Custodial Symmetry Violation in the Georgi-Machacek model*

Abstract
This poster examines the effects of custodial symmetry (CS) violation in the GM model. We assume that CS is exact at some scale, \( \Lambda \) and use the 1st order RGEs (leading log approx.) to run down to the weak scale and calculate the effect on observables. We find that the typical scales of CS can be as high as \( O(10\text{--}200 \text{ TeV}) \) with a maximum of \( \Lambda^6 \). Even with large amounts of running, the CS violating effects (on masses, couplings and decays) are tiny. Outside of special parameters of parameter space (e.g. a degenerate spectrum) they are generally too small to be detected at the LHC although they may be large enough to detect at a future e\(^-\)e\(^+\) collider.

Georgi-Machacek Model (GM)
Add real and complex triplet to SM doublet
Enforce \( SU(2)_L \times SU(2)_R \) symmetry to fix \( \rho = 1 \) at tree-level:
\[
\Phi = \begin{pmatrix} \phi^0 \ + i \phi^+ \ & 0 \\ -i \phi^+ \ & \phi^0 \ - i \phi^+ \end{pmatrix}, \quad X = \begin{pmatrix} x^0 \ + i x^+ \ & 0 \\ -i x^+ \ & x^0 \ - i x^+ \end{pmatrix}
\]
Most general scalar potential that preserves \( SU(2)_L \times SU(2)_R \):
\[
V(\Phi, X) = - \frac{1}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{1}{2} \text{Tr}(X^\dagger X) + \lambda_I \text{Tr}(\Phi^\dagger \Phi)^I + \lambda_J \text{Tr}(X^\dagger X)^J + \lambda_K \text{Tr}(X^\dagger X)^K + \lambda_L \text{Tr}(X^\dagger X)^L
\]
Georgi & Machacek 1985
Chananwiz & Golden 1985

GM Physical spectrum
Physical spectrum arranged according to \( SU(2)_c \) representation:
Bidoublet gives: 2 \( \otimes \) 2 \( \rightarrow \) 3 \( \oplus \) 0
Btriplet gives: 3 \( \otimes \) 3 \( \rightarrow \) 5 \( \oplus \) 1 \( \oplus \) 0

History of custodial violation in GM
Custodial violation in Georgi-Machacek model has had a vibrant history
Gunion, Vega & Wulck 1991: Standard T parameter yields very small effects as a result of uncontrolled UV divergence from hypercharge violating custodial symmetry. Need full gauge invariant potential for counterterm
Englert, Re & Spansowsky 1302.6505 applied S, T parameter constraints by subtracting a counterterm for T
Chiang, Yuku & Yagyu 1804.02633 used measured T parameter as input to fix custodial violating counterterm when calculating h couplings at 1-loop
Blasi, De Curtis & Yagyu 1704.08512 used RGEs to study custodial violation from running up from a custodial symmetric theory at the weak scale

Our approach assume custodial symmetry generated accidentally at some scale \( \Lambda \) in an unspecified UV completion (e.g. composite higgs) and use RGEs to run down to weak scale

Most general gauge potential and custodial symmetry violation
\[
V(\phi, X, \xi) = - \frac{M_0^2}{2} \phi^\dagger \phi + \frac{M_0^2}{2} \chi^\dagger \chi + \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\tilde{\phi}^\dagger \tilde{\phi})^2 + \lambda_4 (\tilde{\xi}^\dagger \tilde{\xi})^2 + \lambda_5 (\phi^\dagger \phi)^2 (\chi^\dagger \chi) + h.c. + \lambda_6 (\phi^\dagger \phi)^2 (\tilde{\phi}^\dagger \tilde{\phi}) + \lambda_7 (\phi^\dagger \phi)^2 (\tilde{\xi}^\dagger \tilde{\xi}) + \lambda_8 (\chi^\dagger \chi)^2 (\tilde{\phi}^\dagger \tilde{\phi}) + \lambda_9 (\chi^\dagger \chi)^2 (\tilde{\xi}^\dagger \tilde{\xi}) + \lambda_{10} (\phi^\dagger \phi) (\chi^\dagger \chi) (\tilde{\phi}^\dagger \tilde{\phi}) + h.c. + \lambda_{11} \chi^\dagger \chi \tilde{\phi}^\dagger \tilde{\phi} + \lambda_{12} \chi^\dagger \chi \tilde{\xi}^\dagger \tilde{\xi} - \frac{1}{2} \left( \frac{\lambda_{13}}{\sqrt{2}} \Delta \phi - \lambda_{14} \Delta \chi \right)
\]
where, \( \phi = (\phi^+ / \sqrt{2}) \) \( \chi = (\chi^+ / \sqrt{2}) \) \( \xi = (\xi^+ / \sqrt{2}) \)
Reduces to GM potential if impose special conditions which will be violated by hypercharge loops
Can only be exact at 1 energy scale, away from scale RGE running will violate relations

Relation to GM potential
Now have 16 parameters which reduce to the CS 9 when they obey:
\[
\begin{align*}
\lambda_1 &= 2 \lambda_3; & \lambda_2 &= 2 \lambda_2; & \lambda_4 &= 4 \lambda_4 \\
\lambda_5 &= 2 \lambda_5; & \lambda_6 &= 2 \lambda_6; & \lambda_7 &= 2 \lambda_7 + 4 \lambda_4; & M_0^2 &= M_0^2 \\
\lambda_8 &= 2 \lambda_8; & \lambda_9 &= 2 \lambda_9; & \lambda_{10} &= 2 \lambda_{10}; & M_0^2 &= M_0^2 \\
\lambda_{11} &= 2 \lambda_{11}; & \lambda_{12} &= 2 \lambda_{12}; & \lambda_{13} &= 2 \lambda_{13}; & M_0^2 &= M_0^2
\end{align*}
\]
Running RGEs with \( g' = 0 \) will respect these relations
Running \( g' \neq 0 \) will violate these relations
Treat violation as perturbation of GM spectrum- express new mass eigenstates in terms of GM eigenstates

Our method
- Choose a set of parameters at the weak scale (parameterized by \( m_0 \)) from the custodial symmetric GM model and solve for spectrum
- Choose a scale of custodial symmetry (subject to constraint from perturbative unitarity of \( \lambda_i \) at scale) and run up to CS scale using RGEs with \( g' = 0 \)
- Use CS relations to set all 16 general parameters at high scale
- Run back down to the weak scale using full RGEs
- Calculate vevs, 5\( F \), and \( m_0 \) adjust inputs \( \lambda_1 \) and \( \mu_2 \) iteratively until match measurements in custodial violating theory
- Compute \( \rho \) and use \( \pm 2 \sigma \) region of \( \rho = 1.00037 \pm 0.00023 \) to place second constraint on scale
- Use constraints to place upper bound on scale, \( \Lambda \)
- Calculate weak scale custodial violating observables using upper bound

Results: Upper bound on \( \Lambda \)
Typical scales in the order of \( 10^{-10} \text{--} 10^6 \text{ TeV} \) but can be as high as \( 10^9 \text{ TeV} \)

Results: Test of custodial symmetry violation in sm-like higgs couplings

Results: Mass splittings within custodial 5-plet
In benchmark plane mass splittings of 5-plet obey hierarchy: \( m_R > m_L > m_T \) in general scan some points have \( m_R < m_L \) but with splittings of no more than 1.5

Results: Induced BR of \( H_0 \) to fermions
CS violation induces mixing between CS states. \( H_0 \) states mixing with doublet induces \( H_0 \) decay to fermions

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3. Based on arXiv: 1807.11511