The implication of gauge-Higgs unification for the hierarchical fermion masses

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(arXiv: 1801.01639[hep-ph])
1. Introduction

The SM has theoretical problems in its Higgs sector:
(1) The hierarchy problem: main motivation of BSM
(2) The Higgs interactions cannot be predicted
← no guiding principle to restrict Higgs interactions

It would be nice if Higgs interactions can be controlled by gauge principle

【Gauge Higgs Unification (GHU)】
The origin of Higgs boson: (higher dimensional) gauge boson (N.S. Manton (’79), Y. Hosotani (’83)):

In 5D space-time, $A_M = (A_\mu, A_y)$ ($H = A_y$)
(1) ⇒ By virtue of (higher dimensional) gauge symmetry, hierarchy problem is solved

【Higgs as AB phase】
Is such Higgs field physical?
Yes! it has a meaning as a AB (Wilson-line) phase:

\[ W = \text{Tr}(e^{i \frac{g}{2} \int A_y dy}) = e^{ig_4 \pi R A_y^{(0)}} = e^{i \frac{g_4}{2} \Phi} \]

(Circle: non-simply-connected: topology)

\( (5D) \rightarrow (4D) \)
(2) ⇒ We will see: the impressive hierarchical structure of fermion masses can be naturally understood (without fine tuning) by geometrical and/or topological nature of magnetic field generated by magnetic monopole (placed inside the extra dimension).

(N.B.)

• The particle limit of (open-) superstring may be regarded as a sort of GHU:10D SUSY (pure-) Y-M theory \((A^{(a)}_A, \lambda^{(a)})\)

• CP violation may be also attributed to the geometry of extra space: For C-Y compactification, defined by
  \[ z_1^5 + z_2^5 + \cdots + z_5^5 - c(z_1z_2\cdots z_5) = 0 \]
  CP is broken for complex \(c\).

  (C.S.L. (‘90), · · · · · ·, w./ N. Maru and K. Nishiwaki (‘10))

• “Gauge inflation”
  ( ⇒ S.C. Park’s talk in this workshop)
2. The implication of GHU for the hierarchical fermion masses

(C.S. L., PTEP 2018(’18),093B02 arXiv:1801.01639[hep-ph])

The Yukawa coupling in GHU is gauge coupling to start with and therefore universal → seems ridiculous

Possible solution in the theory with orbifold $S^1/Z_2$ as the extra space is to invoke "$Z_2$-odd" bulk mass term:

$$\epsilon(y) M_B \bar{\psi}\psi$$ (\(\epsilon(y)\) : sign function)

⇒ exponentially suppressed Yukawa coupling,

$$\sim g (\pi R M_B) e^{-\pi R M_B} (R : the \ radius \ of \ S^1)$$

, though there is no principle to fix $M_B$
The impressive hierarchical structure of fermion masses

\[ \log m_f \]

(By Dr. H. Terao)
The observed impressive hierarchy implies

\[ \log m_f \sim \alpha N_g + \beta \iff m_f = e^{\beta + 3\alpha} e^{-(3-N_g)\alpha} \]

\((N_g : \text{generation number})\)

This feature may be naturally understood in GHU:

♠ Originally all fermion masses are universal

⇒ provides good reasoning of the universal factor \( e^{\beta + 3\alpha} \)

♠ The exponential suppression may be realized by \( e^{-\pi RM_B} \)

♠ The only remaining task is to find a natural mechanism to guarantee “quantized \(Z_2\) - odd mass”

\(M_B \propto 0, 1, 2\)
**Idea**: the quantization of $M_B$ may be attributed to the Dirac quantization condition of magnetic charge.

【Basic framework】6D GHU with $T^2$ as extra space, where a (Dirac) magnetic monopole is placed inside the torus.

$y_5 = 0$

$magnetic\ monopole$

$y_5 = -\pi R_5\quad y_5 = \pi R_5$

$\Rightarrow\ eQ A_6 = \epsilon(y_5) \frac{M}{2R_6}$

quantization condition of magnetic charge

$\Rightarrow\ M : integer.$
(N.B.)

(1) Now $Z_2$-odd bulk mass has been replaced by the flux generated by magnetic monopole, and it cannot be arbitrary, but controlled by the quantization condition.

(2) Again, topology plays essential roles:

$$\pi_2(SU(2)/U(1)) = \mathbb{Z}$$ for ’t Hooft-Polyakov monopole, 

$$\pi_1(U(1)) = \mathbb{Z}$$ for Dirac monopole

(3) The index theorem (Atiyah-Singer) applied for 2D extra space guarantees $M$ generations of chiral fermion:

$$\text{index of Dirac operator} = \frac{1}{2\pi} \int_{\text{torus}} \text{tr} F \ (= M)$$

(“magnetized extra dimension”)

Toy model: 6D U(1) GHU model with one fermion with U(1) ("electric") charge eQ.

Writing a KK zero mode of 4D left-handed fermion, as

$$\psi^{(0)}_L(x^\mu, y_5, y_6) = \psi^{(0)}_L(x^\mu)f_L(y_5, y_6)$$

the mode function should satisfy

$$\left\{ \partial_5 + i(\partial_6 - ieQ\epsilon(y_5)\frac{g}{4\pi R_6}) \right\}f_L(y_5, y_6) = 0.$$
The boundary condition @ \( y_5 = \pm \pi R_5 \)

\[
f_L(\pi R_5, y_6) = e^{i \frac{eQg}{2\pi R_6} y_6} f_L(-\pi R_5, y_6)
\]

with

\[
eQg = 2\pi M \quad (M : \text{ integer}) \quad \text{: Dirac quantization condition}
\]

\[
f_L^{(M,j)}(\hat{y}_5, \hat{y}_6) = c^{(M,j)} \sum_{r=-\infty}^{\infty} e^{-\frac{\pi R_5}{R_6} M(r + \frac{j}{M})^2} e^{\frac{\pi R_5}{R_6} M\{r + \frac{j}{M}\hat{y}_5 - \frac{1}{2}|\hat{y}_5|\}} e^{i\pi M(r + \frac{j}{M})\hat{y}_6}
\]

\[
= c^{(M,j)} e^{-\frac{\pi R_5}{R_6} M\{\left(\frac{j}{M}\right)^2 - \frac{j}{M}(\hat{y}_5 + i\frac{R_6}{R_5}\hat{y}_6) + \frac{1}{2}|\hat{y}_5|^2\}} \left(\frac{\hat{y}_5}{R_5} = \frac{y_5}{\pi R_5}, \quad \frac{\hat{y}_6}{R_6} = \frac{y_6}{\pi R_6}\right)
\]

\[
\cdot \theta_3\left(i\frac{R_5}{R_6} M\left(\frac{j}{M} - \frac{\hat{y}_5 + i\frac{R_6}{R_5}\hat{y}_6}{2}\right) | i\frac{R_5}{R_6} M\right), \quad \text{Jacobi's theta function}
\]

\((j = 0, 1, \ldots, M - 1 \quad \text{: } M \text{ chiral generations})!\)

well-approximated by (under $R_5 \gg R_6$),

$$\frac{1}{\sqrt{\pi R_6}} \sqrt{\frac{M^2}{4} - j^2} \cdot e^{-\frac{\pi R_5}{R_6} \left( \frac{M}{2} |\tilde{y}_5| - j \tilde{y}_5 \right)} \cdot e^{i \pi j \tilde{y}_6} \quad \text{(for} \ 0 \leq j < \frac{M}{2} \text{)}$$

$$\frac{1}{\sqrt{\pi R_6}} \sqrt{\frac{M^2}{4} - (M-j)^2} \cdot e^{-\frac{\pi R_5}{R_6} \left( \frac{M}{2} |\tilde{y}_5| + (M-j) \tilde{y}_5 \right)} \cdot e^{i \pi (M-j) \tilde{y}_6} \quad \text{(for} \ \frac{M}{2} < j < M \text{)}$$

⇒ localization at $y_5 = 0$

The overlap integral of mode functions

$$\pi^2 R_5 R_6 \int_{-1}^{1} d\tilde{y}_5 \int_{-1}^{1} d\tilde{y}_6 f_{L}^{(M,j)}(\tilde{y}_5, \tilde{y}_6) * f_{R}^{(-M,j)}(\tilde{y}_5, \tilde{y}_6)$$

$$\approx 2\pi \frac{R_5}{R_6} \frac{M^2}{4} - j^2 e^{-\frac{\pi R_5}{R_6} \left( \frac{M}{2} - j \right)} \quad \text{(for} \ 0 < j < \frac{M}{2} \text{)},$$

$$\approx 2\pi \frac{R_5}{R_6} \frac{M^2}{4} - (M-j)^2 e^{-\frac{\pi R_5}{R_6} \left( j - \frac{M}{2} \right)} \quad \text{(for} \ \frac{M}{2} < j < M \text{)}$$

now shows exponential suppression \( \propto e^{-\frac{\pi R_5}{R_6} |\frac{M}{2} - j|} \)
(A problem)

To get 3 generation model, we naively expect that one fermion with \( M = 3 \) will work.

Unfortunately, there appear degenerate Yukawa couplings proportional to

\[
e^{-\frac{\pi R_5}{R_6} \frac{3}{2}}, \quad e^{-\frac{\pi R_5}{R_6} \frac{1}{2}}, \quad e^{-\frac{\pi R_5}{R_6} \frac{1}{2}}
\]
【Successful 3 generation models】

Two types of successful models, without degeneracy, are known to exist:

(1) 2 + 1 model

Introduce one $M = 2$ fermion & one $M = 1$ fermion, which lead to exponential suppression factors,

$$e^{-\frac{\pi R_5}{R_6}} (M = 2, \; j = 0), \quad 1 (M = 2, \; j = 1),$$

$$e^{-\frac{\pi R_5}{2 R_6}} (M = 1, \; j = 0)$$

i.e. to successful hierarchical relation

(m1,2,3 : mass eigenvalues):

$$\frac{m_3}{m_2} \sim \frac{m_2}{m_1} \sim e^{\frac{\pi R_5}{2 R_6}}$$
More precisely, including the numerical factor due to normalization factor, double ratio is given as

\[
\left( \frac{m_3}{m_2} \right) \left( \frac{m_2}{m_1} \right) = \frac{1}{\log \{2(m_2/m_1)\}}.
\]

For charged lepton sector,

\[
\left( \frac{m_3}{m_2} \right) \left( \frac{m_2}{m_1} \right) = 0.17,
\]

while the observed value is

\[
\left( \frac{m_\tau}{m_\mu} \right) \left( \frac{m_\mu}{m_e} \right) = 0.075
\]

(2) Model with twisted boundary condition

Take just one \( M = 3 \) fermion, but with “twisted” b.c. :

\[
f_L(y_5, y_6 + 2\pi R_6) = e^{i2\pi^a} f_L(y_5, y_6)
\]
(Remaining issues)

- How to confine the magnetic flux into the inside of torus? (superconductivity? some hints in superstring?)
- The role of ’t Hooft- Polyakov monopole (GHU provides very suitable framework)?