Electroweak vacuum collapse induced by Higgs vacuum fluctuations around evaporating black holes

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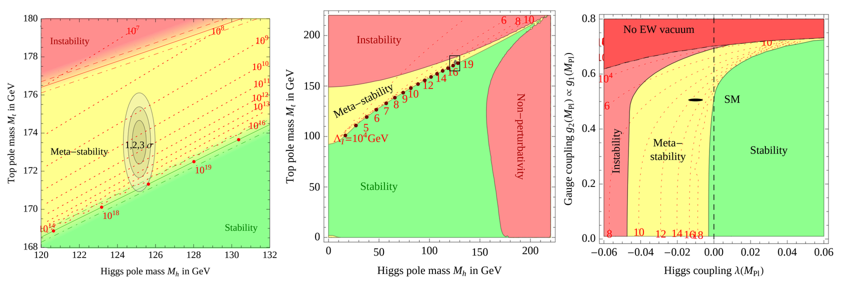
1 Higgs Vacuum Metastability

The recent LHC experiments of the Higgs boson mass $m_h = 125.09 \pm 0.21(\text{stat}) \pm 0.11(\text{syst})$ GeV and the top quark mass $m_t = 172.44 \pm 0.13(\text{stat}) \pm 0.47(\text{syst})$ GeV suggest that the electroweak vacuum is *metastable* and finally cause a catastrophic vacuum decay through quantum tunneling.

Is the electroweak vacuum stable or not ?



[1] D. Buttazzo, G. Degrassic, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, A. Strumia, JHEP 1312 (2013).

Fortunately, the decay timescale is longer than the age of our Universe. However, the strong gravitational background enhances the vacuum decay and the large-scale inflation or evaporating black holes are trouble for the Higgs vacuum stability.

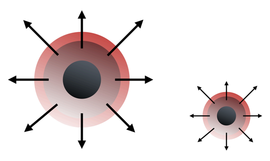
The gravitational Higgs fluctuations $\langle \delta\phi^2 \rangle$

$$\langle \delta\phi^2 \rangle^{1/2} \simeq O(T_{\rm H}) \gtrsim \Lambda_I \approx 10^{11}\ \text{GeV} \Longrightarrow \text{Collapse !!}$$

where $T_{\rm H}$ = is the Gibbons-Hawking temperature.

2 Evaporating Black Holes

Black Hole emits the thermal Hawking radiation



The black holes emitting the Hawking radiation at the Hawking temperature $T_{\rm H} = 1/(8\pi M_{\rm BH})$ reduce the mass and finally evaporate. At the final stage of evaporating black holes, the Hawking temperature is extremely high since the limit $M_{\rm BH} \to 0$ leads to $T_{\rm H} \to \infty$, and the tiny black-hole strongly affects the Higgs vacuum stability.

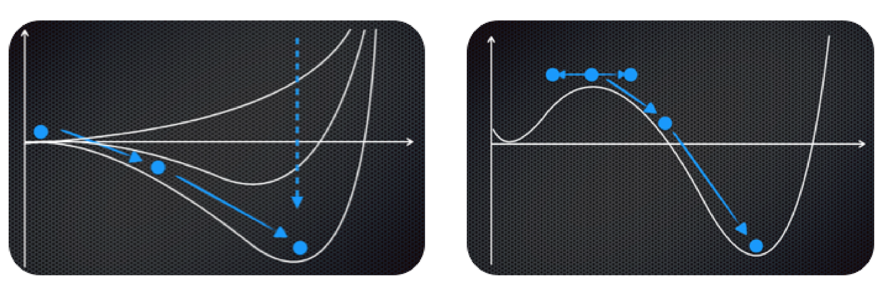
The vacuum fluctuations at event horizon

$$\langle \delta\phi^2 \rangle \approx \langle \delta W^2 \rangle \approx \langle \delta Z^2 \rangle \approx \langle \delta t^2 \rangle \approx \frac{T_{\rm H}^2}{3} - 2T_{\rm H}^2 \int_0^\infty \frac{d\omega \omega \sum_{l=0}^{\infty} (2l+1)|B_l(\omega)|^2}{\omega\left(e^{2\pi\omega/\kappa}-1\right)}.$$

where $\kappa = (4M_{\rm BH})^{-1}$ is surface gravity of black hole.

3 Effective Higgs Potential around Evaporating Black Holes

The two scenarios of the collapsing Higgs vacuum



The vacuum fluctuations of the Higgs field $\langle \delta\phi^2 \rangle$ destabilize the effective Higgs potential $V_{\rm eff}(\phi)$ as the back-reaction or generate true vacuum domains or bubbles. The Higgs fluctuations induce the false vacuum decay around the black-hole. However, W and Z bosons and top quark vacuum fluctuations stabilize the Higgs potential via the interaction.

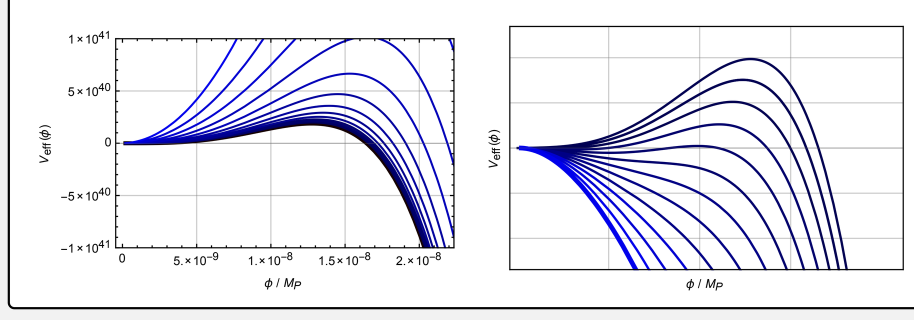
$$V_{\rm eff}(\phi) = \rho_\Lambda(\mu) + \frac{m_\phi^2(\mu)}{2}\phi^2 + \frac{\lambda_\phi(\mu)}{4}\phi^4 + \frac{3\lambda(\mu)}{2}\langle \delta\phi^2 \rangle \phi^2 + \frac{g^2(\mu)}{8}\langle \delta W^2 \rangle \phi^2 + \frac{\left[g^2(\mu) + g'^2(\mu)\right]}{8}\langle \delta Z^2 \rangle \phi^2 + \frac{y_t^2(\mu)}{4}\langle \delta t^2 \rangle \phi^2 + \sum_{i=W,Z,t,G,H} \frac{n_i M_i^4(\phi)}{64\pi^2}\left[\log\frac{M_i^2(\phi)}{\mu^2} - C_i\right],$$

The Higgs potential $V_{\rm eff}(\phi)$ including thermal Hawking radiations around can be approximately written as,

$$V_{\rm eff}(\phi) \simeq O\left(T_{\rm H}^2\right)\phi^2 + \frac{\lambda_{\rm eff}(\phi)}{4}\phi^4, \quad \phi_{\max} \approx (1 \sim 10)\cdot T_{\rm H}$$

where $\lambda_{\rm eff}(\phi)$ is the effective Higgs self-coupling. Hence, thermal Hawking radiation stabilizes the Higgs potential !

Higgs potential around evaporating black hole



4 Higgs Vacuum Collapse around Evaporating Black Holes

We introduce the vacuum decay rate by using the two-point correlation function $\langle \delta\phi^2 \rangle$.

The probability of the local Higgs fields

$$P(\phi) = \frac{1}{\sqrt{2\pi\langle \delta\phi^2 \rangle}} \exp\left(-\frac{\phi^2}{2\langle \delta\phi^2 \rangle}\right).$$

By using the above equation, we obtain the probability not to exceed the hill of the Higgs potential:

$$P(\phi < \phi_{\max}) \equiv \int_{-\phi_{\max}}^{\phi_{\max}} P\left(\phi, \langle \delta\phi^2 \rangle\right) d\phi, = \text{erf}\left(\frac{\phi_{\max}}{\sqrt{2\langle \delta\phi^2 \rangle}}\right),$$

where $\phi_{\max}$ is the maximal field value of the Higgs potential. Considering the probability that the localized Higgs fields go into true vacuum, the vacuum decay ratio is estimated to be

The probability of the Higgs vacuum decay

$$\Gamma_{\rm decay}(\phi) \equiv P(\phi > \phi_{\max}) = 1 - \text{erf}\left(\frac{\phi_{\max}}{\sqrt{2\langle \delta\phi^2 \rangle}}\right), \simeq \frac{\sqrt{2\langle \delta\phi^2 \rangle}}{\pi\phi_{\max}} \exp\left(-\frac{\phi_{\max}^2}{2\langle \delta\phi^2 \rangle}\right).$$

Then, the constraint from the vacuum decay for the Higgs field is represented by

$$\mathcal{N}_{\rm PBH} \cdot \Gamma_{\rm decay}(\phi) \lesssim 1,$$

where $\mathcal{N}_{\rm PBH}$ is the number of the evaporating (or evaporated) primordial black holes during the cosmological history of the Universe. Substituting these equations, we can simplify the constraint of the vacuum stability,

The constraint on the vacuum stability

$$\frac{\langle \delta\phi^2 \rangle}{\phi_{\max}^2} \lesssim \frac{1}{2}\left(\log \mathcal{N}_{\rm PBH}\right)^{-1},$$

where the vacuum fluctuations $\langle \delta\phi^2 \rangle$ around evaporating black holes are given as,

$$\langle \delta\phi^2 \rangle_{\rm ren} \simeq O\left(10^{-2} \sim 10^{-1}\right) \cdot T_{\rm H} \quad (r \to 2M_{\rm BH}).$$

We can estimate a constraint on the number of the evaporating primordial black holes as,

The constraint on the primordial black holes

$$\mathcal{N}_{\rm PBH} \cdot \Gamma_{\rm decay}(\phi) \simeq \frac{\mathcal{N}_{\rm PBH}\sqrt{2\langle \delta\phi^2 \rangle}}{\pi\phi_{\max}} \exp\left(-\frac{\phi_{\max}^2}{2\langle \delta\phi^2 \rangle}\right) \approx \mathcal{N}_{\rm PBH} \cdot e^{-O(10^{2\sim3})} \lesssim 1,$$

which provides a new bound on the number of the primordial black holes to be $\mathcal{N}_{\rm PBH} \lesssim O\left(10^{43\sim434}\right)$.

5 The upper bound on the PBH

The upper bound on the yield of the PBHs

$$Y_{\rm PBH} = \frac{n_{\rm PBH}}{s} = \frac{\mathcal{N}_{\rm PBH}}{s_0/H_0^3} \lesssim O\left(10^{-43}\right).$$

where s_0 denotes the entropy density at present ($\approx (3\times 10^{-4}\ \text{eV})^3$), and H_0 is the current Hubble constant ($\approx 10^{-33}$ eV). It is convenient to transform this bound into an upper bound on β, which is defined by taking values at the formation of the PBH to be

$$\beta \equiv \left.\frac{\rho_{\rm PBH}}{\rho_{\rm tot}}\right|_{\rm formation},$$

where $\rho_{\rm PBH}$ and $\rho_{\rm tot}$ are the energy density of the PBHs and the total energy density of the Universe including the PBHs at the formation, respectively. Then we have a relation,

$$\beta \sim 10^{30}\frac{n_{\rm PBH}}{s}\left(\frac{m_{\rm PBH}}{10^{15}\text{g}}\right)^{3/2}.$$

Combining this relation, we obtain

The upper bound on β

$$\beta \lesssim O\left(10^{-21}\right)\left(\frac{m_{\rm PBH}}{10^9\text{g}}\right)^{3/2}.$$

which is stronger than the known one for $m_{\rm PBH} \lesssim 10^9$g.

6 The effects of the BSM and QG on the Higgs vacuum instability

The corrections of the BSM and QG strongly affects the Higgs vacuum stability around the black hole. The Higgs potential including higher dimension operators ϕ^6 and ϕ^8 is given by,

$$V_{\rm eff}(\phi) = \frac{\lambda_{\rm eff}(\phi)}{4}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{\Lambda_{\rm UV}^2} + \frac{\lambda_8}{8}\frac{\phi^8}{\Lambda_{\rm UV}^4} + \cdots,$$

where λ_6 and λ_8 are dimensionless coupling constants. They are usually negligible, but in the final stage of the black hole evaporation $T_{\rm H} \to M_{\rm P}$, they strongly affect the vacuum stability.

The corrections of the BSM and QG

$$V_{\rm eff}(\phi) \simeq O\left(T_{\rm H}^2\right)\phi^2 + \frac{\lambda_{\rm eff}(\phi)}{4}\phi^4 + \frac{\lambda_6 \cdot O\left(T_{\rm H}^4\right)}{6\Lambda_{\rm UV}^2}\phi^2 + \frac{\lambda_8 \cdot O\left(T_{\rm H}^6\right)}{8\Lambda_{\rm UV}^4}\phi^2 + \cdots,$$

which would destabilize at $T_{\rm H} \approx \Lambda_{\rm UV}$ when these higher-dimension operators are negative $\lambda_6, \lambda_8 < 0$.

7 Conclusion and Discussion

We have investigated the electroweak vacuum stability around evaporating black holes. We have provide a new approach to investigate the false vacuum decay around the black hole using the vacuum fluctuations $\langle \delta\phi^2 \rangle$. Clearly, we have shown how evaporating black holes induce a collapse of the electroweak vacuum and get the following conditions,

Black-hole Higgs Vacuum Instability

- $m_h \approx 125.09$ GeV and $m_t \approx 172.44$ GeV
- $\mathcal{N}_{\rm PBH} \gtrsim O\left(10^{43\sim434}\right)$
- $\beta \gtrsim O\left(10^{-21}\right)\left(m_{\rm PBH}/10^9\text{g}\right)^{3/2}$
- The higher-dimension operators of $\lambda_6, \lambda_8 < 0$ destabilize the Higgs potential at $T_{\rm H} \approx \Lambda_{\rm UV}$,

$$V_{\rm eff}(\phi) \simeq O\left(T_{\rm H}^2\right)\phi^2 + \frac{\lambda_{\rm eff}(\phi)}{4}\phi^4 + \frac{\lambda_6 \cdot O\left(T_{\rm H}^4\right)}{6\Lambda_{\rm UV}^2}\phi^2 + \frac{\lambda_8 \cdot O\left(T_{\rm H}^6\right)}{8\Lambda_{\rm UV}^4}\phi^2 + \cdots,$$

$\Longrightarrow$ Electroweak Vacuum Collapse !!