Higgs potential, New Physics, and Stability of the EW Vacuum

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VB, E. Messina, A. Platania JHEP 1409 (2014) 182
VB, E. Messina, M. Sher, Phys. Rev. D91 (2015) 1, 013003
VB, E. Messina, EPL 117 (2017) 61002
E. Bentivegna, VB, F. Contino, D. Zappalà, JHEP 1712 (2017) 100
VB, F. Contino, P.M. Ferreira, JHEP 1811 (2018) 107
... Work in progress ...

Workshop HPNP 2019
18 - 22 February 2019, Osaka
Stability analysis of the EW vacuum (few references)


After the discovery of the Higgs boson ... renewed interest ...


... many other references ...
Stability analysis of the EW vacuum

Key tool: Higgs Effective Potential

Top loop-corrections destabilize the EW Vacuum...

EW = \( v \sim 246 \text{ GeV} \); For \( M_H \sim 125 \text{ GeV} \), \( M_t \sim 173 \text{ GeV} \): Instability \( \sim 10^{11} \text{ GeV} \)
One-Loop Higgs Effective Potential $V_{1l}(\phi)$

$$V_{1l}(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{24} \phi^4 + \frac{1}{64\pi^2} \left[ \left( m^2 + \frac{\lambda}{2} \phi^2 \right)^2 \left( \frac{m^2 + \frac{\lambda}{2} \phi^2}{\mu^2} \right)^2 \left( \ln \left( \frac{m^2 + \frac{\lambda}{2} \phi^2}{\mu^2} \right) - \frac{3}{2} \right) \right. \\
+ 3 \left( m^2 + \frac{\lambda}{6} \phi^2 \right)^2 \left( \ln \left( \frac{m^2 + \frac{\lambda}{6} \phi^2}{\mu^2} \right) - \frac{3}{2} \right) + 6 \frac{g_1^4}{16} \phi^4 \left( \ln \left( \frac{1}{4} g_1^2 \phi^2 \right) - \frac{5}{6} \right) \\
+ 3 \frac{(g_1^2 + g_2^2)}{16} \phi^4 \left( \ln \left( \frac{1}{4} \frac{(g_1^2 + g_2^2)}{\mu^2} \phi^2 \right) - \frac{5}{6} \right) - 12 h_t^4 \phi^4 \left( \ln \frac{g_2^2 \phi^2}{\mu^2} - \frac{3}{2} \right) \left. \right]$$
Running the RG eqs. for the SM couplings \( \Rightarrow \) RGI Potential:

Depending on \( M_H \) and \( M_t \), the second minimum can be:

1. **lower** than the EW minimum (as in the figure): This is the case for \( M_H \sim 125 \text{ GeV}, M_t \sim 173 \text{ GeV} \) (central values);
2. at the **same height** ... ;
3. **higher** ...

Case 1 (figure): **EW vacuum Metastable**
If the EW vacuum lifetime larger than the age of the Universe ...

... we may well live in such a Metastable Vacuum ....
Technically - Tunneling Rate (= inverse tunneling time) obtained as:

$$\Gamma = \frac{1}{\tau} = De^{-\Delta(S[\phi_b]-S[\phi_{fv}] )} \equiv De^{-B}$$

$\phi_b(r)$ Bounce: Solution to the Euclidean EOM with appropriate b.c.

**Euclidean equations of motion (O(4) Symmetry)**

$$- \partial_\mu \partial_\mu \phi + \frac{dV(\phi)}{d\phi} = - \frac{d^2\phi}{dr^2} - \frac{3}{r} \frac{d\phi}{dr} + \frac{dV(\phi)}{d\phi} = 0$$

 Boundary conditions: $\phi'(0) = 0 , \quad \phi(\infty) = v \rightarrow 0$.

A well known example: $V(\phi) = \frac{\lambda}{4} \phi^4$ with constant and negative $\lambda$

Bounce (Fubini instanton): $\phi_b(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2 + R^2} \quad (R =$ size$)$

**Degeneracy**

$S[\phi_b] = \frac{8\pi^2}{3|\lambda|}$  Bounce Action does not depend on $R$

**Classical Scale Invariance**

**Degeneracy removed at the Quantum Level**
\[ \Gamma = \frac{1}{\tau} = De^{-(S[\phi_b]-S[\phi_{iv}]}) \equiv De^{-S[\phi_b]} \]

A good estimate for \( \Gamma \) is obtained by approximating the prefactor \( D \) in terms of the bounce size \( R \), defined as the value of \( r \) such that:

\[ \phi_b(R) = \frac{1}{2} \phi_b(0) \]

and the age of the universe \( T_U \).

For the EW vacuum lifetime \( \tau = \Gamma^{-1} \) we get:

\[ \tau \simeq \left( \frac{R^4}{T_U^3} \right) e^{S[\phi_b]} \]
For SM the instability occurs at large values of $\phi$

$\Rightarrow$ $V_{eff}(\phi)$ well approximated by keeping only the quartic term

$$V_{SM}(\phi) \sim \frac{\lambda(\phi)}{24} \phi^4$$

$\lambda(\phi)$ depends on $\phi$ essentially as $\lambda(\mu)$ depends on $\mu$

For large values of $\phi$, the coupling $\lambda$ becomes negative and almost constant in the region of interest ... close to the above example ... In fact people used analytical approximations, but we can do better ... we can calculate the bounce numerically ...
1. Stability Analysis in Flat Spacetime Background

Euclidean action $S[\phi] = \int d^4x \left[ \frac{1}{2} (\partial_\mu \phi)^2 + V_{SM}(\phi) \right]$

Bounce Solution

EW vacuum lifetime $\tau_{flat} \sim \left( \frac{R^4}{T_U^3} \right) e^B \approx 10^{639} T_U$

Obtained for $M_H \sim 125 \text{ GeV}$ and $M_t \sim 173 \text{ GeV}$
More generally: Stability Diagram in the $M_H - M_t$ plane

Stability region: $V_{eff}(v) < V_{eff}(\phi_{min}^{(2)})$.

Meta-stability region: $V_{eff}(\phi_{min}^{(2)}) < V_{eff}(v)$ and $\tau > T_U$.

Instability region: $V_{eff}(\phi_{min}^{(2)}) < V_{eff}(v)$ and $\tau < T_U$.

Stability line: $V_{eff}(v) = V_{eff}(\phi_{min}^{(2)})$.

Instability line: $M_H$ and $M_t$ such that $\tau = T_U$. 
2. Stability Analysis in Curved Spacetime Background

Including the Einstein-Hilbert term, the Euclidean action is:

\[
S[\phi, g_{\mu\nu}] = \int d^4x \sqrt{g} \left[ -\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V_{SM}(\phi) \right]
\]

\[ R = \text{Ricci scalar, } G = \text{Newton constant.} \]

Requiring again \(O(4)\) symmetry, the (Euclidean) metric:

\[
ds^2 = dr^2 + \rho^2(r) d\Omega^2_3
\]

\(d\Omega^2_3 = \text{unit 3-sphere line element, } \rho(r) = \text{volume radius of the 3-sphere at fixed } r.\)

The **bounce** now: \((\phi_b(r), \rho_b(r))\), solutions of the coupled equations: \((\kappa \equiv 8\pi G)\):

\[
\ddot{\phi} + 3 \frac{\dot{\rho}}{\rho} \dot{\phi} = \frac{dV_{SM}(\phi)}{d\phi} \quad \rho^2 = 1 + \frac{\kappa \rho^2}{3} \left( \frac{1}{2} \dot{\phi}^2 - V_{SM}(\phi) \right)
\]

**First equation:** replaces the equivalent equation in flat spacetime;

**Second equation:** the only Einstein equation left by the symmetry.

For the decay of a Minkowski false vacuum to a true AdS vacuum (the case of interest to us) the boundary conditions are:

\[
\phi_b(\infty) = 0 \quad \dot{\phi}_b(0) = 0 \quad \rho_b(0) = 0.
\]
Profile of $\phi_b(r)$ and of the difference between $\rho(r)$ and its asymptotic value, $\rho(r) - r$ (asymptotically $\rho_b(r)$ reaches the Minkowskian $\rho_M(r) \sim r + Const$).

**EW vacuum lifetime** \( \tau_{\text{grav}} \simeq 10^{661} T_U \)

Obtained for \( M_H \sim 125 \text{ GeV} \) and \( M_t \sim 173 \text{ GeV} \)
Crucial point - Calculation of $\tau$ under the assumption that ...

... Even though New Physics Interactions are certainly present at the Planck scale, they have no impact on the EW vacuum lifetime, so they are neglected when computing $\tau$.

Argument: Instability scale, $\Lambda_{\text{inst}} \sim 10^{11}$ GeV, much lower than $M_P \Rightarrow$

$\Rightarrow$ suppression $\left(\frac{\Lambda_{\text{inst}}}{M_P}\right)^n$ expected
... However, things are more subtle ... 

The Stability Diagram is not universal

New Physics at Planck scale can strongly modify this Stability Diagram


E. Bentivegna, VB, F. Contino, D. Zappalà, JHEP 1712 (2017) 100
Soon after the appearance of the first of these papers, several other authors confirmed these results ... and were also inspired for applications and developments ...

Just to mention a few:

Lalak, Lewicki, Olszewski, JHEP 1405 (2014) 119
Eichhorn, Gies, Jaeckel, Plehn, Schrerer, Sondenheimer, JHEP 1504 (2015) 022
Burda, Gregory, Moss, JHEP 1508 (2015) 114

.........
... Let’s add New Physics around $M_P$ ...
New Physics around $M_P$

$$V(\phi) = \frac{\lambda(\phi)}{4} \phi^4 + \frac{\lambda_6}{6} \frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8} \frac{\phi^8}{M_P^4}$$

Yellow line: Potential with $\lambda_6 = -0.4$ and $\lambda_8 = 2$.

Blue line: SM alone.
**Bounce profiles in Flat Spacetime Background**

![Graph showing bounce profiles](image)

**Blue curve**: bounce obtained for the potential with $\lambda_6 = 0$ and $\lambda_8 = 0$ (SM alone).

**Yellow curve**: bounce for $\lambda_6 = -0.3$ and $\lambda_8 = 0.3$.

**Green curve**: bounce for $\lambda_6 = -0.01$ and $\lambda_8 = 0.01$. 
## Tunneling times for different values of $\lambda_6$ and $\lambda_8$

<table>
<thead>
<tr>
<th>$\lambda_6$</th>
<th>$\lambda_8$</th>
<th>$\tau_{\text{flat}}/T_U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$10^{639}$</td>
</tr>
<tr>
<td>-0.05</td>
<td>0.1</td>
<td>$10^{446}$</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.2</td>
<td>$10^{317}$</td>
</tr>
<tr>
<td>-0.3</td>
<td>0.3</td>
<td>$10^{-52}$</td>
</tr>
<tr>
<td>-0.45</td>
<td>0.5</td>
<td>$10^{-93}$</td>
</tr>
<tr>
<td>-0.7</td>
<td>0.6</td>
<td>$10^{-162}$</td>
</tr>
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<td>-1.2</td>
<td>1.0</td>
<td>$10^{-195}$</td>
</tr>
<tr>
<td>-2.0</td>
<td>2.1</td>
<td>$10^{-206}$</td>
</tr>
</tbody>
</table>

Remember:

$$\tau \sim e^{S[\phi_b]}$$

New bounce $\phi_b^{(\text{new})}(r)$, New action $S[\phi_b^{(\text{new})}]$, New $\tau$
Bounce profiles in Curved Spacetime Background

Left Panel - Blue curve: profile of the bounce solution with $\lambda_6 = 0$ and $\lambda_8 = 0$, i.e. in the absence of new physics. Yellow curve: profile of the bounce solution for $\lambda_6 = -0.03$ and $\lambda_8 = 0.03$. Green curve: profile of the bounce solution for $\lambda_6 = -0.04$ and $\lambda_8 = 0.04$.

Right Panel - Profile of the difference between $\rho(r)$ and its asymptotic value: $\rho(r) - r$. 
Tunneling times for different values of $\lambda_6$ and $\lambda_8$

<table>
<thead>
<tr>
<th>$\lambda_6$</th>
<th>$\lambda_8$</th>
<th>$\tau_{\text{flat}} / T_U$</th>
<th>$\tau_{\text{grav}} / T_U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$10^{639}$</td>
<td>$10^{661}$</td>
</tr>
<tr>
<td>$-0.05$</td>
<td>0.1</td>
<td>$10^{446}$</td>
<td>$10^{653}$</td>
</tr>
<tr>
<td>$-0.1$</td>
<td>0.2</td>
<td>$10^{317}$</td>
<td>$10^{598}$</td>
</tr>
<tr>
<td>$-0.3$</td>
<td>0.3</td>
<td>$10^{-52}$</td>
<td>$10^{287}$</td>
</tr>
<tr>
<td>$-0.45$</td>
<td>0.5</td>
<td>$10^{-93}$</td>
<td>$10^{173}$</td>
</tr>
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<td>$-0.7$</td>
<td>0.6</td>
<td>$10^{-162}$</td>
<td>$10^{47}$</td>
</tr>
<tr>
<td>$-1.2$</td>
<td>1.0</td>
<td>$10^{-195}$</td>
<td>$10^{-58}$</td>
</tr>
<tr>
<td>$-2.0$</td>
<td>2.1</td>
<td>$10^{-206}$</td>
<td>$10^{-121}$</td>
</tr>
</tbody>
</table>

Gravity tends to stabilize the EW vacuum ($\tau_{\text{grav}}$ always higher than $\tau_{\text{flat}}$). However, New Physics has always a strong (that can be even devastating) impact.
In the blue region $\tau > T_U$ both for the flat and curved spacetime analysis. In the yellow region $\tau < T_U$ for the flat spacetime background. In the red region $\tau < T_U$ in both cases.
Stability Diagram in the \((M_H, M_t)\) - plane for \(\lambda_6 = -0.2\) and \(\lambda_8 = 0.5\).

The strips move downwards \ldots \ Central values no longer at 3\(\sigma\) from the stability line \ldots

\ldots The Stability Diagram depends on new physics \ldots
Stability Diagram in the \((M_H, M_t)\) - plane
for \(\lambda_6 = -0.4\) and \(\lambda_8 = 0.7\)

... The Stability Diagram depends on new physics ...
As previously said ... These results came as a surprise ...

It was thought that New Physics at the Planck scale should have no impact on the EW vacuum lifetime ... on the Stability Diagram

How comes that New Physics at $M_P$ has such an impact on $\tau$ ?

How comes that decoupling arguments do not apply ?
• As $\Lambda_{\text{inst}} \sim 10^{11}$ GeV, a decoupling effect was expected: the contribution of higher dimension operators $\frac{\phi^n M_n}{M_P^2}$ was expected to be suppressed as $(\frac{\Lambda_{\text{inst}}}{M_P})^n$.

• However: Tunnelling is a non-perturbative phenomenon. We first select the saddle point, i.e. compute the bounce (tree level). Then, on the top of that, we compute the quantum fluctuations (loop corrections).

• Suppression in terms of inverse powers of $M_P$ (power counting theorem) concerns the loop corrections, not the selection of the saddle point (tree level).

Remember:

$$\tau \sim e^{S[\phi_b]} \Rightarrow$$

New bounce $\phi_b^{(\text{new})}(r) \Rightarrow$ New bounce action $S[\phi_b^{(\text{new})}] \Rightarrow$

New $\tau$
... It seems that the problem is there ...

... Rescue ? ...

\[ S[\phi] = \int d^4x \sqrt{g} \left[ -\frac{R}{2\kappa} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V_{SM}(\phi) + \frac{1}{2} \xi \phi^2 R \right] \]

Again \(O(4)\) symmetry:

\[
\ddot{\phi} + 3 \frac{\dot{\rho}}{\rho} \dot{\phi} = \frac{dV}{d\phi} + \xi \phi R \quad \rho^2 = 1 - \frac{\kappa}{3} \rho^2 \frac{-\frac{1}{2} \dot{\phi}^2 + V(\phi) - 6\xi \frac{\dot{\phi}}{\rho} \phi}{1 - \kappa \xi \phi^2},
\]

with \(R\) given by:

\[
R = \kappa \frac{\dot{\phi}^2(1 - 6\xi) + 4V(\phi) - 6\xi \phi dV/d\phi}{1 - \kappa \xi (1 - 6\xi) \phi^2}.
\]

For \(\xi = 0\) these Equations become the minimal coupling ones.

Asymptotics: For \(r \to \infty\), \(\rho_b^2 = 1\), so \(\rho(r)\) approaches the flat spacetime metric. In the same limit, \(R \to 0\).
SM Potential. Non-minimal coupling. $S_{\text{bounce}} \equiv B(\xi)$  

$B$ very sensitive to $\xi$. Outside the range $[\xi = 0, \xi = 1/3]$, $B(\xi)$ is greater than $B(\xi = 0)$, and non-minimal coupled gravity stabilizes the EW vacuum more than minimally coupled gravity.

Minimum at $\xi_{\text{min}} \approx 0.17$, close to the conformal value $\xi = 1/6$. Actually for the scale invariant potential $V(\phi) = \frac{1}{4} \phi^4$ (constant $\lambda$) the minimum is reached at $\xi = 1/6$. 
What happens if we Add New Physics at $M_P$?
Add New Physics: $\lambda_6 \phi^6$ and $\lambda_8 \phi^8$

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$(\tau/T_U)_{SM}$</th>
<th>$(\tau/T_U)_{NP}$</th>
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<tbody>
<tr>
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<td>$10^{736}$</td>
<td>$10^{736}$</td>
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<td>$10^{726}$</td>
<td>$10^{726}$</td>
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<td>$10^{661}$</td>
<td>$10^{-58}$</td>
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<table>
<thead>
<tr>
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<th>$(\tau/T_U)_{SM}$</th>
<th>$(\tau/T_U)_{NP}$</th>
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<td>$10^{725}$</td>
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<tr>
<td>$15$</td>
<td>$10^{735}$</td>
<td>$10^{735}$</td>
</tr>
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</table>

Values of $\tau$ with and without New Physics for different values of $\xi$, where $\lambda_6 = -1.2$ and $\lambda_8 = 1.$
Tunneling exponent $B(\xi)$ as a function of $\xi$

Yellow: $B(\xi)$ when the SM potential alone is considered. Blue: $B(\xi)$ when the New Physics potential with $\lambda_6 = -1.2$ and $\lambda_8 = 1$ is considered.
Stability diagrams in the \((\lambda_6, \lambda_8)\) plane with non-minimal coupling: \(\xi = -0.2\) (left), \(\xi = 0.9\) (right). In both cases, for the range of \(\lambda_6\) and \(\lambda_8\) considered, the EW vacuum is always stable \((\tau > T_U)\), unlike the minimal coupling case ... see next page ...
The $\xi = 0$ case for comparison

The same range of values of $\lambda_6$ and $\lambda_8$ as in the previous slide.
... Protection from Non-Minimal Coupling ...

The dimension four operator $\xi \phi^2 R$ naturally arises when quantization is carried out in a curved space-time background ... in the SM the term $\xi R H H^*$ is required in order to make the theory multiplicatively renormalizable in curved spacetime.

In view of the above remarks, and of the enormous stabilizing effect induced by the $\xi \phi^2 R$ term for values of $\xi$ outside the tiny range of values $-1 \lesssim \xi \lesssim 1$, under the assumption that the physical (yet unknown) value of $\xi$ lies outside this range, we could be lead to formulate the following ...

“Direct Coupling Stability Conjecture”

The quantum nature of physical laws and the very existence of gravity provide a model-independent stabilization mechanism that protects the EW vacuum against the destabilization that could come from unknown Planckian New Physics
Conclusions

- New Physics at Planckian scales, generically parametrized with the help of higher order operators in the Higgs potential ($\phi^6$ and $\phi^8$) can destabilize the EW vacuum.

- This result was first established in a flat spacetime background, and later confirmed by performing the analysis in a curved spacetime background (minimal coupling).

- Gravity shows a tendency toward stabilization. However, for large portions of the New Physics parameter space, the destabilizing impact of the latter wins against the competing stabilization tendency of gravity.

- Irrespectively of the specific model that might be responsible for generating the instability of the EW vacuum, if the Higgs field is non-minimally coupled to gravity (with the exception a tiny range of values of $\xi$) the destabilizing effect of unknown Planckian New Physics is washed out.

- “Direct Coupling Stabilization Conjecture”.
BACK UP SLIDES
Non-Renormalizable New Physics → Renormalizable New Physics

... It was also argued that the fact that New Physics was parametrized in terms of Non-Renormalizable operators actually could invalidate these results ...
New Physics around $M_P$ in terms of renormalizable operators

Add to the SM potential a “New Boson $S$” and a “New Fermion $\psi$” :

$$\Delta V(\phi, S, \psi) = \frac{M_S^2}{2} S^2 + \frac{\lambda S}{4} S^4 + \frac{g_S}{4} \phi^2 S^2 + M_f \bar{\psi} \psi + \frac{g_f}{\sqrt{2}} \phi \bar{\psi} \psi$$

with $M_f \sim 10^{17}$ GeV and $M_S \sim 10^{18}$ GeV.

Integrating out this new scalar and fermion fields we get the

Modified Higgs Potential

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{1}{64\pi^2} \left( M_S^2 + \frac{g_S}{2} \phi^2 \right)^2 \left[ \ln \left( \frac{M_S^2 + \frac{g_S}{2} \phi^2}{\mu^2} \right) - \frac{3}{2} \right]$$

$$- \frac{1}{16\pi^2} \left( M_f^2 + \frac{g_f^2}{2} \phi^2 \right)^2 \left[ \ln \left( \frac{M_f^2 + \frac{g_f^2}{2} \phi^2}{\mu^2} \right) - \frac{3}{2} \right]$$
The values of the parameter are: $M_S = 2.0 \times 10^{-1} M_P$, $M_f = 10^{-3} M_P$, $g_S = 0.95$, $g_f^2 = 0.4$. 
Profile of the bounce solutions $\varphi(x)$ relative to the four cases: $M_S = 2.5 \times 10^{-1}$, $M_f = 3 \times 10^{-4}$, $g_S = 0.96$, $g_f^2 = 0.5$ (yellow); $M_S = 2.0 \times 10^{-1}$, $M_f = 10^{-4}$, $g_S = 0.9$, $g_f^2 = 0.5$ (blue); $M_S = 2.0 \times 10^{-1}$, $M_f = 10^{-3}$, $g_S = 0.95$, $g_f^2 = 0.4$ (green); $M_S = 1.5 \times 10^{-1}$, $M_f = 5 \times 10^{-3}$, $g_S = 0.92$, $g_f^2 = 0.4$ (red).
Bounce profiles for the **Curved Spacetime Case**

Left panel: Profile of the bounce solutions $\varphi(x)$ relative to the four cases:
- $M_S = 2.5 \times 10^{-1}, \ M_f = 3 \times 10^{-4}, \ g_S = 0.96, \ g_f^2 = 0.5$ (yellow);
- $M_S = 2.0 \times 10^{-1}, \ M_f = 10^{-4}, \ g_S = 0.9, \ g_f^2 = 0.5$ (blue);
- $M_S = 2.0 \times 10^{-1}, \ M_f = 10^{-3}, \ g_S = 0.95, \ g_f^2 = 0.4$ (green);
- $M_S = 1.5 \times 10^{-1}, \ M_f = 5 \times 10^{-3}, \ g_S = 0.92, \ g_f^2 = 0.4$ (red).

Right panel: difference between the curvature radius and its asymptotic value, $a(x) - x$, for the same parameters as in the left panel.
Tunneling times for different values of the parameters

<table>
<thead>
<tr>
<th>$M_S$</th>
<th>$M_f$</th>
<th>$g_S$</th>
<th>$g_f^2$</th>
<th>$\tau_{\text{flat}}/T_U$</th>
<th>$\tau_{\text{grav}}/T_U$</th>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$10^{639}$</td>
<td>$10^{661}$</td>
</tr>
<tr>
<td>$1.5 \times 10^{-1} M_P$</td>
<td>$5 \times 10^{-3} M_P$</td>
<td>0.92</td>
<td>0.4</td>
<td>$10^{293}$</td>
<td>$10^{307}$</td>
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<tr>
<td>$2.0 \times 10^{-1} M_P$</td>
<td>$10^{-3} M_P$</td>
<td>0.95</td>
<td>0.4</td>
<td>$10^{80}$</td>
<td>$10^{94}$</td>
</tr>
<tr>
<td>$2.5 \times 10^{-1} M_P$</td>
<td>$3 \times 10^{-4} M_P$</td>
<td>0.96</td>
<td>0.5</td>
<td>$10^{-80}$</td>
<td>$10^{-65}$</td>
</tr>
<tr>
<td>$2.0 \times 10^{-1} M_P$</td>
<td>$10^{-4} M_P$</td>
<td>0.9</td>
<td>0.5</td>
<td>$10^{-103}$</td>
<td>$10^{-93}$</td>
</tr>
</tbody>
</table>

As for the case of the parametrization of New Physics with

$$V_{NP}(\phi) = \frac{\lambda_6}{6} \frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8} \frac{\phi^8}{M_P^4}$$

we again observe that Gravity tends to stabilize the EW vacuum ($\tau_{\text{grav}}$ always higher than $\tau_{\text{flat}}$). However, New Physics has always a strong (that can be even devastating) impact.
Degeneracy removed at the Quantum Level

Transition rate as a function of $R$ : ($\mu \sim \frac{1}{R}$)

$$p = \max_R \frac{V_U}{R^4} \exp \left[ -\frac{8\pi^2}{3|\lambda(\mu)|} - \Delta S \right]$$

False Vacuum Decay

Coleman analysis in flat space-time (1977)

Later (1980) Coleman - De Luccia considered the impact of gravity

In both cases... “Thin Wall” ...

47
In a gravitational background - Thin Wall Approximation

Comparing the action $B$ in the gravitational background with the action $B_0$ in flat space-time

$$B = \frac{B_0}{\left[1 - \left(\frac{\xi_0}{2\Lambda}\right)^2\right]^2}$$

with

$$\Lambda = \left(8\pi G \cdot \Delta U/3\right)^{-1/2}$$

and

$$\Delta U = U(\phi_f) - U(\phi_t)$$
Thin-Wall
Out of Thin Wall

![Graph showing the function U(\phi) for \phi ranging from -2 to 2. The y-axis represents U(\phi) ranging from -0.4 to 0.6. The graph exhibits a peak at \phi = 0 and a deep valley at \phi = \pm 1.5.](image-url)
Comparing the action $B$ in the gravitational background with the action $B_0$ in flat space-time

In the Thin Wall Approximation and Out of “Thin Wall”

\[
\frac{B}{B_0} \quad 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0
\]

\[
0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5
\]

\[x \quad 0 \quad 2 \quad L\]

\[
\frac{\bar{\xi}_0}{2 \Lambda}
\]
LESSON

When $U(\phi_{fv}) - U(\phi_{tv})$ is not small, the intuition that we have developed from the Coleman-DeLuccia analysis on the Impact of Gravity does not apply!

It is no longer true that when the Bounce becomes larger and larger, the probability of materialization of the bounce becomes smaller and smaller ... eventually vanishing ...
... “Old Ideas” ...


“For most of the relevant values of the top and Higgs masses, the instability scale \( \Lambda_{\text{inst}} \) is sufficiently smaller than the Planck mass, justifying the hypothesis of neglecting effects from unknown Planckian physics.”


“The SM potential is eventually stabilized by unknown new physics around \( M_P \) : because of this uncertainty, we cannot really predict what will happen after tunnelling has taken place. Nevertheless, a computation of the tunnelling rate can still be performed, this result does not depend on the unknown new physics at the Planck scale.”
Turning points...

![Diagram of a potential function with turning points](image)

This is QFT with “very many” dof, not 1 dof QM ⇒ the potential is not $V(\phi)$ in figure with 1 dof, but...

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} (\vec{\nabla} \phi)^2 - V(\phi) = \frac{1}{2} \dot{\phi}(\vec{x}, t)^2 - U(\phi(\vec{x}, t))$$

where $U(\phi(\vec{x}, t))$ is: $U(\phi(\vec{x}, t)) = V(\phi(\vec{x}, t)) + \frac{1}{2} (\vec{\nabla} \phi(\vec{x}, t))^2$

Very many dof, not 1 dof... The Potential is: $\sum_{\vec{x}} U(\phi(\vec{x}, t))$

The bounce is not a constant configuration ... Gradients do matter a lot.