High-energy precision

Gauthier Durieux
(Technion)

GD, Martín Perelló, Marcel Vos, Cen Zhang, 1807.02121,

Top-Quark Physics at the CLIC Electron-Positron Linear Collider, 1807.02441,

GD, Oleksii Matsedonskyi, 1807.10273,

The CLIC Potential for New Physics, 1812.02093.

CLIC Workshop 2019
CERN, 24 January

Introduction

High-energy precision?

One can trade precision for energy
when new-physics effects grow with energy
faster than SM ones.

In EFT language, if linear effects scale as SM $\times\, C\dfrac{E^2}{\Lambda^2}$:

$$\left|\frac{C}{\Lambda^2}\right| \lesssim \frac{\mathcal{O}(10\%)}{(\text{TeV})^2} \quad\Longleftrightarrow\quad \left|\frac{C}{\Lambda^2}\right| \lesssim \frac{\mathcal{O}(0.1\%)}{(100\,\text{GeV})^2}$$

Exploited at the LHC, e.g. paper titles:

- *Strong tW Scattering at the LHC*, '15
- *Energy helps accuracy: electroweak precision tests at hadron colliders*, '16
- *Precision Probes of QCD at High Energies*, '17
- *Electroweak Precision Tests in High-Energy Diboson Processes*, '17
- *Probing Electroweak Precision Physics via boosted Higgs-strahlung at the LHC*, '18
- *New Physics from High Energy Tops*, '18
- *Higgs Couplings without the Higgs*, '18,
- ...

CLIC pushes on both precision and energy,
e.g. with $\sigma(e^+e^- \to t\,\bar{t})$ measurement to $\mathcal{O}(1\%)$ up to 3 TeV.

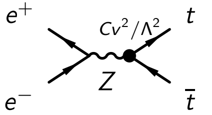
Energy growing effects?

Naive expectation from dimension-six operator: SM $\times\, CE^2/\Lambda^2$

- $1/\Lambda^2$ to get a dimension-four Lagrangian
- E^2, the dominant energy scale in the problem

In many cases, however, one gets: SM $\times\, Cv^2/\Lambda^2$

- *Explicit vev* E.g. $(\bar{u}\gamma^\mu u)(\phi^\dagger \overleftrightarrow{D}_\mu \phi) \ni (\bar{u}\gamma^\mu u)\dfrac{ev^2 Z_\mu}{2 s_W c_W}$



- *Helicity selection rules* Leading-order 2-to-2 linear-EFT amplitudes involving transverse gauge bosons don't have the same helicity configurations as SM ones. [Azatov et al. '16]

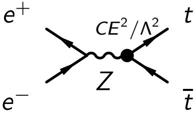
  → v/E-suppressed inclusive interferences
  - go differential in the decay products [Panico et al '17]
  - add radiation/extra leg [Azatov et al '17]
  - add a loop [Baglio et al. '17] [Chiesa et al. '18] [Azatov et al '19]

- ...

Growing faster than the SM (and unitarity)

- Adding derivatives

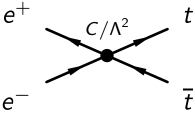
  E.g. $(\bar{u}\gamma^\mu u)(D^\nu B_{\mu\nu})$



- Removing propagators

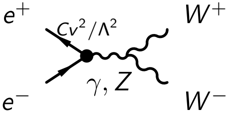
  E.g. $(\bar{u}\gamma^\mu u)(\bar{e}\gamma_\mu e) \ni (\bar{u}\gamma^\mu u)(D^\nu B_{\mu\nu})$ by EOM

  Dirac spinor $u_\xi(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi \\ \sqrt{p\cdot\bar{\sigma}}\xi \end{pmatrix} \sim \sqrt{E}$



- Upsetting $SU(2)_L$ cancellations

$$k = (E_k, 0, 0, k)$$
$$\epsilon_k^T = (0,1,0,0), (0,0,1,0)$$
$$\epsilon_k^L = (k/m, 0, 0, E_k/m) \sim E/m$$

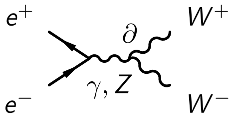


- And still circumventing helicity selection rules

$SU(2)_L$ cancellations?

E.g. $e_R e_R \to W_{L,T} W_{L,T}$

- Naive SM high-energy behaviour



pol.	naive SM	SM	EFT
RRLL			
RRLT			
RRTT			

E power counting:	$1/2 + 1/2$	two Dirac spinors
	-2	one vector propagator
	$+1$	one derivative VVV coupling
	$+n$	n longitudinal V

$SU(2)_L$ cancellations?

E.g. $e_R e_R \to W_{L,T} W_{L,T}$

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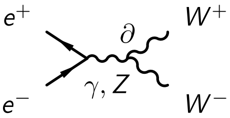

pol.	naive SM	SM	EFT
RRLL	E^2/v^2		
RRLT	E/v		
RRTT	1		

E power counting:	$1/2 + 1/2$	two Dirac spinors
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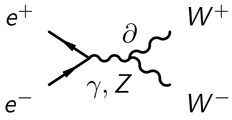

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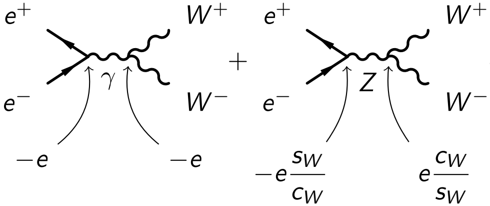
- Pedestrian computation

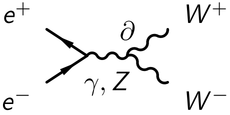


$$\sim \frac{e^2}{E^2} - \frac{e^2}{E^2 - m_Z^2} \overset{E \gg v}{\sim} \frac{1}{E^2}\frac{v^2}{E^2}$$

$SU(2)_L$ cancellations?

E.g. $e_R e_R \to W_{L,T} W_{L,T}$

- Naive SM high-energy behaviour



pol.	naive SM	SM	EFT
RRLL	E^2/v^2	1	
RRLT	E/v	v/E	
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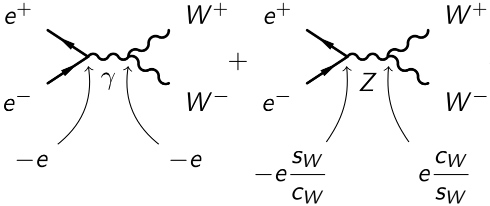
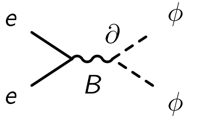


$$\sim \frac{e^2}{E^2} - \frac{e^2}{E^2 - m_Z^2} \overset{E \gg v}{\sim} \frac{1}{E^2}\frac{v^2}{E^2}$$

$SU(2)_L$ cancellations?

E.g. $e_R e_R \to W_{L,T} W_{L,T}$

- Unbroken SM phase
  (and Goldstone eq. th.: $V_L \to \phi$)
  - RRLL



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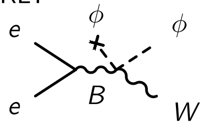
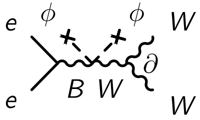
  - RRLT



$\sim v/E$

  - RRLT



$\sim v^2/E^2$

pol.	naive SM	SM	EFT
RRLL	E^2/v^2	1	
RRLT	E/v	v/E	
RRTT	1	v^2/E^2	

reproduces the high-energy behaviour of the SM in a less pedestrian way

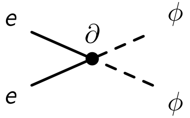
$SU(2)_L$ cancellations?

E.g. $e_R e_R \to W_{L,T} W_{L,T}$

- EFT in the unbroken phase

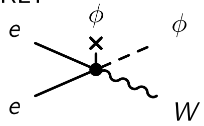
  E.g. $\frac{C_{\phi e}}{\Lambda^2} (\bar{e}\gamma^\mu e)(\phi^\dagger \overleftrightarrow{D}_\mu \phi)$

  - RRLL



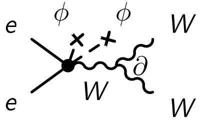
$\sim E^2/\Lambda^2$

  - RRLT



$\sim Ev/\Lambda^2$

  - RRLT



$\sim v^2/\Lambda^2$

pol.	naive SM	SM	EFT
RRLL	E^2/v^2	1	
RRLT	E/v	v/E	
RRTT	1	v^2/E^2	

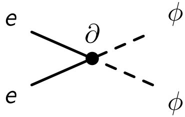
$SU(2)_L$ cancellations?

E.g. $e_R e_R \to W_{L,T} W_{L,T}$

- EFT in the unbroken phase

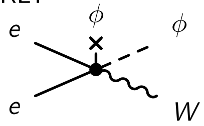
  E.g. $\frac{C_{\phi e}}{\Lambda^2} (\bar{e}\gamma^\mu e)(\phi^\dagger \overleftrightarrow{D}_\mu \phi)$

  - RRLL



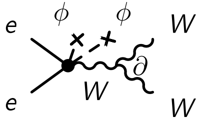
$\sim E^2/\Lambda^2$

  - RRLT



$\sim Ev/\Lambda^2$

  - RRLT



$\sim v^2/\Lambda^2$

pol.	naive SM	SM	$C_{\phi e}$
RRLL	E^2/v^2	1	E^2/Λ^2
RRLT	E/v	v/E	Ev/Λ^2
RRTT	1	v^2/E^2	v^2/Λ^2

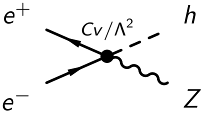
The EFT can upset $SU(2)_L$ cancellations and restore the naive high-energy behaviour of the SM.

$SU(2)_L$ cancellations must at least occur when the naive high-energy violates unitarity.

More e^+e^- examples of energy growth

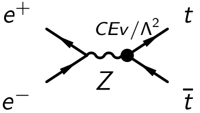
Fewer propagators

- $(\bar{e}\gamma^\mu e)(\phi^\dagger \overleftrightarrow{D}_\mu \phi) \ni (\bar{e}\gamma^\mu e)\frac{eZ_\mu}{2s_W c_W}(h+v)^2$



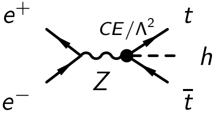
Derivative & decay angles

- $(\bar{q}\sigma^{\mu\nu}u)\phi B_{\mu\nu}$
  more difficult reconstruction than total rate
  only SM $\times$ CEv/Λ^2



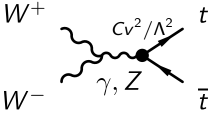
Derivative & fewer propagators & extra leg

- $(\bar{q}\sigma^{\mu\nu}u)\phi B_{\mu\nu}$
  lower rates than $e^+e^- \to t\bar{t}$
  SM $\times$ CE^2/Λ^2



Upset $SU(2)_L$ cancellations

- $(\bar{u}\gamma^\mu u)(\phi^\dagger \overleftrightarrow{D}_\mu \phi) \ni (\bar{u}\gamma^\mu u)\frac{eZ_\mu}{2s_W c_W}(h+v)^2$



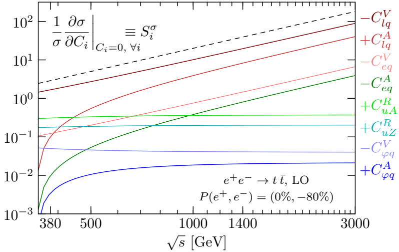
$e^+e^- \to t\,\bar{t}$ application

GD, Martín Perelló, Marcel Vos, Cen Zhang, 1807.02121

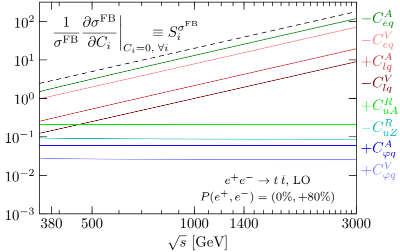
Top-Quark Physics at the CLIC Electron-Positron Linear Collider, 1807.02441

Operator sensitivities as functions of energy

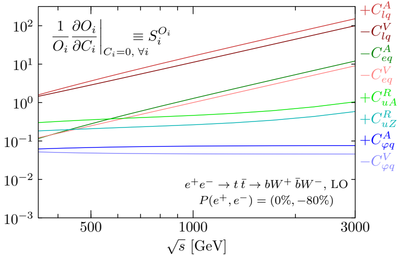
Total cross section (left pol.)

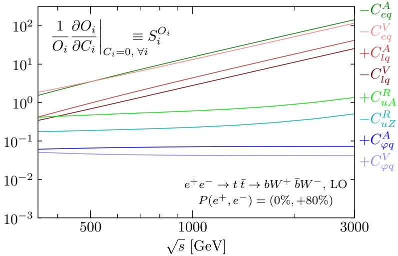


FB-integrated cross section (right pol.)



Statistically optimal observable (left/right pol.)





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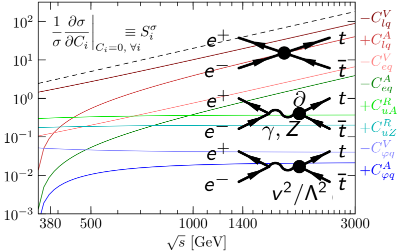
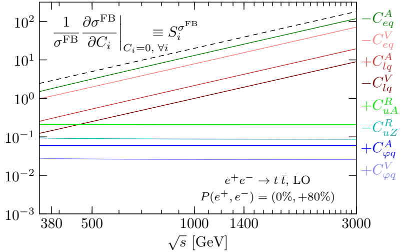


FB-integrated cross section (right pol.)



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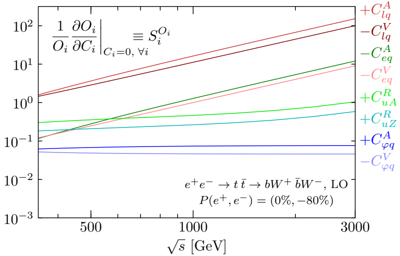
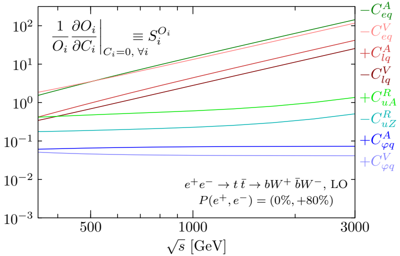
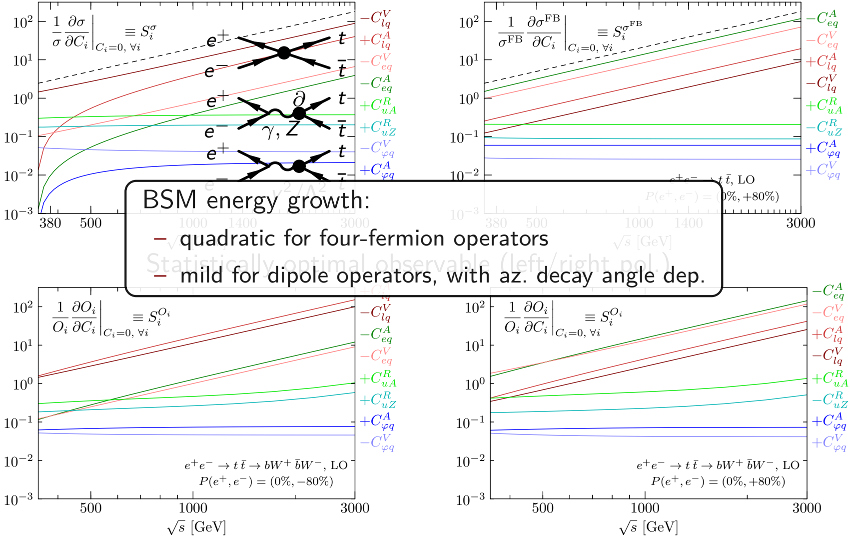




Operator sensitivities as functions of energy

Total cross section (left pol.) FB-integrated cross section (right pol.)

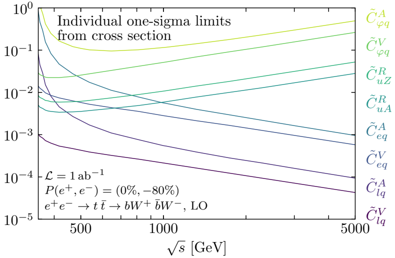


BSM energy growth:

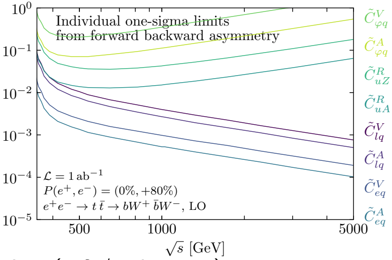
- quadratic for four-fermion operators
- mild for dipole operators, with az. decay angle dep.

Individual limits as functions of energy, for $1\,\mathrm{ab}^{-1}$

Total cross section (left pol.)



FB-integrated cross section (right pol.)



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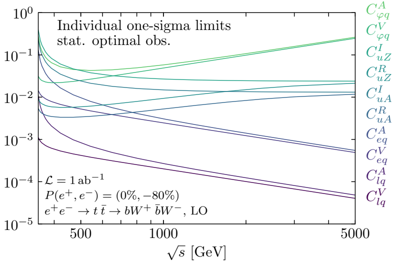
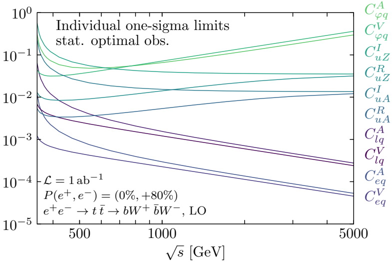
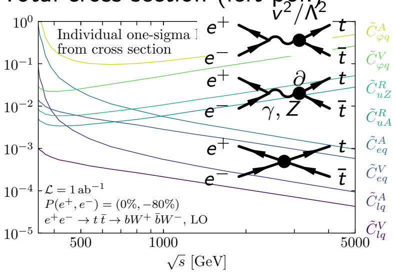


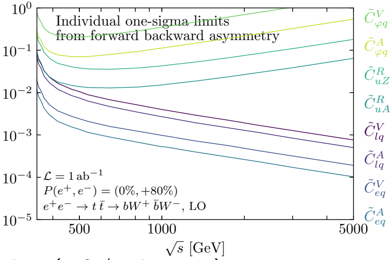


Individual limits as functions of energy, for $1\,\mathrm{ab}^{-1}$

Total cross section (left pol.)



FB-integrated cross section (right pol.)



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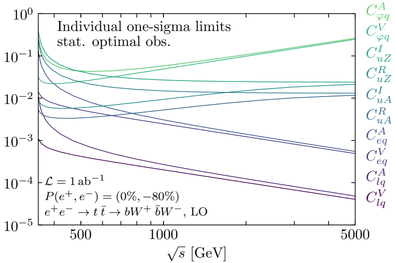

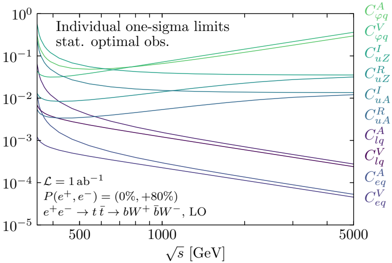

Individual limits as functions of energy, for $1\,\mathrm{ab}^{-1}$

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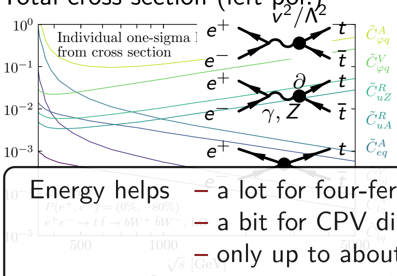

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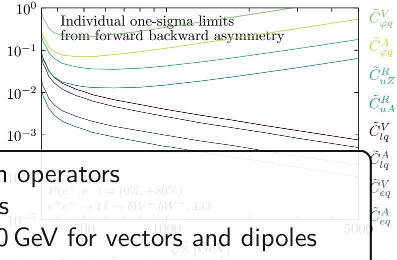
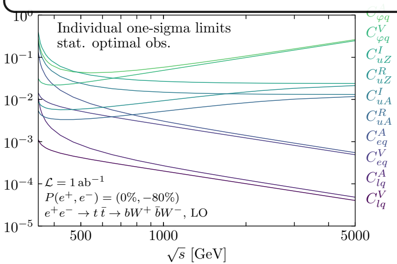
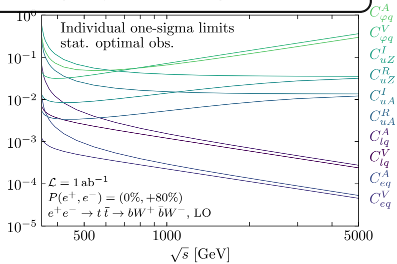


Energy helps
- a lot for four-fermion operators
- a bit for CPV dipoles
- only up to about 500 GeV for vectors and dipoles

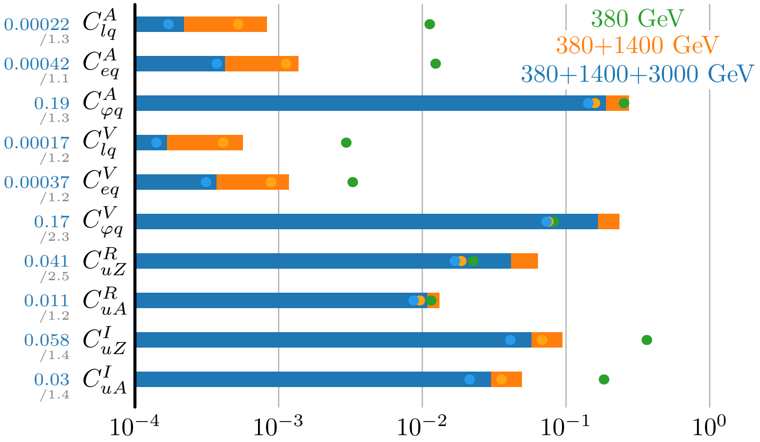
(A luminosity growing linearly with $\sqrt{s}$ only changes this picture qualitatively above 3 TeV.)

Statistically optimal observable (left/right pol.)





Global reach



$\Lambda = 1$ TeV, $\Delta\chi^2 = 1$, $1 \oplus 2.5 \oplus 5\,\text{ab}^{-1}$
from statistically optimal observables
global (bars) and individual (dots) constraints
too few constraints with 380 GeV run only to simultaneously probe all directions

Impact on
composite Higgs models

GD, Oleksii Matsedonskyi, 1807.10273

The CLIC Potential for New Physics, 1812.02093

Framework

- The Higgs is composite, pNGB of a new strong sector
- Typical strong sector coupling and mass: $g_\star$, $m_\star$
- Linear mixings between SM states and composite ones: ϵ_u, ϵ_q

$\rightarrow$ dimensional analysis for strong-sector operator coefficients

preserving $\hbar$ and mass dimensions

$$\frac{m_\star^4}{g_\star^2}\, O^{\text{dim-6}}\left(\epsilon_\psi \frac{g_\star}{m_\star^{3/2}}\psi,\ \frac{g}{m_\star^2}F^{\mu\nu},\ \frac{g_\star}{m_\star}\phi\right)$$

up to order-one factors or justified suppressions

in particular $y_t \simeq \epsilon_u \epsilon_q g_\star$

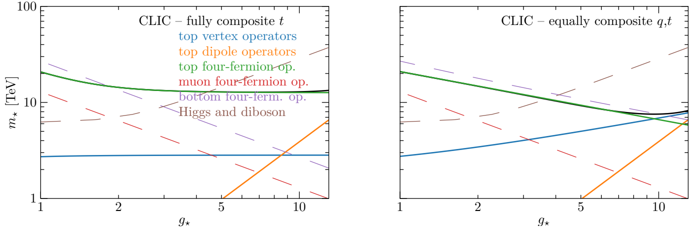
fix either $\epsilon_u = \epsilon_q \simeq \sqrt{\frac{y_t}{g_\star}}$: equally composite top left and right

$\epsilon_u = 1,\ \epsilon_q \simeq \frac{y_t}{g_\star}$: fully composite top right

CLIC prospects $(0.5 \oplus 1.5 \oplus 3\,\text{ab}^{-1})$

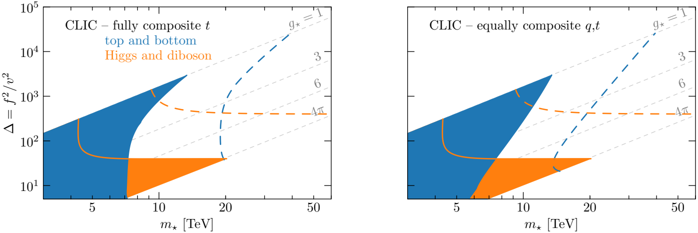
[Higgs fit from 1704.02333]

Different five-sigma sensitivities in (coupling,mass) plane



power counting exactly satisfied

Final five-sigma discovery reach in (mass,tuning) plane



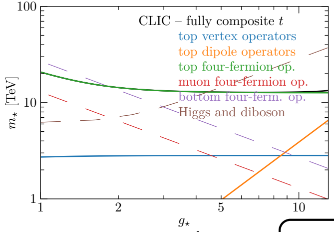
power counting satisfied up to $\pm[1/2, 2]$ → filled: conservative

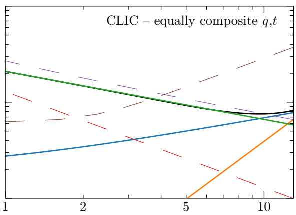
→ dashed: optimistic

CLIC prospects $(0.5 \oplus 1.5 \oplus 3\,\text{ab}^{-1})$

[Higgs fit from 1704.02333]

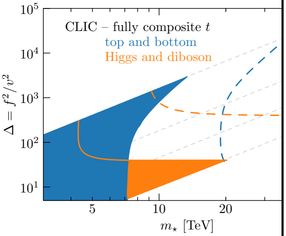
Different five-sigma sensitivities in (coupling,mass) plane





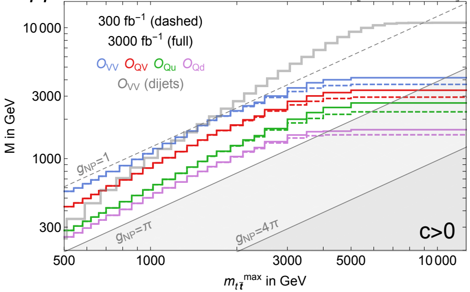
power counting exac

Final five-sigma discovery



power counting satis

$pp \to t\bar{t}$ [1811.04084]



$\to m_\star \lesssim 2\,\text{TeV}\ (t\bar{t})$ and $8\,\text{TeV}/g_\star\ (jj)$

Summary

High-energy precision

Precision can be traded for energy
when new physics grows faster than the SM:

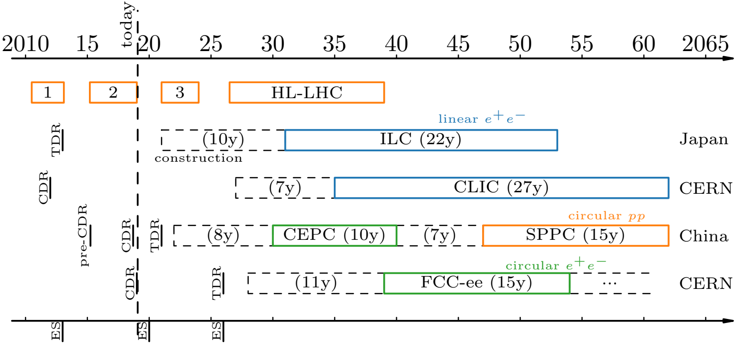
- derivative couplings
- multipoint interactions
- $SU(2)_L$ cancellations upset
- helicity selection rules escaped

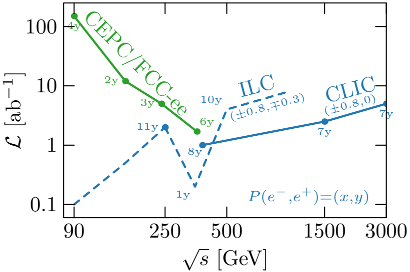
CLIC would actually deliver high precision at high energy.

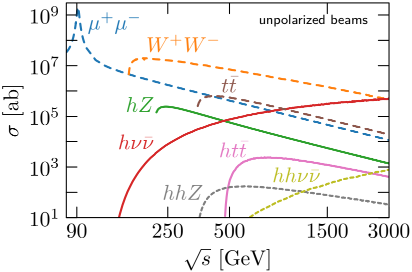
It allows to probe indirectly scales beyond the HL-LHC reach
(both direct and indirect), in composite Higgs models.

Backup

Future lepton colliders







Up-sector SMEFT

[Grzadkowski et al '10]

Two-quark operators:

$$\mathcal{L}_{\text{EFT}} = \sum_i \frac{C_i}{\Lambda^2} O_i$$

Scalar: $O_{u\varphi} \equiv \bar{q} u \,\tilde{\varphi}\;\; \varphi^\dagger\varphi,$

Vector:
$O^1_{\varphi q} \equiv \bar{q}\gamma^\mu q \;\; \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi \equiv O^+_{\varphi q} + O^V_{\varphi q} - O^A_{\varphi q},$
$O^3_{\varphi q} \equiv \bar{q}\gamma^\mu \tau^I q \;\; \varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi \equiv O^+_{\varphi q} - O^V_{\varphi q} + O^A_{\varphi q}$ (CC also)
$O_{\varphi u} \equiv \bar{u}\gamma^\mu u \;\; \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi \equiv O^V_{\varphi q} + O^A_{\varphi q}$
$O_{\varphi ud} \equiv \bar{u}\gamma^\mu d \;\; \tilde{\varphi}^\dagger i\overleftrightarrow{D}_\mu \varphi,$ (CC only, m_b int.)

Tensor:
$O_{uB} \equiv \bar{q}\sigma^{\mu\nu} u \,\tilde{\varphi}\, g_Y B_{\mu\nu}, \equiv O_{uA} - \tan\theta_W O_{uZ}$
$O_{uW} \equiv \bar{q}\sigma^{\mu\nu}\tau^I u \,\tilde{\varphi}\, g_W W^I_{\mu\nu}, \equiv O_{uA} + \text{cotan}\,\theta_W O_{uZ}$ (CC also)
$O_{dW} \equiv \bar{q}\sigma^{\mu\nu}\tau^I d \,\tilde{\varphi}\, g_W W^I_{\mu\nu},$ (CC only, m_b int.)
$O_{uG} \equiv \bar{q}\sigma^{\mu\nu} T^A u \,\tilde{\varphi}\, g_s G^A_{\mu\nu}.$ (NLO only)

Two-quark–two-lepton operators:

Scalar:
$O^S_{lequ} \equiv \bar{l} e \;\; \varepsilon \;\; \bar{q} u,$ (CC also, m_e int.)
$O_{ledq} \equiv \bar{l} e \;\; \bar{d} q,$ (CC only, m_e int.)

Vector:
$O^1_{lq} \equiv \bar{l}\gamma_\mu l \;\; \bar{q}\gamma^\mu q \equiv O^+_{lq} + O^V_{lq} - O^A_{lq},$
$O^3_{lq} \equiv \bar{l}\gamma_\mu \tau^I l \;\; \bar{q}\gamma^\mu \tau^I q \equiv O^+_{lq} - O^V_{lq} + O^A_{lq},$ (CC also)
$O_{lu} \equiv \bar{l}\gamma_\mu l \;\; \bar{u}\gamma^\mu u \equiv O^V_{lq} + O^A_{lq},$
$O_{eq} \equiv \bar{e}\gamma^\mu e \;\; \bar{q}\gamma_\mu q \equiv O^V_{eq} - O^A_{eq},$
$O_{eu} \equiv \bar{e}\gamma_\mu e \;\; \bar{u}\gamma^\mu u \equiv O^V_{eq} + O^A_{eq},$

Tensor:
$O^T_{lequ} \equiv \bar{l}\sigma_{\mu\nu} e \;\; \varepsilon \;\; \bar{q}\sigma^{\mu\nu} u.$ (CC also, m_e int.)

Statistically optimal observables

[Atwood,Soni '92]
[Diehl,Nachtmann '94]

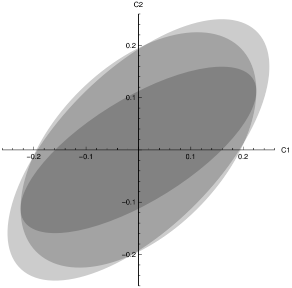
minimize the one-sigma ellipsoid in EFT parameter space

(*joint efficient* set of estimators, saturating the Cramér-Rao bound: $V^{-1} = I$,like MEM)

For small C_i, with a phase-space distribution $\sigma(\Phi) = \sigma_0(\Phi) + \sum_i C_i\, \sigma_i(\Phi)$,
the stat. opt. obs. are the average values of $O_i(\Phi) = n\, \sigma_i(\Phi)/\sigma_0(\Phi)$.

The associated covariance at $C_i = 0,\ \forall i$ is

$$\text{cov}(C_i, C_j)^{-1} = \epsilon\mathcal{L} \int \text{d}\Phi\ \frac{\sigma_i(\Phi)\sigma_j(\Phi)}{\sigma_0(\Phi)}.$$



e.g. $\sigma(\phi) = 1 + \cos(\phi) + C_1 \sin(\phi) + C_2 \sin(2\phi)$

1. asymmetries: $O_i \sim \text{sign}\{\sin(i\phi)\}$
2. moments: $O_i \sim \sin(i\phi)$
3. statistically optimal: $O_i \sim \dfrac{\sin(i\phi)}{1+\cos\phi}$

$\Longrightarrow$ area ratios 1.9 : 1.7 : 1

Previous applications in $e^+e^- \to t\,\bar{t}$, on different distributions:
[Grzadkowski, Hioki '00] [Janot '15] [Khiem et al '15]

Helicity amplitude decomposition in $bW^+\bar{b}W^-$ [Jacob,Wick '59]

Production amplitudes: [Schmidt '95]

$$++ : A_1 \sim \frac{2m_t}{\sqrt{s}} V + \sqrt{s}\,(D - \beta\tilde{D})$$
$$-- : A_2 \sim \frac{2m_t}{\sqrt{s}} V + \sqrt{s}\,(D + \beta\tilde{D})$$
$$+- : A_3 \sim (V + \beta A) + 2m_t D$$
$$-+ : A_4 \sim (V - \beta A) + 2m_t D$$

In terms of $\Omega = \{\theta_0, \theta_1, \phi_1, \theta_2, \phi_2\}$ helicity angles:

$\frac{d\sigma}{d\Omega} \propto$

$+3/4$		$(\lvert A_3\rvert^2 + \lvert A_4\rvert^2)$	$\lvert a_2\rvert^2 + \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 + \lvert b_3\rvert^2$	$(1+\cos^2\theta_0)$			
$+3/4$		$(\lvert A_3\rvert^2 - \lvert A_4\rvert^2)$	$\lvert a_2\rvert^2 + \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$(1+\cos^2\theta_0)$		$\cos\theta_2$	
$+3/4$		$(\lvert A_3\rvert^2 - \lvert A_4\rvert^2)$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 + \lvert b_3\rvert^2$	$(1+\cos^2\theta_0)$	$\cos\theta_1$		
$+3/4$		$(\lvert A_3\rvert^2 + \lvert A_4\rvert^2)$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$(1+\cos^2\theta_0)$	$\cos\theta_1$	$\cos\theta_2$	
$-3/2$		$(\lvert A_3\rvert^2 - \lvert A_4\rvert^2)$	$\lvert a_2\rvert^2 + \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 + \lvert b_3\rvert^2$	$\cos\theta_0$			
$-3/2$		$(\lvert A_3\rvert^2 + \lvert A_4\rvert^2)$	$\lvert a_2\rvert^2 + \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\cos\theta_0$		$\cos\theta_2$	
$-3/2$		$(\lvert A_3\rvert^2 + \lvert A_4\rvert^2)$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 + \lvert b_3\rvert^2$	$\cos\theta_0$	$\cos\theta_1$		
$-3/2$		$(\lvert A_3\rvert^2 - \lvert A_4\rvert^2)$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\cos\theta_0$	$\cos\theta_1$	$\cos\theta_2$	
$+3/2$		$(\lvert A_1\rvert^2 + \lvert A_2\rvert^2)$	$\lvert a_2\rvert^2 + \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 + \lvert b_3\rvert^2$	$\sin^2\theta_0$			
$-3/2$		$(\lvert A_1\rvert^2 - \lvert A_2\rvert^2)$	$\lvert a_2\rvert^2 + \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin^2\theta_0$		$\cos\theta_2$	
$+3/2$		$(\lvert A_1\rvert^2 - \lvert A_2\rvert^2)$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 + \lvert b_3\rvert^2$	$\sin^2\theta_0$	$\cos\theta_1$		
$-3/2$		$(\lvert A_1\rvert^2 + \lvert A_2\rvert^2)$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin^2\theta_0$	$\cos\theta_1$	$\cos\theta_2$	
$+3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_1^*A_4\}$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$\sin\theta_1$	$(1+\cos\theta_2)$	$\cos\phi_1$
$+3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_1^*A_4\}$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$\sin\theta_1$	$(1-\cos\theta_2)$	$\cos\phi_1$
$+3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_2^*A_3\}$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$\sin\theta_1$	$(1+\cos\theta_2)$	$\cos\phi_1$
$+3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_2^*A_3\}$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_3\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$\sin\theta_1$	$(1-\cos\theta_2)$	$\cos\phi_1$
$-3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_2^*A_4\}$	$\lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$(1+\cos\theta_1)$	$\sin\theta_2$	$\cos\phi_2$
$-3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_2^*A_4\}$	$\lvert a_2\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$(1-\cos\theta_1)$	$\sin\theta_2$	$\cos\phi_2$
$-3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_1^*A_3\}$	$\lvert a_2\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$(1+\cos\theta_1)$	$\sin\theta_2$	$\cos\phi_2$
$-3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_1^*A_3\}$	$\lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$(1-\cos\theta_1)$	$\sin\theta_2$	$\cos\phi_2$
-3		$\mathrm{Re}\{A_1^*A_2\}$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin^2\theta_0$	$\sin\theta_1$	$\sin\theta_2$	$\cos(\phi_1+\phi_2)$
$-3/2$		$\mathrm{Re}\{A_3^*A_4\}$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin^2\theta_0$	$\sin\theta_1$	$\sin\theta_2$	$\cos(\phi_1-\phi_2)$
$+3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_1^*A_4\}$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$\sin\theta_1$	$(1+\cos\theta_2)$	$\sin\phi_1$
$+3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_1^*A_4\}$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$\sin\theta_1$	$(1-\cos\theta_2)$	$\sin\phi_1$
$-3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_2^*A_3\}$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_1\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$\sin\theta_1$	$(1+\cos\theta_2)$	$\sin\phi_1$
$-3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_2^*A_3\}$	$\lvert a_2\rvert^2 - \lvert a_4\rvert^2$	$\lvert b_3\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$\sin\theta_1$	$(1-\cos\theta_2)$	$\sin\phi_1$
$+3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_2^*A_4\}$	$\lvert a_4\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$(1+\cos\theta_1)$	$\sin\theta_2$	$\sin\phi_2$
$+3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_2^*A_4\}$	$\lvert a_2\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$(1-\cos\theta_1)$	$\sin\theta_2$	$\sin\phi_2$
$-3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_1^*A_3\}$	$\lvert a_2\rvert^2$	$\lvert b_1\rvert^2 - \lvert b_3\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$(1+\cos\theta_1)$	$\sin\theta_2$	$\sin\phi_2$