The implication of gauge-Higgs unification for the hierarchical fermion masses

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(arXiv: 1801.01639[hep-ph])

1. Introduction

The SM has theoretical problems in its Higgs sector:

(1) The hierarchy problem : main motivation of BSM

(2) The Higgs interactions cannot be predicted

$\leftarrow$ no guiding principle to restrict Higgs interactions

It would be nice if Higgs interactions can be controlled by gauge principle

【Gauge Higgs Unification (GHU)】

The origin of Higgs boson: (higher dimensional) gauge boson (N.S. Manton ('79), Y. Hosotani ('83)):

In 5D space-time, $A_M = (A_\mu,\ A_y) \quad (H = A_y)$

(1) ⇒ By virtue of (higher dimensional) gauge symmetry,
hierarchy problem is solved
(with/ H. Hatanaka and T. Inami, Mod. Phys. Lett. A13 (1998) 2601)

【Higgs as AB phase】

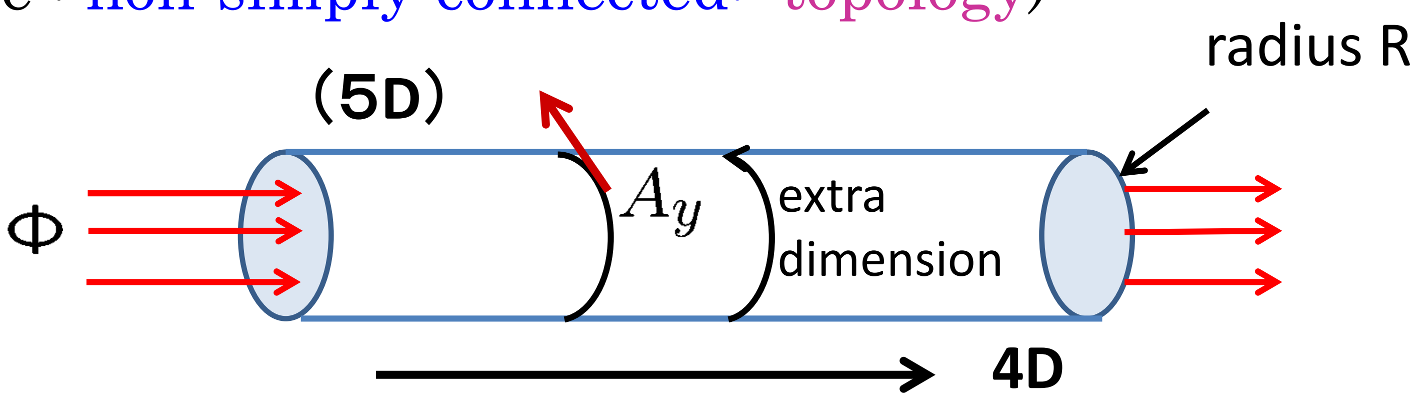
Is such Higgs field physical ?

Yes ! it has a meaning as a AB (Wilson-line) phase:

$$W = \mathrm{Tr}\left(e^{i\frac{g}{2}\oint A_y dy}\right) = e^{i g_4 \pi R A_y^{(0)}} = e^{i\frac{g_4}{2}\Phi}$$

Higgs

(Circle : non-simply-connected: topology)



(2) $\Rightarrow$ We will see: the impressive hierarchical structure of fermion masses can be naturally understood (without fine tuning) by geometrical and/or topological nature of magnetic field generated by magnetic monopole (placed inside the extra dimension).

(N.B.)

- The particle limit of (open-) superstring may be regarded as a sort of GHU:10D SUSY (pure-) Y-M theory

$$(A_M^{(a)},\ \lambda^{(a)})$$

- CP violation may be also attributed to the geometry of extra space: For C-Y compactification, defined by

$$z_1^5 + z_2^5 + \cdots + z_5^5 - c(z_1 z_2 \cdots z_5) = 0$$

CP is broken for complex c .

(C.S.L. ('90), ······, w./ N. Maru and K. Nishiwaki (' 10))

- "Gauge inflation"

($\Rightarrow$ S.C. Park's talk in this workshop)

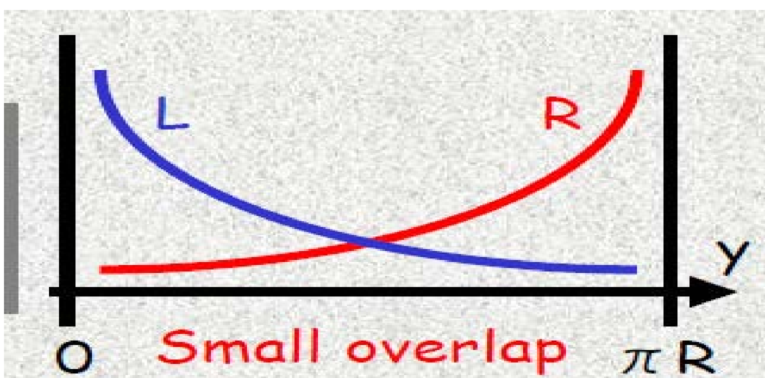
2. The implication of GHU for the hierarchical fermion masses

(C.S. L. , PTEP 2018('18),093B02 arXiv:1801.01639[hep-ph])

The Yukawa coupling in GHU is gauge coupling to start with and therefore universal $\rightarrow$ seems ridiculous

Possible solution in the theory with orbifold S^1/Z_2 as the extra space is to invoke "Z_2- odd" bulk mass term:

$$\epsilon(y) M_B \bar{\psi}\psi \quad (\epsilon(y) : \text{sign function})$$

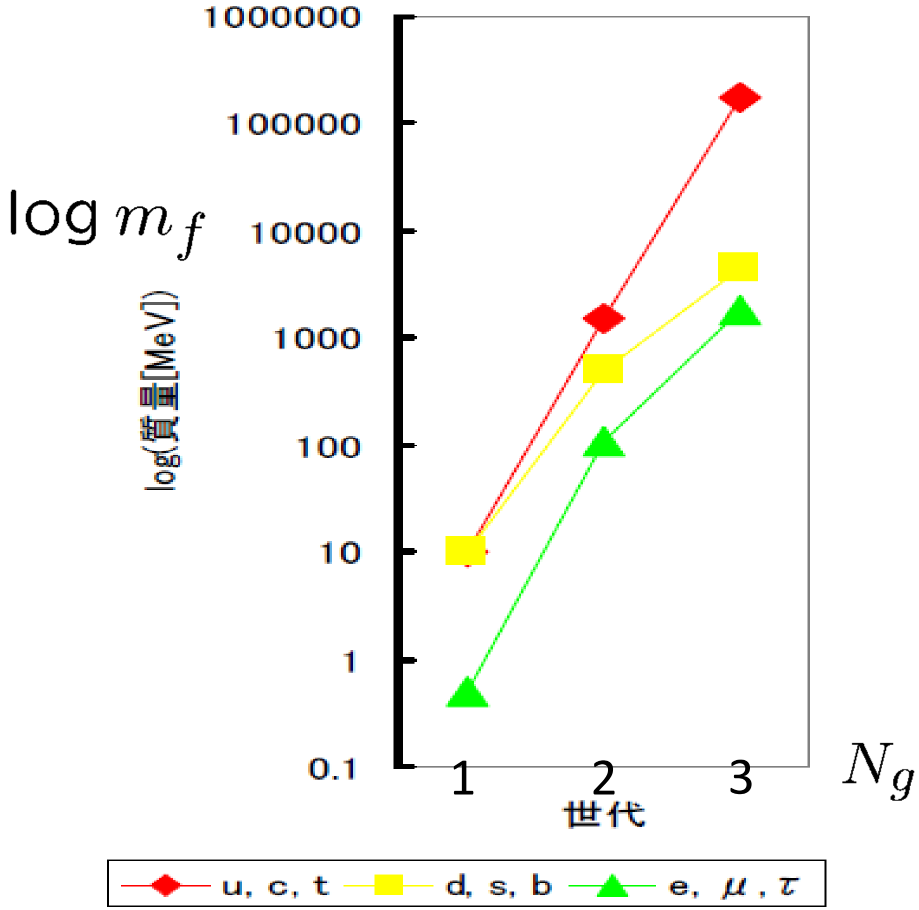


$\Rightarrow$ exponentially suppressed Yukawa coupling,

$$\sim g\ (\pi R M_B) e^{-\pi R M_B}\ (R : \text{the radius of } S^1)$$

, though there is no principle to fix M_B

【 The impressive hierarchical structure of fermion masses 】



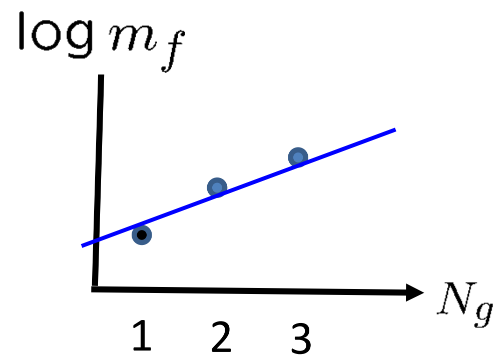
(By Dr. H. Terao)

The observed impressive hierarchy implies

$$\log m_f \sim \alpha N_g + \beta \leftrightarrow m_f = e^{\beta+3\alpha} e^{-(3-N_g)\alpha}$$

(N_g : generation number)

universal integer



This feature may be naturally understood in GHU:

♠ Originally all fermion masses are universal

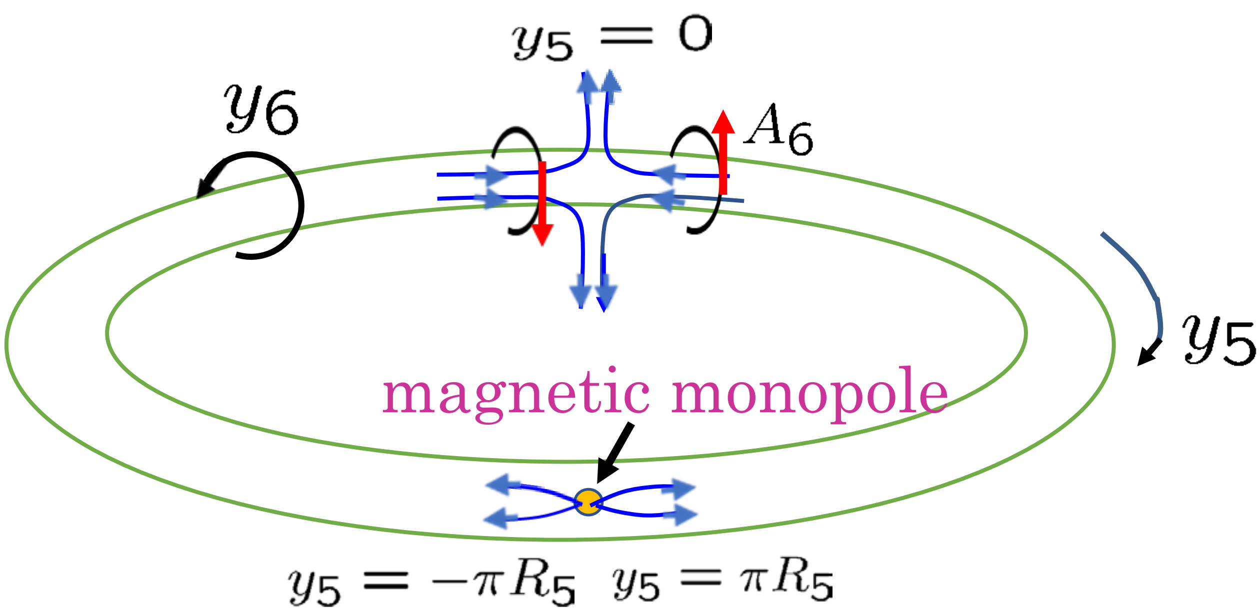
$\Rightarrow$ provides good reasoning of the universal factor $e^{\beta+3\alpha}$

♠ The exponential suppression may be realized by $e^{-\pi R M_B}$

♠ The only remaining task is to find a natural mechanism to guarantee "quantized Z_2 - odd mass" $M_B \propto 0, 1, 2$:

Idea: the quantization of M_B may be attributed to the Dirac quantization condition of magnetic charge

【Basic framework】 6D GHU with T^2 as extra space, where a (Dirac) magnetic monopole is placed inside the torus.



$$T^2 = S^1 \times S^1$$

R_5 R_6

$$R_5 \gg R_6$$

$$\Rightarrow \ eQA_6 = \epsilon(y_5)\frac{M}{2R_6}$$

quantization condition of magnetic charge

$\Rightarrow$ M : integer.

(N.B.)

(1) Now Z_2-odd bulk mass has been replaced by the flux generated by magnetic monopole, and it cannot be arbitrary, but controlled by the quantization condition.

(2) Again, topology plays essential roles:

$\pi_2(SU(2)/U(1)) = Z$ for 't Hooft-Polyakov monopole,

$\pi_1(U(1)) = Z$ for Dirac monople

(3) The index theorem (Atiyah-Singer) applied for 2D extra space guaranties M generations of chiral fermion:

$$\text{index of Dirac operator} = \frac{1}{2\pi}\int_{\text{torus}} \text{tr} F \quad (= M)$$

(" magnetized extra dimension ")

D. Cremades, L.E. Ibanez, F. Marchesano ('04); T. Kobayashi H. Abe, T. Kobayashi, and H. Ohki ('08); Y. Fujimoto, T. Kobayashi, T. Miura, K. Nishiwaki, and M. Sakamoto ('13); T. Kobayashi, K. Nishiwaki, and Y. Tatsuta ('17) …)

【 Mode functions for Kaluza-Klein (KK) zero mode fermion 】

Toy model: 6D U(1) GHU model with one fermion with U(1) ("electric") charge eQ .

Writing a KK zero mode of 4D left-handed fermion, as

$$\psi_L^{(0)}(x^\mu, y_5, y_6) = \psi_L^{(0)}(x^\mu) f_L(y_5, y_6)$$

the mode function should satisfy

$$\{\partial_5 + i(\partial_6 - ieQ\epsilon(y_5)\frac{g}{4\pi R_6})\} f_L(y_5, y_6) = 0.$$

Z_2-odd bulk mass generated by the monopole

The boundary condition @ $y_5 = \pm\pi R_5$

$$f_L(\pi R_5, y_6) = e^{i\frac{eQg}{2\pi R_6}y_6} f_L(-\pi R_5, y_6)$$

with

$eQg = 2\pi M \ (M: \ \text{integer})$: Dirac quantization condition

$$\begin{aligned} f_L^{(M,j)}(\hat{y_5}, \hat{y_6}) &= c^{(M,j)} \sum_{r=-\infty}^{\infty} e^{-\frac{\pi R_5}{R_6}M(r+\frac{j}{M})^2} e^{\frac{\pi R_5}{R_6}M\{(r+\frac{j}{M})\hat{y_5} - \frac{1}{2}|\hat{y_5}|\}} e^{i\pi M(r+\frac{j}{M})\hat{y_6}} \\ &= c^{(M,j)} e^{-\frac{\pi R_5}{R_6}M\{(\frac{j}{M})^2 - \frac{j}{M}(\hat{y}_5 + i\frac{R_6}{R_5}\hat{y}_6) + \frac{|\hat{y}_5|}{2}\}} \quad (\hat{y_5} = \frac{y_5}{\pi R_5}, \ \hat{y_6} = \frac{y_6}{\pi R_6}) \\ &\cdot \theta_3(i\frac{R_5}{R_6}M(\frac{j}{M} - \frac{\hat{y}_5 + i\frac{R_6}{R_5}\hat{y}_6}{2}) \mid i\frac{R_5}{R_6}M), \end{aligned}$$

Jacobi's theta function

$(j = 0, 1, \ldots, M-1 \quad : M$ chiral generations !)

well-approximated by (under $R_5 \gg R_6$) ,

$$\frac{1}{\sqrt{\pi}R_6}\sqrt{\frac{\frac{M^2}{4}-j^2}{M}} \cdot e^{-\frac{\pi R_5}{R_6}(\frac{M}{2}|\hat{y_5}|-j\hat{y_5})} \cdot e^{i\pi j\hat{y}_6} \qquad \text{(for } 0 \le j < \frac{M}{2} \text{)}$$

$$\frac{1}{\sqrt{\pi}R_6}\sqrt{\frac{\frac{M^2}{4}-(M-j)^2}{M}} \cdot e^{-\frac{\pi R_5}{R_6}(\frac{M}{2}|\hat{y_5}|+(M-j)\hat{y_5})} \cdot e^{i\pi(M-j)\hat{y}_6} \qquad \text{(for } \frac{M}{2} < j < M \text{)}$$

$\Rightarrow$ localization at $y_5 = 0$

The overlap integral of mode functions

$$\pi^2 R_5 R_6 \int_{-1}^{1} d\hat{y_5} \int_{-1}^{1} d\hat{y}_6 f_L^{(M,j)}(\hat{y_5},\hat{y_6})^* f_R^{(-M,j)}(\hat{y_5},\hat{y_6})$$

$$\simeq 2\pi\frac{R_5}{R_6}\frac{\frac{M^2}{4}-j^2}{M}e^{-\frac{\pi R_5}{R_6}(\frac{M}{2}-j)} \quad \text{(for } 0<j<\frac{M}{2}\text{)},$$

$$\simeq 2\pi\frac{R_5}{R_6}\frac{\frac{M^2}{4}-(M-j)^2}{M}e^{-\frac{\pi R_5}{R_6}(j-\frac{M}{2})} \quad \text{(for } \frac{M}{2}<j<M\text{)}$$

now shows exponential suppression $\propto e^{-\frac{\pi R_5}{R_6}|\frac{M}{2}-j|}$

(A problem)

To get 3 generation model, we naively expect that one fermion with $M=3$ will work.

Unfortunately, there appear degenerate Yukawa couplings proportional to

$$e^{-\frac{\pi R_5}{R_6}\frac{3}{2}},\ e^{-\frac{\pi R_5}{R_6}\frac{1}{2}},\ e^{-\frac{\pi R_5}{R_6}\frac{1}{2}}$$

【Successful 3 generation models】

Two types of successful models, without degeneracy, are known to exist:

(1) 2 + 1 model

Introduce one $M = 2$ fermion & one $M = 1$ fermion, which lead to exponential suppression factors,

$$e^{-\frac{\pi R_5}{R_6}} \ (M = 2,\ j = 0), \quad 1 \ (M = 2,\ j = 1),$$
$$e^{-\frac{\pi R_5}{R_6}\frac{1}{2}} \ (M = 1.\ j = 0)$$

i.e. to successful hierarchical relation ($m_{1,2,3}$: mass eigenvalues) :

$$\frac{m_3}{m_2} \sim \frac{m_2}{m_1} \sim e^{\frac{\pi R_5}{2R_6}}$$

More precisely, including the numerical factor due to normalization factor, double ratio is given as

$$\frac{\left(\frac{m_3}{m_2}\right)}{\left(\frac{m_2}{m_1}\right)} = \frac{1}{\log\{2(\frac{m_2}{m_1})\}}.$$

For charged lepton sector, $\frac{\left(\frac{m_3}{m_2}\right)}{\left(\frac{m_2}{m_1}\right)} = 0.17,$

while the observed value is $\frac{\left(\frac{m_\tau}{m_\mu}\right)}{\left(\frac{m_\mu}{m_e}\right)} = 0.075$

(2) Model with twisted boundary condition

Take just one $M = 3$ fermion, but with "twisted" b.c. :

$$f_L(y_5, y_6 + 2\pi R_6) = e^{i2\pi a} f_L(y_5, y_6)$$

(Remaining issues)

- How to confine the magnetic flux into the inside of torus ? (superconductivity ? some hints in superstring ?)
- The role of ’t Hooft- Polyakov monopole (GHU provides very suitable framework) ?