Basis-invariant road to 3HDMs with symmetries

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based on:

I. P. Ivanov, C. Nishi, J. P. Silva, A. Trautner, PRD99 (2019) 015039
I. P. Ivanov, C. Nishi, A. Trautner, arXiv:1901.11472

and work in progress

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1. Why basis invariant methods?
2. Adjoint space approach to 3HDM
3. Detecting symmetries in 3HDM
4. Conclusions

Is there life beyond the SM Higgs?

The minimal Higgs sector of the SM is overstretched. As a result:

- does not explain fermion masses and mixing, neutrino masses, CP-violation;
- has boring flavor properties: no tree-level FCNCs;
- does not help explain DM or baryon asymmetry.

These issues can be successfully addressed in models with extended scalar sectors.

A conservative but rich class of models: N-Higgs-doublet models (NHDMs).

2HDM has been our playground for decades, time to move on!

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3HDM

What's new in 3HDM compared to 2HDM:

- richer pheno (both scalar and fermion sectors);
- combining nice features of 2HDM, e.g. NFC + CPV [Weinberg, 1976; Branco, 1979], scalar DM + CPV [Grzadkowski et al, 2009];
- new options for *CP* violation, e.g. geometrical CPV [Branco, Gerard, Grimus, 1984],
- *CP* symmetry of order 4 (CP4) [Ivanov, Silva, 2015]:
  - mass degeneracy, *CP* eigenstates beyond *CP*-even/odd [Ivanov, Silva, 2015; Haber et al, 2018];
  - DM stabilized by CP4: [Koepke, 2018; Ivanov, Laletin, 2018];
  - quark/neutrino patterns from CP4: [Ferreira et al, 2017; Ivanov, 2018];
  - solution to strong *CP* problem: [Cherchiglia, Nishi, 2019].
- symmetries, lots of symmetries in the 3HDM scalar sector!

Symmetries in 3HDM

Particular examples of 3HDMs with symmetries begin in 1970's;
full classification only recently.

- abelian groups: [Ferreira, Silva, 1012.2874; Ivanov, Keus, Vdovin, 1112.1660]

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2\times\mathbb{Z}_2, \quad U(1), \quad U(1)\times\mathbb{Z}_2, \quad U(1)\times U(1)\,.$$

- discrete non-abelian groups: [Ivanov, Vdovin, 1210.6553]

$$S_3, \quad D_4, \quad A_4, \quad S_4, \quad \Delta(54), \quad \Sigma(36)\,.$$

- symmetry breaking patterns $G \to G_v$: [Ivanov, Nishi, 1410.6139]
- interplay between G and CP [many classical works].

Symmetries in 3HDM: flavour physics connection

- The original idea from 1970's:
  - extent G to fermion sector,
  - arrange for spontaneous violation $G \to G_v$,
  - derive masses/mixing/CPV.
- Many combinations of G + irreps + vevs were tested, but
  - if G is large $\to$ severe problems in the quark sector; A_4/S_4 illustrations in [Gonzales Felipe et al, 1302.0861, 1304.3468];
  - if G is small $\to$ too many free parameters, no predictive power.
- The fundamental obstacle [Leurer, Nir, Seiberg, 1993; Gonzales Felipe et al, 1401.5807]:

  If the (active) Higgs sector is equipped with G, then vevs must break G completely in order to produce physical m_q's and CKM.

  But for large G, this is algebraically impossible.

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Proximity to a symmetric 3HDM

For large G:

- imposing an exact $G \to$ some observables $= 0$;
- a 3HDM in the vicinity, ϵ, of an exact $G \to$ observables depend as ϵ^α.
- a 3HDM can be close to several distinct symmetric situations $\to$ competing symmetries.

Challenge

When scanning the 3HDM parameter space,

one must detect (proximity to) a G-symmetric situations.

Basis-invariant methods

Large freedom of basis changes: $\phi_a \mapsto U_{ab}\phi_b$, $U \in U(N)$.

Physics does not change upon basis changes!

A symmetry can be evident in one basis and hidden in another $\to$ challenge!

The goal

Detecting structural properties of NHDMs irrespective of the basis choice!

General recipe [Botella, Silva, 1995]:

- write down all couplings as tensors under basis changes,
- take their product and contract all indices $\to$ basis invariants J_k,
- find algebraically independent J_k,
- link them to the phenomenon you study.

Explicit CP conservation in 2HDM scalar sector

The most general 2HDM potential:

$$V = Y_{ab}(\phi_a^\dagger\phi_b) + Z_{ab,cd}(\phi_a^\dagger\phi_b)(\phi_c^\dagger\phi_d)\,,$$

or, in the explicit form,

$$\begin{aligned} V &= -\frac{1}{2}\left[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2\,*}(\phi_2^\dagger\phi_1)\right] \\ &+ \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &+ \left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \mathrm{h.c.}\right] \end{aligned}$$

It contains $4+10=14$ free parameters.

General 2HDM scalar sector

Checking explicit CP-conservation [Davidson, Haber, 2005; Gunion, Haber, 2005; Branco, Rebelo, Silva-Marcos, 2005]:

- There exists of a basis with all coefs real $\to$ symmetry $\phi_a \to \phi_a^*$.
- Construct invariants with Y_{ab} and $Z_{ab,cd}$ and establish independent ones;
- Basis-invariant criterion: check the following four invariants

$$\mathrm{Im}(Z^{(1)}_{ac} Z^{(1)}_{eb} Z_{be,cd} Y_{da}) = 0\,, \qquad \mathrm{Im}(Y_{ab} Y_{cd} Z_{ba,df} Z^{(1)}_{fc}) = 0\,,$$
$$\mathrm{Im}(Z_{ab,cd} Z^{(1)}_{bf} Z^{(1)}_{dh} Z_{fa,jk} Z_{kj,mn} Z_{nm,hc}) = 0\,,$$
$$\mathrm{Im}(Z_{ac,bd} Z_{ce,dg} Z_{eh,fq} Y_{ga} Y_{hb} Y_{qf}) = 0\,, \quad \text{where} \quad Z^{(1)}_{ac} \equiv Z_{ab,bc}\,.$$

Basis invariants

Drawbacks:

- non-intuitive, relies on computer algebra; one needs to find the generating set of the ring of symmetry-related invariants;
  NB! [Trautner, 1812.02614] shows how to derive them in 2HDM.
- becomes even more complicated beyond 2HDM; conditions for *CP* symmetry in 3HDM via basis invariants still not established [Varzielas et al, 1603.06942];
- not all information can be easily retrieved! *CP*-odd basis invariants in 3HDM cannot tell the usual *CP* from CP4 (order-4 *CP* symmetry).

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Bilinears in 3HDM

Geometric constructions in the adjoint space [Nachtmann et al, 2004–2007; Ivanov, 2006–2007; Nishi, 2006–2008]. V is built of 9 bilinears $\phi_a^\dagger \phi_b$.

$$r_0 = \frac{1}{\sqrt{3}}\phi_a^\dagger \phi_a\,, \quad r_i = \phi_a^\dagger (t^i)_{ab}\phi_b\,, \quad i = 1, \dots, 8\,,$$

where $t_i = \lambda_i/2$ are $SU(3)$ generators satisfying

$$[t_i, t_j] = i f_{ijk} t_k\,, \quad \{t_i, t_j\} = \frac{1}{3}\delta_{ij}\mathbf{1}_3 + d_{ijk} t_k\,.$$

The orbit space:

$$r_0 \geq 0\,, \quad r_0^2 - r_i^2 \geq 0\,, \quad \sqrt{3} d_{ijk} r_i r_j r_k + (r_0^2 - 3r_i^2) r_0/2 = 0\,.$$

Basis changes $\to$ $SO(8)$ rotations of r_i.

$SU(3) \subset SO(8) \Rightarrow$ not all $SO(8)$ rotations are basis changes!

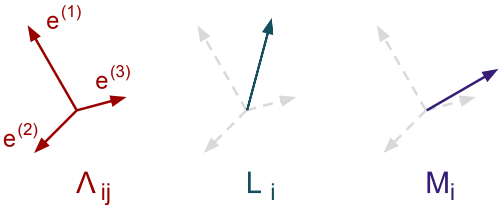
Adjoint space

The NHDM potential takes the simple form

$$V = -M_0 r_0 - M_i r_i + \Lambda_{00} r_0^2 + L_i r_0 r_i + \Lambda_{ij} r_i r_j \,,$$

with vectors $M, L \in \mathbb{R}^{N^2-1}$ and an $(N^2-1)\times(N^2-1)$ matrix Λ.

In 2HDM: 3×3 matrix Λ can be always diagonalized by basis change.



Orientation of M and L with respect to eigenvectors of Λ $\Rightarrow$ symmetries.

Adjoint space

In 3HDM, we lack the full $SO(8)$ rotation group:

- directions in $\mathbb{R}^8$ are not equivalent!
- Λ is not in general diagonalizable by basis change.

We need to make sense of the adjoint space.

The toolbox

Suppose vectors $a, b \in \mathbb{R}^8$. Define new products:

$$F_i^{(ab)} = f_{ijk} a_j b_k\,, \quad D_i^{(ab)} = \sqrt{3} d_{ijk} a_j b_k\,, \quad D_i^{(aa)} = \sqrt{3} d_{ijk} a_j a_k\,.$$

Applied to the eigenvectors of Λ, these products help detect basis-invariant structures in $\Lambda \Rightarrow$ symmetries in 3HDM.

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Detecting special subspaces

- **Test-(8)**. Consider $a \in \mathbb{R}^8$, $|a| = 1$. Compute vector $D^{(aa)}$.
  If $D^{(aa)} = -a$, then there is a basis in which a is along x_8.
  If an eigenvector of Λ passes Test-(8), then in this basis

$$\Lambda = \begin{pmatrix} \square_{7\times 7} & 0 \\ 0 & \Lambda_{88} \end{pmatrix}.$$

- **Test-(38)**. Consider $a, b \in \mathbb{R}^8$, $|a| = |b| = 1$.
  If $F^{(ab)} = 0$, then there is a basis in which $a, b \in (x_3, x_8)$.
  If two eigenvectors of Λ pass Test-(38), then in this basis

$$\Lambda = \begin{pmatrix} \square_{6\times 6} & 0 \\ 0 & \square_{2\times 2} \end{pmatrix}.$$

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Detecting further splitting of Λ

Test-(12)(45)(67)

Suppose Λ passes Test-(38). Then, in a certain basis, it has a generic 6×6 block within the subspace

$$V_6 = (x_1, x_2;\ x_4, x_5;\ x_6, x_7)\,.$$

Take 6 eigenvectors from this subspace. If they break into three pairs such that each pair of eigenvectors a', b' satisfies

$$D^{(a'b')} = 0 \quad \text{and} \quad D^{(a'a')} = D^{(b'b')} \in (x_3, x_8)\,,$$

then Λ splits into four 2×2 blocks within subspaces

$$(x_3, x_8)\,, \quad (x_1, x_2)\,, \quad (x_4, x_5)\,, \quad (x_6, x_7)\,.$$

Detecting special subspaces

- Such Tests give necessary and sufficient conditions for the corresponding features to occur.
- They can be checked in any basis.
- One just needs to relate them to symmetries.

Symmetries in 3HDM

The NHDM potential

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

may be invariant under global symmetries:

- family symmetries: $\phi_a \to U_{ab}\phi_b$, with $U \in U(N)$,
- GCP symmetries: $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$, with $X \in U(N)$.

Each symmetry group G and its breaking by vevs $G_v \subseteq G$ lead to a characteristic phenomenology (scalars, DM candidates, fermion masses, mixing, sources of CPV, etc).

In 3HDM, a novel form of CP-symmetry (CP4) [Ivanov, Silva, 1512.09276] which is physically distinct from the usual CP (CP2) [Haber, Ogreid, Osland, Rebelo, 1808.08629].

Explicit CP2 conservation

CP2: there exists a basis in which it takes the standard form: $\phi_a \to \phi_a^*$.

In the adjoint space, the standard CP is the following reflection:

- vectors from $V_+ = (x_3, x_8, x_1, x_4, x_6)$ stay unchanged,
- vectors from $V_- = (x_2, x_5, x_7)$ flip signs.

3HDM potential is explicitly CP2-invariant if there exists a basis in which:

- Λ has the block-diagonal form:

$$\Lambda = \begin{pmatrix} \Box_{5\times 5} & 0 \\ 0 & \Box_{3\times 3} \end{pmatrix}$$

  with generic blocks within V_+ and V_-.

- vectors $M, L \in V_+$,

Detecting explicit CP2 conservation

Detecting $\square_{3\times 3}$ in (x_2, x_5, x_7):

- There exist three mutually orthogonal eigenvectors a, b, c such that

$$2F^{(ab)} = c\,, \quad 2F^{(bc)} = a\,, \quad 2F^{(ca)} = b\,.$$

- vectors M, L are orthogonal to these a, b, c.

Derived first in [Nishi, hep-ph/0605153].

Explicit CP4 conservation

CP4 leads in a certain basis in the bilinear space to

$$x_8 \to x_8\,, \quad (x_1, x_2, x_3) \to -(x_1, x_2, x_3)$$
$$x_4 \to x_6\,, \quad x_6 \to -x_4\,, \quad x_5 \to -x_7\,, \quad x_7 \to x_5\,.$$

3HDM potential is explicitly CP4-invariant iff there exists a basis in which

- the matrix Λ is

$$\Lambda = \begin{pmatrix} \square_{3\times 3} & 0 & 0 \\ 0 & \square_{4\times 4} & 0 \\ 0 & 0 & \Lambda_{88} \end{pmatrix}$$

  with a specific pattern in the 4×4 block,

- all possible vectors M, L, $(\Lambda^n)L$, $K_i \equiv d_{ijk}\Lambda_{jk}, \dots$ are all parallel to x_8 (complete alignment).

Detecting explicit CP4 conservation

Basis invariant necessary and sufficient conditions for explicit CP4 conservation [Ivanov, Nishi, Silva, Trautner, 1810.13396]:

- Λ passes Test-(8): three exists an eigenvector $e^{(8)}$ such that $D^{(88)} = -e^{(8)}$;
- There exist three other eigenvectors a, b, c such that

$$F^{(a8)} = F^{(b8)} = F^{(c8)} = 0\,,$$

  which guarantees the 3×3 block within (x_1, x_2, x_3) subspace.
- M, L, $K_i = d_{ijk}\Lambda_{jk}$, and $K_i^{(2)} = d_{ijk}(\Lambda^2)_{jk}$ are aligned with $e^{(8)}$.

Weinberg's model

Weinberg's model ($\mathbb{Z}_2 \times \mathbb{Z}_2$):

- Λ passes Test-(38) and Test-(12)(45)(67);
- $M, L \in (x_3, x_8)$.

If, in addition, there are degenerate eigenvalues within V_6:

- if the degeneracy pattern is $1+1+2+2 \to U(1) \times \mathbb{Z}_2$;
- if the degeneracy pattern is $2+2+2 \to U(1) \times U(1)$.

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Conclusions

Done:

- Efficient parameter space scans in multi-Higgs models must be able to detect symmetries in a basis invariant way.
- We found a way how to do it in the scalar sector of 3HDM: via subspace detection techniques applied to eigenvectors of Λ.

To do:

- Implement the algorithms in a working computer code.
- Go beyond 3HDM.
- Apply the idea to the fermion sector.