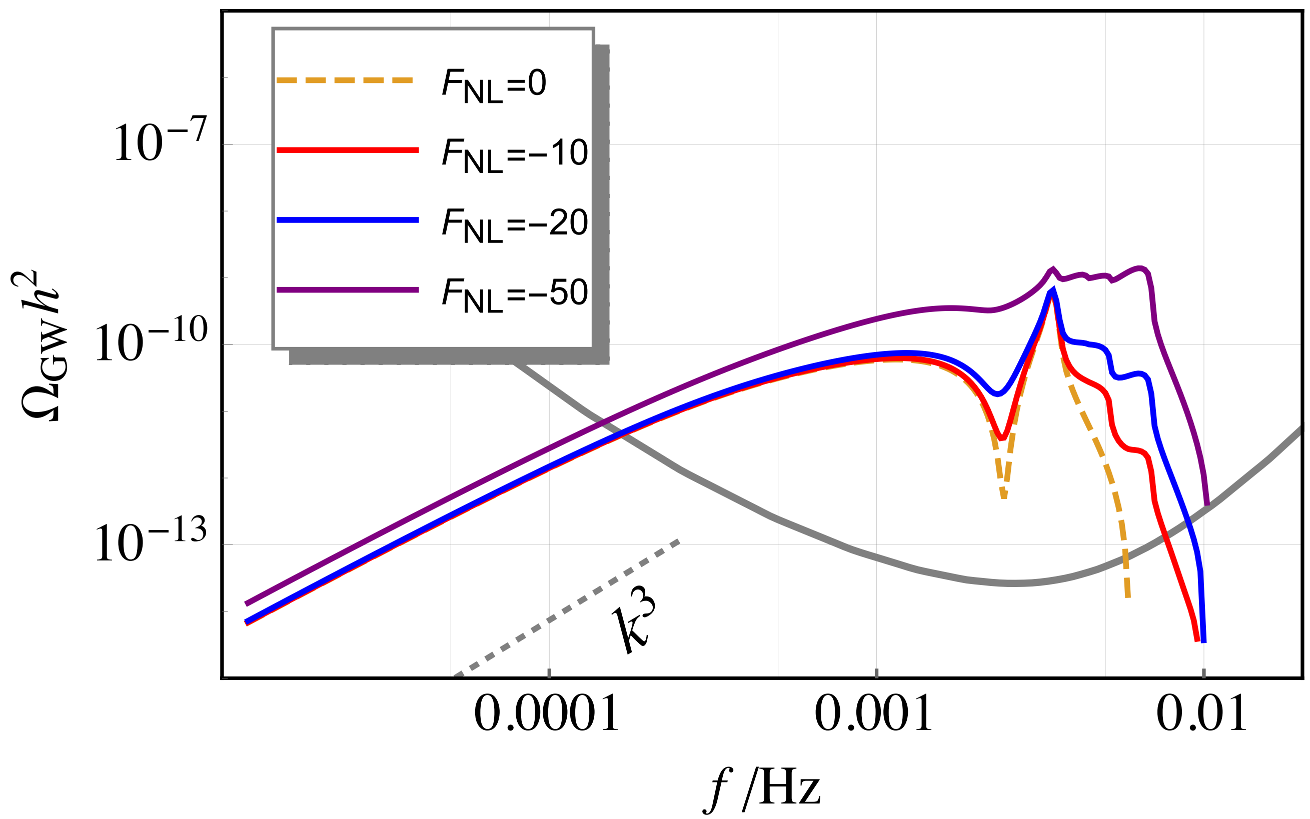
Saito & Yokoyama,arXiv:0912.5317

Effects of non-Gaussianity

Cai, Pi & MS, '18

- Up: $F_{NL} > 0$, and we fix the PBH abundance to be 1.
- Down: $F_{NL} < 0$, and we fix the peak amplitude to be $\mathscr{A}_{\mathscr{R}} = 10^{-2}$
- Gray curve: LISA
- Frequency: PBH window <—> LISA band
- Coincidence, but fortunate for our universe.





Summary

- 2-field inflation models can provide PBH-as-CDM scenario.

$N_1 \sim 35 - 40$ after CMB scale left the horizon

$\Leftrightarrow M_{\rm PBH} \sim 10^{19} - 10^{22}$g

- GWs are generated from large scalar perturbations:

k^3 - slope, multiple peaks, cutoff

- If PBHs = CDM, induced GWs must be detectable by LISA, indep of non-Gaussianity $f_{\rm NL}$.

- Conversely if LISA doesn't detect the induced GWs, it constrains the PBH abundances on mass range $M_{\rm PBH} \sim 10^{19}$ -10^{22}g where no other experiment can explore.