Investigating the use of Quantum Computers for Final State Radiation

Benjamin Nachman

Lawrence Berkeley National Laboratory

Based in part on 1904.03196 in collaboration with D. Provasoli, C. Bauer, and W. de Jong

with support from the QuantISED HEP initiative
I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world. - Feynman
I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world. - Feynman

~ Outline ~

Quantum machines (“quantum computers”)
I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world. - Feynman

~ Outline ~

Quantum machines ("quantum computers")

A simple, but real model
I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world. - Feynman

~ Outline ~

Quantum machines (“quantum computers”)  
A simple, but real model  
The future
Goal: implement our system’s Hamiltonian (e.g. the SM) in a proxy system (“quantum computer”) and let it evolve.

What can be a proxy system?

…any quantum system, like a collection of spins.
Analog versus Digital Quantum Circuits

Goal: implement our system’s Hamiltonian (e.g. the SM) in a proxy system (“quantum computer”) and let it evolve.

The best quantum computer is the one that looks just like the system you are trying to model!

Image credit: https://www3.physik.uni-stuttgart.de/TR21/en/about/research.php
Analog versus Digital Quantum Circuits

Goal: implement our system’s Hamiltonian (e.g. the SM) in a proxy system (“quantum computer”) and let it evolve.

The best quantum computer is the one that looks just like the system you are trying to model!

Not always possible!
Analog versus Digital Quantum Circuits

Goal: implement our system’s Hamiltonian (e.g. the SM) in a proxy system (“quantum computer”) and let it evolve.

In this setup, the possibilities are endless; the key is efficiency.
There is no consensus on architecture, but most efforts for universal quantum computing use superconductors.

I’m not going to talk about hardware, though it is an exciting topic.
Modern Universal Quantum Computers

There is no consensus on architecture, but most efforts for universal quantum computing use superconductors.

I’m not going to talk about hardware, though it is an exciting topic.
State-of-the-art quantum computers

The best quantum computers have $O(10)$ qubits with $O(1)$ connections per qubit and can stay coherent for $O(100)$ of operations.

A qubit is an abstract representation of a quantum system that can be in a superposition of two states (often thought of as a spin).

This is one of IBM’s 20-qubit quantum computers. Lines represent connections.
Just like a classical computer, one can write programs for a universal quantum computer.
Just like a classical computer, one can write programs for a universal quantum computer.
Just like a classical computer, one can write programs for a universal quantum computer.

Initialize in the ground state.
Just like a classical computer, one can write programs for a universal quantum computer.

Apply unitary matrix $U_1$ to the third qubit
Just like a classical computer, one can write programs for a universal quantum computer.

Apply unitary matrix $U_2$ to the second qubit when the third is 0, else apply $U_3$. 

|0⟩ ——— $U_1$ ——— |0⟩ ——— $U_2$ ——— |0⟩ ——— $U_3$ ——— |0⟩ ——— $U_4$ ——— |0⟩ ——— $U_5$ ——— |0⟩ ——— $U_4$ ———
Just like a classical computer, one can write programs for a universal quantum computer.

Apply unitary matrix $U_4$ to both the first and second quits when the third is 0.
Just like a classical computer, one can write programs for a universal quantum computer.
Challenges with current computers

In practice: only controlled operation that is allowed is CNOT (swap if 1 otherwise do nothing) … need to decompose.

There is no compiler … need to do circuit decomposition by hand (!)
Challenges with current computers

In practice: only controlled operation that is allowed is CNOT (swap if 1 otherwise do nothing) … need to decompose.

Circuit implementation is architecture-dependent

*need to know what connections are available*

(can swap, but CANNOT clone qubits!)
Challenges with current computers

In practice: only controlled operation that is allowed is CNOT (swap if 1 otherwise do nothing) … need to decompose.

Circuit implementation is architecture-dependent

Computers are super noisy. Need to minimize number of operations.

Most importantly: current quantum computers are super noisy. Need to minimize number of operations.

FIG. 4. The probability of finding an e+ pair in the two-dimensional spatial-site Schwinger model from the initial empty state following time evolution with CNOT \( T \), in the unshaded region, with \( T \) = \( 3 \) saturates to classical probability of 0.5 after a small number of temporal extent, the quantum hardware. Using the reported gate specifica-

\( CNOT \rightarrow CNOT^3 \)

\( CNOT \rightarrow CNOT^5 \)

\( CNOT \rightarrow CNOT^7 \)

N.B. \( CNOT^2 \) = identity

Our work has identified key areas of future develop-

ment needed to robustly explore quantum field theories of Energy, O

Operating on quantum computers. By enforcing Gauss's law, momentum information and Matter and Oak Ridge National Laboratory for kind hospitality during this work. MS

John Preskill, Larry McLerran, Aidan Murran, Ken-
Potential of quantum computers

Caveats aside, there is a good reason to be excited.

There have been impressive leaps in hardware, “firmware”, & algorithms in the last years and interest has exploded.

Will you have a QPU in your laptop 5 years from now?

No. But you may be able to run on a QPU in 5 years that allows you to make a calculation that was not possible before (!)
Our goal

There are many ongoing efforts to do full QFT calculations with a QPU lattice*.

Our goal is more focused: many aspects of QFT calculations can be performed well on classical computers (e.g. automated NLO with MadGraph … N.B. high energy part hardest for lattice)

Can a piece of the calculation that is hard/impossible with classical computers & accelerate it on a QPU?

*for a great perspective piece, see Preskill’s recent Lattice2018 talk:1811.10085
One challenge: Final state radiation

FSR is a complex many-body quantum system.

Perhaps quantum tools can be used to incorporate quantum degrees of freedom!
A simple model with complex pheno

\[ \mathcal{L} = \bar{f}_1 i(\bar{\phi} + m_1) f_1 + \bar{f}_2 (i\bar{\phi} + m_2) f_2 + (\partial_\mu \phi)^2 \]

\[ + g_1 \bar{f}_1 f_1 \phi + g_2 \bar{f}_2 f_2 \phi + g_{12} [\bar{f}_1 f_2 + \bar{f}_2 f_1] \phi \]

(like the SM Higgs when \( g_{12} \sim m/\nu \) and \( g_1 = g_2 = 0 \))

aka baby electroweak shower … not QCD because Sudakov independent of spin
The quantum algorithm

Table 1: All of the registers in the quantum circuit with the number of qubits they require after each step.

<table>
<thead>
<tr>
<th>Register</th>
<th>Purpose</th>
<th># of qubits</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>p\rangle)</td>
<td>Particle state</td>
</tr>
<tr>
<td>(</td>
<td>h\rangle)</td>
<td>Emission history</td>
</tr>
<tr>
<td>(</td>
<td>e\rangle)</td>
<td>Did emission happen?</td>
</tr>
<tr>
<td>(</td>
<td>n_\phi\rangle)</td>
<td>Number of bosons</td>
</tr>
<tr>
<td>(</td>
<td>n_a\rangle)</td>
<td>Number of (f_a)</td>
</tr>
<tr>
<td>(</td>
<td>n_b\rangle)</td>
<td>Number of (f_b)</td>
</tr>
</tbody>
</table>
For each particle in the state numbers in the count registers where the operation transforms the system of particles from the original basis to the final basis, we apply the counting operation as follows:

$$U_e^{(m)} = \left( \begin{array}{cc} \Delta^{(m)}(\theta_m) & -\sqrt{1 - \Delta^{(m)}(\theta_m)} \\ \sqrt{1 - \Delta^{(m)}(\theta_m)} & \Delta^{(m)}(\theta_m) \end{array} \right)$$

$$\Delta_i(\theta_m, \theta_{m+1}) = e^{-\Delta \theta P_i(\theta_m)}$$

(Sudakov factor)

$$\Delta^{(m)}(\theta_m) = \Delta_n^{\phi}(\theta_m) \Delta^{n_{f1}}(\theta_m) \Delta^{n_{f2}}(\theta_m)$$

I’ll show you the circuit when the splitting is turned off in a moment, but for fun, let’s talk about one element.
The circuit without scalar splitting

In words: rotate to the basis where there is no interference, “emit” scalars (at the amplitude level), and then rotate back to the physical basis at the end.
Note: $|\phi_i\rangle$ is not touched after timestep i and so one can reuse qubits ... only need 2 total qubits (!)

Fine print: (1) re-measurement is not a feature of most current quantum computers and (2) this led us to a classical algorithm that can capture the full interference effects (but is not the naive MCMC).
Some numerical results

Figure 1: The normalized differential cross section for $\log(\theta_{\text{max}})$ (left) and the number of emissions (right). Interference effects are turned on ($g_{12} = 0$) and off ($g_{12}=0$), where the classical simulations/calculations are expected to agree with the quantum simulations and measurements. As a demonstration of the full circuit with $\bar{f}f$ is also included with two simulated steps both with $g_{12}=0$ and $g_{12}=1$.

$\frac{1}{\sigma} \frac{d\sigma}{d\log(\theta_{\text{max}})}$

$\frac{1}{\sigma} \frac{d\sigma}{dN}$

angle of maximum emission

number of emissions

no interference

with interference

$\left(g_1, g_2, \varepsilon\right) = (2, 1, 10^{-3})$
The future

There is a long road ahead, but quantum algorithms are very promising for modeling high energy scattering processes.

Today I gave you a small taste of what is possible - stay tuned for more!

Still serious challenges: scalability, noise, etc.

…in the mean time, note that there is an impressive effort to add in quantum effects to parton showers as corrections.

see e.g. this pioneering work: Nagy and Soper, JHEP 09 (2007) 114
Figure 2: The number of standard qubit gates as a function of the number of states, using the formulae given in Eqs. (41), (43), (46) and (48). The asymptotic behavior is illustrated with a fit to $N^5 \log(N)$. 
<table>
<thead>
<tr>
<th>Operation</th>
<th>Scaling</th>
<th>(N = 4)</th>
<th>(N = 24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>count particles ([U_{\text{count}}])</td>
<td>(N \ln N)</td>
<td>(4.93 \times 10^2)</td>
<td>(5.45 \times 10^4)</td>
</tr>
<tr>
<td>decide emission ([U_e])</td>
<td>(N^4 \ln N)</td>
<td>(9.29 \times 10^3)</td>
<td>(8.75 \times 10^6)</td>
</tr>
<tr>
<td>create history ([U_h])</td>
<td>(N^5 \ln N)</td>
<td>(1.69 \times 10^5)</td>
<td>(1.19 \times 10^9)</td>
</tr>
<tr>
<td>adjust particles ([U_p])</td>
<td>(N^2 \ln N)</td>
<td>(5.01 \times 10^3)</td>
<td>(3.37 \times 10^5)</td>
</tr>
</tbody>
</table>
The quantum circuit introduced in this paper has a total of $6$ registers. The first register, $|h\rangle$, holds the information about which particle emitted a particle at a particular time. The second register, $|p\rangle$, contains the flavor information about each particle. Each particle $p_i$ is represented as a three-qubit state $|n_{p_i}\rangle = \begin{pmatrix} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \end{pmatrix} = \begin{pmatrix} 0 \\ \phi \\ - \\ - \\ f_1/f_a \\ f_2/f_b \\ \bar{f}_1/\bar{f}_a \\ \bar{f}_2/\bar{f}_b \end{pmatrix}$ where the third and fourth states are not used and one chooses $\phi$ to be the initial state of the system.

The initial state consists of $6$ particles (which can be fermions or bosons) in the system can be in one of $6$ states $|0\rangle, |\phi\rangle, |\bar{f}_1\rangle, |\bar{f}_2\rangle, |f_1\rangle, |f_2\rangle$. Since there can be up to $n$ particles in the system, one needs a total of $a/b$ to encode these. To encode these particles, one chooses a total of $f$ particles before and after the operation discussed in the Methods. Then, a series of operations evolving the particles states are applied: the number of particles of each type are counted using a simple unitary.

To determine if an emission occurred, we have $\text{count}_i = |\langle n_{p_i} | R^{(m)} | 0 \rangle|^2$ where each splitting function is represented through a splitting matrix as $g$. The complexity of taking this into account is $12$ splittings) for one of the particles (which can be fermions or bosons). The initial state for the emission algorithm (now including the structure of the splitting function) needs to be retained. The full matrix calculation is given by the following diagram:

$|p\rangle \rightarrow R^{(m)} \rightarrow P \rightarrow U'_{p} \rightarrow R^{(m)\dagger}$