Muon g-2 and Supersymmetry

Summer Institute 2019
Gangneung, Korea
Aug 19, 2019

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The Standard Model (~1967)

↓

Higgs discovery (~2012)

↓

No evidence of new physics

magnetic dipole moment of muon

$$\hat{\mu} = g_s(\frac{e}{2m})\hat{s}$$

electron, muon	$g_s \simeq 2$
proton	$g_s \simeq 5.6$
neutron	$g_s \simeq -3.3$

Lande g factor

Anomalous magnetic dipole moment of muon

$$\mu = (1+a)\frac{e\hbar}{2m} = g_s\frac{e\hbar}{4m}$$

Dirac moment (1928)

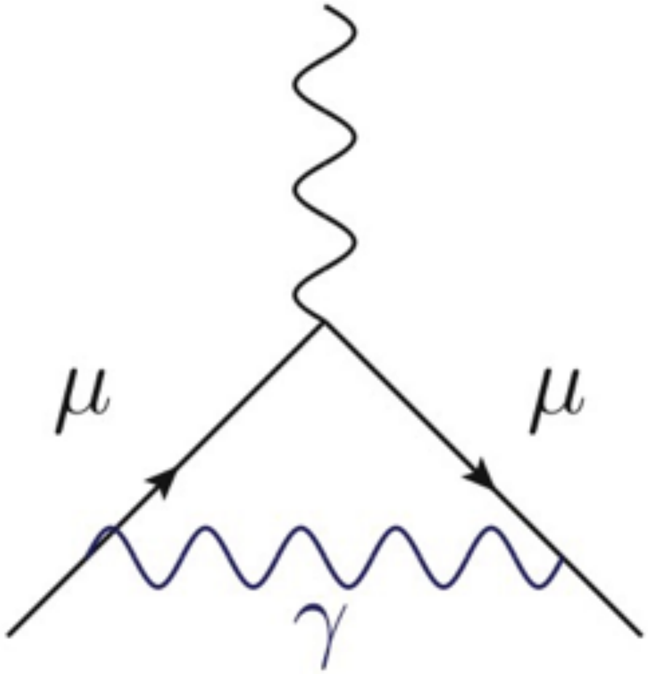
$$1 + a = \frac{g_s}{2} \longrightarrow a = \frac{g_s - 2}{2}$$

Pauli moment

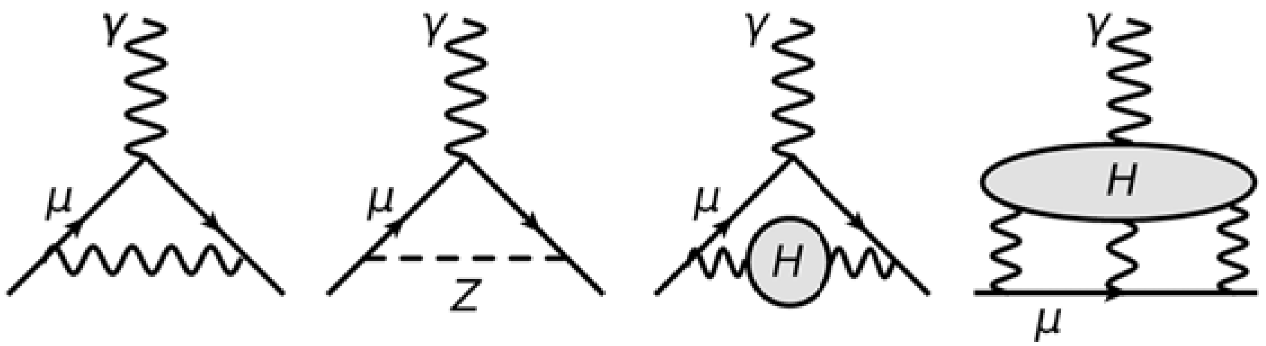
radiative correction (1948)

$$\longrightarrow a = \frac{\alpha}{2\pi} = 0.00116\cdots$$

$\frac{\alpha}{2\pi}$
JULIAN SCHWINGER
2·12·1918 — 7·16·1994
CLARICE CARROL SCHWINGER
9·23·1917 — 1·9·2011



Standard Model contributions to a_μ … updates → 3.6 σ



QED	Weak	HVP	HLbL
Known	Known	Data	Models/Lattice

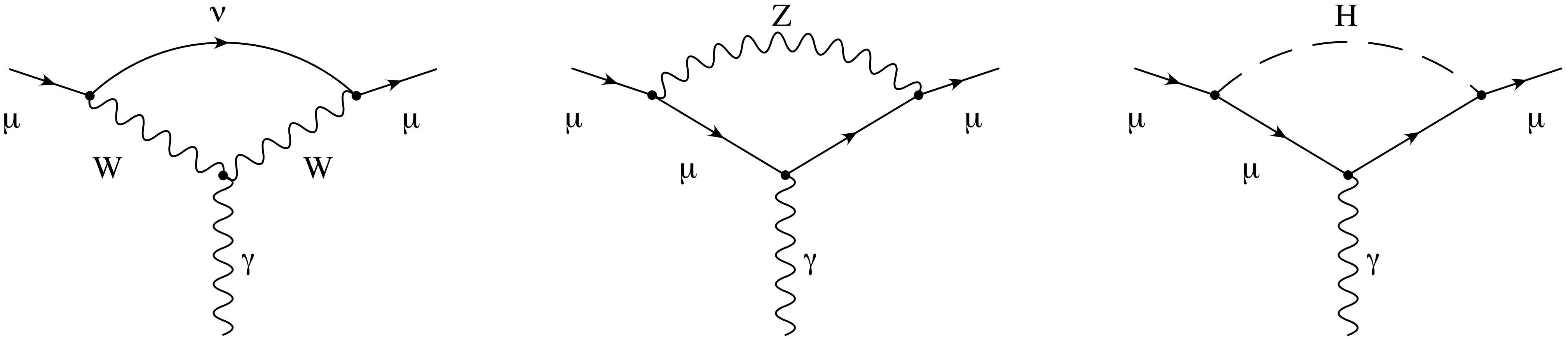
	VALUE ($\times 10^{-10}$) UNITS
QED ($\gamma + \ell$)	$11\,658\,471.8951 \pm 0.0009 \pm 0.0019 \pm 0.0007 \pm 0.0077_\alpha$
HVP(lo) Davier17	692.6 ± 3.33
HVP(lo)KNT2017	693.9 ± 2.6
HVP(ho) KNT2017	-9.84 ± 0.07
HLbL Glasgow	10.5 ± 2.6
EW	15.4 ± 0.1
Total SM Davier17	$11\,659\,181.7 \pm 4.2$
Total SM KNT17	$11\,659\,182.7 \pm 3.7$

← This is a fancy guess; it will improve →

BNL E821 δa_μ(Expt) = ± 6.3

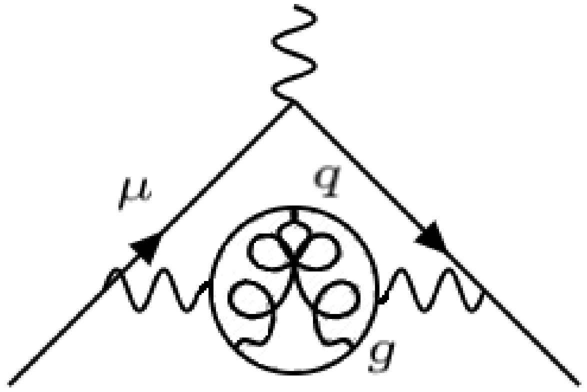
from David Hertzog

Electroweak contribution to muon g-2

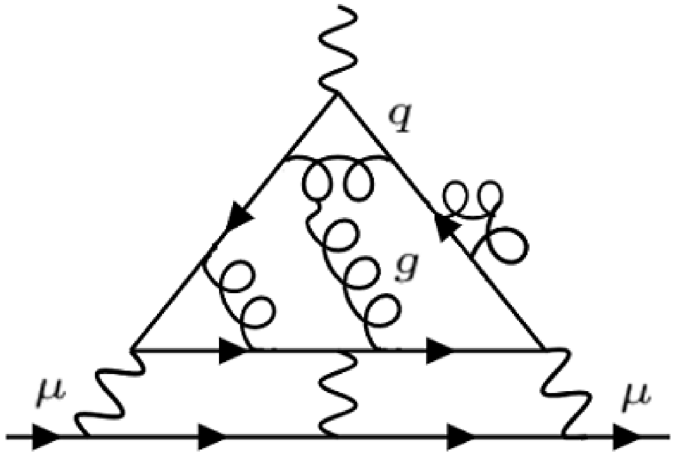


$$a_\mu^{\mathrm{EW}} = 1.5 \times 10^{-9}$$

Contribution	**Value** $[10^{-11}]$	**Date**	**Reference**
a_μ^{QED}	116584718.971 ± 0.07	18 December 2017	[4]
a_μ^{EW}	153.6 ± 1.0	9 September 2013	[7]
a_μ^{HVP}	6846.8 ± 24.2	25 July 2018	[8]
$a_\mu^{H\,LbL}$	98 ± 26	19 September 2016	[9]
a_μ^{SM} total	116591820.4 ± 35.6	10 November 2018	[10]

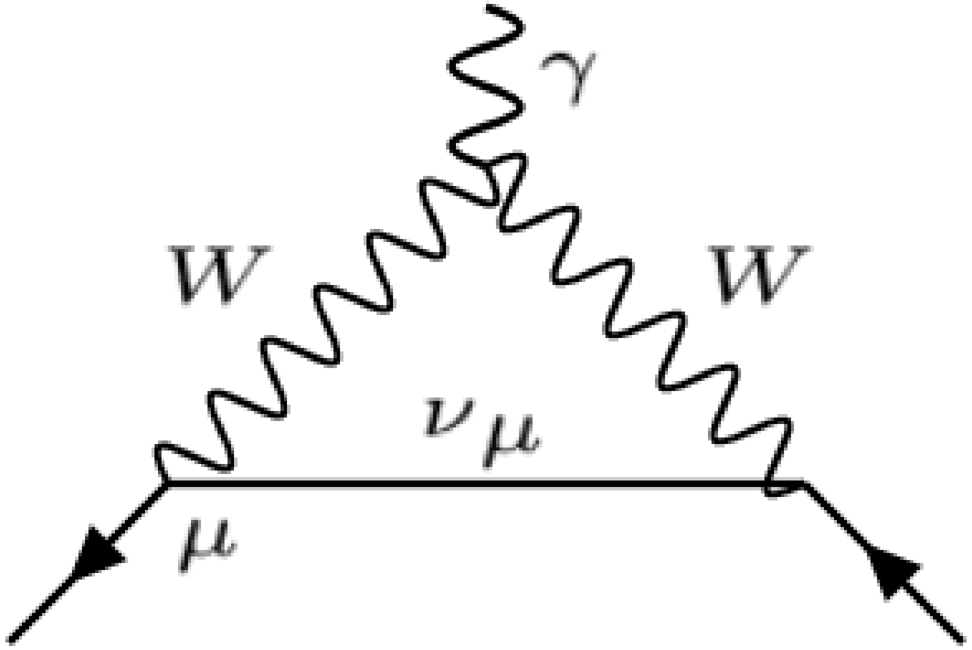


Hadronic (vacuum polarization, light by light)



from David Susic

Electroweak contribution to muon g-2



+ diagram with Z boson

$$a_\mu^{\rm EW} = 1.5 \times 10^{-9}$$

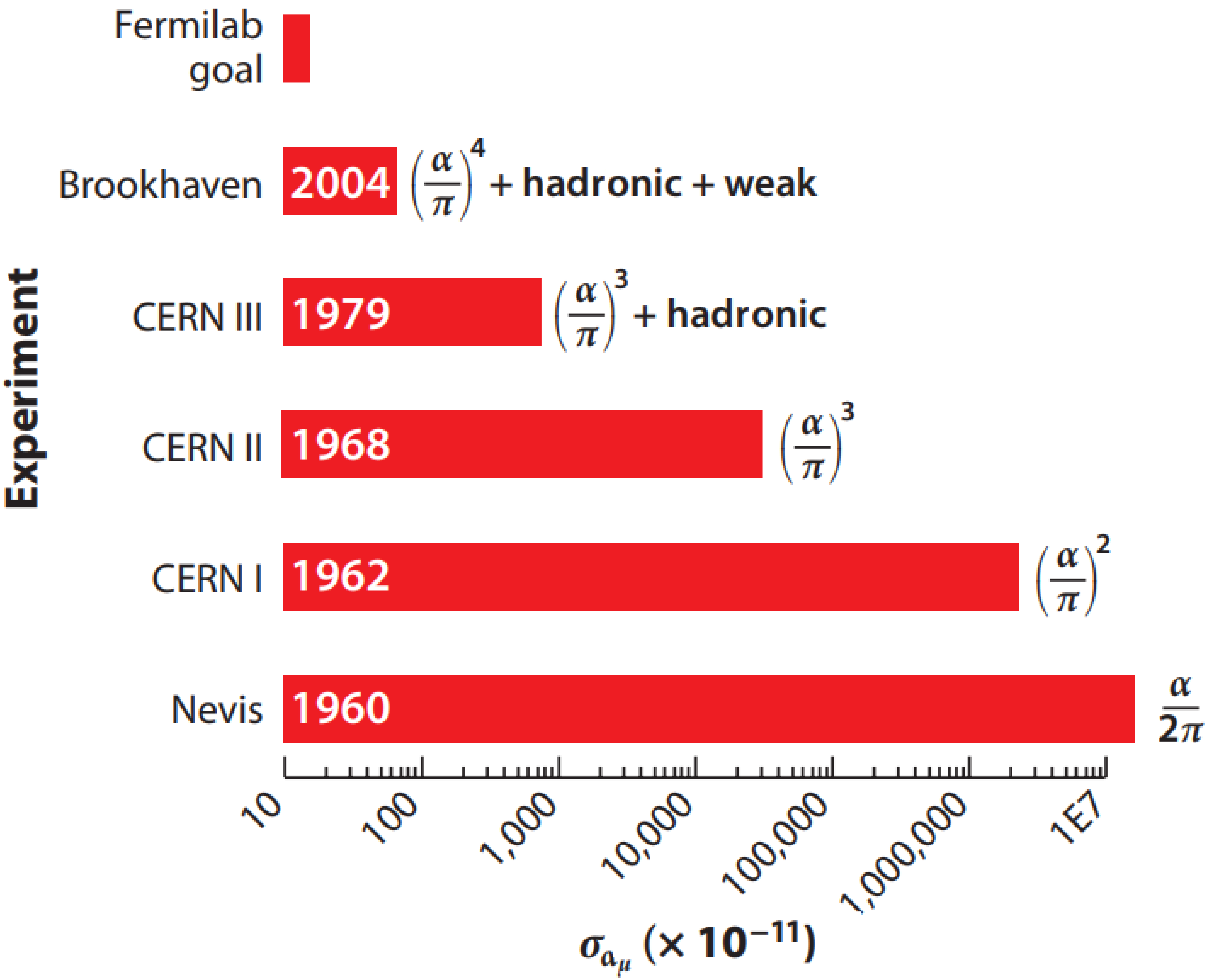
$$\downarrow$$

$$100\ \mathrm{GeV}$$

$$a_\mu^W \sim \frac{g^2}{32\pi^2}\frac{m_\mu^2}{m_W^2} I$$

(I : loop function)

from David Susic



from David Susic

a_μ

exp : 116592091(63) → (16) FNAL E989

th : 116591820(36)

$$\Delta a_\mu = (27 \pm 7) \times 10^{-10}$$

$$\mathbf{3.7\sigma}$$

$$(3.5\sigma \sim 3.9\sigma)$$

FNAL E989 Run I over, Run II soon

$0.54\text{ppm} \rightarrow 0.14\text{ppm}$

$7\sigma \sim 8\sigma$ expected

scattering of the lepton by an external magnetic field

$$e\bar{u}(p_{\rm out})[\gamma^\mu F_1(q^2) + \frac{i}{2m}\sigma^{\mu\nu}q_\nu F_2(q^2)]u(p_{\rm in})A_\mu^{\rm ext}(q^2)$$

Gordon identity $\downarrow$

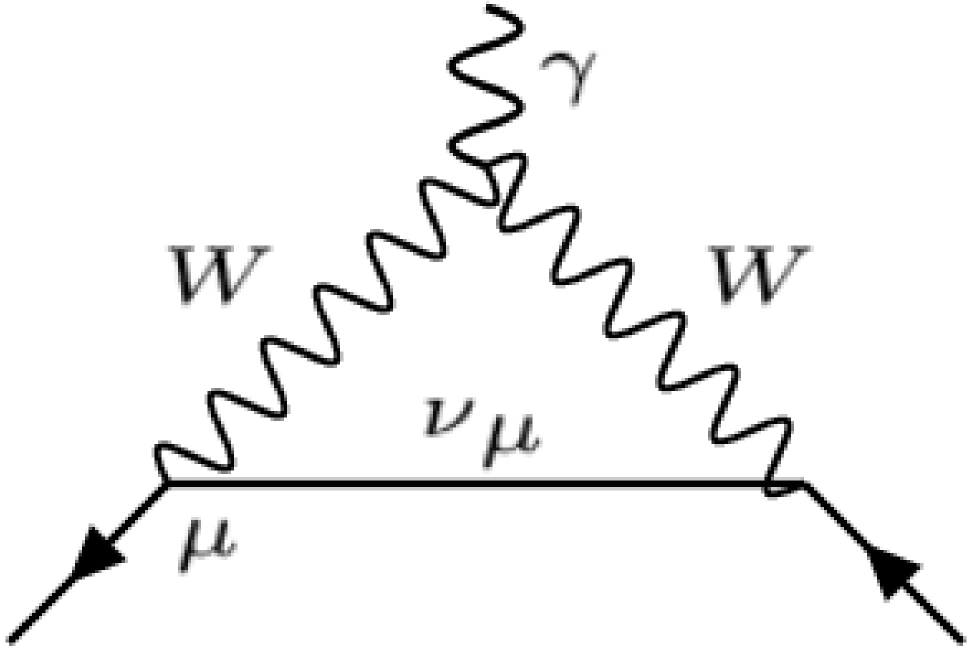
$$e\bar{u}(p_{\rm out})[\frac{p^\mu}{2m}F_1(q^2) + \frac{i}{2m}\sigma^{\mu\nu}q_\nu F_1(q^2) + F_2(q^2)]u(p_{\rm in})A_\mu^{\rm ext}(q^2)$$

$q^2 \to 0$ $\downarrow$

$$-\frac{e}{2m}(1 + F_2(0))\psi^\dagger\vec{\sigma}\cdot\vec{B}\psi$$

$$a_\mu = F_2(0) \longrightarrow \boxed{\frac{e}{2m}aF_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi}$$

Electroweak contribution to muon g-2



+ diagram with Z boson

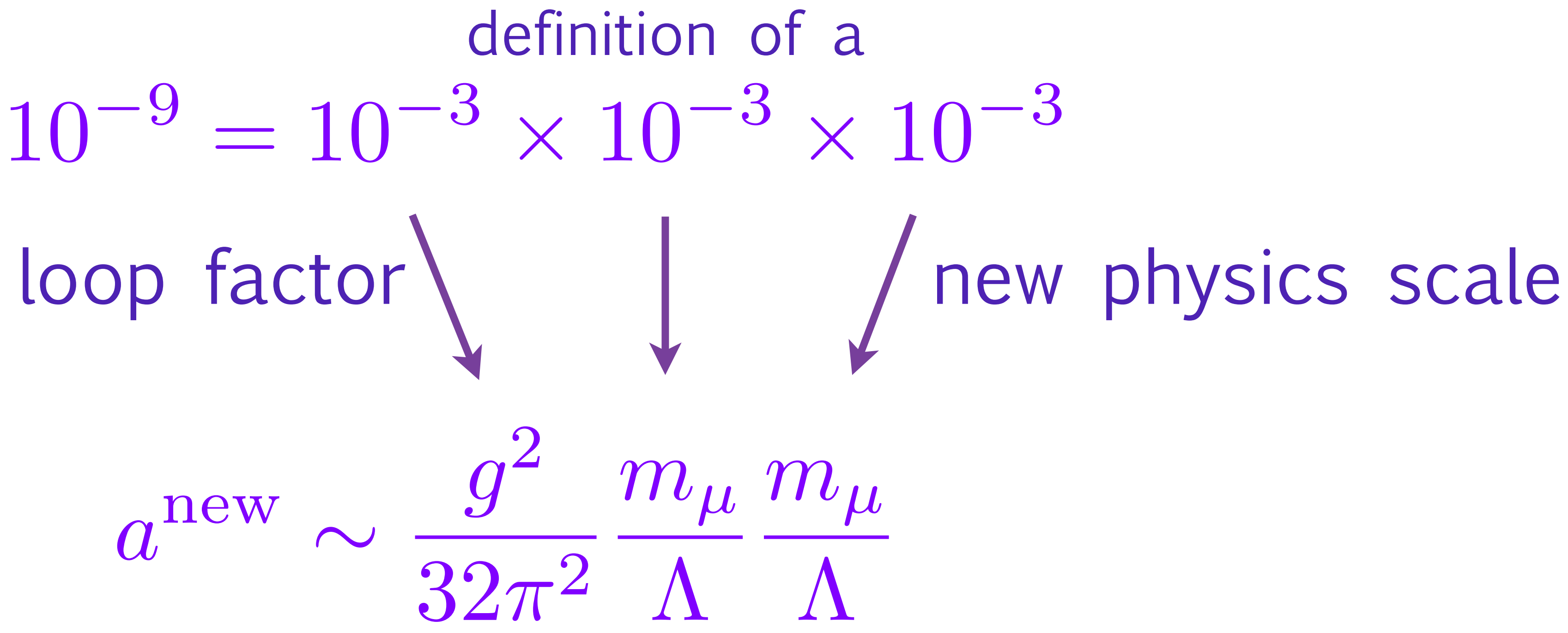
$$2 \times 10^{-9}$$

$$\downarrow$$

100 GeV

$$a_\mu^W \sim \frac{g^2}{32\pi^2}\frac{m_\mu^2}{m_W^2} I$$

$$\frac{eg^2}{32\pi^2}\frac{m_\mu}{m_W^2} F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi$$



definition of a

$$10^{-9} = 10^{-3} \times 10^{-3} \times 10^{-3}$$

loop factor

new physics scale

$$a^{\text{new}} \sim \frac{g^2}{32\pi^2} \frac{m_\mu}{\Lambda} \frac{m_\mu}{\Lambda}$$

to explain the anomaly of muon g-2

$\Lambda \sim 100\ \text{GeV}$ for $\dfrac{eg^2}{32\pi^2} \dfrac{m_\mu}{\Lambda^2} F_{\mu\nu} \bar{\psi} \sigma^{\mu\nu} \psi$

e.g. smuon and gaugino/higgsino in supersymmetry

frequently cited expression for
supersymmetry smuon diagram

$$a_\mu^{\mathrm{SUSY}} = \pm 13 \times 10^{-10} (\frac{100\ \mathrm{GeV}}{M_{\mathrm{SUSY}}})^2 \tan\beta$$

typically 100 GeV to 500 GeV smuon

large $\tan\beta$ ⟶ $1\ \mathrm{TeV}$ smuon

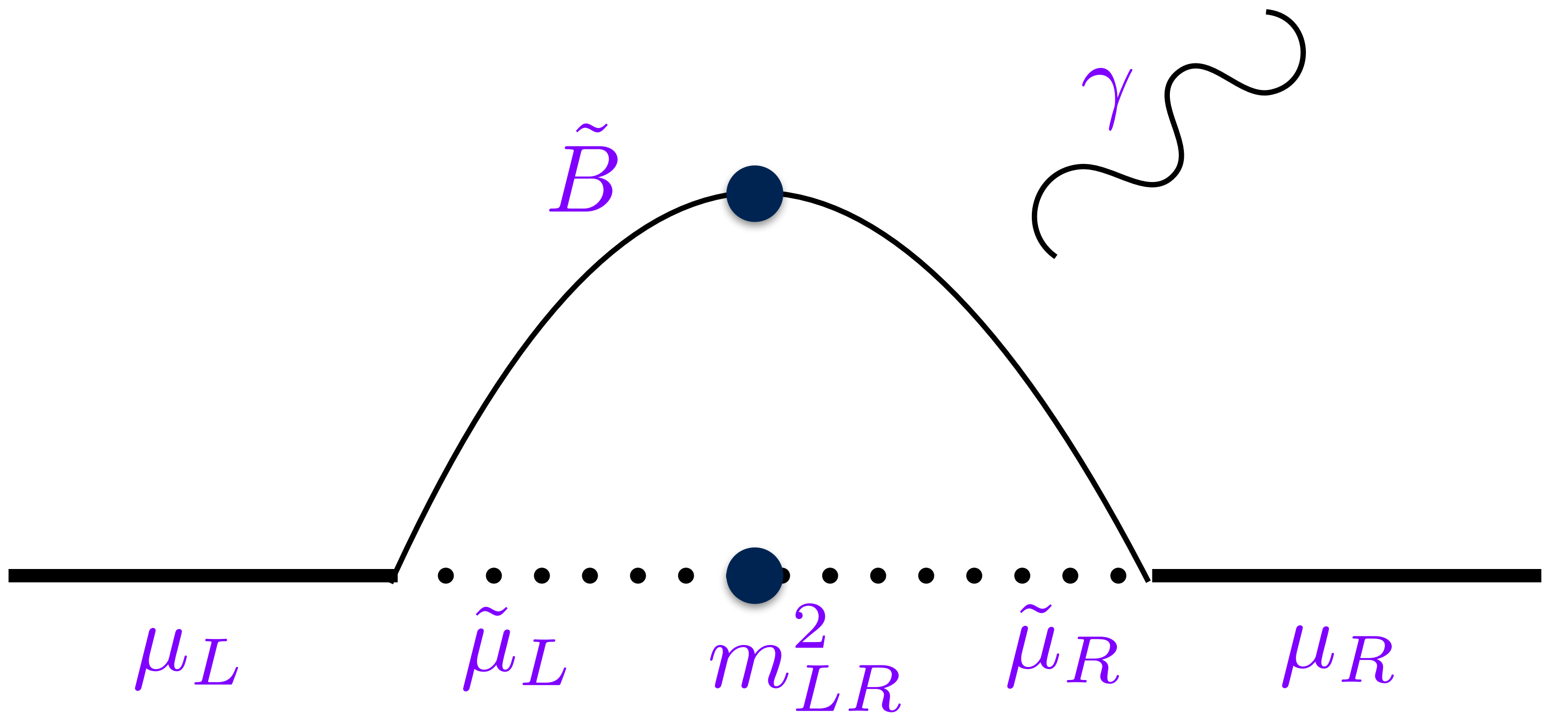
can be consistent with muon g-2 anomaly

Is the Standard Model ruled out by muon g-2?

Is split supersymmetry ruled out by muon g-2?

What is the largest smuon mass or bino mass
to be consistent with muon g-2?

(For large mu, wino and higgsino diagrams are suppressed by mu)



$$m^2_{LR} = m_\mu(A - \mu \tan\beta)$$

Large mixing or maximal mixing of smuon would be interesting as there is no suppression of muon mass

$$A \sim \frac{M_{\rm SUSY}}{m_\mu} M_{\rm SUSY} \quad \longrightarrow \quad m^2_{LR} \sim M^2_{\rm SUSY}$$

smuon mass matrix

$$m_L^2 = m_R^2 \qquad r = \frac{|m_{LR}^2|}{m_L^2}$$

$$\begin{pmatrix} m_L^2 & m_{LR}^2 \\ m_{LR}^2 & m_R^2 \end{pmatrix} = \frac{M^2}{1-r}\begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}$$

smuon mass eigenvalue

$$M^2, (\frac{1+r}{1-r})M^2$$

bino mass $M_1 = M$

$M_1 = m_{\tilde{\mu}_1} \ll m_{\tilde{\mu}_2}$ for r close to 1

Muon suppression once or twice?

$$\Delta a_\mu(\tilde{\mu}_L, \tilde{\mu}_R, \tilde{B}) = \frac{\alpha_Y}{4\pi} \frac{m_\mu^2 M_1 \mu}{m_{\tilde{\mu}_L}^2 m_{\tilde{\mu}_R}^2} \tan\beta \cdot f_N\left(\frac{m_{\tilde{\mu}_L}^2}{M_1^2}, \frac{m_{\tilde{\mu}_R}^2}{M_1^2}\right)$$

mass insertion approximation

$$= \frac{\alpha_Y}{4\pi} m_\mu \frac{M_1 m_{\tilde{\mu}_{LR}}^2}{m_{\tilde{\mu}_L}^2 m_{\tilde{\mu}_R}^2} f_N\left(\frac{m_{\tilde{\mu}_L}^2}{M_1^2}, \frac{m_{\tilde{\mu}_R}^2}{M_1^2}\right),$$

$$\frac{m_\mu}{(100\ \mathrm{GeV})^2} = \frac{1}{M}$$

smuon $10\ \mathrm{TeV} \sim 100\ \mathrm{TeV}$ can be possible

$$M \sim (\frac{100\ \mathrm{GeV}}{m_\mu}) 100\ \mathrm{GeV} \sim 100\ \mathrm{TeV}$$

Maximal mixing of smuon
(when r is close to 1)

$$\underset{\substack{\uparrow \\ 10^{-9}}}{a_\mu(\mathrm{SUSY})} = \underset{\substack{\uparrow \\ 10^{-4}}}{\frac{g_1^2}{192\pi^2}} \underset{\substack{\uparrow \\ 10^{-5}}}{\frac{m_\mu}{M}} I(r)$$

$$a_\mu(\mathrm{SUSY}) = 10^{-9} \quad \longrightarrow \quad M \sim 5 \text{ TeV}$$

$$M \sim \frac{5}{\Delta a_\mu \times 10^9} \text{ TeV}$$

Threshold correction to muon Yukawa coupling after integrating out smuon

$$y_{\rm UV} v_d (1+\Delta) = m_\mu \qquad y_\mu^{\rm IR} = y_\mu^{\rm UV}(1+\Delta)$$

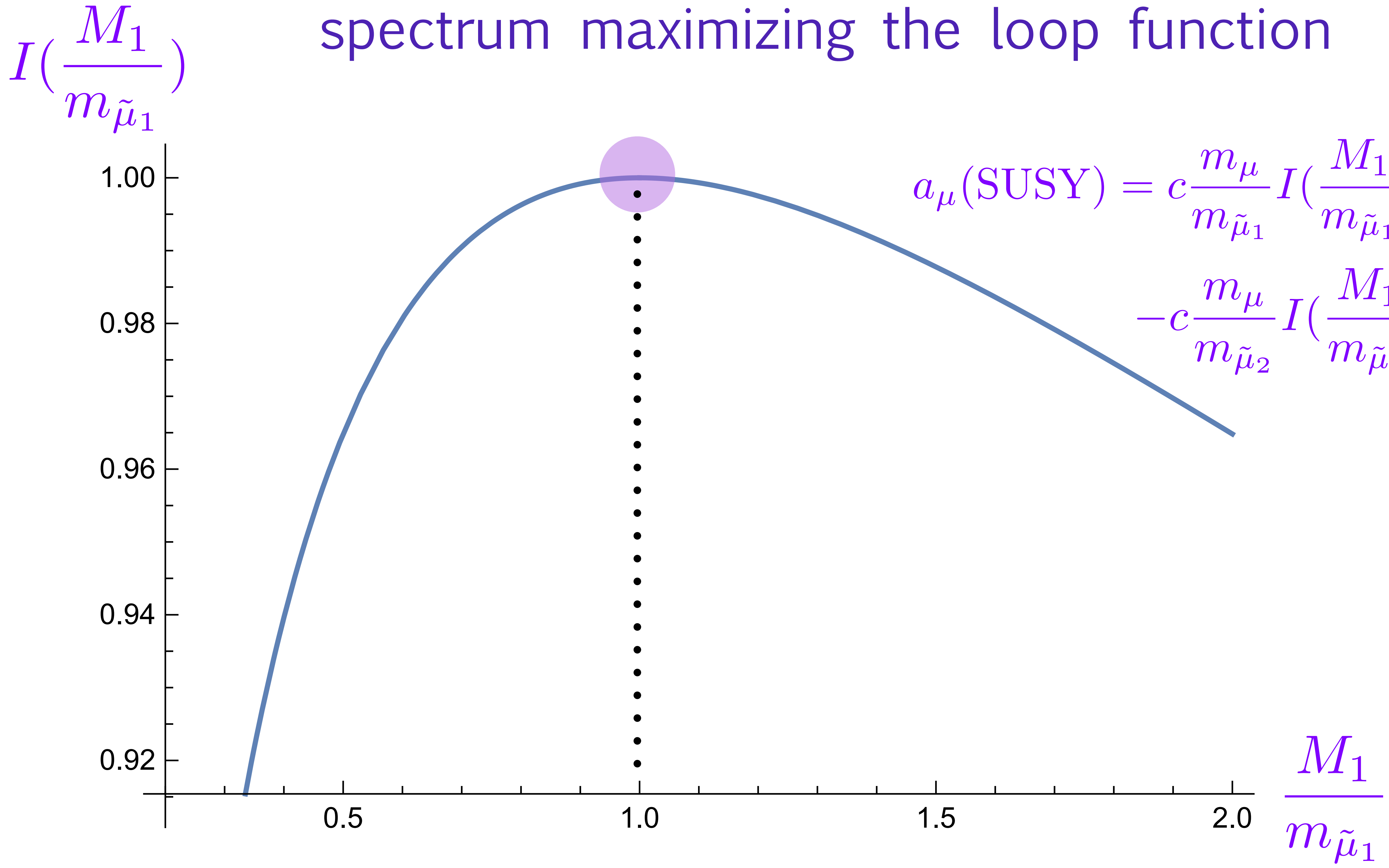
$$m_{LR}^2 = \quad y_{\rm UV} v_d A = \quad \frac{m_\mu}{1+\Delta} A.$$

$$A < 0, \Delta > 0, \Delta a_\mu > 0$$

$$|\Delta| > 1 \to A > 0, \Delta < 0, 1+\Delta < 0, \Delta a_\mu > 0$$

both sign of A (or mu) allowed

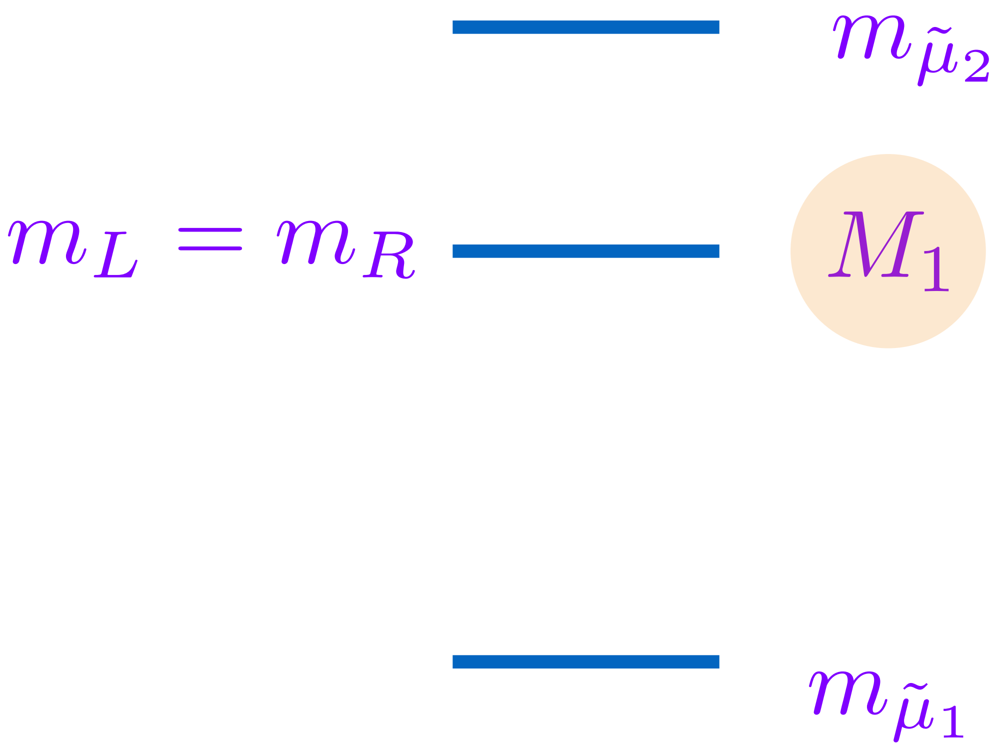
spectrum maximizing the loop function



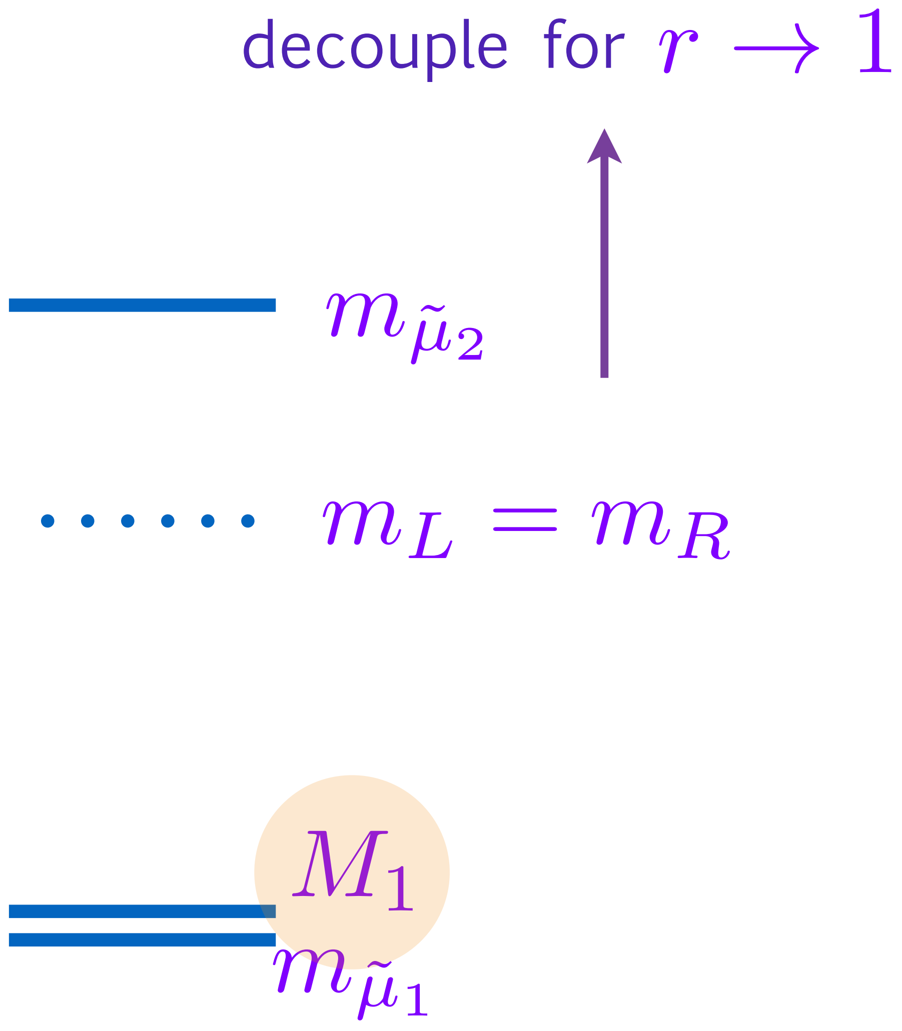
$$a_\mu(\mathrm{SUSY}) = c\frac{m_\mu}{m_{\tilde{\mu}_1}} I(\frac{M_1}{m_{\tilde{\mu}_1}})$$

$$-c\frac{m_\mu}{m_{\tilde{\mu}_2}} I(\frac{M_1}{m_{\tilde{\mu}_2}})$$

spectrum maximizing the loop function



previous one was not the optimal spectrum



bino=light smuon

$$m_{\tilde{\mu}_1} \qquad M$$

$$M_1 \qquad M$$

$$m_{\tilde{\mu}_2} \qquad \sqrt{\frac{1+r}{1-r}}M$$

decouple for $r \to 1$

light smuon is as heavy as $3(4.5)$ TeV for $1(2)\sigma$

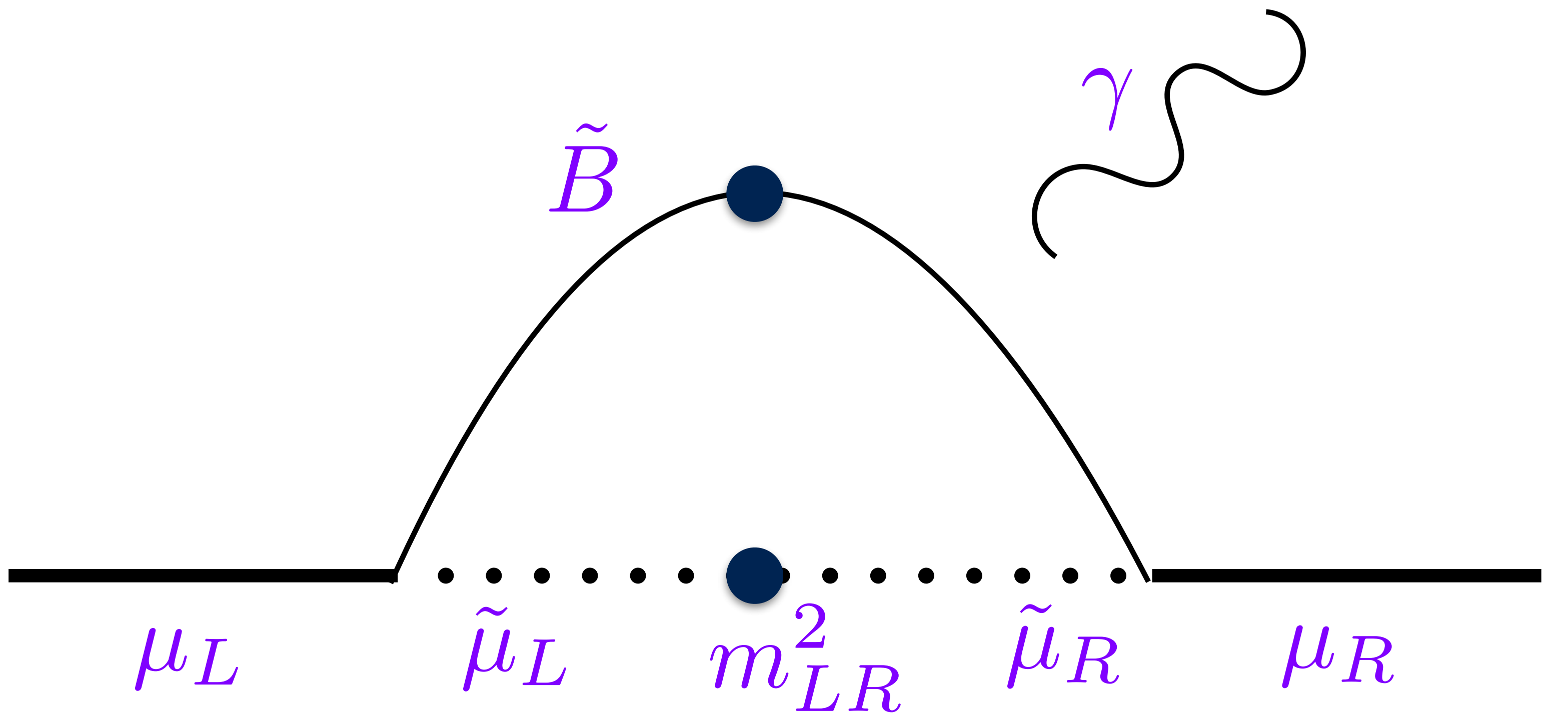
different from
1309.3065
Endo et al

$$m_{\tilde{\mu}_1} \qquad \sqrt{1-r}M$$

$$M_1 \qquad M$$

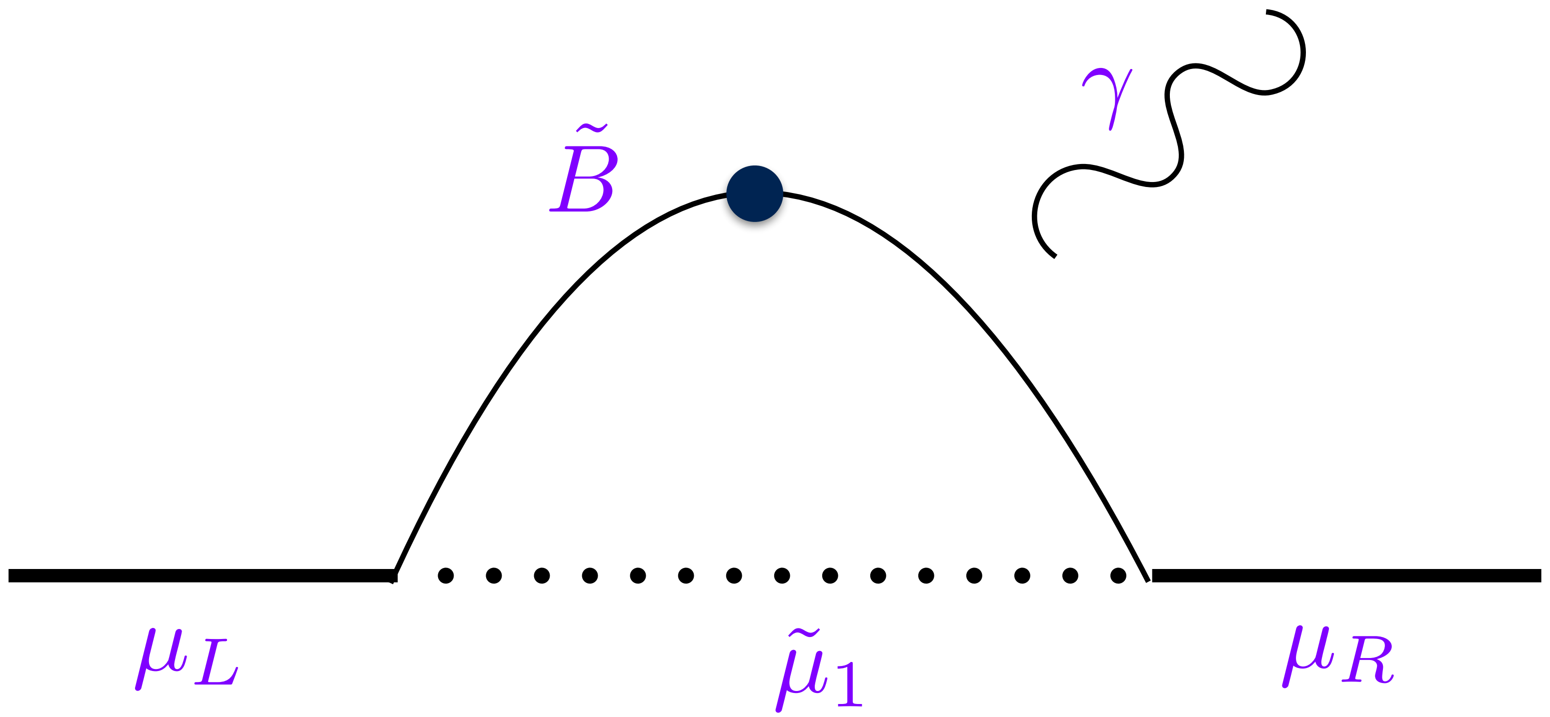
$$m_{\tilde{\mu}_2} \qquad \sqrt{1+r}M$$

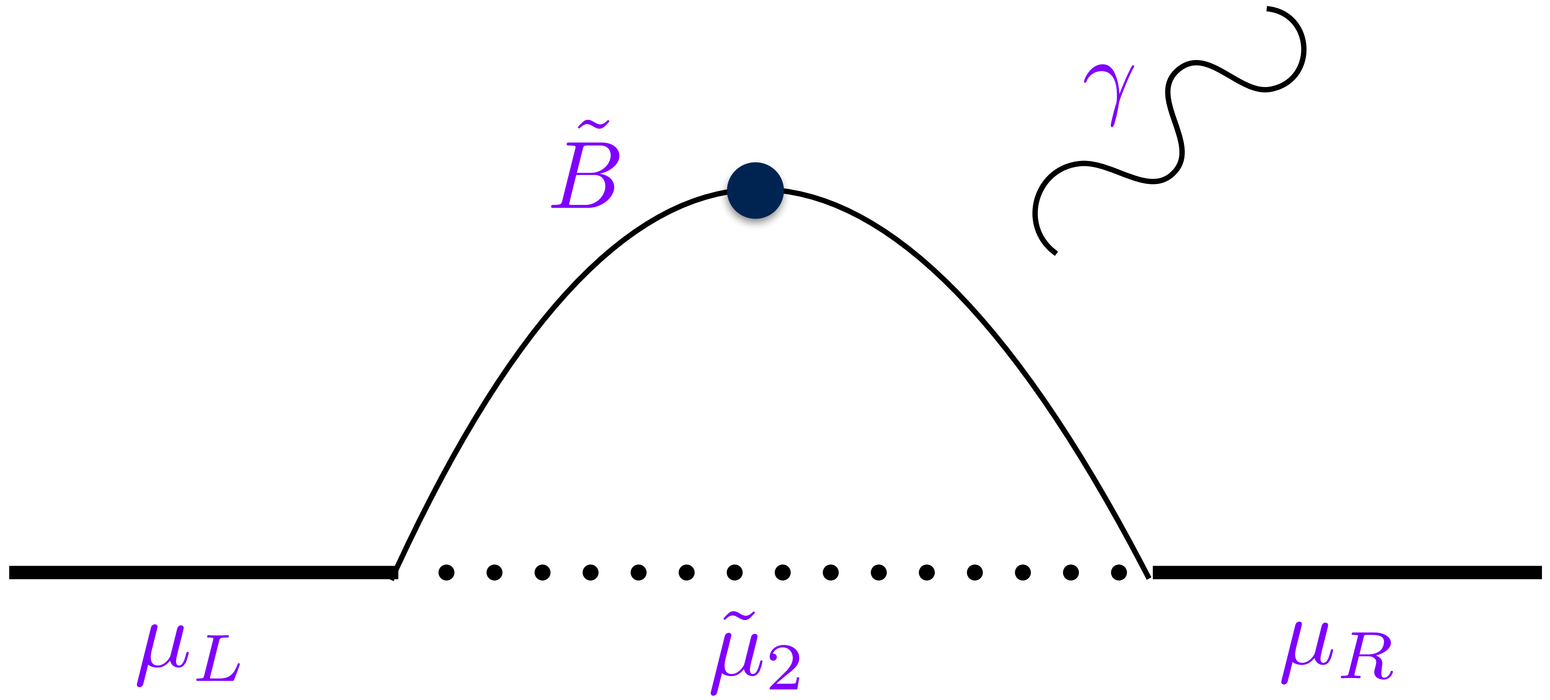
light smuon is as heavy as $1.4(1.9)$ TeV for $1(2)\sigma$



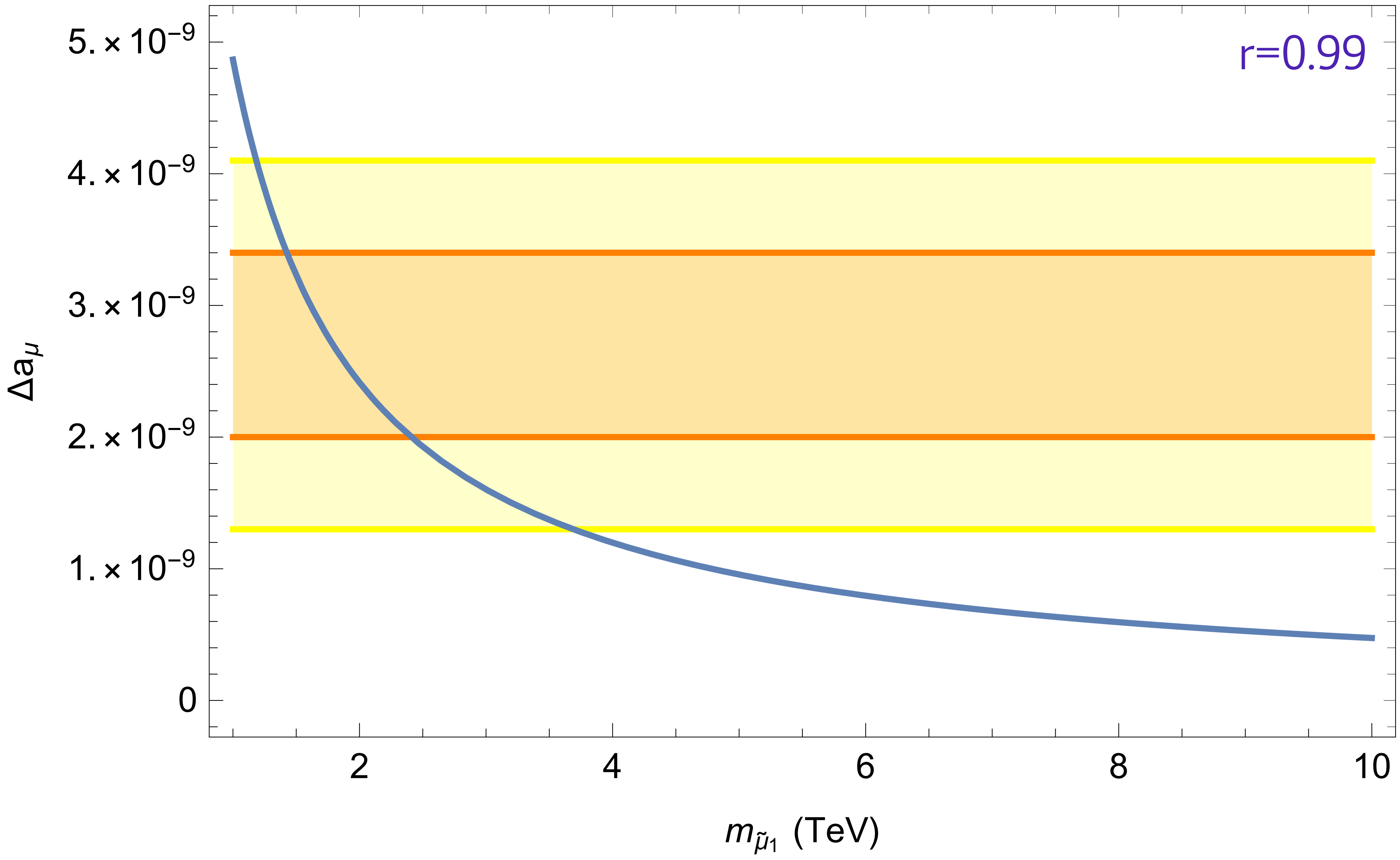
When r is small, the mass insertion approximation is good

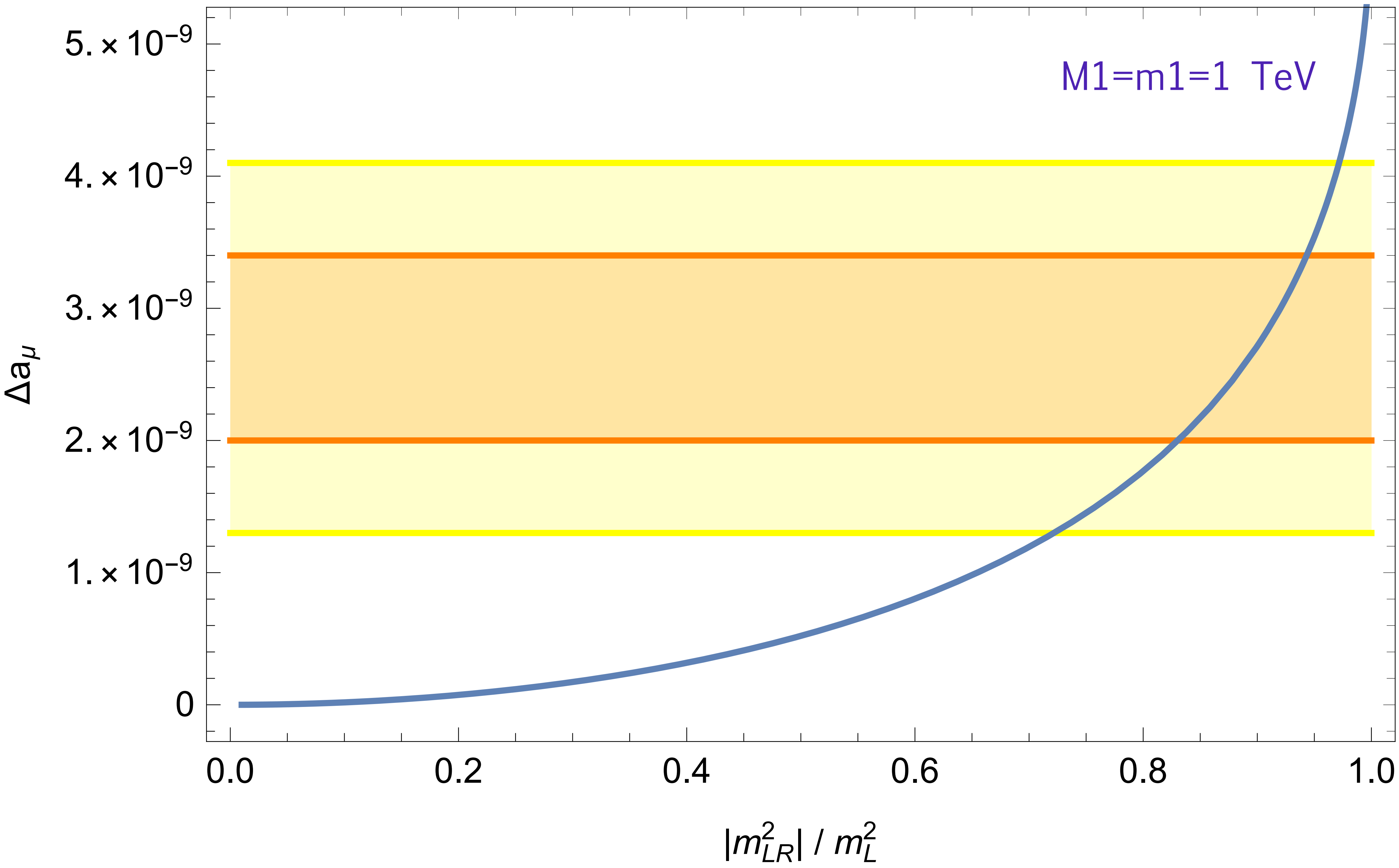
When r is close to 1, heavy smuon diagram decouples

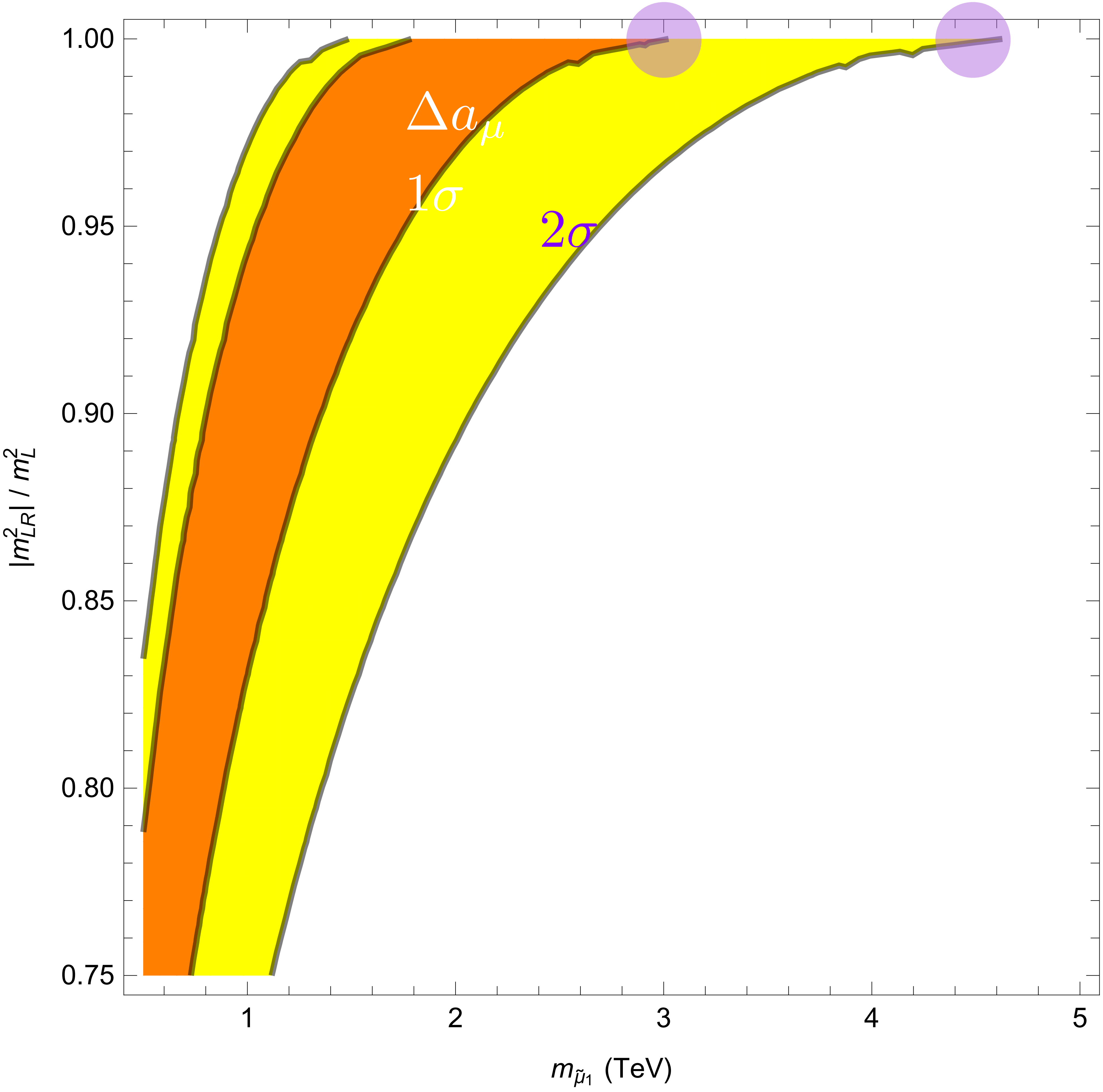




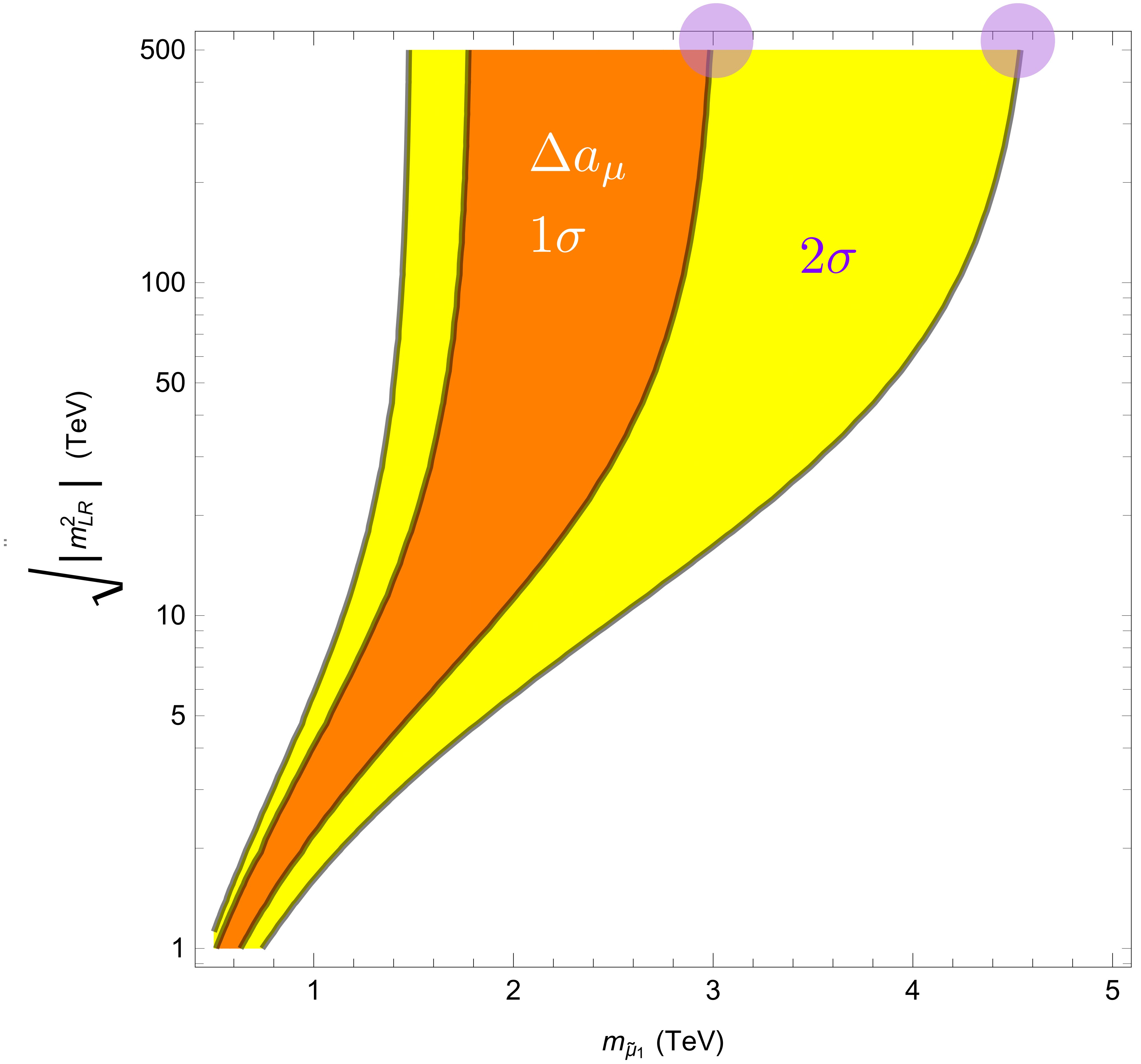
the canceling contribution from heavy smuon is more suppressed as heavy smuon becomes heavier


r=0.99
Δa_μ
$5.\times10^{-9}$
$4.\times10^{-9}$
$3.\times10^{-9}$
$2.\times10^{-9}$
$1.\times10^{-9}$
0
2
4
6
8
10
$m_{\tilde{\mu}_1}$ (TeV)


M1=m1=1 TeV
Δa_μ
$5.\times10^{-9}$
$4.\times10^{-9}$
$3.\times10^{-9}$
$2.\times10^{-9}$
$1.\times10^{-9}$
0
0.0
0.2
0.4
0.6
0.8
1.0
$|m^2_{LR}|\ /\ m^2_L$

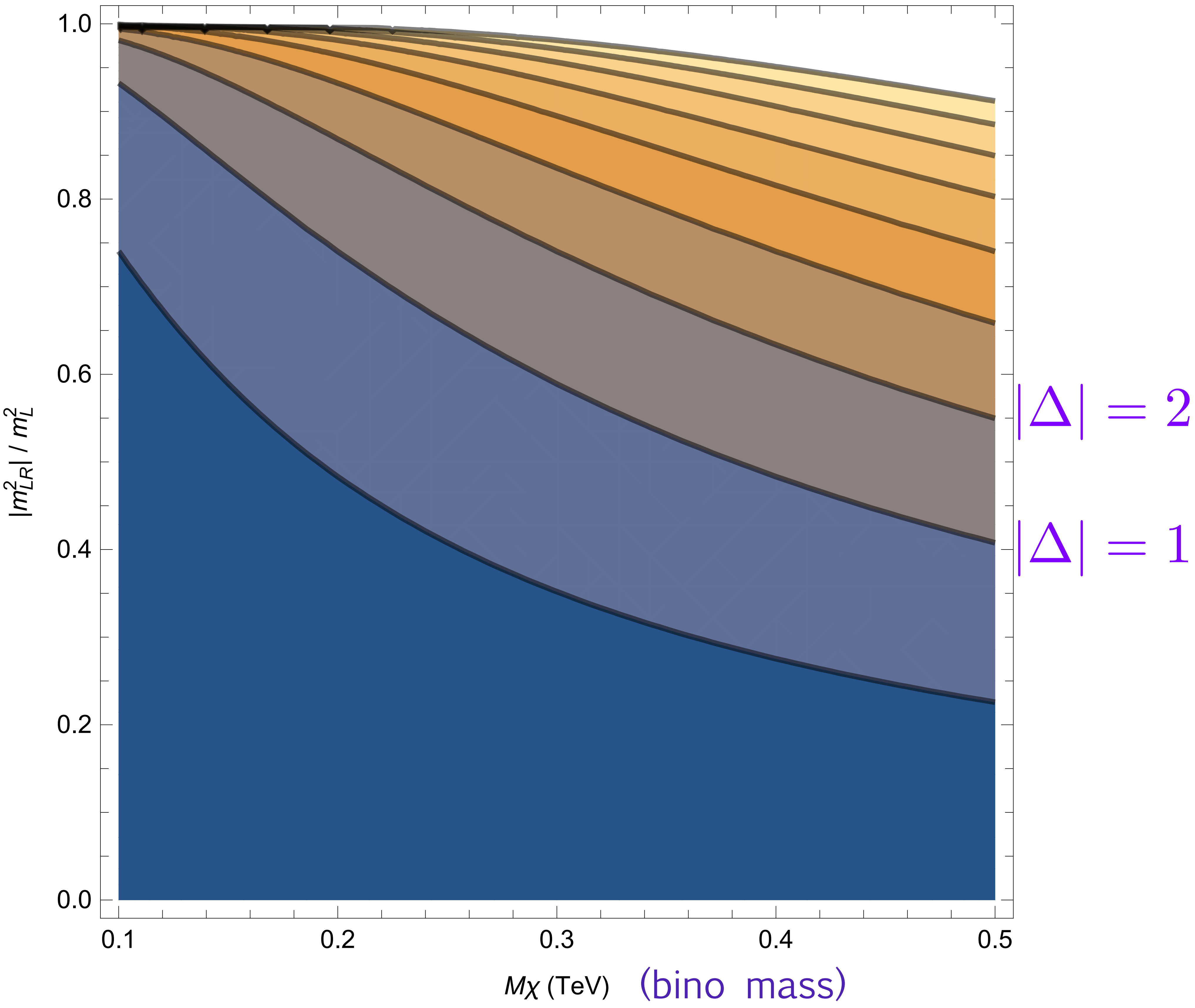


close to
maximal mixing



$$\frac{m_{\tilde{\mu}_2}}{m_{\tilde{\mu}_1}} \sim 140$$

Threshold correction to muon Yukawa coupling



Vacuum stability

When Hu and Hd does not mix
and Hd is very heavy,
the electroweak vacuum can live long enough

Difference of large A and large mu (tan beta)

Detailed study is in progress

Summary

Maximal smuon mixing allows the explanation of muon g-2 anomaly in terms of heavy smuon (a few TeV)

light smuon (and bino) is as heavy as $3(4.5)$ TeV for $1(2)\sigma$ explanation of Δa_μ

No discovery of smuon up to 3 or 4 TeV does not rule out the supersymmetric explanation of muon g-2 anomaly

$$a_\mu(\mathrm{SUSY}) \propto \frac{m_\mu^2}{M^2}$$

one power from the definition,
the other power from smuon mass mixing

smuon mass mixing needs not be suppressed

$$M \sim \frac{M_Z^2}{m_\mu}$$

heavy smuon can explain muon g-2 anomaly

$$\underset{\overset{\uparrow}{10^{-9}}}{a_\mu(\mathrm{SUSY})} = \underset{\overset{\uparrow}{10^{-4}}}{\frac{g_1^2}{192\pi^2}} \underset{\overset{\uparrow}{10^{-5}}}{\frac{m_\mu}{M}} I(r)$$