Generation of particle number asymmetry with inflaton background (arXiv:1609:02990; 1709.08781)

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Abstract

A possible scenario for generation of particle number asymmetry with inflaton background has been investigated. We study a simple model which may generate asymmetry through interaction with inflaton field. This model has CP violating and particle number violating features. We compute time evolution of particle number asymmetry in pre-heating era by using quantum field theory combined with density matrix.

Research background

- One of unsolved problem in both particle physics and inflationary cosmology: Baryon Asymmetry
- The origin of the asymmetry: oscillating inflaton field couples with a complex scalar field carries U(1) charge; particle number asymmetry (PNA).
- A possible scenario: the asymmetry was generated during preheating era. During this era, an inflaton decays into the another (lighter) particles and the universe turns into the reheating era.
- In this work, we study a simple model which generates PNA through "interactions" and compute its time evolution using quantum field theory with density operator.

The Model

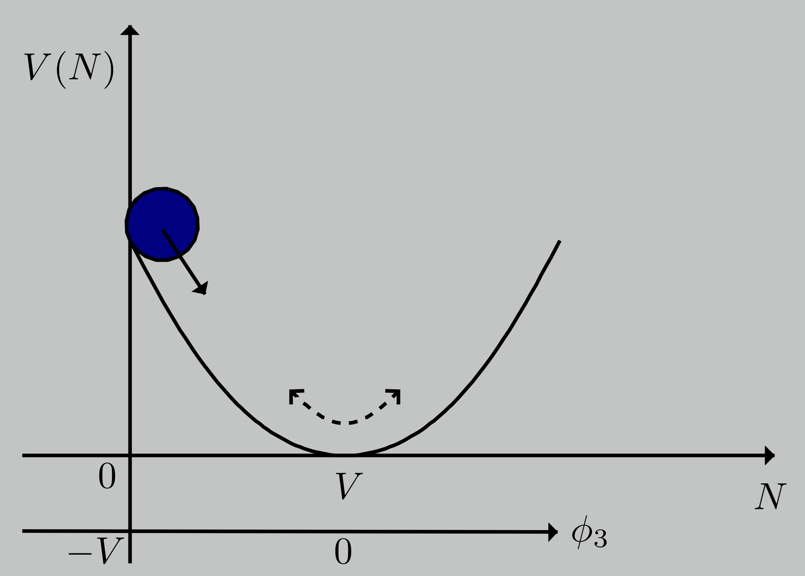
The action is given by

$$S=\int d^4x\sqrt{-g}\left(\mathcal{L}_{\rm free}+\mathcal{L}_{\rm int}\right),$$

$$\mathcal{L}_{\rm free}=g^{\mu\nu}\nabla_\mu\phi^\dagger\nabla_\nu\phi-m_\phi^2|\phi|^2+\frac{1}{2}\nabla_\mu N\nabla^\mu N-\mathcal{V}(N)+\underbrace{\frac{B^2}{2}(\phi^2+\phi^{\dagger 2})+\left(\frac{\alpha_2}{2}\phi^2+h.c.\right)R}_{\text{particle number violating terms}}+\alpha_3|\phi|^2R,$$

$$\mathcal{L}_{\rm int}=\underbrace{A\phi^2N+A^*\phi^{\dagger 2}N}_{\text{CP violating terms}}+A_0|\phi|^2N,$$

$$g_{\mu\nu}=(1,-a^2(x^0),-a^2(x^0),-a^2(x^0))\quad\text{(FRW metric)}$$



The potential reads,

$$\mathcal{V}(N)=\frac{M_N^2}{2}(N-V)^2$$

$$\phi_3\equiv N-V\ \text{(shifted field)}$$

Decomposed complex scalar: real and imaginary.

$$\phi\equiv\frac{\phi_1+i\phi_2}{\sqrt{2}}$$

Effective masses:

$$\tilde{m}_{1,2}^2=m_\phi^2\mp B^2,\qquad \tilde{m}_3^2=\tilde{m}_N^2=\omega_{3,0}^2,$$

$$\mathcal{L}_{\rm free}=\frac{1}{2}\sqrt{-g}[g^{\mu\nu}\nabla_\mu\phi_i\nabla_\nu\phi_i-\tilde{m}_i^2(x^0)\phi_i^2]$$

$$\mathcal{L}_{\rm int}=\sum_{ijk=1}^{3}\frac{1}{3}A_{ijk}\phi_i\phi_j\phi_k+V\sum_{ij=1}^{2}A_{ij3}\phi_i\phi_j$$

Table: The cubic interactions and their properties

$A_{113}=\frac{A_0}{2}+\mathrm{Re.}(A)$	–
$A_{223}=\frac{A_0}{2}-\mathrm{Re.}(A)$	–
$A_{113}-A_{223}=2\mathrm{Re.}(A)$	U(1) violation
$A_{123}=-\mathrm{Im.}(A)$	U(1), CP violation

Current expectation value and the initial conditions

- The current:

$$j_\mu=\frac{1}{2}\left(\phi_2\overleftrightarrow{\partial}_\mu\phi_1-\phi_1\overleftrightarrow{\partial}_\mu\phi_2\right)$$

- Current expectation value written density operator:

$$\langle j_0(x)\rangle=\mathrm{Tr}(j_0(x)\rho(t_0))=\mathrm{Re}\left[\left(\frac{\partial}{\partial x^0}-\frac{\partial}{\partial y^0}\right)G_{12}^{12}(x,y)|_{y\to x}+\bar{\phi}_2^{2,*}(x)\overleftrightarrow{\partial}_0\bar{\phi}_1^1(x)\right]$$

$$\phi=\bar{\phi}+\varphi\quad\text{and}\quad G_{12}^{12}(x,y)=\mathrm{Tr}(\varphi_2(y)\varphi_1(x)\rho(t_0))$$

- $G_{ij}(x,y)$ and $\bar{\phi}_i$ are obtained from 2PI CTP EA (References: 2PI formalism [Calzetta and Hu,2008], 2PI in curved space [Ramsey and Hu,1997]).
- Initial condition: The state is given by a density operator ($x^0=t_0$),

$$\rho(t_0)=\frac{e^{-\beta H}}{\mathrm{tr}\ e^{-\beta H}},\quad \beta=\frac{1}{T}$$

$$H=\frac{1}{2}\sum_{i=1}^{3}\int d^3x\ a(t_0)^3\left[\pi_{\phi_i}\pi_{\phi_i}+\frac{\nabla\phi_i\cdot\nabla\phi_i}{a(t_0)^2}+\tilde{m}_i^2(\phi_i-v_i)^2\right]$$

- The initial condition for Green's function and field ($\bar{\phi}(t_0)\equiv\langle\phi(t_0)\rangle$):

$$\begin{pmatrix}\bar{\phi}_1(t_0)\\ \bar{\phi}_2(t_0)\\ \bar{\phi}_3(t_0)\end{pmatrix}=\begin{pmatrix}0\\0\\v_3\end{pmatrix}=\begin{pmatrix}0\\0\\-V\end{pmatrix};\quad G_{ij,t_0t_0}^{ab}(\mathbf{k})=\delta_{ij}\frac{1}{2\omega_i(\mathbf{k})a_{t_0}^3}\left[\begin{matrix}\sinh\beta\omega_i(\mathbf{k})\\ \hline \cosh\beta\omega_i(\mathbf{k})-1\end{matrix}\right]$$

Since $N(t_0)=0$, $\bar{\phi}_3(t_0)\equiv\langle\phi_3(t_0)\rangle=v_3=-V$.

- Temperature limit, $T\to 0\Leftrightarrow\beta\to\infty$: In the preheating era, we assume that all of temperature for the fields is zero. Although we introduce density operator with the finite temperature, one can take zero temperature limit at the end calculation which correspond to the case that the initial state is the vacuum contribution.

The inflaton field equation at finite time $T:=x^0-t_0$

$$\left[\frac{\partial^2}{\partial x^{0 2}}+\{3H(x^0)+\Gamma_{\rm tot}(T)\}\frac{\partial}{\partial x^0}+\Omega_3^2(T)\right]\bar{\phi}_3(x^0)=F(T)$$

$$\Omega_3^2(T):=\omega_{3,0}^2-\frac{1}{2}\sum_{i,j=1}^{2}A_{ij3}^2\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\omega_{i,\mathbf{k}}\omega_{j,\mathbf{k}}}\times\left(\frac{1-\cos[(\omega_{3,0}+\omega_{i,\mathbf{k}}+\omega_{j,\mathbf{k}})T]}{\omega_{3,0}+\omega_{i,\mathbf{k}}+\omega_{j,\mathbf{k}}}+\frac{1-\cos[(\omega_{i,\mathbf{k}}+\omega_{j,\mathbf{k}}-\omega_{3,0})T]}{\omega_{i,\mathbf{k}}+\omega_{j,\mathbf{k}}-\omega_{3,0}}\right)$$

$$\Gamma_{\rm tot}(T):=\frac{1}{2\omega_{3,0}}\sum_{i,j=1}^{2}A_{ij3}^2\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\omega_{i,\mathbf{k}}\omega_{j,\mathbf{k}}}\left(\frac{\sin[(\omega_{3,0}+\omega_{i,\mathbf{k}}+\omega_{j,\mathbf{k}})T]}{\omega_{i,\mathbf{k}}+\omega_{j,\mathbf{k}}+\omega_{3,0}}-\frac{\sin[(\omega_{i,\mathbf{k}}+\omega_{j,\mathbf{k}}-\omega_{3,0})T]}{\omega_{i,\mathbf{k}}+\omega_{j,\mathbf{k}}-\omega_{3,0}}\right)$$

$$F(T):=\sum_{i=1}^{2}A_{ii3}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{2\omega_{i,\mathbf{k}}}+V\sum_{i,j=1}^{2}A_{ij3}^2\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\omega_{i,\mathbf{k}}\omega_{j,\mathbf{k}}}\frac{\{1-\cos[(\omega_{i,\mathbf{k}}+\omega_{j,\mathbf{k}})T]\}}{\omega_{i,\mathbf{k}}+\omega_{j,\mathbf{k}}}$$

Time evolution of PNA

We consider the case that:

$$a(s)=a(t_0)e^{H_0s};\quad \Omega=\omega_{1,\mathbf{k}}+\omega_{2,\mathbf{k}};\qquad s=t'-t_0;\quad t=x^0-t_0$$

$$\lim_{\beta\to\infty}\langle j_0(x^0)\rangle=-\frac{VA_{123}(\tilde{m}_2^2-\tilde{m}_1^2)V}{4\pi^2e^{3H_0t}}\int_{\tilde{m}_2+\tilde{m}_1}^{\infty}\frac{d\Omega}{\Omega}\sqrt{\left(1-\frac{(\tilde{m}_1+\tilde{m}_2)^2}{\Omega^2}\right)\left(1-\frac{(\tilde{m}_2-\tilde{m}_1)^2}{\Omega^2}\right)}\left[\frac{\sin[\Omega t]}{\Omega}-\frac{1}{2}\left(\frac{\frac{3}{2}H_0\{\cos[\Omega t]-e^{-\frac{3}{2}H_0t}\cos\omega_{3,0}t\}+(\omega_{3,0}+\Omega)\{e^{-\frac{3}{2}H_0t}\sin\omega_{3,0}t+\sin[\Omega t]\}}{(\frac{3}{2}H_0)^2+(\omega_{3,0}+\Omega)^2}+\frac{\frac{3}{2}H_0\{\cos[\Omega t]-e^{-\frac{3}{2}H_0t}\cos\omega_{3,0}t\}+(\omega_{3,0}-\Omega)\{e^{-\frac{3}{2}H_0t}\sin\omega_{3,0}t-\sin[\Omega t]\}}{(\frac{3}{2}H_0)^2+(\omega_{3,0}-\Omega)^2}\right)\right]$$

For non zero PNA, we remark several requirements:

- $VA_{123}(\tilde{m}_1-\tilde{m}_2)\neq 0$ (non-degeneracy of masses for $\bar{\phi}_1$ and $\bar{\phi}_2$)
- Non zero CP and U(1) coupling, $A_{123}\neq 0$
- Non zero $\omega_{3,0}$ (angular frequency, mass of inflaton)

Numerical Results (preliminary)

As for numerical purpose, we define the following dimensionless quantities:

$$z:=\frac{\Omega}{(\tilde{m}_1+\tilde{m}_2)};\quad \tau:=(\tilde{m}_1+\tilde{m}_2)t;\quad M:=\frac{\omega_{3,0}}{(\tilde{m}_1+\tilde{m}_2)}\quad \delta m_{21}:=\frac{(\tilde{m}_2-\tilde{m}_1)}{(\tilde{m}_1+\tilde{m}_2)};\quad h:=\frac{H(t_0)}{(\tilde{m}_1+\tilde{m}_2)}$$

$$\lim_{\beta\to\infty}\langle j_0(x^0)\rangle=\frac{A_{123}V(\tilde{m}_2-\tilde{m}_1)}{4\pi^2}\times f(\tau,M,\delta m_{21},h)$$

$$f(\tau,M,\delta m_{21},h)=-e^{-3h\tau}\int_1^\infty\frac{dz}{z}\sqrt{\left(1-\frac{1}{z^2}\right)\left(1-\frac{(\delta m_{21})^2}{z^2}\right)}\left[\frac{\sin[z\tau]}{z}-\frac{1}{2}\left\{\frac{\frac{3}{2}h\left(\cos[z\tau]-e^{-\frac{3}{2}h\tau}\cos[M\tau]\right)+(M+z)\left(e^{-\frac{3}{2}h\tau}\sin[M\tau]+\sin[z\tau]\right)}{(\frac{3}{2}h)^2+(M+z)^2}+\frac{\frac{3}{2}h\left(\cos[z\tau]-e^{-\frac{3}{2}h\tau}\cos[M\tau]\right)+(M-z)\left(e^{-\frac{3}{2}h\tau}\sin[M\tau]-\sin[z\tau]\right)}{(\frac{3}{2}h)^2+(M-z)^2}\right\}\right]$$

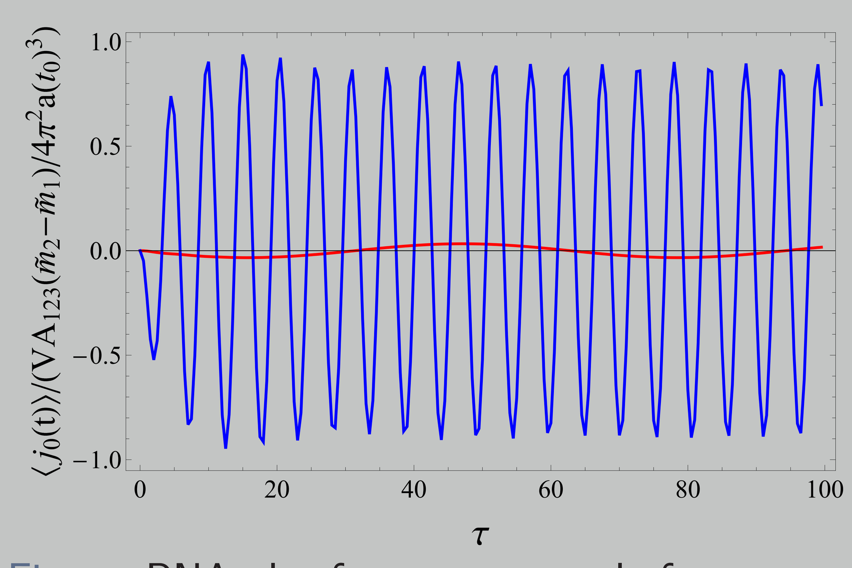


Figure: PNA plot for constant scale factor: $(M,\delta m_{21})=(0.1,0.2),(1.2,0.2)$ and $0\leq\tau\leq 100$. The amplitude of PNA decreases as the inflaton mass becomes smaller and vice versa.

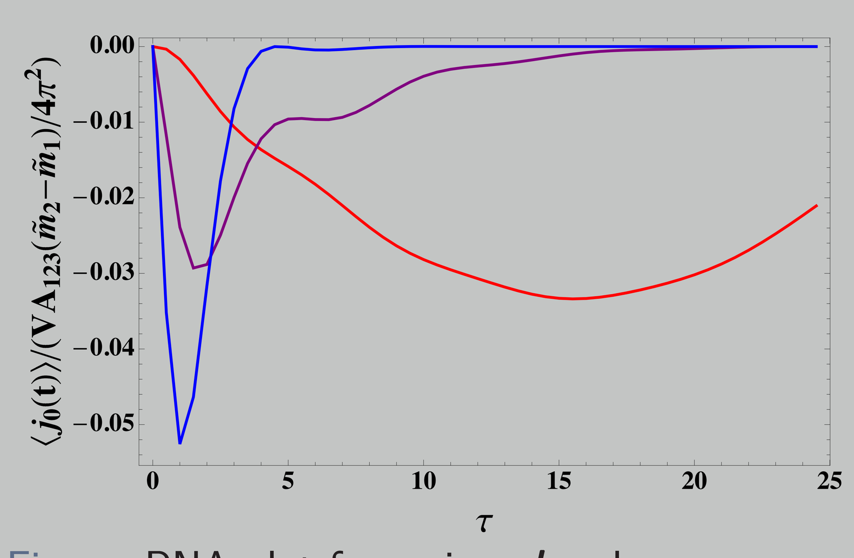


Figure: PNA plot for various h values (0.0,0.05,0.2) and $(M,\delta m_{21})=(0.1,0.2)$ are fixed.

Summary and Conclusion

- We study an interacting model in which particle number asymmetry is generated through oscillating inflaton field coupled with a complex scalar field.
- The current for the particle and anti-particle asymmetry is given up to the first order of cubic coupling A.