Positivity Constraints on Vector Boson Scattering

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Based on 1808.00010 with Shuang-Yong Zhou

TH framework

- We study the anomalous Quartic Gauge-boson Couplings (aQGC) parametrized by 18 dim-8 operators. [O. J. P. Eboli et al. '06]

$$
\begin{aligned}
O_{S,0} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi] \\
O_{S,1} &= [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi] \\
O_{S,2} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi] \\
O_{M,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] \\
O_{M,1} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] \\
O_{M,2} &= \left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] \\
O_{M,3} &= \left[\hat{B}_{\mu\nu}\hat{B}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] \\
O_{M,4} &= \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\mu\Phi\right] \times \hat{B}^{\beta\nu} \\
O_{M,5} &= \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\nu\Phi\right] \times \hat{B}^{\beta\mu} (+h.c.) \\
O_{M,7} &= \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}\hat{W}^{\beta\mu} D^\nu\Phi\right]
\end{aligned}
$$

$$
\begin{aligned}
O_{T,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right] \\
O_{T,1} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right] \\
O_{T,2} &= \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right] \\
O_{T,5} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
O_{T,6} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\mu\beta}\hat{B}^{\alpha\nu} \\
O_{T,7} &= \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \\
O_{T,8} &= \hat{B}_{\mu\nu}\hat{B}^{\mu\nu} \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
O_{T,9} &= \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta} \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha},
\end{aligned}
$$

- TGC and QGC are fully correlated at dim-6. To parametrize independent QGC couplings not constrained by TGC measurements, we need the above operators.

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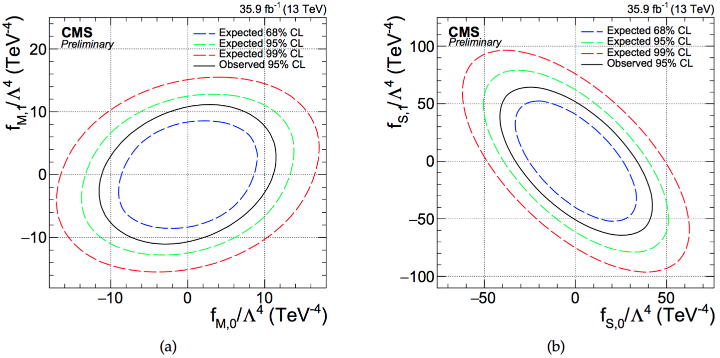


Figure 5: Two-dimensional observed 95% CL limits (solid contour) and expected 68 and 95% CL limits (dashed contour) on the selected aQGC parameters. The values of couplings outside of contours are excluded at the corresponding confidence level.

What are positivity constraints

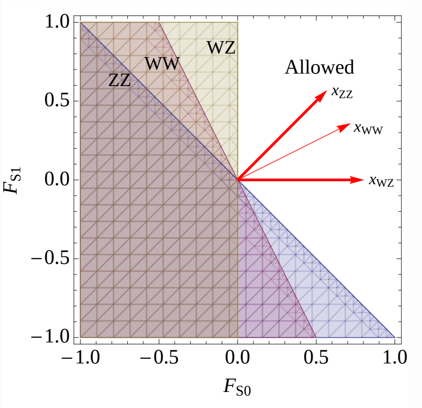
- In this work, we derive a set of "positivity constraints" on the 18 operator coefficients, by assuming that the SMEFT has a UV completion.
- What are "positivity constraints":
  - A linear combination of coefs. $(F_{S,0}, F_{S,1}, F_{S,2}, \cdots)$ must be positive.
  - Or equivalently, consider a vector $\vec{c} = (F_{S,0}, F_{S,1}, F_{S,2}, \cdots)$. Positivity says that $\vec{c}$ has to be positive upon projection on a certain direction $\vec{x}_i$, i.e.

$$\vec{c} \cdot \vec{x}_i \geq 0$$

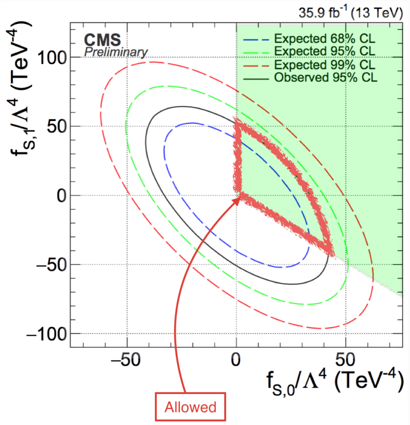
  - $\vec{x}_i$ come from the requirements that the VBS amplitudes $(WW, ZZ, \ldots)$ satisfy the fundamental principles of QFT (analyticity, unitarity, etc.), i.e. we have $\vec{x}_{WW}, \vec{x}_{ZZ}, \vec{x}_{WZ}, \ldots$
    - But the resulting constraint apply regardless of the amplitude from which it is derived.

Implications on EXP results

$\vec{x}_{WW,ZZ,WZ}$ and positivity bounds



Combined with measurements



Outline

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Positivity approach

- First established in [A. Adams et al. JHEP '06]: dispersion relation + optical theorem, forward 2-to-2 scattering.
- Non-forward generalization: [C. de Rham et al. Phys.Rev.D '17], [C. de Rham et al. JHEP '18]
- Application in collider pheno:
  - ZZ and $Z\gamma$: *[B. Bellazzini and F. Riva '18]*
  - Implications in Higgs physics under ceratin assumptions: *[I. Low et al. '09]* *[A. Falkowski et al. '12]*
- In general the approach has strong implication on SMEFT dim-8 operators, which are important for the interpretation of VBS, so we should understand the constraints.

Analytic dispersion relation

- As an simplified version: consider the forward scattering ($t = 0$) of two identical particles with mass m, with possible heavy new physics.

  (see [C. Cheung and G. N. Remmen '16] for a quick overview)

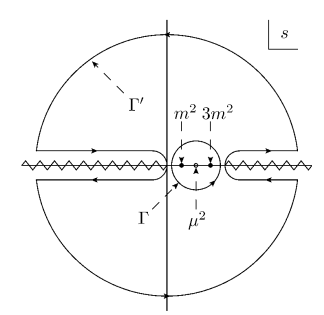
- If the UV completion exists, the amplitude $M(s, t = 0)$
  - is analytic and
  - satisfies Froissart unitarity bound $M(s, 0) \leq \mathcal{O}(s \ln^2 s)$.

Analytic dispersion relation

- Consider the contour integral:

$$f = \frac{1}{2\pi i}\oint_\Gamma ds \frac{M(s,0)}{(s-\mu^2)^3}$$

- Deform Γ to Γ' and notice that boundary contribution vanishes due to Froissart bound:



$$f = \frac{1}{2\pi i}\oint_\Gamma ds \frac{M(s,0)}{(s-\mu^2)^3} = \frac{1}{2\pi i}\left(\int_{-\infty}^{0} + \int_{4m^2}^{\infty}\right) ds \frac{\mathrm{Disc}M(s,0)}{(s-\mu^2)^3}$$

i.e. [sum of residues at low energy] =

[discontinuity along +x axis] + [discontinuity along -x axis]

- Note that BSM (above Λ) enters the discontinuity, as poles (tree level) or branch cuts (heavy loops).

Derivation of positivity

- discontinuity along $\pm$x axis must positive, because of optical theorem (disc. = xsec >0) (plus crossing symmetry for $-x$)
- $\Rightarrow$ sum of residues at low energy is positive.

We started with the amplitude in the full theory, but have reached a conclusion that only involves low energy, which can be computed in SMEFT:

$$\boxed{\text{sum of residues at low energy}} = \frac{\mathrm{d}^2 M(s,0)}{\mathrm{d}s^2} = \sum_i c_i^{(8)} x_i + \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j} > 0$$

- Conclusion: the above positivity condition must be satisfied, if
  - SMEFT has a UV completion, that satisfies unitarity, Lorentz symmetry, is analytic.
  - At low energy, the SMEFT is valid and tree level calculation is a good approximation, which anyway need to be assumed in a real measurement.

Dim-6 contributions can be removed

$$\sum_i c_i^{(8)} x_i \geq -\sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j}$$

- In general, we expect dim-6 to be better constrained by other processes.
- But in any case, dim-6 doesn't matter, because by explicit calculation the RHS is positive.
- E.g. from WZ scattering:

$$\text{R.H.S} \propto a_3^2 b_3^2 \left[e^2 C_{DW} - s_W^2 c_W^2 C_{\varphi D} - 4 s_W^3 c_W C_{\varphi WB}\right]^2 + 36(a_1 b_1 + a_2 b_2)^2 e^2 s_W^2 c_W^2 C_W^2$$

- and from WW:

$$\text{R.H.S} \propto a_3^2 b_3^2 s_W^2 \left(e^2 C_{DB} + c_W^2 C_{\varphi D}\right)^2 + e^2 c_W^2 \left[6(a_1 b_1 + a_2 b_2) s_W C_W + a_3 b_3 e C_{DW}\right]^2$$

$$\boxed{\sum_i c_i^{(8)} x_i \geq -\sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j} \geq 0}$$

or simply: $\boxed{\vec{c} \cdot \vec{x}_i \geq 0}$

Explicitly:

- Polarization matters. We will use

$$\vec{a} = (a_1, a_2, a_3)$$
$$\vec{b} = (b_1, b_2, b_3)$$

to denote the polarization of the two vector boson being scattered.

- As an example, $ZZ \to ZZ$ gives the following constraint:

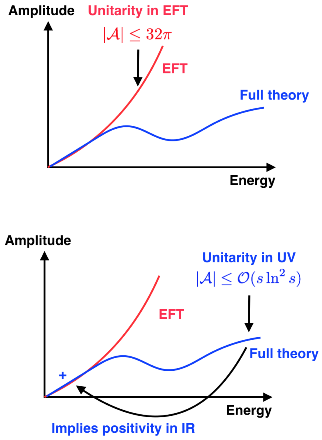
$$8a_3^2 b_3^2 t_W^4 \left(F_{S,0} + F_{S,1} + F_{S,2}\right) + \left[a_3^2 \left(b_1^2 + b_2^2\right) + \left(a_1^2 + a_2^2\right) b_3^2\right] t_W^2 \left(-t_W^4 F_{M,3} + t_W^2 F_{M,5} - 2F_{M,1} + F_{M,7}\right) + \left[\left(a_1 b_1 + a_2 b_2\right)^2 + \left(a_1^2 + a_2^2\right)\left(b_1^2 + b_2^2\right)\right]\left(2t_W^8 F_{T,9} + 4t_W^4 F_{T,7} + 8F_{T,2}\right) + 8\left(a_1 b_1 + a_2 b_2\right)^2 \left[t_W^4 \left(t_W^4 F_{T,8} + 2F_{T,5} + 2F_{T,6}\right) + 4F_{T,0} + 4F_{T,1}\right] \geq 0$$

- Depending on $\vec{a}$, $\vec{b}$, there is a infinite number of constraints from ZZ ...
- Other constraints from $W^\pm Z$, $W^\pm W^\pm$, $W^\pm W^\mp$, $W^\pm\gamma$, $Z\gamma$, $\gamma\gamma$.
- These are the key results of this work.

"Unitarity"

- It is well-known that unitarity violation can be a problem in SMEFT.
  - In VBS, unitarization techniques are needed. (two previous talks.)
  - However, here unitarity problem concerns only the prediction of the SMEFT, and only signals the breakdown of EFT.
- Our bounds are derived from a different information, i.e. the Froissart unitarity bound. This unitarity refers to the behaviour of the UV theory at large energy.
  - This is then connected to the IR (EFT) of the theory by the dispersion relation

i.e. Unitarity in UV (full theory) $\Rightarrow$ Positivy in IR (EFT)



Outline

Example: simplified model

Consider the simplified model in [Brass, Fleper, Kilian, Reuter, Sekulla '18] (i.e. talk by Prof. Kilian)

In the present paper, we do not refer to a specific scenario. We construct a simplified model with transverse couplings of a generic heavy resonance σ. The effective Lagrangian takes the following form,

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma(m_\sigma^2 - \partial^2)\sigma + \sigma(J_{\sigma\parallel} + J_{\sigma\perp}) \tag{19a}$$

$$J_{\sigma\parallel} = F_{\sigma H}\,\mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger(\mathbf{D}^\mu\mathbf{H})\right] \tag{19b}$$

$$J_{\sigma\perp} = g^2 F_{W\sigma}\sigma\,\mathrm{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] + g'^2 F_{B\sigma}\sigma\,\mathrm{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] \tag{19c}$$

with three independent coupling parameters.

In the low-energy limit, the scalar resonance can be integrated out, and we obtain the SMEFT Lagrangian with the following nonzero coefficients of the dimension-8 operators at leading order:

$$F_{S_0} = F_{\sigma H}^2/2m_\sigma^2 \tag{20a}$$

$$F_{M_0} = -F_{\sigma H}F_{\sigma W}/m_\sigma^2 \tag{20b}$$

$$F_{M_2} = -F_{\sigma H}F_{\sigma B}/m_\sigma^2 \tag{20c}$$

$$F_{T_0} = F_{\sigma W}^2/2m_\sigma^2 \tag{20d}$$

$$F_{T_5} = F_{\sigma W}F_{\sigma B}/m_\sigma^2 \tag{20e}$$

$$F_{T_8} = F_{\sigma B}^2/2m_\sigma^2. \tag{20f}$$

Example: simplified model

If we plug in the dim-8 coefficients into our positivity constraints, we see:

$$
\begin{aligned}
ZZ &: (a_1 b_1 + a_2 b_2)^2 \left(s_W^4 F_{\sigma B} + 2 c_W^4 F_{\sigma W}\right)^2 + a_3^2 b_3^2 s_W^4 c_W^4 e^{-4} F_{\sigma H}^2 > 0 \\
W^\pm Z &: a_3^2 b_3^2 F_{\sigma H}^2 > 0 \\
W^\pm W^\pm &: (a_1 b_1 + a_2 b_2)^2 F_{\sigma W}^2 + \left[(a_1 b_1 + a_2 b_2) F_{\sigma W} + a_3 b_3 s_W^2 e^{-2} F_{\sigma H}\right]^2 > 0 \\
W^\pm W^\mp &: (a_1 b_1 + a_2 b_2)^2 F_{\sigma W}^2 + \left[(a_1 b_1 + a_2 b_2) F_{\sigma W} - a_3 b_3 s_W^2 e^{-2} F_{\sigma H}\right]^2 > 0 \\
ZA &: (a_1 b_1 + a_2 b_2)^2 \left[s_W^2 F_{\sigma B} - 2 c_W^2 F_{\sigma W}\right]^2 > 0 \\
WA &: \text{none} \\
AA &: (a_1 b_1 + a_2 b_2)^2 (F_{\sigma B} + 2 F_{\sigma W})^2 > 0
\end{aligned}
$$

*up to factors of 2 that can be absorbed in the definitions of $F_{\sigma X}$

All inequalities are satisfied, as they are all sum of squares.

- In a top-down approach, positivity is automatically true, in different models, different ways — by asking for positivity, we are not restricting the UV models.
- In a bottom-up approach, we can derive the same constraints, but without using model details, and therefore we restrict the parameter space without losing model-independence.

1D limits

- Consider one operator at a time:

$f_{S,0}$	$f_{S,1}$	$f_{S,2}$	$f_{M,0}$	$f_{M,1}$	$f_{M,2}$	$f_{M,3}$	$f_{M,4}$	$f_{M,5}$
+	+	+	✗	−	O	−	O	✗

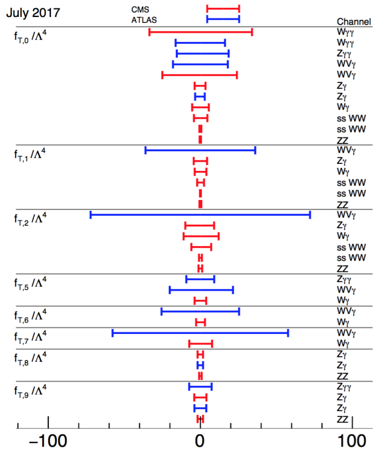
$f_{M,7}$	$f_{T,0}$	$f_{T,1}$	$f_{T,2}$	$f_{T,5}$	$f_{T,6}$	$f_{T,7}$	$f_{T,8}$	$f_{T,9}$
+	+	+	+	✗	+	✗	+	+

+: positive −: negative
O: free ✗: forbidden

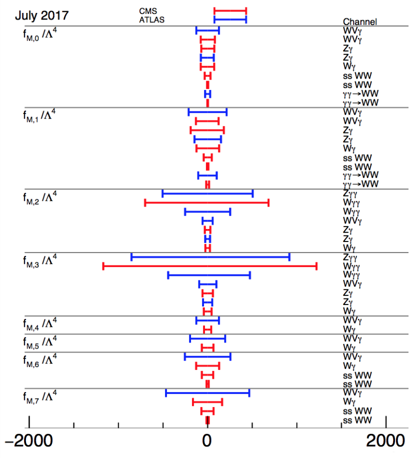
- Note there are coefficients that are not individually allowed.
  - E.g. F_{T5}. In the simplified model $F_{T5} \propto F_{\sigma W} F_{\sigma B}$, cannot take nonzero value independent of $F_{T0} \propto F_{\sigma W}^2$ and $F_{T8} \propto F_{\sigma B}^2$.

1D limits: EXP

Individual limits on transversal coefficients



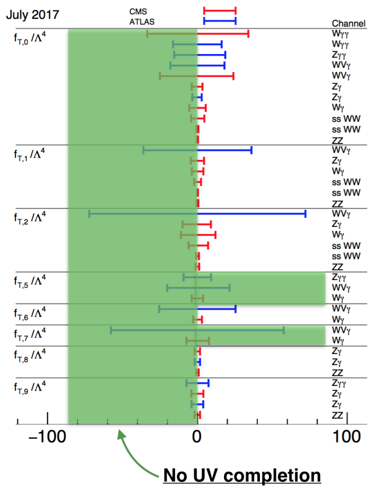
Individual limits on mixed coefficients



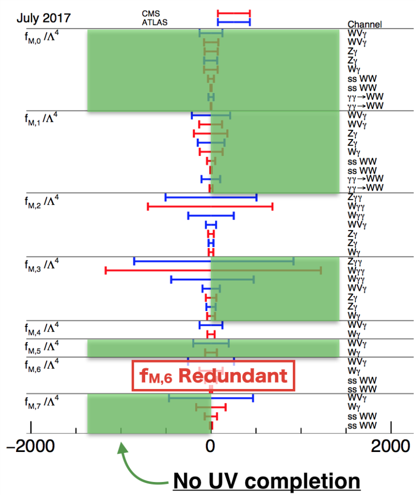
https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSMPaTGC#aQGC_Results

1D limits: EXP+positivity

Transversal coefficients, positivity



Mixed coefficients, positivity



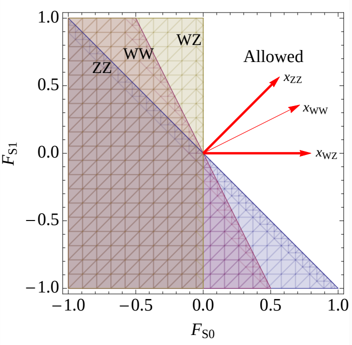
2D limits: Longitudinal case

As a first example, consider $O_{S,0}$ and $O_{S,1}$.

- Coef. vector: $\vec{c} \equiv (F_{S,0}, F_{S,1})$. Positivity: $\vec{c} \cdot \vec{x}_i \geq 0$.
- There are 3 useful constraints, from WW, ZZ, WZ scattering

$$\vec{x}_{WZ} = a_3^2 b_3^2 (1,0), \quad \vec{x}_{WW} = a_3^2 b_3^2 (2,1), \quad \vec{x}_{ZZ} = a_3^2 b_3^2 (1,1)$$

$\vec{x}$ and corresponding exclusion



- Note that $\vec{x}_{WW}$ is between $\vec{x}_{ZZ}$ and $\vec{x}_{WZ}$, so positivity on $\vec{x}_{WW}$ does not give new independent information.
- **In general: if $\vec{x}$ is a positive linear combination of a set of $\vec{x}_i$, i.e.**

$$\vec{x} = \alpha_i \vec{x}_i, \quad \alpha_i \geq 0 \quad \forall\, i$$

Then positivity along $\vec{x}$ does not lead to additional exclusion.

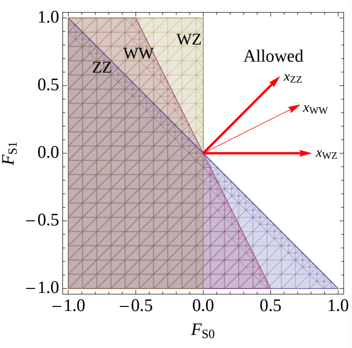
2D limits: longitudinal case

As a first example, consider $O_{S,0}$ and $O_{S,1}$.

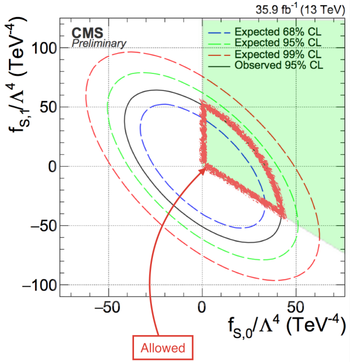
- Coef. vector: $\vec{c} \equiv (F_{S,0}, F_{S,1})$. Positivity: $\vec{c} \cdot \vec{x}_i \geq 0$.
- There are 3 useful constraints, from WW, ZZ, WZ

$$\vec{x}_{WZ} = a_3^2 b_3^2 (1,0), \quad \vec{x}_{WW} = a_3^2 b_3^2 (2,1), \quad \vec{x}_{ZZ} = a_3^2 b_3^2 (1,1)$$

$\vec{x}$ and corresponding exclusion



Combined with EXP



2D limits: longitudinal and transversal case

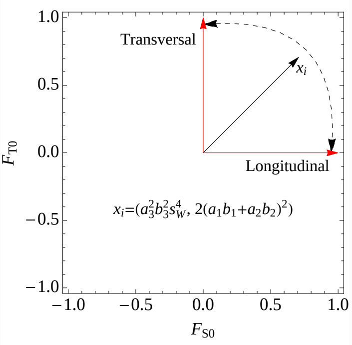
As a second example, consider $O_{S,0}$ and $O_{T,0}$.

- The constraint from WW is

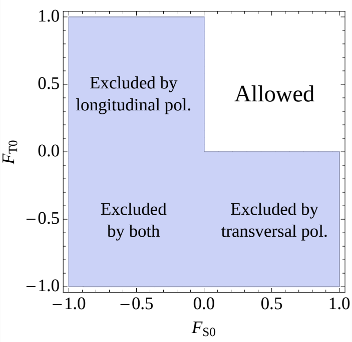
$$\vec{x}_{WW} = (a_3^2 b_3^2 s_W^4, 2(a_1 b_1 + a_2 b_2)^2)$$

- Depending on the chosen $\vec{a}$, $\vec{b}$, $\vec{x}_{WW}$ can take any direction in the first quadrant.

$\vec{x}$ can vary in the 1st quadrant



Corresponding exclusion



2D limits: mixed case

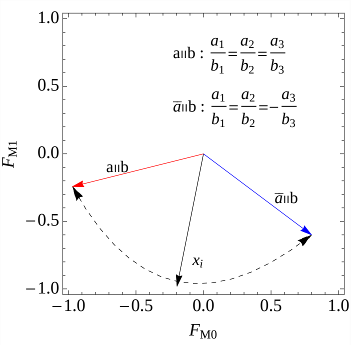
As a third example, consider $O_{M,0}$ and $O_{M,1}$.

- The most useful constraint is from WW

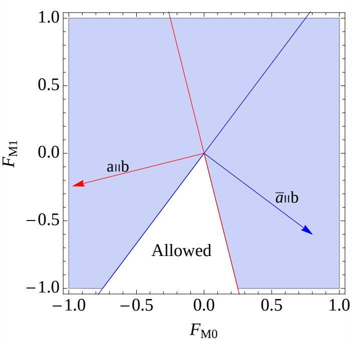
$$\vec{x}_{WW} = -\left(4(a_1 b_1 + a_2 b_2)a_3 b_3,\ (a_1^2 + a_2^2)b_3^2 - (a_1 b_1 + a_2 b_2)a_3 b_3 + (b_1^2 + b_2^2)a_3^2\right)$$

- Varies between $(-4,-1)$ and $(4,-3)$.

$\vec{x}$ sits between red and blue arrows



Corresponding exclusion

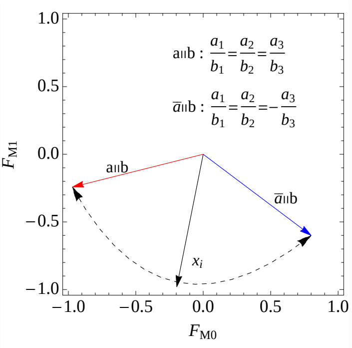


2D limits: mixed case

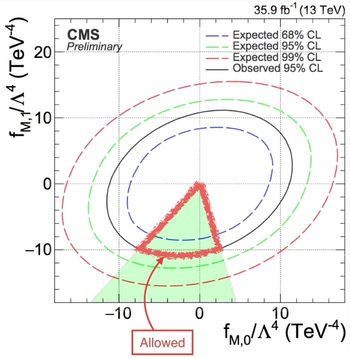
As a third example, consider $O_{M,0}$ and $O_{M,1}$.

- The most useful constraint is from WW
$$\vec{x}_{WW} = -\Big(4(a_1b_1 + a_2b_2)a_3b_3,\ (a_1^2 + a_2^2)b_3^2 - (a_1b_1 + a_2b_2)a_3b_3 + (b_1^2 + b_2^2)a_3^2\Big)$$
- Varies between $(-4, -1)$ and $(4, -3)$.

$\vec{x}$ sits between red and blue arrows



Corresponding exclusion



2D limits: summary

In 2D case, constraints are given by minimally two key vectors.

- $F_{S0} - F_{S1}$:

$$\vec{x}_{WZ} = (1, 0)$$
$$\vec{x}_{ZZ} = (1, 1)$$

- $F_{S0} - F_{T0}$:

$$\vec{x}_{WW}(trans.) = (0, 1)$$
$$\vec{x}_{WW}(long.) = (1, 0)$$

- $F_{M0} - F_{M1}$:

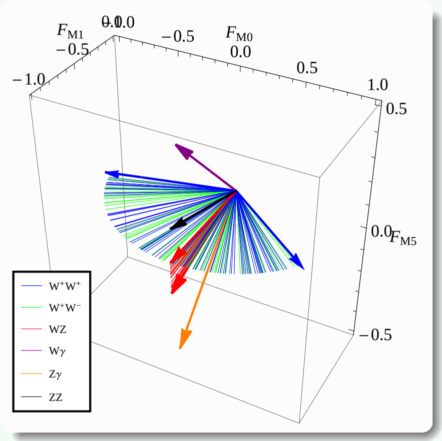
$$\vec{x}_{WW}(\vec{a} \parallel \vec{b}) = (-4, -1)$$
$$\vec{x}_{WW}(\vec{a}\overline{\|}\vec{b}) = (4, -3)$$

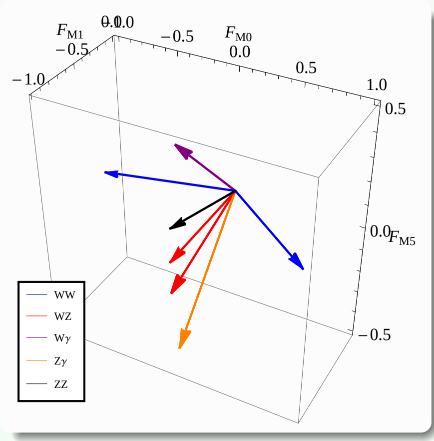
What happens in higher dimension parameter space?

3D limits

As a last example, consider $O_{M,0}$, $O_{M,1}$ and $O_{M,5}$.

- Scan the polarization space by randomly generating $\vec{a}$ and $\vec{b}$.
- **$\vec{x}$ within the pyramid formed by other $\vec{x}_i$ does not give new info!**





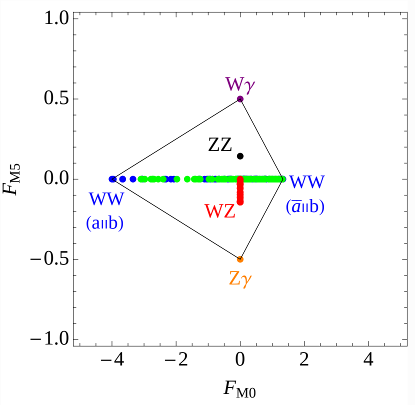
3D limits

How to find the “key vectors” that characterize the bounds in general:

- Scan all possible polarizations $\vec{a}$, $\vec{b}$.
- Project to a 2D plane ($f_{M0} - f_{M5}$).
- Take the endpoints of $\vec{x}$.
- **Find the convex hull** of the set of points.
- The vertices corresponds to the key vectors:

$$\begin{aligned} \vec{x}_{W_\gamma} &= (0, -2, 1), \\ \vec{x}_{Z_\gamma} &= (0, -2, -1), \\ \vec{x}_{WW}(\vec{a} \parallel \vec{b}) &= (-4, -1, 0), \\ \vec{x}_{WW}(\vec{a} \nparallel \vec{b}) &= (4, -3, 0). \end{aligned}$$

$\vec{x}_i$ projected on 2D plane



3D limits

- Allowed region is given by

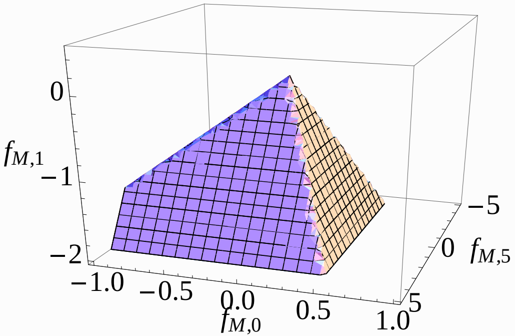
$$-2F_{M1} + F_{M5} \geq 0,$$
$$-2F_{M1} - F_{M5} \geq 0,$$
$$-4F_{M0} - F_{M1} \geq 0,$$
$$4F_{M0} - 3F_{M1} \geq 0.$$

  - Note that

$$F_{M5} \in [-2|F_{M1}|, 2|F_{M1}|],$$
$$F_{M0} \in [-\frac{3}{4}|F_{M1}|, \frac{1}{4}|F_{M1}|]$$

- In principle same approach applies for higher-D case: *the problem is equivalent to finding a* ***D-1 dimensional convex hull***.

3D allowed region given by a pyramid



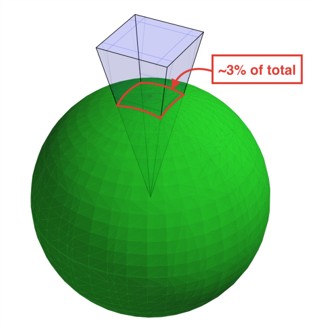
Recall:

$f_{M,0}$	$f_{M,1}$	$f_{M,2}$	$f_{M,3}$	$f_{M,4}$	$f_{M,5}$
✗	–	O	–	O	✗

Volume in full parameter space

When all 18 parameters are turned on, how much of the parameter space is excluded by positivity?

- By brutal force, randomly through points on a 18D sphere, uniformly distributed, and count how many of the them fall within constraints for all polarizations.
- We find that **only** $\sim 3\%$ **parameter space** is left.



How does positivity affect future experimental search?

We don't know, but some thoughts...

- It might help MC generation? Because less space need to be scanned? Does it help to move to higher dim? (i.e. towards global EFT fit)
  - 2D/3D benchmarks?
  - Or simply a MATHEMATICA script...

```
ForAll[{a1, a2, a3, b1, b2, b3}, PositivityConditions]
```

- Does it provide enough info to support a "guided search"? Instead of blindly searching for BSM in 18D space, can we make use of the fact that we know BSM only exists in the 3% parameter space?

- Presentation, e.g. 1D limits should be presented in a more reasonable form.

Outline

Conclusion

- Dim-8 aQGC operator coefficients satisfy a set of positivity constraints, if they are generated by a UV completion.
- They have strong implication, e.g. 18D parameter space reduced to 3%, independent of experimental precision.
- The shape of the allowed parameter space shows interesting structure.
- They should be taken into account for future aQGC studies.

Thank you!