Heavy Ion Interactions

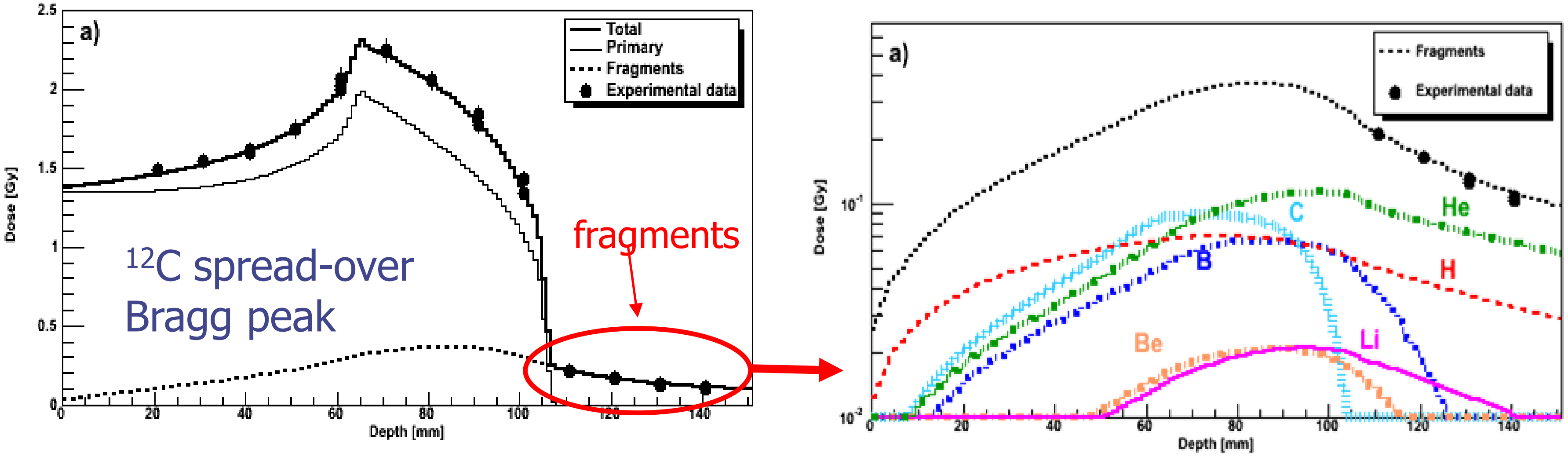
21st FLUKA Beginner's Course
ALBA – Barcelona, Spain
08 – 12 April 2019

Overview

Introduction	
The models	DPMJET RQMD BME
Input options	Beam definition Transport thresholds

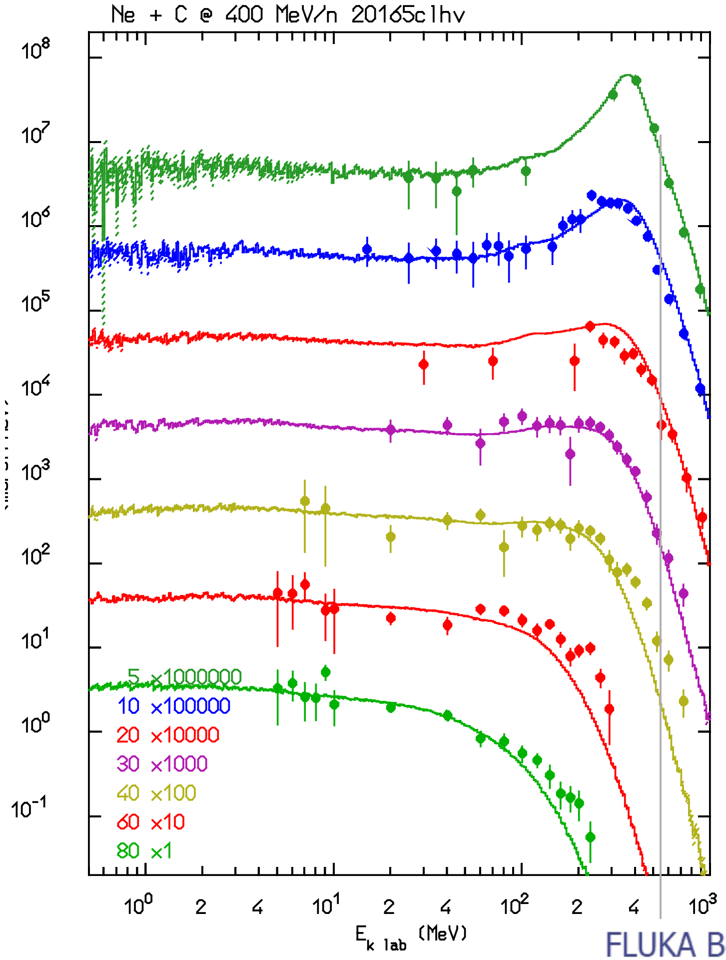
Introduction - 1

- In hadron – nucleus interactions, reaction products and residuals come mostly from the TARGET nucleus
- In nucleus-nucleus interactions, reaction products and residuals come from both TARGET and PROJECTILE nuclei.
- Indeed, except for complete fusion, one often refers to "projectile-like" and "target-like" fragments
- ➔ projectile-like fragments travel with the projectile speed, thus they can be energetic, and travel longer /shorter than the average projectile range (range ≈÷ A/Z^2 at given β)



Introduction - 2

Left : Ne+C at 400 MeV/A, right: p+C at 256 MeV, neutron energy spectra at different angles. Note the high energy (>E/A) tails, and the different shape. Also, different “effect”of reaction stages: in A-A, evaporation products can be fast (from proj like)!



Heavy ion interaction models in FLUKA

E > 5 GeV/n

Dual Parton Model (DPM)

DPMJET-III (original code by R.Engel, A.Fedynitch, J.Ranft and S.Roesler,
FLUKA-implemenation by T.Empl *et al.*)

~0.1 GeV/n < E < 5 GeV/n

Relativistic Quantum Molecular Dynamics Model (RQMD)

RQMD-2.4 (original code by H.Sorge *et al.,*
FLUKA-implementation by A.Ferrari *et al.*)

E < ~0.1 GeV/n

Boltzmann Master Equation (BME) theory

BME (original code by E.Gadioli *et al.,*
FLUKA-implementation by F.Cerutti *et al.*)

At ALL Energies: Electro-Magnetic dissociation, native for FLUKA

DPMJET

E > 5 GeV/n

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DPMJET – Overview

DPMJET = Dual Parton Model and JETs

Monte Carlo **event-generator** for the simulation of high-energy hadronic interactions

DPMJET - Version III.1

- hadron-nucleus collisions
  nucleus-nucleus collisions
  photon-nucleus collisions off nuclei

It uses PHOJET for

- hadron-hadron collisions
  photon-hadron collisions

energy range: 5 GeV/nucleon – 10^{11} GeV/nucleon
programming language: Fortran 77
size of the code: about 180000 lines
authors: R.Engel, A.Fedynitch, J.Ranft, S.Roesler

DPMJET – Basic physics (review)

DPMJET: (as well as the FLUKA high energy h–A generator) is based on the Dual Parton Model in connection with the Gribov-Glauber formalism.

Parton model: to analyze high-energy hadron collisions.
Hadrons are considered made of "partons".

Glauber formalism: elastic, quasi-elastic and absorption hadron-nucleus (h-A) cross sections are derived from the hadron-nucleon (h-N) cross sections.
Inelastic interactions are equivalent to multiple interactions of the projectile with the target nucleons.

Gribov theory: the elastic hadron-nucleus (h-A) amplitude is obtained by the Glauber model (multiple elastic rescatterings) plus all possible diffractive excitations of the initial hadron.

DPMJET – Main steps (1)

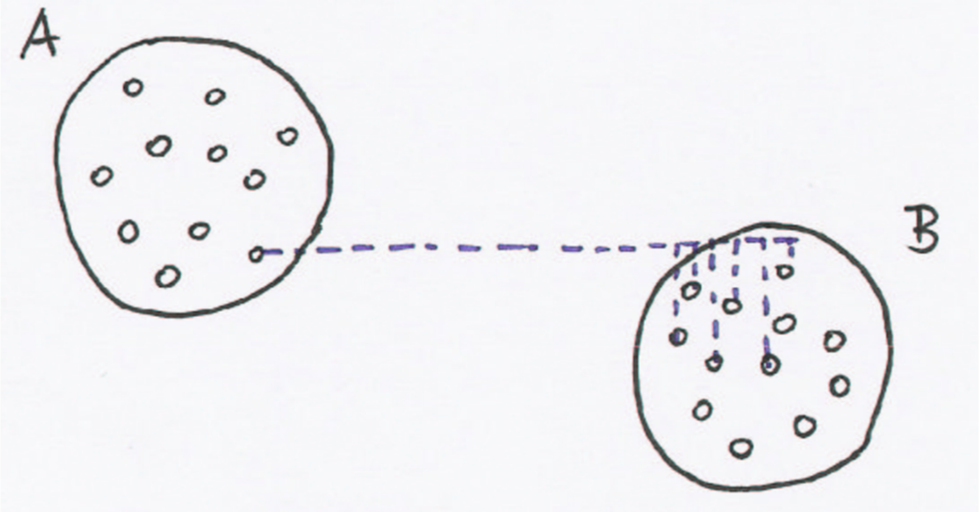
1. **Interaction of high energy nuclei:**

    Individual nucleon-nucleon scatterings

    Dual Parton Model as a two components model using:

    Gribov's reggeon field theory for soft and perturbative interactions
    QCD improved parton model for hard interactions

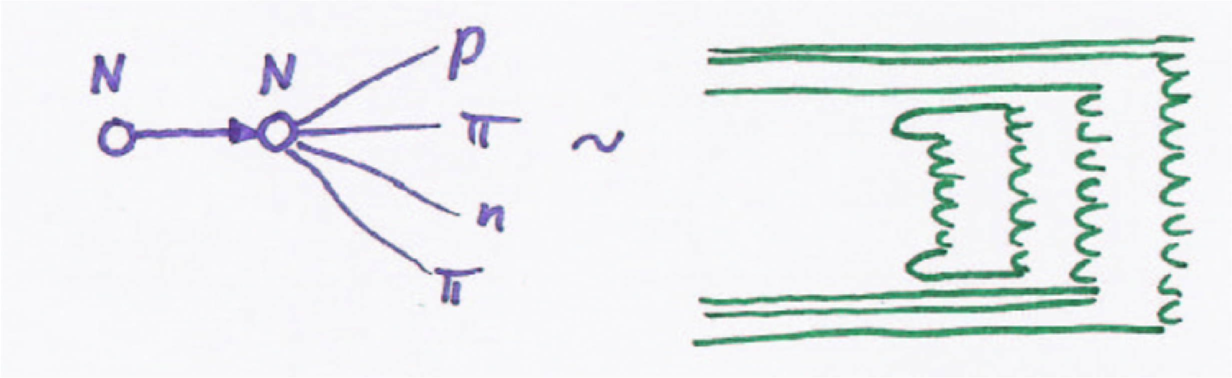
→ Formation of strings between valence and sea partons (quarks, gluons)



DPMJET – Main steps (2)

2. Hadronization process

→ Creation of hadrons / resonances from string fragmentation

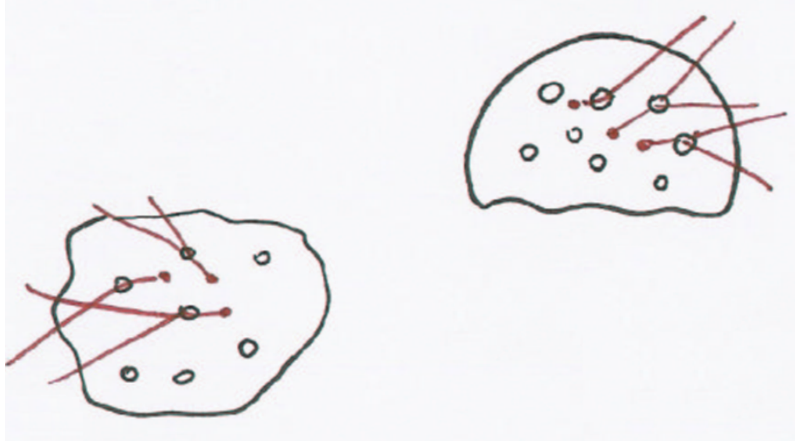


Results are **hadrons** and **spectators**, where the latter ones are nucleons from the projectile and target nuclei that did not take part in the interactions

DPMJET – Main steps (3)

3. Intranuclear Cascade

Secondary low-energy interactions of hadrons with spectator nucleons



Hadrons are followed in space and time as straight trajectories

Hadrons may re-interact after certain *formation time*

→ Emission of nucleons
→ Spread of excitation energy

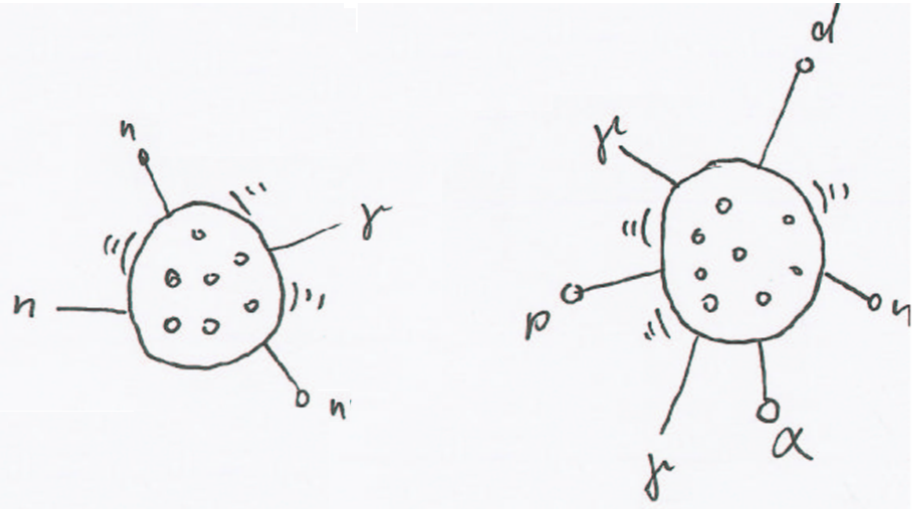
Note: DPMJET has its own implementation of intranuclear cascade. It is similar in its fundamental ideas to the one of FLUKA but it is much more simplified.

DPMJET – Main steps (5)

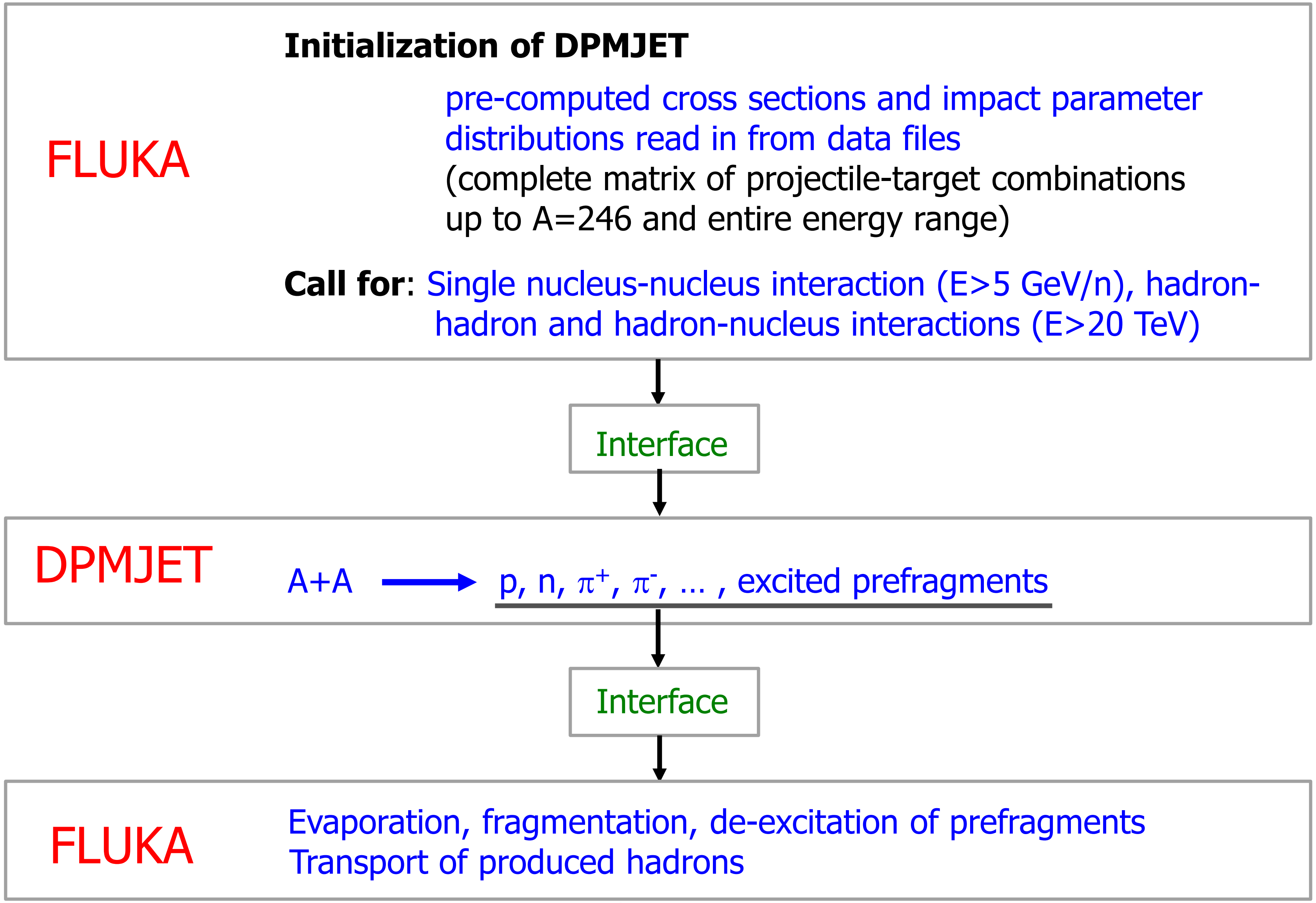
4. Break-up of excited spectator nuclei

Excited fragments are treated by PEANUT in FLUKA by

- nuclear evaporation ⟶ emission of nucleons and light fragments
- Fermi break-up for light residual nuclei
- high-energy fission
- γ-deexcitation ⟶ production of final residual nuclei



DPMJET – Interface to FLUKA



FLUKA

Initialization of DPMJET

pre-computed cross sections and impact parameter distributions read in from data files (complete matrix of projectile-target combinations up to A=246 and entire energy range)

Call for: Single nucleus-nucleus interaction (E>5 GeV/n), hadron-hadron and hadron-nucleus interactions (E>20 TeV)

Interface

DPMJET

A+A → p, n, π^+, π^-, … , excited prefragments

Interface

FLUKA

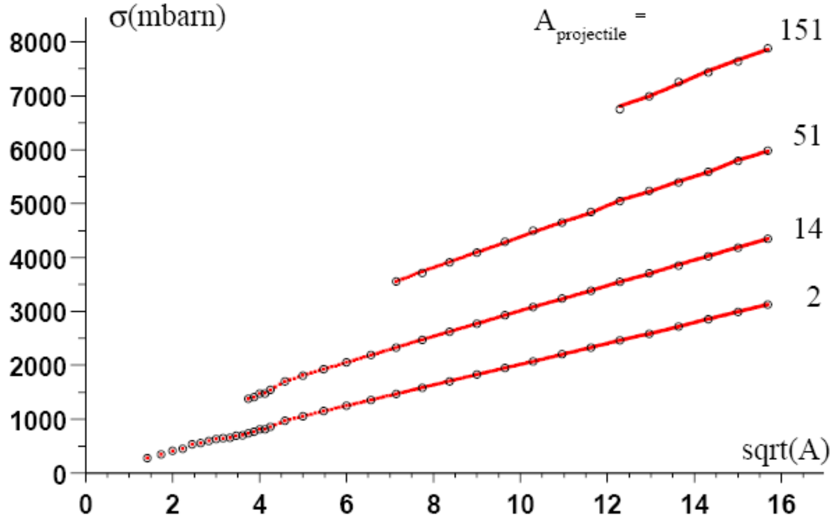
Evaporation, fragmentation, de-excitation of prefragments
Transport of produced hadrons

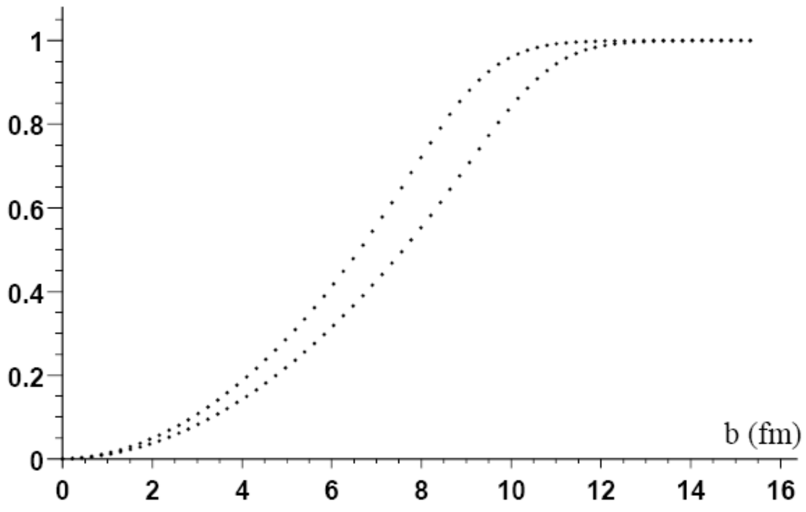
DPMJET – Pre-computed parameters

Examples for pre-initialised data:

Inelastic cross sections

Impact parameter distribution

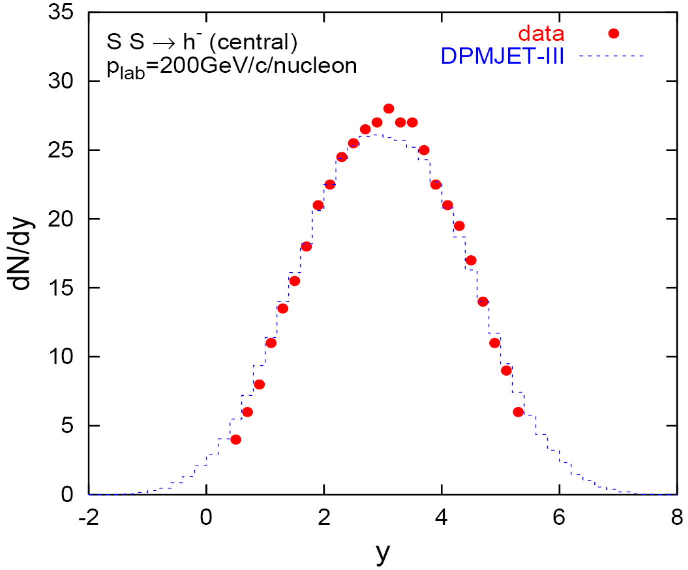


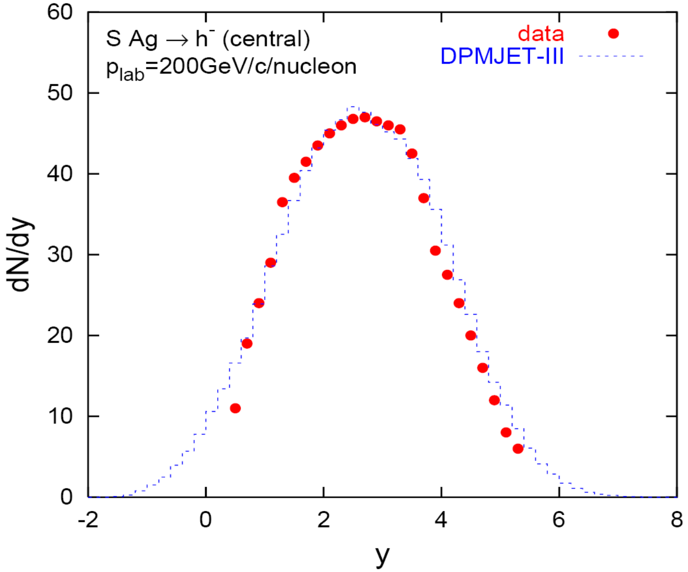


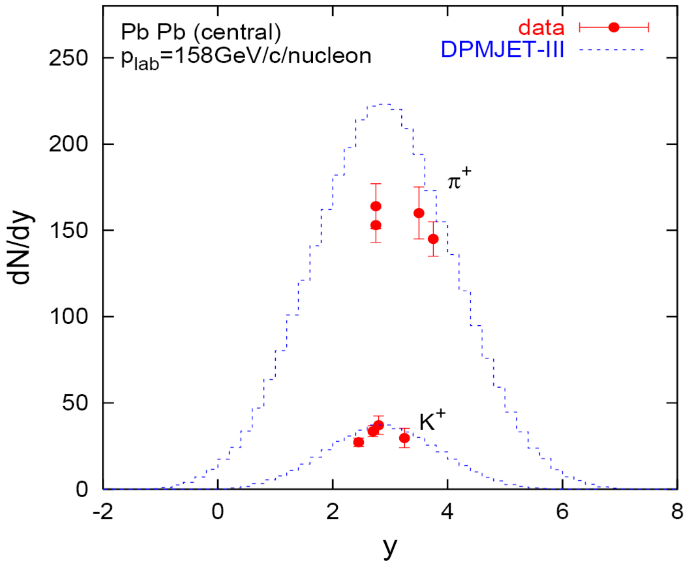
$E_{Lab} = 6.3 \times 10^9$ GeV/nucleon

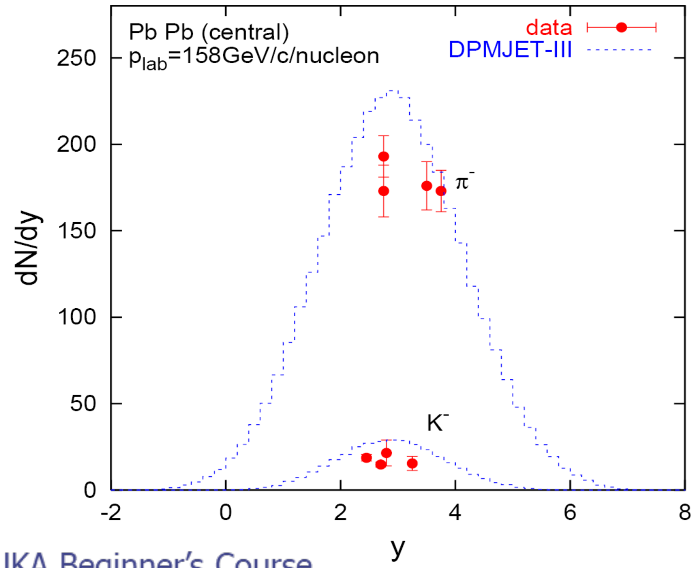
e.g., for highest and lowest energy at one fixed projectile-target configuration

DPMJET – Comparison to data (1)





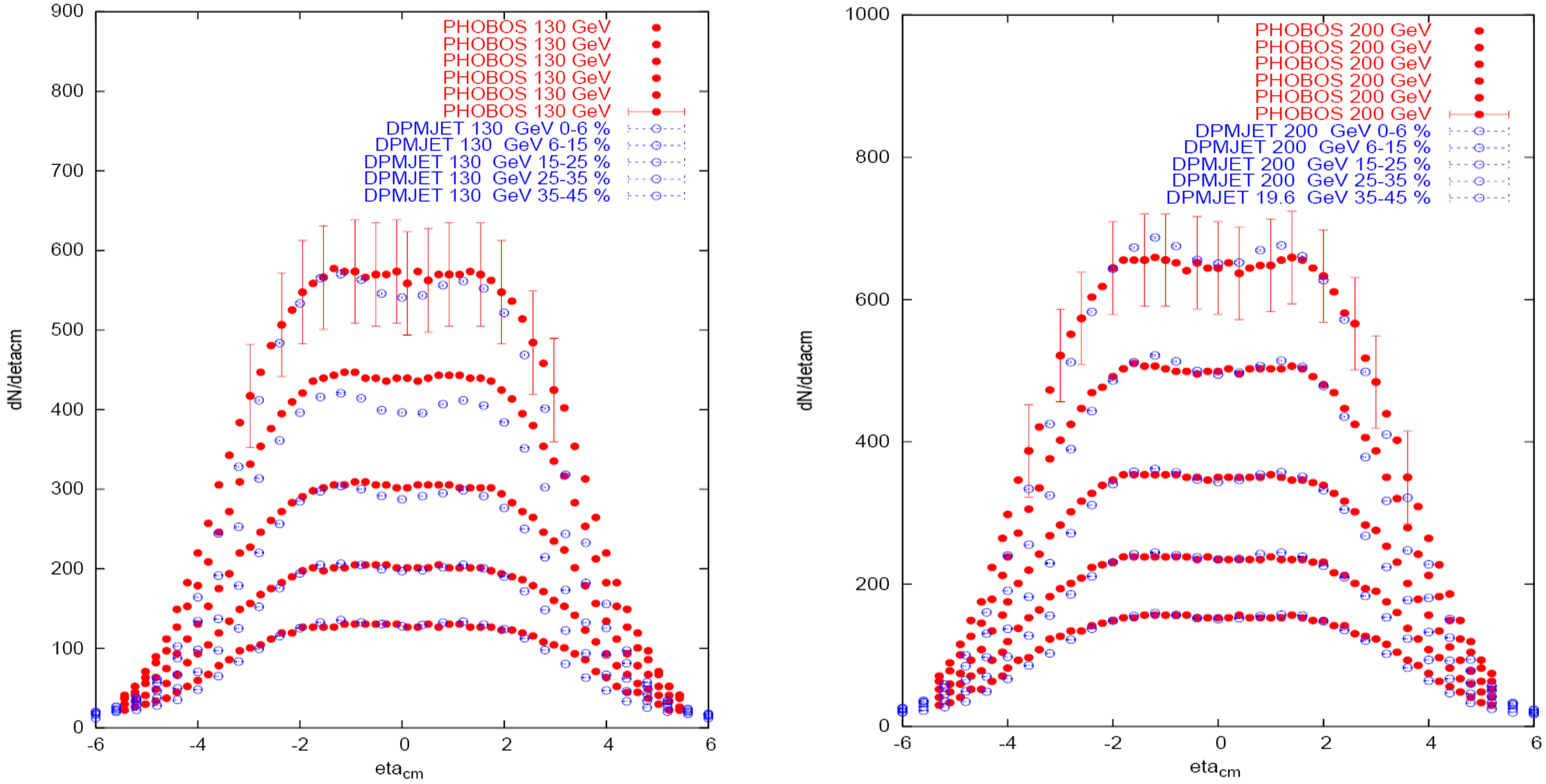




$$\text{Rapidity} = y = \frac{1}{2}\ln\left(\frac{E + p_{\parallel}}{E - p_{\parallel}}\right)$$

DPMJET – Comparison to data (2)

$$\text{pseudorapidity} = \eta = -\ln\left(\tan\frac{\vartheta}{2}\right) = \frac{1}{2}\ln\left(\frac{p+p_{\parallel}}{p-p_{\parallel}}\right)$$

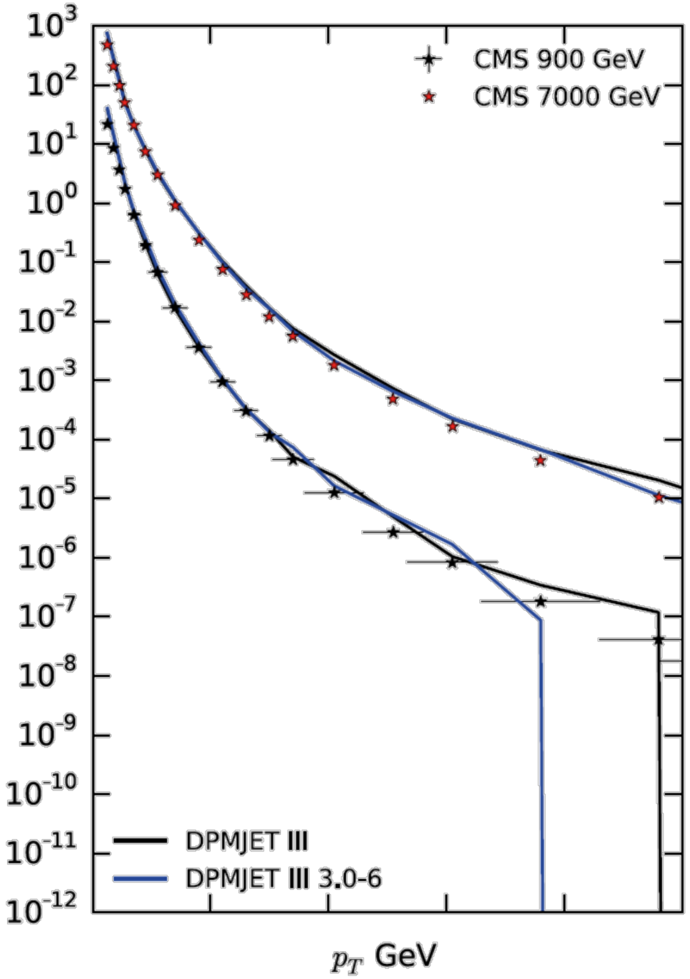


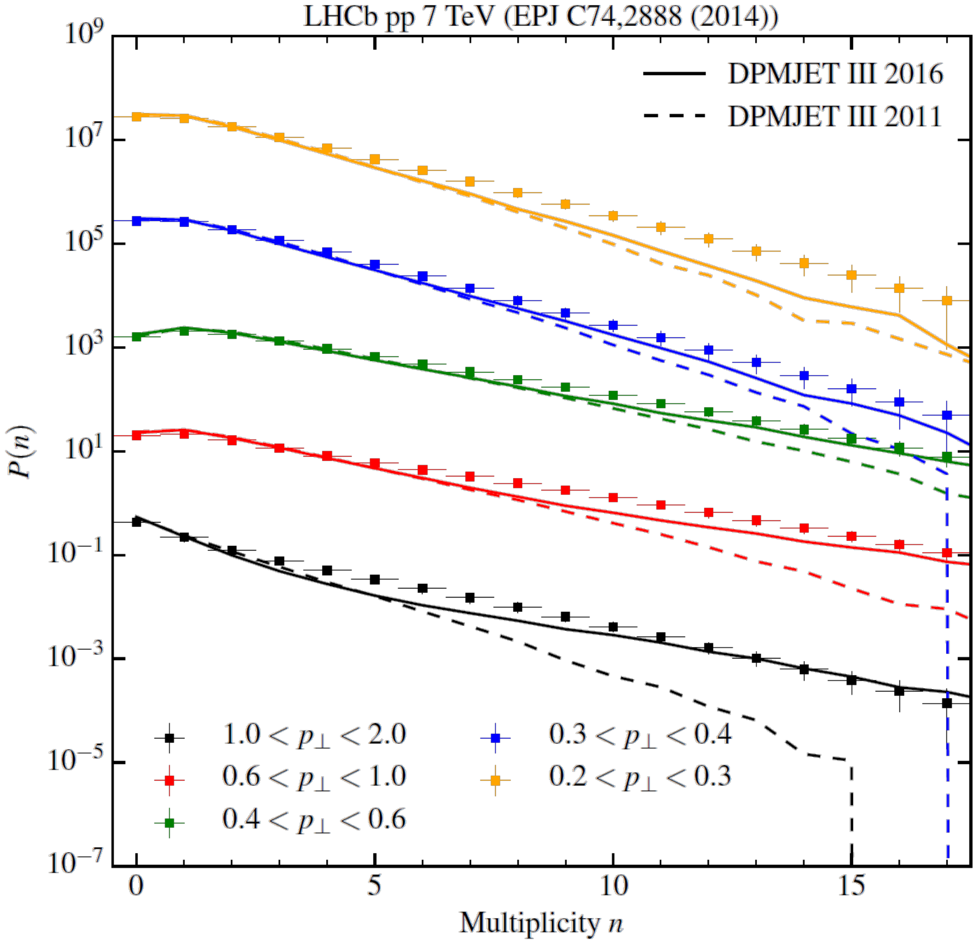
Pseudorapidity distribution of charged hadrons produced in Au-Au collisions at a c.m. energy of 130GeV/A (left) and 200GeV/A (right) for different ranges of centralities.

Exp. data: PHOBOS-Collaboration

J.Ranft, in Proceedings of the Hadronic Shower Simulation Workshop, CP896, Batavia, Illinois (USA), 6-8 September 2006

PHOJET / DPMJET vs LHC results

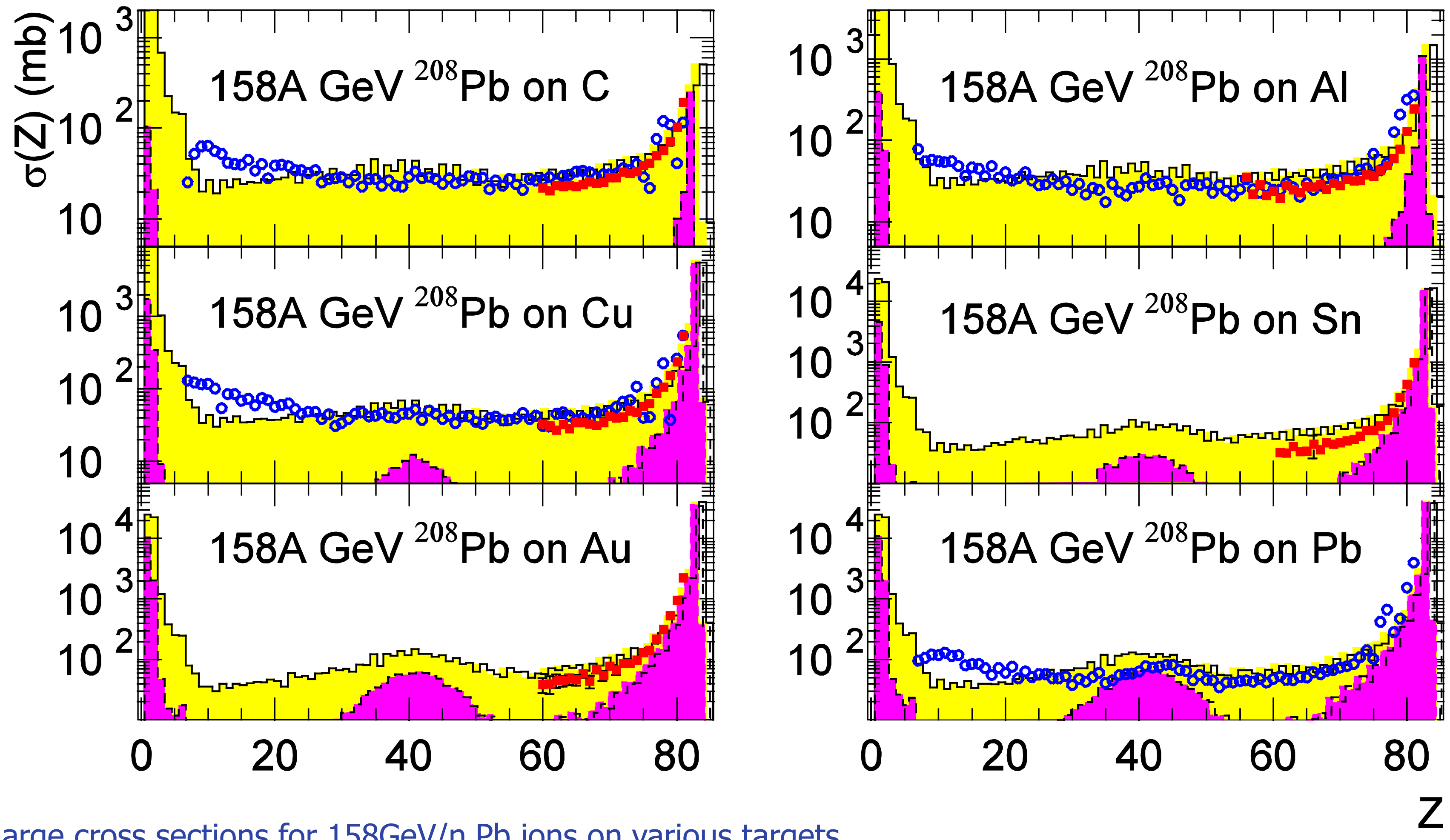




Invariant cross section for charged particles as a function of transverse momentum for pp collisions at various centre-of-mass energies

Charged particle multiplicity distribution for different p_T ranges in the forward region ($2<\eta<4.5$) as measured by LHCb @ $\sqrt{s}$=7 TeV

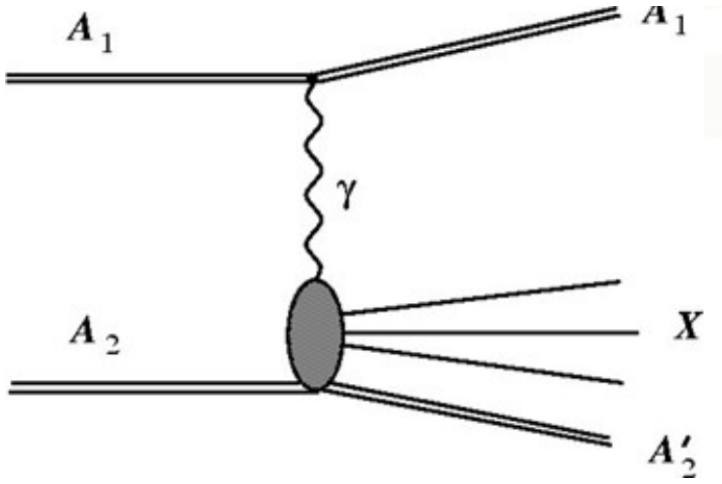
DPMJET – FLUKA benchmarks



Fragment charge cross sections for 158GeV/n Pb ions on various targets.
FLUKA: solid histogram (purple contribution from electromagnetic dissociation)
Exp. data: NPA662, 207 (2000), NPA707, 513 (2002) (blue circles), C.Scheidenberger et al. PRC70, 014902 (2004), (red squares)

Electromagnetic dissociation

- Very peripheral collisions
- Break-up of one of the colliding nuclei in the electromagnetic field of the other nucleus



```
PHYSICS            2.0       0.0       0.0       0.0       0.0       0.0EM-DISSO
```

PHYSICS Type: EM-DISSO ▼ EM Disso: Proj&Target EM-Disso ▼

```
WHAT(1)  : flag for activating ion electromagnetic-dissociation

        =< -1.0 : resets to default (no em-dissociation)
        =   0.0 : ignored
        =   1.0 : (default) no em-dissociation
        =   2.0 : projectile and target em-dissociation activated
        =   3.0 : projectile only em-dissociation activated
        =   4.0 : target only em-dissociation activated

WHAT(2)-WHAT(6): not used
```

RQMD

E > 5 GeV/n

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RQMD – FLUKA implementation (1)

- RQMD, a relativistic QMD model, adapted to FLUKA: RQMD-2.4

H. Sorge, Phys. Rev. **C 52**, 3291 (1995);

H. Sorge, H. Stöcker, and W. Greiner, Ann. Phys. **192**, 266 (1989), Nucl. Phys. **A 498**, 567c (1989)

- QMD: Follows the Time evolution of the combined A+A system performing n-n interactions considering mean field effects and short range interactions

Re-calculation of the nuclear potentials from sum of two-body fields
  fields due to the nucleons of the same nuclei
  fields due to the nucleons of the other particle

⇨ Dynamic approach

- In FLUKA used in its faster cascade-like version

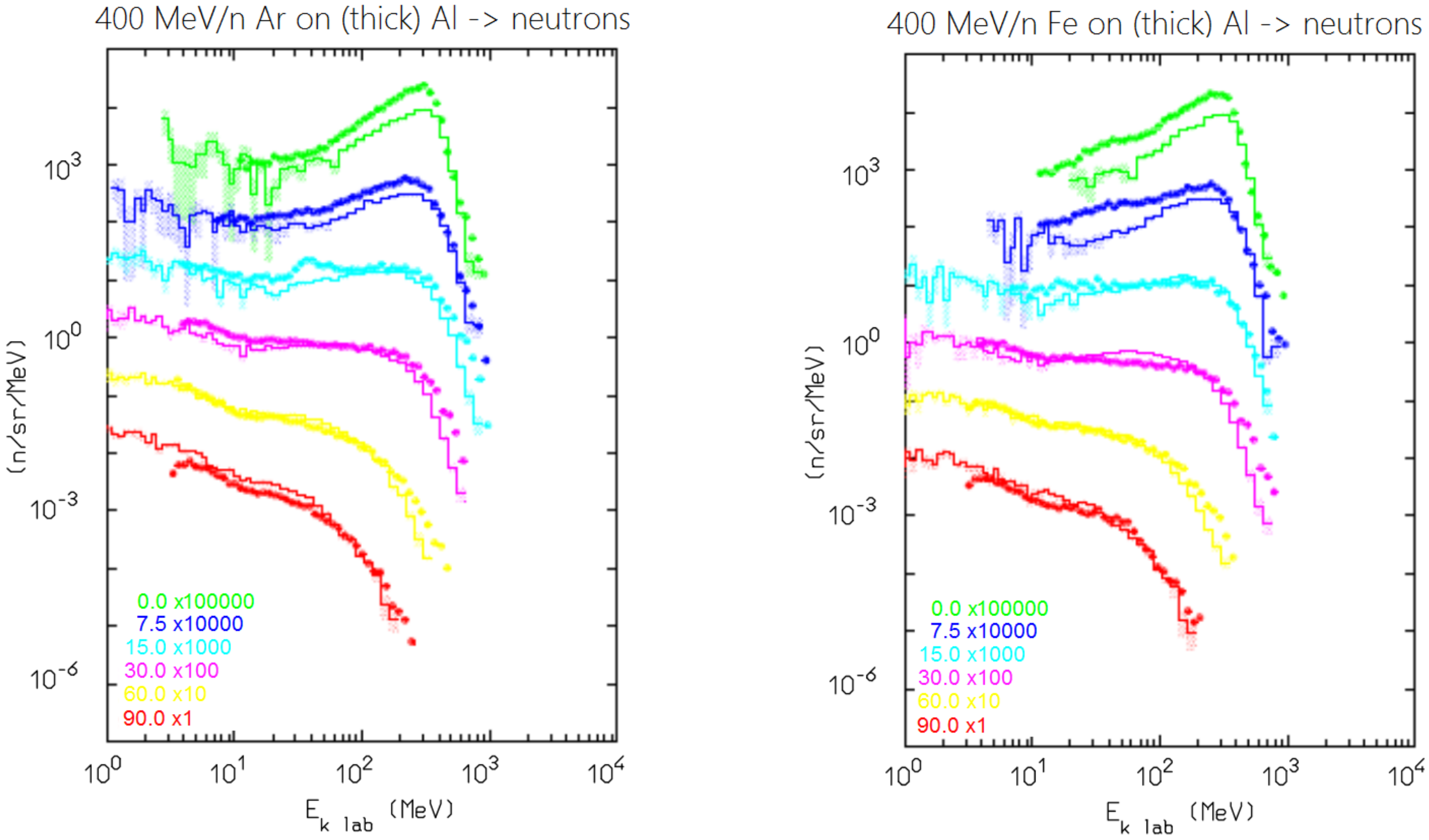
RQMD – FLUKA implementation (2)

- A-posteriori identification of residual fragments and their excitation was not provided by the original RQMD: added in the FLUKA implementation. Fragment de-excitation (evaporation, fission, Fermi break-up) is performed in PEANUT

  ⇨ Statistical approach

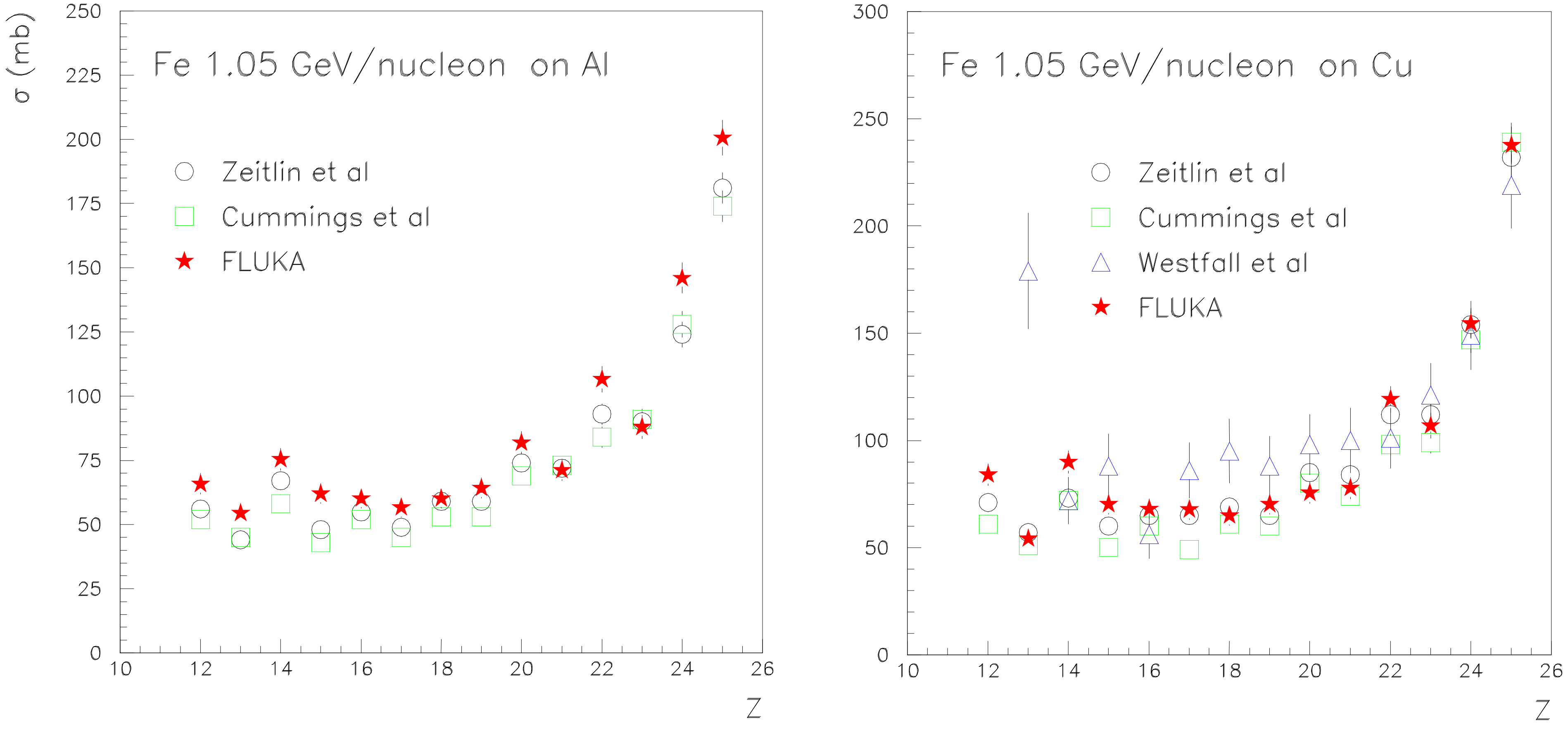
- Correct energy/momentum conservation:
  - Nuclear final state reworked out of the information on spectators
  - Excitation energy deduced from the holes left
  - Use of experimental binding energies

RQMD – FLUKA benchmarks (1)



Exp. Data from T. Kurosawa *et al*, Phys. Rev. **C 62**, 044615 (2000)

RQMD – FLUKA benchmarks (2)



Fragment charge cross section for 1.05 GeV/n Fe ions on Al (left) and Cu (right).
Exp. data from PRC56, 338 (1996) , PRC42,5208(1990) and PRC19, 1309 (1979)

BME

E > 5 GeV/n

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E < ~0.1 GeV/n

Boltzmann Master Equation (BME) theory
BME (original code by E.Gadioli *et al.,*
FLUKA-implementation by F.Cerutti *et al.*)

BME - References

interface to a Monte Carlo code founded on the BME theory (E. Gadioli et al.):

M. Cavinato *et al.*, Nucl. Phys. **A 679**, 753 (2001),

M. Cavinato *et al.*, Phys. Lett. **B 382**, 1 (1996)

BME – The nuclear interaction processes


Higher impact parameter

- **Complete Fusion**: projectile and target nuclei interact and merge in a composite nucleus (P+T → C)
- **Transfer**: pickup reaction where the smaller nucleus is fully overlapped by the density distribution of the bigger one and collects some of the partner nucleons
- **3body:** projectile and target nuclei interact with partial overlap of the density distributions, a hot region is produced (middle source X) and 3 outgoing fragments result (P+T → B+Y+X, with B and Y proj- and target-like)
- **Incomplete FUSion**: as 3 body, with the middle source absorbed by one nucleus (P+T → B+W or P+T → Z+Y)
- **“Inelastic” collisions**: either the projectile or the target loses a single nucleon, possibly absorbed by the partner nucleus

BME – The implemented code

pre-equilibrium de-excitation of the produced fragment(s) according to the BME theory (where available) or the PEANUT exciton model

NB interface to PEANUT pre-eq not yet distributed!

In order to get the multiplicities of the pre-equilibrium particles and their double differential spectra, the BME theory is applied to several representative systems at different bombarding energies and the results are parameterized.

^{12}C + ^{8}Li, ^{8}B, ^{12}C, ^{27}Al, ^{40}Ca

^{16}O + ^{6}Li, ^{8}Li, ^{8}B, ^{10}B, ^{12}C, ^{14}N, ^{16}O, ^{19}F, ^{20}Ne, ^{24}Mg, ^{27}Al, ^{56}Fe, ^{197}Au

at 12, 30, 50, 70, 100 MeV/n

Work is ongoing to extend it to more massive systems, e.g.

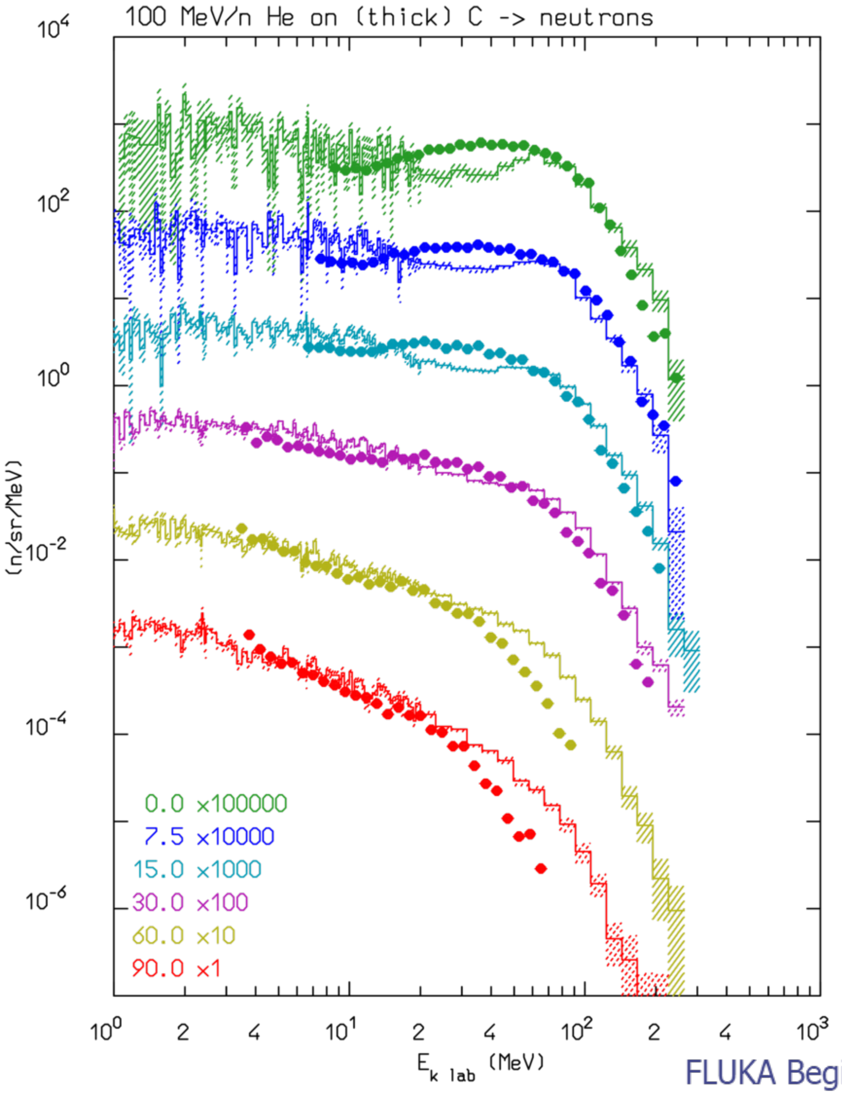
^{40}Ca + ^{120}Sn ^{56}Fe + ^{28}Si, ^{40}Ca, ^{48}Ca, ^{120}Sn

and consequently review the fitting functions
and the extrapolation recipes over a significantly larger mass range

FLUKA evaporation/fission/fragmentation/gamma de-excitation

BME – Benchmarking (1)

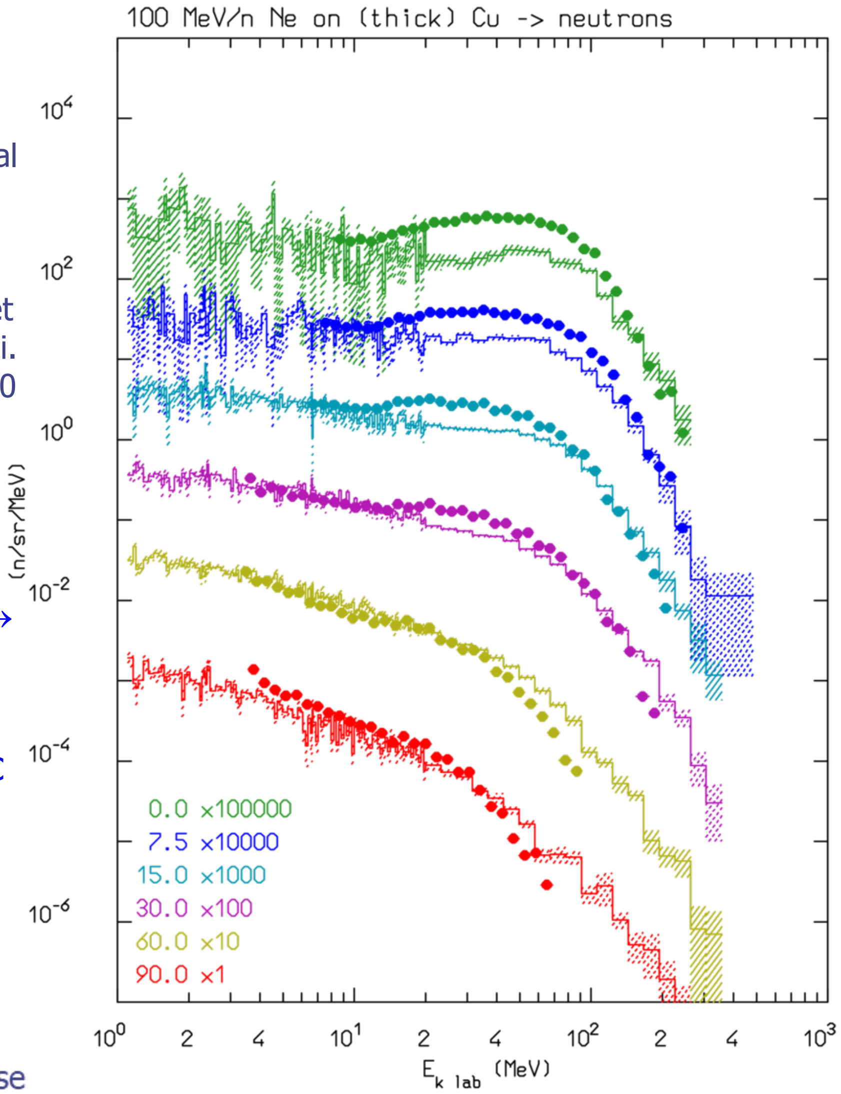
Double differential neutron yields from 100 MeV/n beams on thick targets



FLUKA
(histos) vs
experimental
data
(symbols)
from T.
Kurosawa et
al., Nucl. Sci.
Eng. 132, 30
(1999)

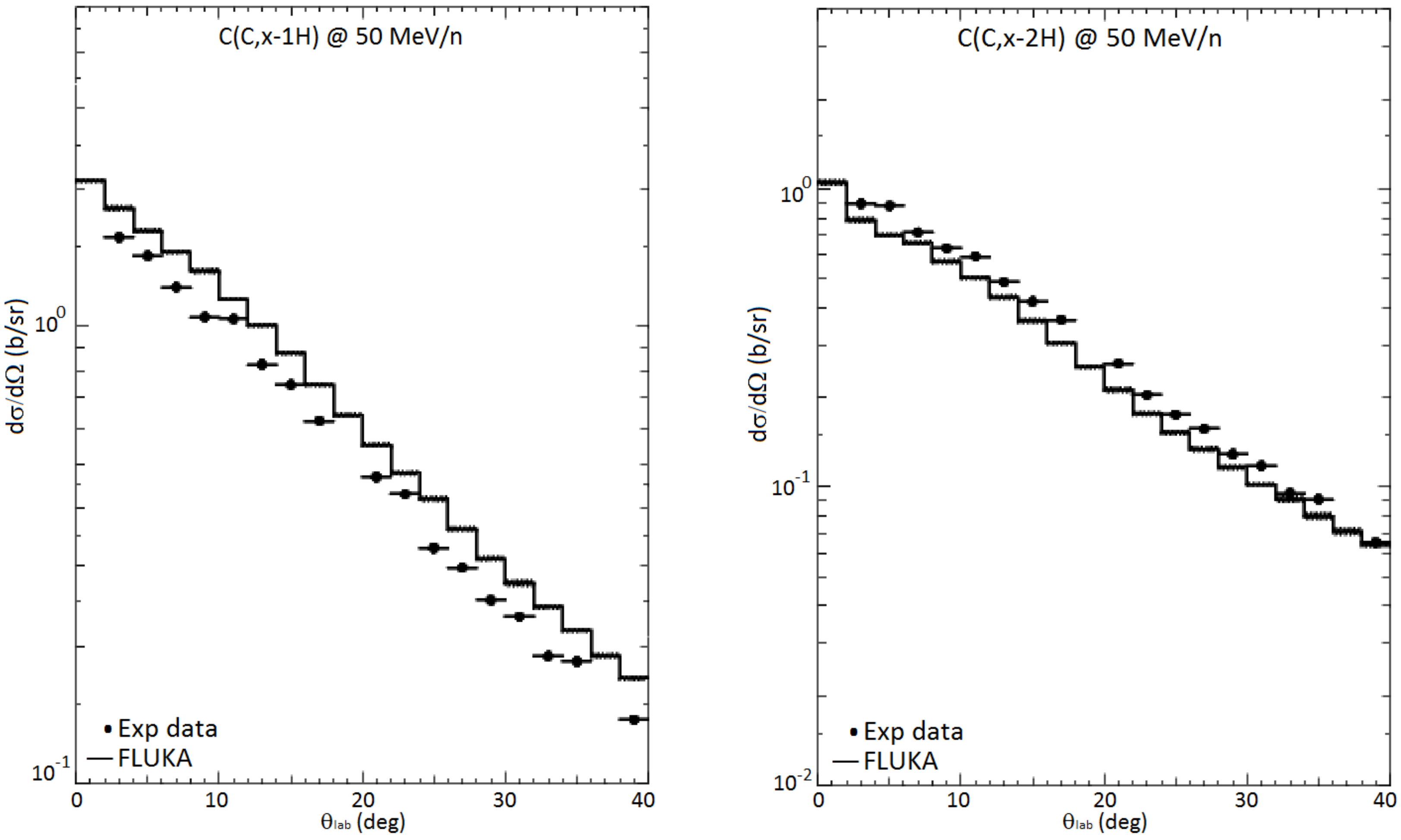
Ne on Cu →

← He on C



BME – Benchmarking (2)

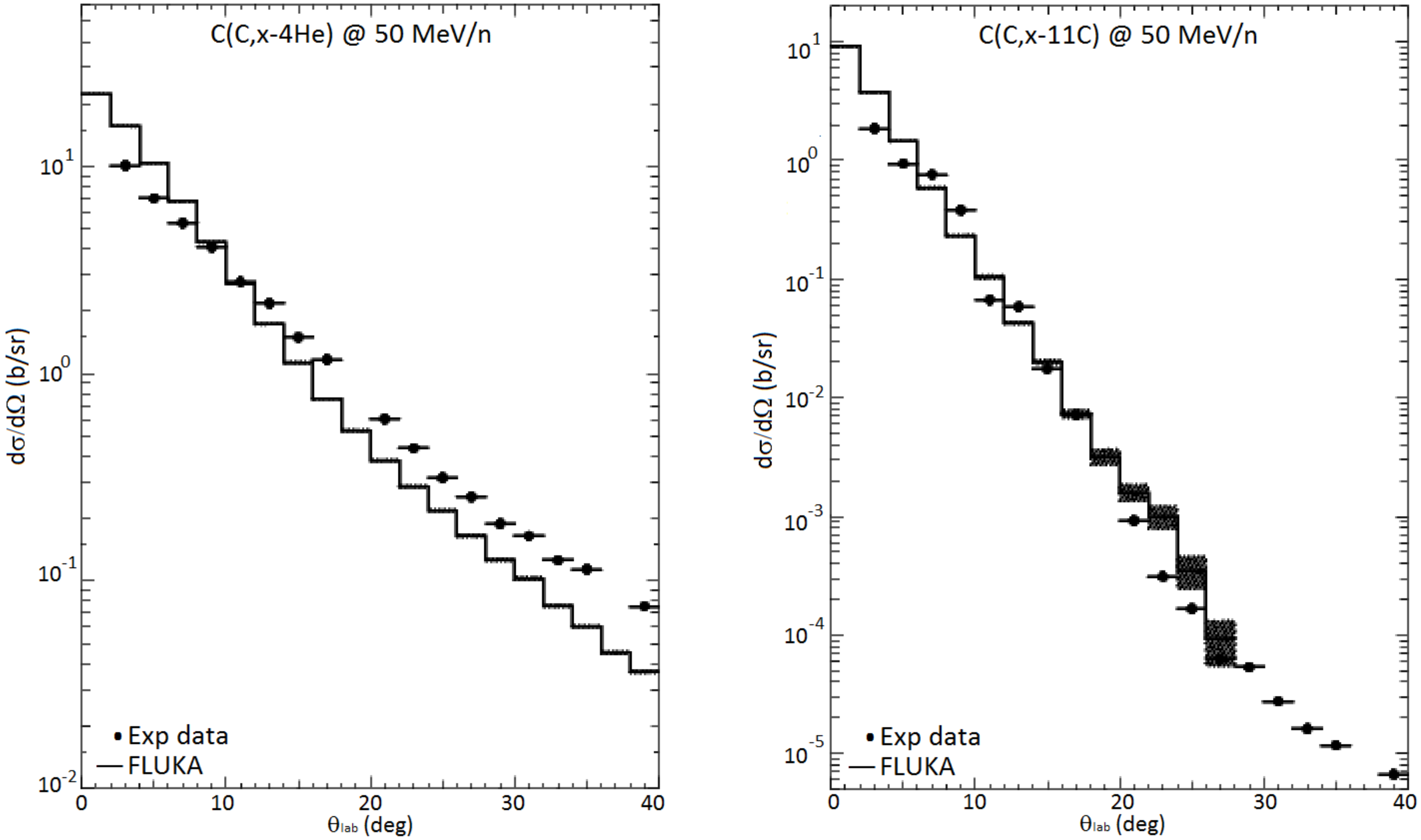
Single differential fragment spectra from C+C at **50 MeV/n**



Experimental data from Divay *et al* (2017) Phys Rev **C 95** 044602

BME – Benchmarking (3)

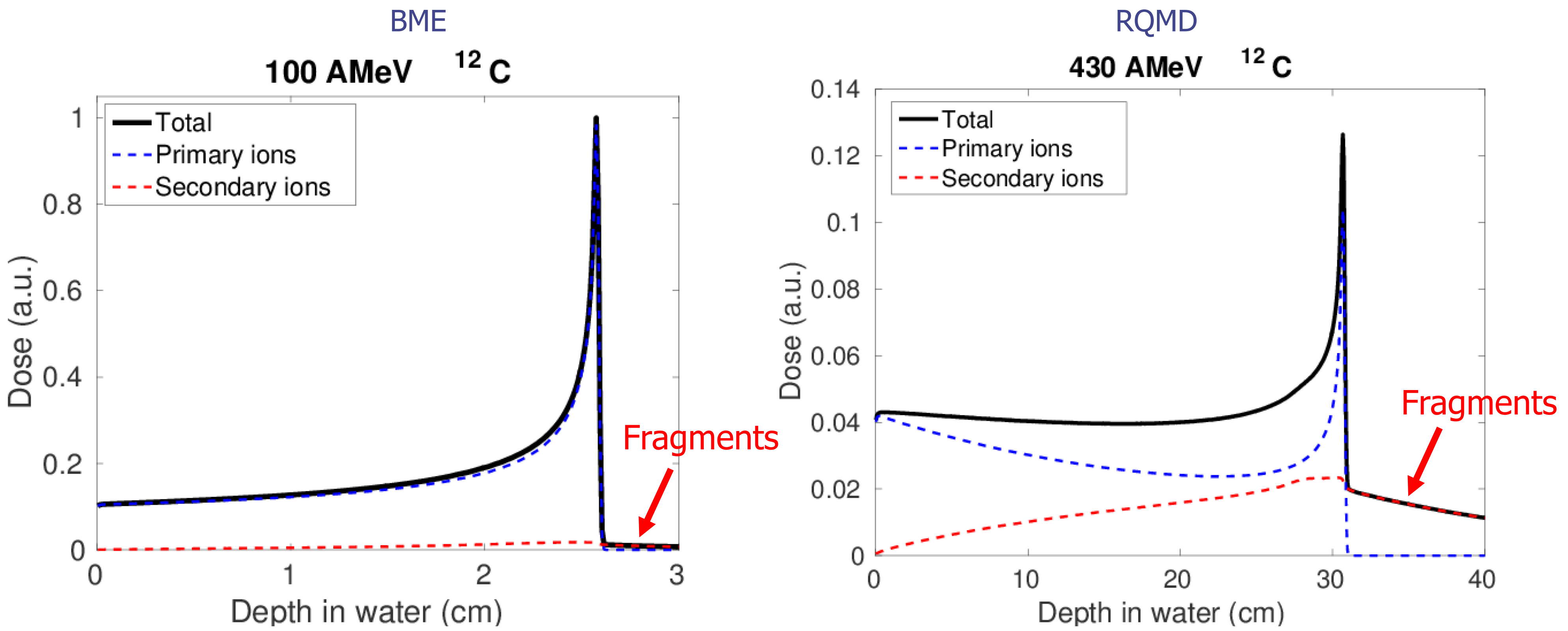
Single differential fragment spectra from C+C at **50 MeV/n**



Experimental data from Divay *et al* (2017) Phys Rev **C 95** 044602

BME – Medical applications

Carbon ion Bragg peaks in water

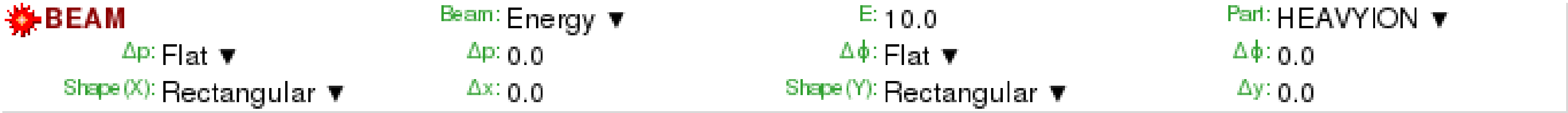


Primary ion fragmentations affect the dose delivered in the patient.
Fragments has to be considered in the simulations for accurate dose calculations.

Input options - 1

a) define momentum / energy

```
BEAM           -10.0       0.0       0.0       0.0       0.0       0.0HEAVYION
```



BEAM
Beam: Energy ▼
E: 10.0
Part: HEAVYION ▼
Δp: Flat ▼
Δp: 0.0
Δϕ: Flat ▼
Δϕ: 0.0
Shape (X): Rectangular ▼
Δx: 0.0
Shape (Y): Rectangular ▼
Δy: 0.0

WHAT(1) > 0.0 : average beam momentum (GeV/*c*)
< 0.0 : average beam kinetic energy (GeV)

Note: **for SDUM = HEAVYION units <u>per nucleon</u> (in fact per *nmu*)**
for SDUM = 4-HELIUM, *etc.* per nucleus

WHAT(2) beam <u>momentum</u> spread (GeV/*c*)

WHAT(3)-WHAT(6) (as for any other particle)

SDUM = HEAVYION

also	**4-HELIUM**	alpha
	3-HELIUM	3-helium
	TRITON	tritium
	DEUTERON	deuterium

Input options - 2

b) define charge and mass (required for BEAM/SDUM=HEAVYION)

If:

```
BEAM            -10.0      0.0       0.0       0.0       0.0       0.0HEAVYION
```

by default: ^{12}C

```
WHAT(1)   = Atomic number Z of the heavy ion, Default: 6.0
WHAT(2)   = Mass number A of the heavy ion, Default: 12.0
WHAT(3)   = if < 0 isomeric state of the heavy ion
```

Otherwise, to define another heavy ion (e.g. ^{79}Au)

```
 HI-PROPE         79.0     197.0       0.0       0.0       0.0       0.0
```

HI-PROPE Z: 79.0 A: 197.0 Isom: 0.0

Input options - 3

b) Switch on heavy ion transport

```
IONTRANS         -2.0
```

IONTRANS Transport: Full transport ▼

(For BEAM : **SDUM = HEAVYION** , full transport of all ions is enabled by default)

```
WHAT(1) >= 1 : no transport, full or approximate, of any light/heavy ion
         = 0 : ignored
        = -1 : approximate transport (without interactions) of all light and heavy ions
        = -2 : full transport of all light and heavy ions
        >= -6 and <= -3 : full transport of light ions with FLUKA id >= WHAT(1)
              (-3=d,-4=t,-5=3-He,-6=4-He), and approximate transport of all other ions

Default: 0.0 (no ion transport, unless a ion beam is requested by the BEAM card

WHAT(2-6) and SDUM not used.
```

When requested, interactions at energies larger than 100MeV/n are performed *provided that the external event generators DPMJET and RQMD are linked* (through the script $FLUPRO/flutil/ldpmqmd).

IMPORTANT:

- the DPMJET/RQMD event generators are EXTERNAL, they are distributed with FLUKA but not included in the main library neither in the standard executable

- **Don't forget** to **link** the DPMJET/RQMD event generators for enabling **ion-ion interactions above 125MeV/n** either using FLAIR or the script

  $FLUPRO/flutil/ldpmqmd

- The BME event generator, covering the low energy range up to 150MeV/n does not need to be linked since it's already embedded in the main $FLUPRO/libflukahp.a library and linked in the standard $FLUPRO/flukahp executable

Card: PHYSICS

Please activate the following two cards if residual nuclei are of interest:

switch to activate evaporation of heavy fragments (up to A=24,CPU expensive)

```
PHYSICS          3.0                                                     EVAPORAT
PHYSICS          1.0                                                     COALESCE
```

special options for coalescence treatment

Warning: deuterons

- Deuteron interactions are NOT modelled in BME, therefore

  ** NO DEUTERON interactions are available in FLUKA below a few hundreds MeV ***
- RQMD performs the interaction, however reliability is not ensured due to the “special” nature of deuteron interactions

Current Francesc Salvat-Pujol’s work:

- Reaction cross sections for d-A collisions
- Elastic break-up
- Stripping to bound state
- Stripping into continuum (→ PEANUT)

Transport thresholds

- The transport momentum threshold for ions ($p_{th,HI}$) is linked to that of alphas ($p_{th,\alpha}$)

$$p_{th,HI} = p_{th,\alpha} \times m_{HI}/m_{\alpha} \quad (GeV/c)$$

- The transport threshold for light ions (alpha, He-3, t, d) is set equal to total kinetic energy = 10 MeV (100 keV) if DEFAULTS=NEW-DEFA (PRECISIO).
- To change the transport threshold use the PART-THR card (requiring GeV and not GeV per nucleon)
- When the energy of an ion becomes lower than the transport threshold, and if such threshold is lower than 100 MeV/n, the ion is not stopped, but it is ranged out to rest

Additional information

DPMJET – The Gribov-Glauber formalism

Fundamental idea: nucleus-nucleus collision expressed in terms of individual nucleon-nucleon interactions

- nucleus-nucleus

  - *total cross section*

$$\sigma^{tot}_{AB}(s) = 4 \int d^2\vec{B}\, \Im[A_{AB}(s,\vec{B})]$$

  - *elastic cross section*

$$\sigma^{el}_{AB}(s) = 4 \int d^2\vec{B}\, |A_{AB}(s,\vec{B})|^2$$

  - *scattering amplitude*

$$A_{AB} = \frac{i}{2}\left[1 - \exp(\chi_{AB})\right]$$

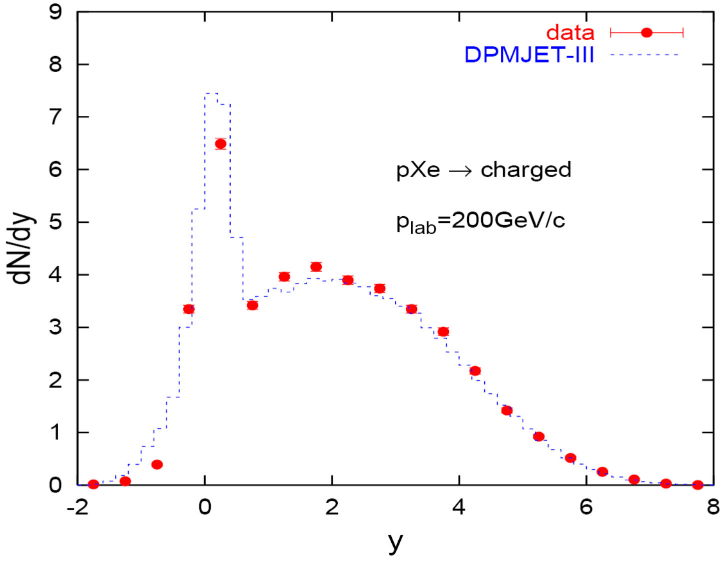
eikonal function $\chi_{AB} = \sum_{k,l} \chi_{N_k N_l}$

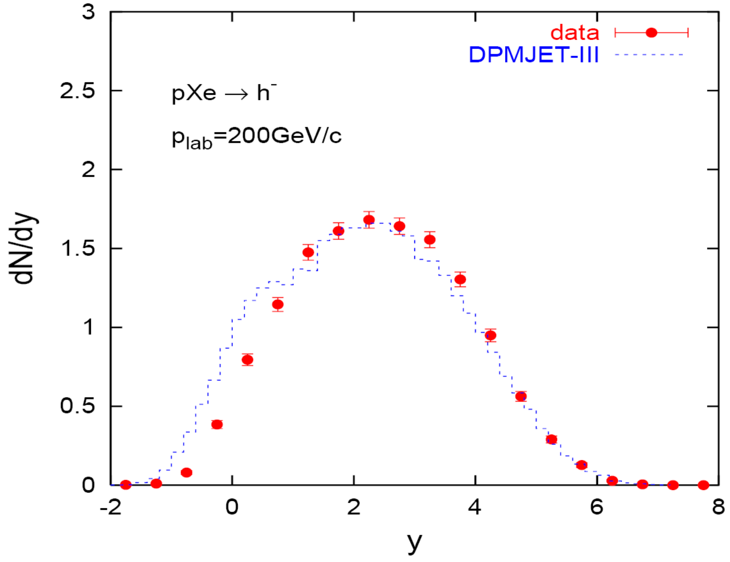
- nucleon-nucleon

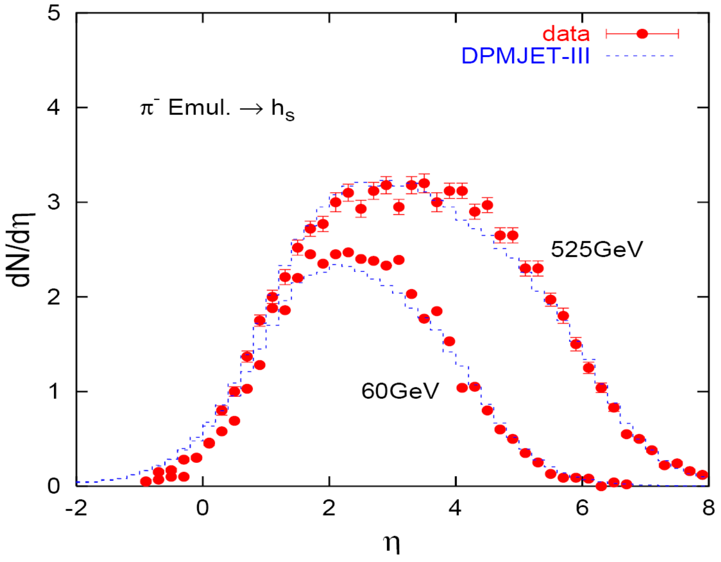
  - *scattering amplitude*

$$a_{N_k N_l} = \frac{i}{2}\left[1 - \exp(\chi_{N_k N_l})\right]$$

DPMJET – Comparison to data (hadron-nucleus)

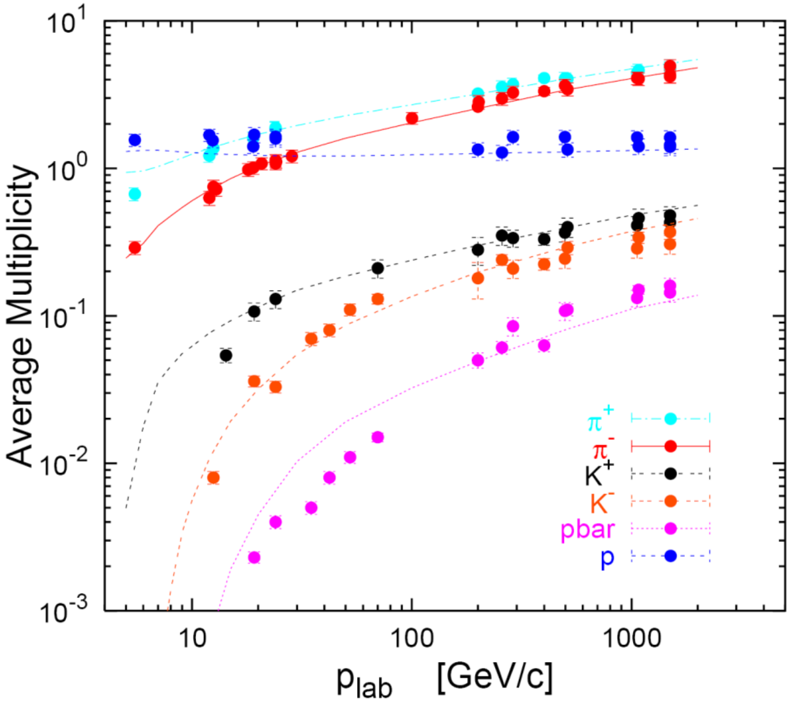






		Exp.	DPMJET-III
14.6 GeV	p Al	1.57 ±0.23	1.52
	p Au	2.15 ±0.33	1.92
200 GeV	p S	5.0 ± 0.2	4.98
	p Xe	6.84 ±0.13	6.67
360 GeV	p Al	6.8 ±0.6	5.87
	p Au	8.9 ±0.4	8.77

DPMJET – Comparison to data (hadron-hadron)

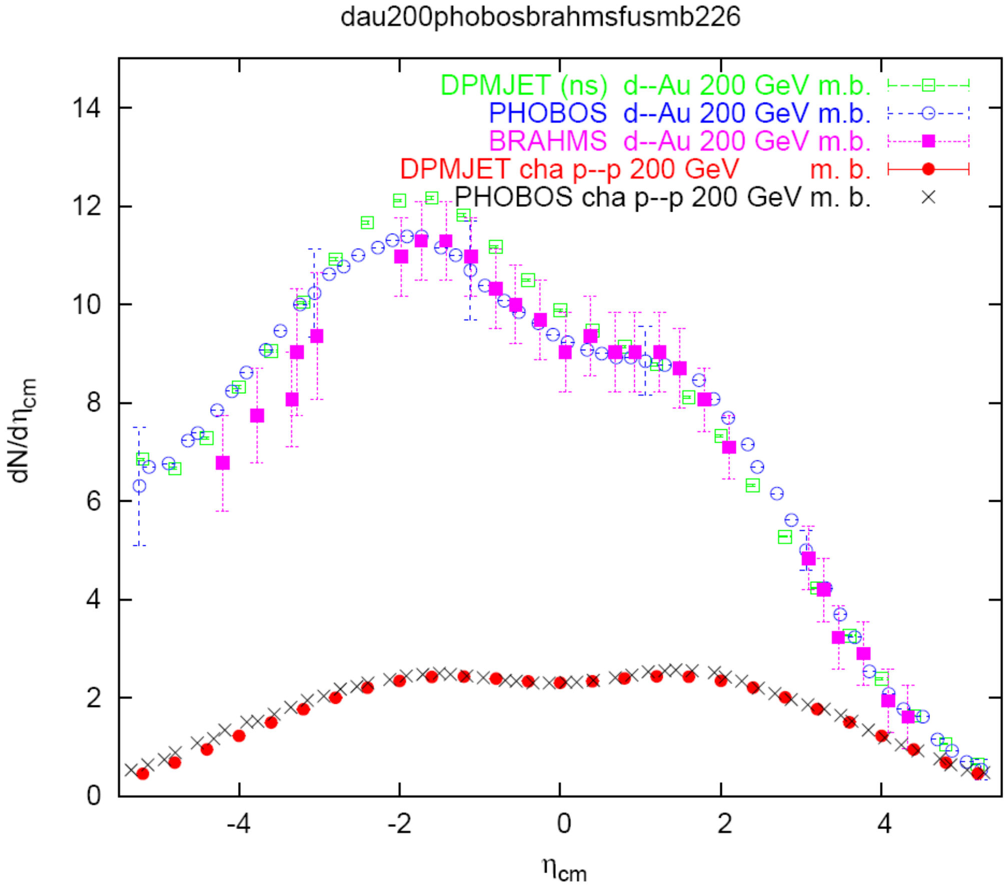

proton - proton

proton - proton, $E_{lab} = 200\text{GeV}$

	Exp.	DPMJET-III
charged	7.69 ± 0.06	7.64
neg.	2.85 ± 0.03	2.82
p	1.34 ± 0.15	1.26
n	0.61 ± 0.30	0.66
π^+	3.22 ± 0.12	3.20
π^-	2.62 ± 0.06	2.55
K^+	0.28 ± 0.06	0.30
K^-	0.18 ± 0.05	0.20
Λ	0.096 ± 0.01	0.10
$\bar{\Lambda}$	0.0136 ± 0.004	0.0105

DPMJET – Comparison to data (nucleus-nucleus)

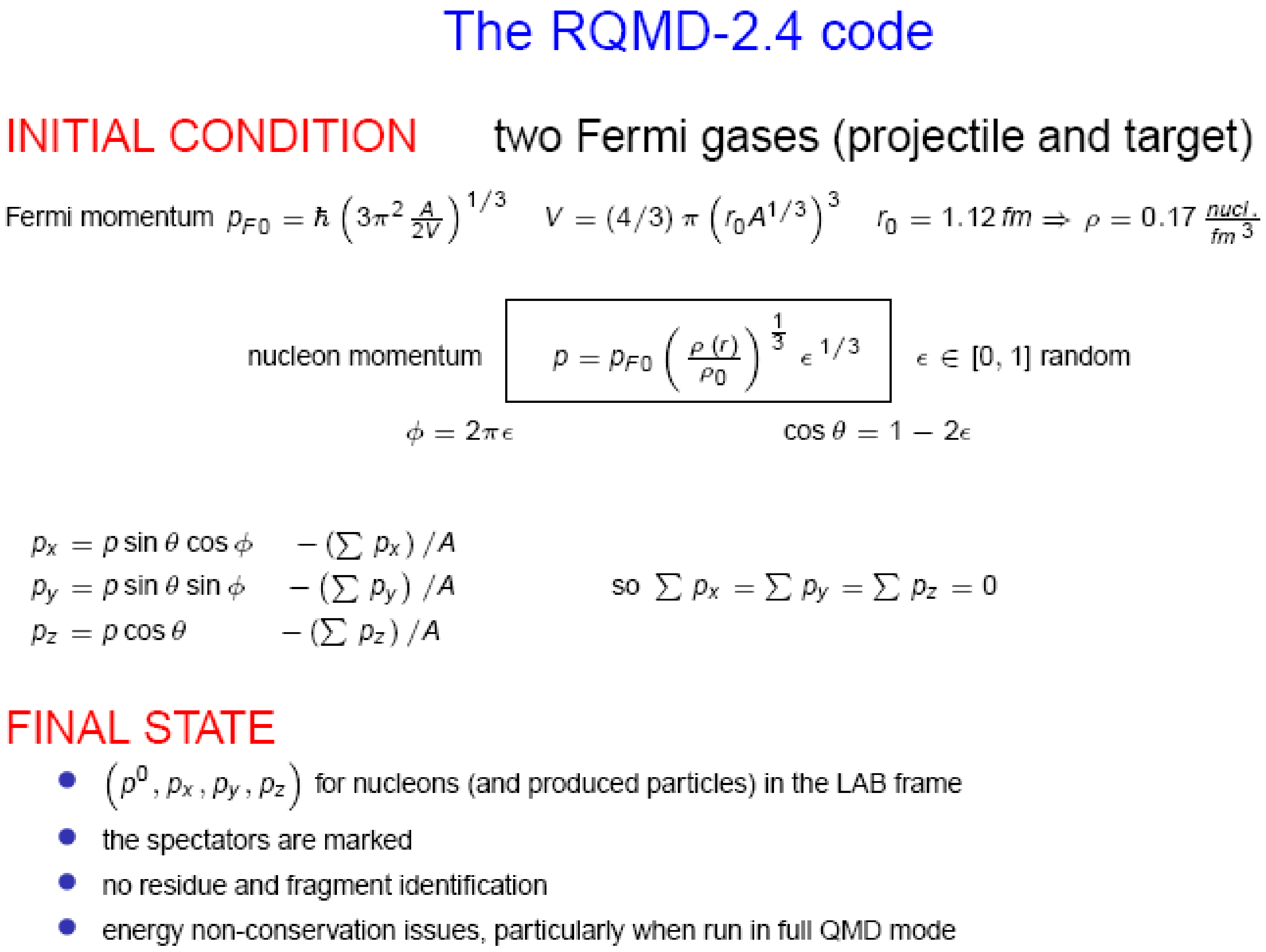
Pseudorapidity distribution of charged hadrons produced in minimum bias d-Au and p-p collisions at a c.m. energy of 200GeV/A.



Exp. data: BRAHMS- and PHOBOS-Collaborations

J.Ranft, in Proceedings of the Hadronic Shower Simulation Workshop, CP896, Batavia, Illinois (USA), 6-8 September 2006

RQMD – The original code



The RQMD-2.4 code

INITIAL CONDITION two Fermi gases (projectile and target)

Fermi momentum $p_{F0} = \hbar \left(3\pi^2 \frac{A}{2V}\right)^{1/3}$ $V = (4/3)\,\pi\,\left(r_0 A^{1/3}\right)^3$ $r_0 = 1.12\,fm \Rightarrow \rho = 0.17\,\frac{nucl.}{fm^3}$

nucleon momentum $\boxed{p = p_{F0}\left(\frac{\rho(r)}{\rho_0}\right)^{\frac{1}{3}}\epsilon^{1/3}}$ $\epsilon \in [0, 1]$ random

$\phi = 2\pi\epsilon$ $\cos\theta = 1 - 2\epsilon$

$$\begin{aligned} p_x &= p\sin\theta\cos\phi - \left(\sum p_x\right)/A \\ p_y &= p\sin\theta\sin\phi - \left(\sum p_y\right)/A \\ p_z &= p\cos\theta - \left(\sum p_z\right)/A \end{aligned}$$

so $\sum p_x = \sum p_y = \sum p_z = 0$

FINAL STATE

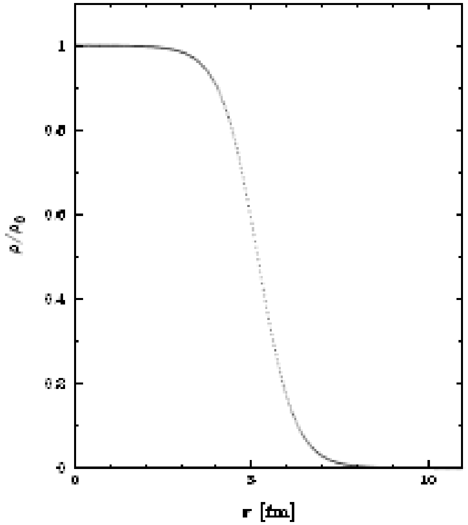
- $\left(p^0, p_x, p_y, p_z\right)$ for nucleons (and produced particles) in the LAB frame
- the spectators are marked
- no residue and fragment identification
- energy non-conservation issues, particularly when run in full QMD mode

RQMD – The interfaced code

Implemented developments

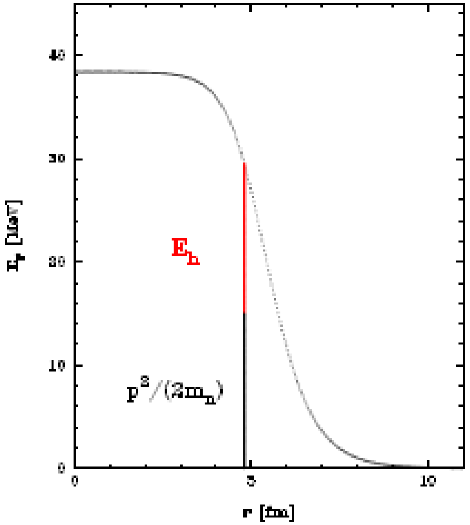
- construct the **projectile- and target-like** nuclei by gathering *spectator* nucleons,

assuming $E^{*}_{PL} = \sum_{pa.\ P} E_h \qquad (TL)$



$$\rho(r) \propto \left(1+\exp\left(\frac{r-R}{a}\right)\right)^{-1}$$

$$R = 1.19\,A^{1/3} - 1.61\,A^{-1/3}\ fm \qquad a = 0.52\ fm$$



$$E_h = \frac{1}{2m_n}\left\{\left[p_{F0}\left(\rho(r)/\rho_0\right)^{1/3}\right]^2 - p^2\right\}$$

$$r, p\,(t=0)$$

- fix the remaining energy-momentum conservation issues taking into account **experimental binding energies**
- use the FLUKA evaporation/fission/fragmentation module

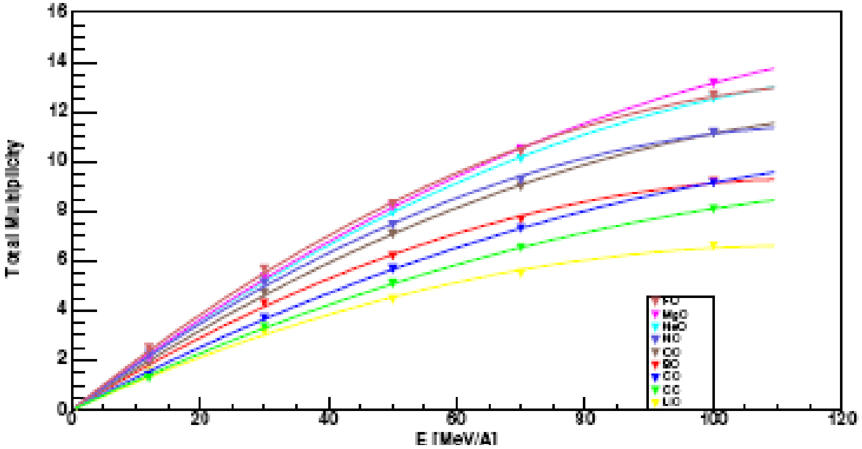
BME – The database for the pre-equilibrium emissions

In order to get the multiplicities of the pre-equilibrium particles and their double differential spectra, the BME theory is applied to several representative systems at different bombarding energies and the results are parameterized.

^{16}**O** + ^{6}Li, ^{8}Li, ^{8}B, ^{10}B, ^{12}C, ^{14}N, ^{16}O, ^{19}F, ^{20}Ne, ^{24}Mg, ^{27}Al, ^{56}Fe, ^{197}Au

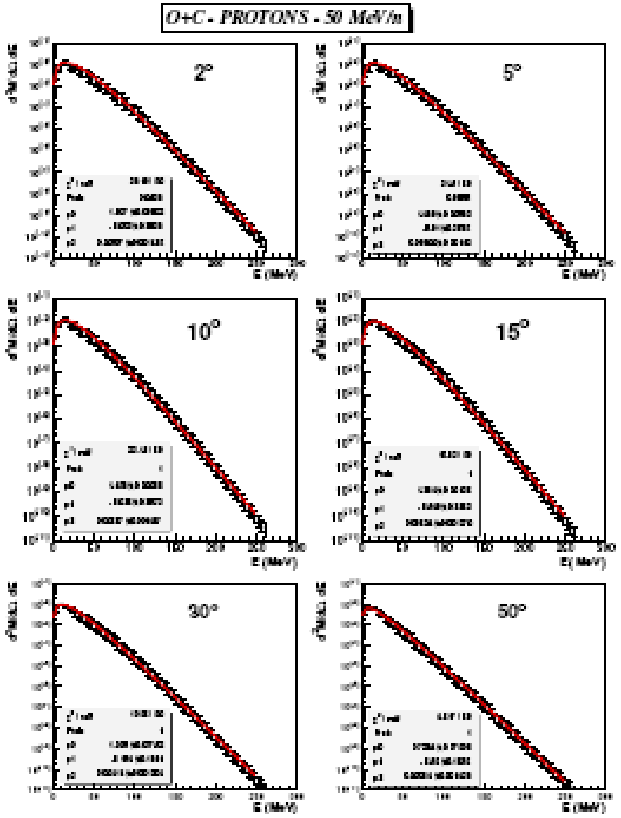
^{12}**C** + ^{8}Li, ^{8}B, ^{12}C, ^{27}Al, ^{40}Ca

@ 12, 30, 50, 70, 100 MeV/n



total multiplicity

$$M = P_1 E_{nucl} - P_2 E_{nucl}^2$$



energy spectra

$$d^2M/(dEd\Omega) = E^{P_0(\theta)} \exp\left(-P_1(\theta) - P_2(\theta)E\right)$$

Work is ongoing to extend it to more massive systems, i.e.

^{40}Ca + ^{120}Sn

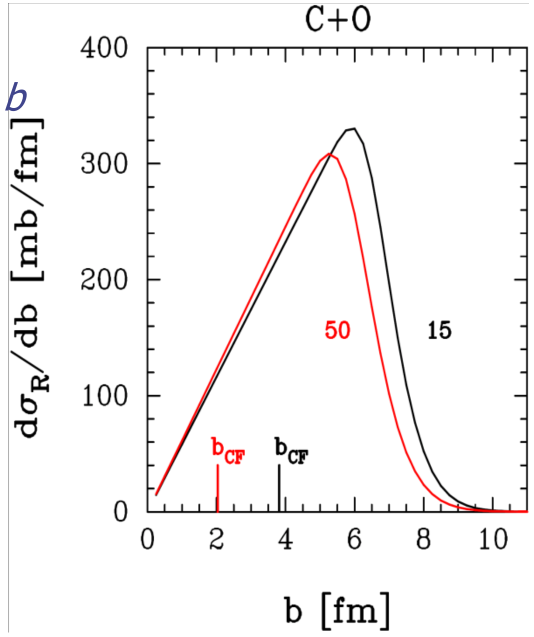
^{56}Fe + ^{28}Si, ^{40}Ca, ^{48}Ca, ^{120}Sn

and consequently review the fitting functions
and the extrapolation recipes over a significantly larger mass range

BME – Peripheral collisions

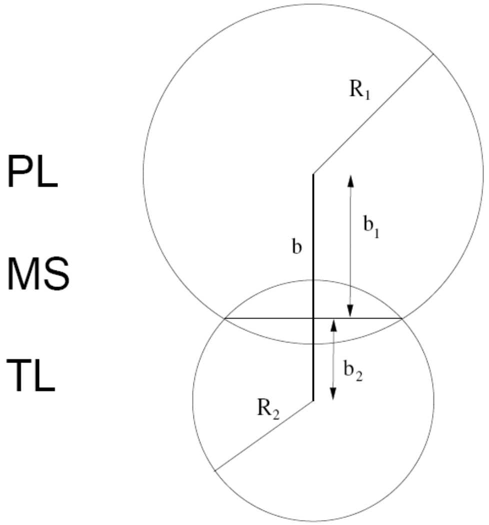
We integrate the nuclear densities of the projectile and the target over their overlapping region, as a function of the impact parameter, and obtain a preferentially excited “middle source” and two fragments (projectile- and target-like). The kinematics is suggested by break-up studies.

i. selection of the *impact parameter b*



ii. kinematics determination

θ_{PL}, θ_{TL} chosen according to $[d\sigma/d\Omega]_{cm} \sim \exp(-k\theta_{cm})$
θ_{MS} momentum conservation
p_{PL}, p_{TL} chosen according to a given energy loss distribution
p_{MS} momentum conservation
ϕ_{PL} free
ϕ_{TL}, ϕ_{MS} same reaction plane



iii. excitation energy sharing

$$E^\star_{MS} = (A_{MS}/A_{tot})E^\star_{tot}\sum_{n=0}^{k}(1 - A_{MS}/A_{tot})^n$$

$$E^\star_{PL} = f(A_{PL}, A_{TL})\left(E^\star_{tot} - E^\star_{MS}\right)$$

forced on the experimental values in the discrete level region

$$E^\star_{TL} = \left(E^\star_{tot} - E^\star_{MS} - E^\star_{PL}\right)$$

BME – Theoretical framework

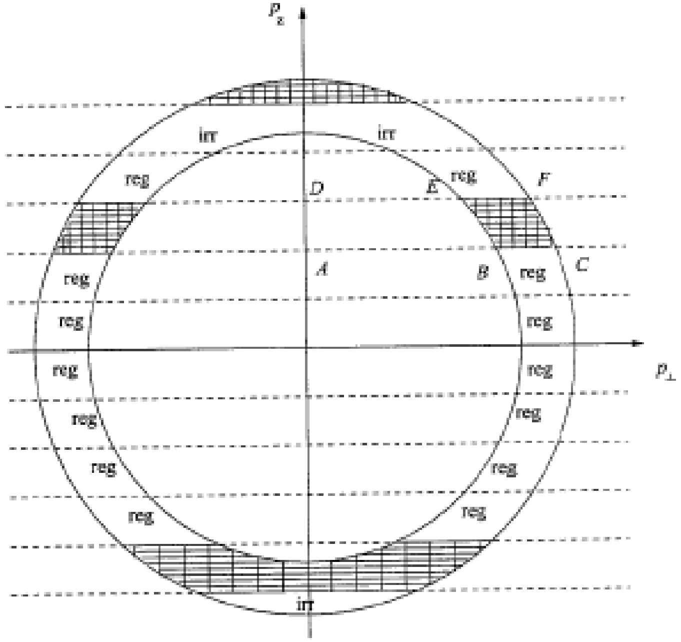
Calculation of preequilibrium for the composite nucleus

proton and neutron momentum spaces divided into bins

$$\left\{ (p_x, p_y, p_z) : p_z \in [\, p_{zi}, p_{zi} + \Delta p_z),\ \varepsilon = (2m)^{-1}\,(p_x^2 + p_y^2 + p_z^2) \in [\, \varepsilon_i\,, \varepsilon_i + \Delta\varepsilon) \right\}$$

(**Z** is the beam direction)

of volume $2\pi m\,\Delta\varepsilon\,\Delta p_z$



BME – Theoretical framework

The BME system

$$N_i = n_i \; g_i$$

nucleon number

occupation probability

number of states in bin i

$$\begin{aligned}\frac{d(n_i^\pi g_i^\pi)}{dt} = & \sum_{jlm} [\omega^{\pi\pi}_{lm\to ij}\, g_l^\pi\, n_l^\pi\, g_m^\pi\, n_m^\pi\,(1-n_i^\pi)(1-n_j^\pi) \\ & -\omega^{\pi\pi}_{ij\to lm}\, g_i^\pi\, n_i^\pi\, g_j^\pi\, n_j^\pi\,(1-n_l^\pi)(1-n_m^\pi)] \\ & +\sum_{jlm} [\omega^{\pi\nu}_{lm\to ij}\, g_l^\pi\, n_l^\pi\, g_m^\nu\, n_m^\nu\,(1-n_i^\pi)(1-n_j^\nu) \\ & -\omega^{\pi\nu}_{ij\to lm}\, g_i^\pi\, n_i^\pi\, g_j^\nu\, n_j^\nu\,(1-n_l^\pi)(1-n_m^\nu)] \\ & -n_i^\pi g_i^\pi \omega^\pi_{i\to i'}\, g_{i'}^\pi\, \delta(\varepsilon_i^\pi - \varepsilon_{i'}^\pi - \varepsilon_F^\pi - B^\pi) - \frac{dD_i^\pi}{dt}\end{aligned}$$

BME – Theoretical framework

Multiplicity spectra

of emitted nucleons

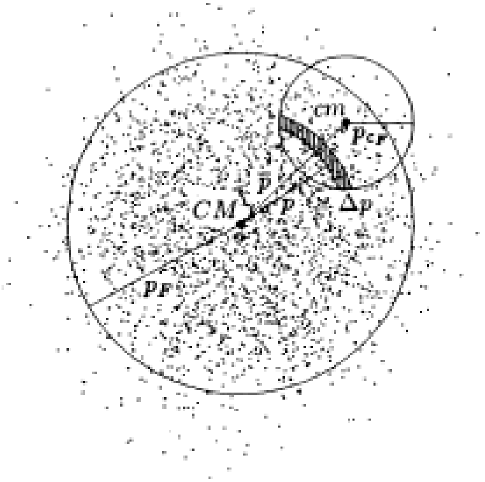
$$\frac{d^2 M(\varepsilon', \theta)}{d\varepsilon' \, d\Omega} = \frac{1}{2\pi \sin\theta} \int_0^{t_{eq}} n(\varepsilon, \theta, t) \, \frac{\sigma_{inv} \, v}{V} \, \rho(\varepsilon', \theta) \, dt$$

of a cluster *c*

$$\frac{d^2 M_c (E_c', \theta_c)}{dE_c' \, d\Omega} = \frac{R_c}{2\pi \sin\theta} \int_0^{t_{eq}} N_c (E_c, \theta_c, t) \, \frac{\sigma_{inv,c} \, v_c}{V} \, \rho_c \left(E_c', \theta_c\right) \, dt$$

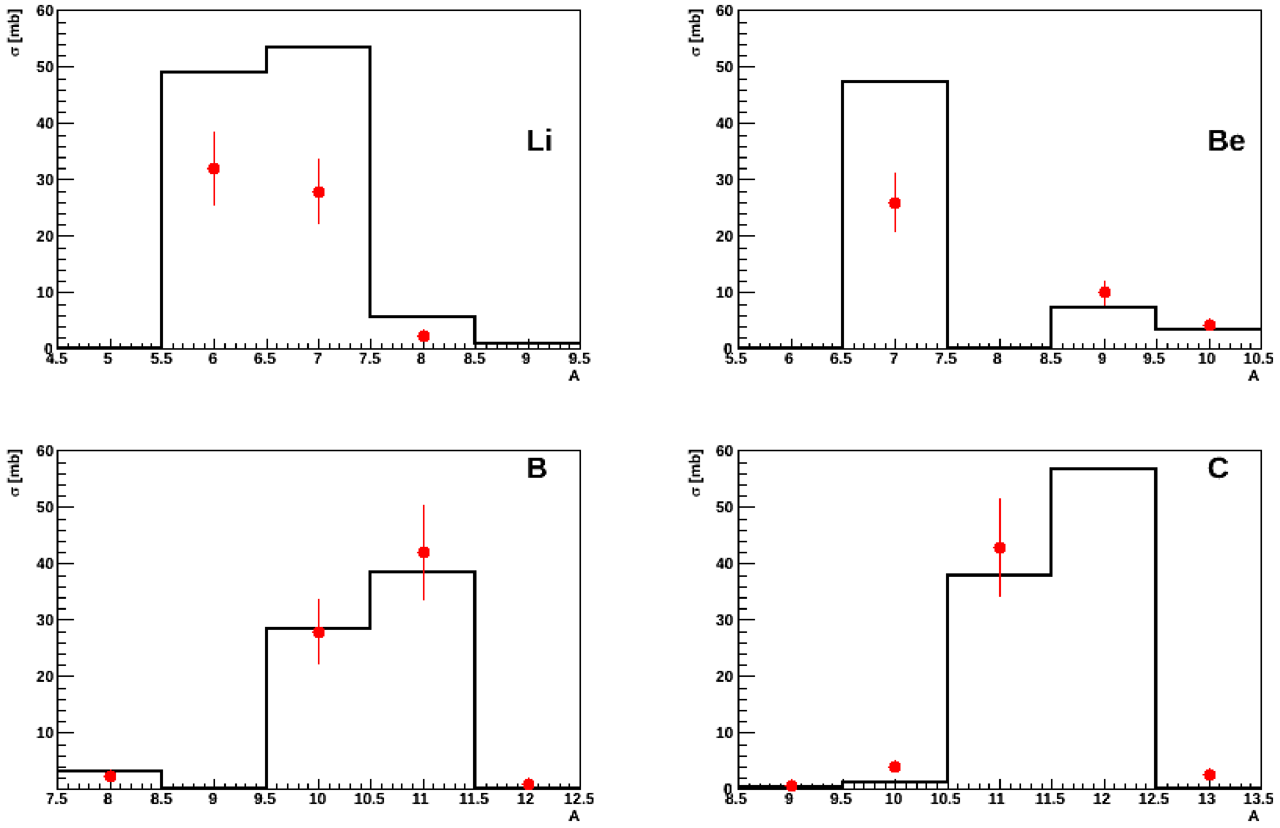
$$N_c (E_c, \theta_c, t) = \prod_i \left(n_i^\pi (\varepsilon, \theta, t)\right)^{P_i(E_c, \theta_c) Z_c} \cdot \prod_i \left(n_i^\nu (\varepsilon, \theta, t)\right)^{P_i(E_c, \theta_c) N_c}$$

joint probability



BME – Benchmarking

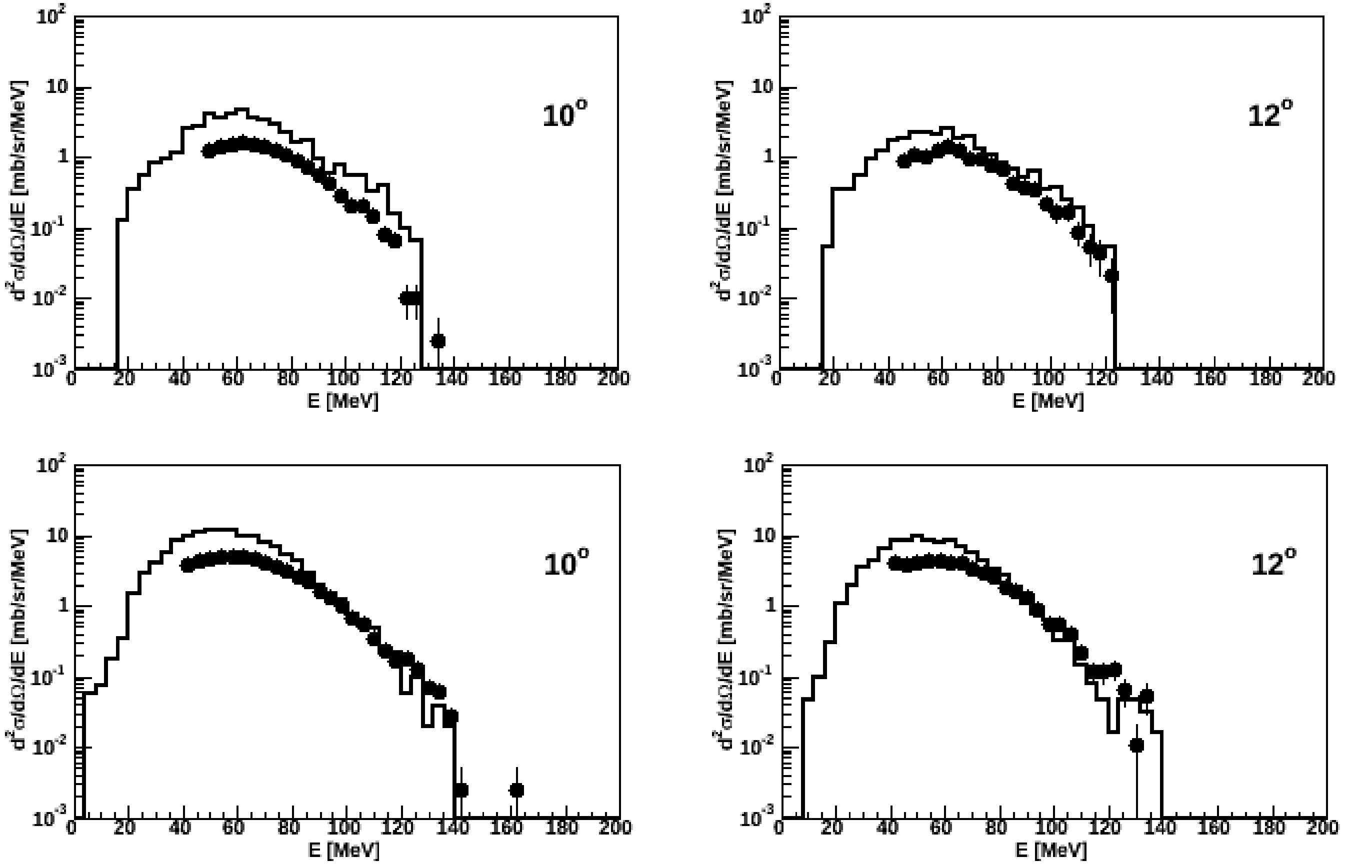
ISOTOPE YIELDS
FROM C+C at **86 MeV/n**



experimental data from H. Ryde, Physica Scripta T5, 114 (1983)

BME – Benchmarking

DOUBLE DIFFERENTIAL **FRAGMENT SPECTRA**
FROM C+C at **13 MeV/n**



Fluorine

Oxygen

experimental data by courtesy of S. Fortsch et al., iThemba Labs, South Africa