Unitarity constraints on general Higgs sectors

Mark D. Goodsell

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

cnrs

SORBONNE UNIVERSITÉ

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Overview

- Introduction
- Unitarity constraints on general theories
- Example in the THDM
- Colourful unitarity

Constraining the electroweak sector of BSM theories

Absence of new particles has led to renewed interest in non-SUSY model building, mostly with purely phenomenological motivation, e.g.

- THDM variants
- SSM, Z_2SSM, inert doublets, singlets, etc
- Georgi-Machacek

These are then hit by the typical toolbox:

- Collider constraints give us information up to a couple of TeV for coloured particles, but much less for electroweak (few hundred GeV).
- Flavour constraints are very powerful, but again mainly for coloured states. E.g. 10s of TeV vs 600 GeV for charged Higgs from $b \to s\gamma$ in THDM-II.
- SMEFT program attempts to constrain models from precision (including EWPT).

But we can gain lots of information from the renormalisable terms that are not being (well) exploited:

- The Higgs mass
- Stability or instability scale of the electroweak vacuum
- Unitarity

Unitarity

- Everyone learns that the Higgs is necessary for unitarisation of $WW \to WW$ scattering.
- Slightly less well known, but still famously, Lee, Quigg and Thacker used this to place an upper bound on the Higgs mass of

$$M_H < \sqrt{\frac{8\pi\sqrt{2}}{3G_F}} \lesssim 1 \text{ TeV}$$

- $2 \to 2$ scattering of scalars is sometimes used to constrain BSM theories: e.g. constraints worked out for THDM [Kanemura, Kubota, Takasugi]
- Doing the full calculation is rather messy. It's far easier to use the Goldstone boson equivalence theorem. From the Ward ID:

$$\frac{k_\mu}{m_V} T(W^\mu, ...) = T(G, ...)$$

but $\epsilon_L^\mu \underset{E \gg m}{\to} \frac{k^\mu}{m_V}$, so we can just calculate $T(G, ...)$

- This requirement for $E \gg m_V$ might be why people typically only consider the $s \to \infty$ limit of unitarity constraints. But this is not necessary!

Unitarity basics

Basically:

- Say $S = 1 + iT$, then for unitary theory $(1 + iT)(1 - iT^\dagger) = 1$.
- For $2 \to 2$ scattering

$$-i(\mathcal{M}^{2\to 2}_{b\,a} - (\mathcal{M}^{2\to 2}_{b\,a})^\dagger) = \sum_c \frac{1}{2^{\delta_c}} \frac{|\mathbf{p}_c|}{16\pi^2\sqrt{s}} \int d\Omega \mathcal{M}^{2\to 2}_{c\,a} \overline{\mathcal{M}}^{2\to 2}_{c\,b} + \underbrace{\sum_{n>2} d\Pi_n\, d\Omega \mathcal{M}^{2\to n}_{c\,a} \overline{\mathcal{M}}^{2\to n}_{c\,b}}_{\geqslant 0}.$$

- We decompose the matrices into partial waves:

$$\mathcal{M}_{ca} = 16\pi \sum (2J+1) P_J(z_c)\, \hat{a}_J(s) \to \hat{a}_0^{ca} = \frac{1}{32\pi} \int_{-1}^{1} dz \mathcal{M}_{ca}$$

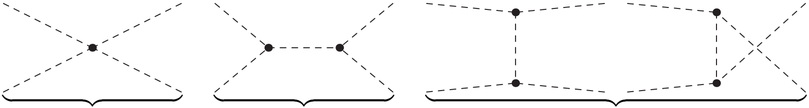
- In general have $a_J^{ba} \equiv \sqrt{\frac{4|\mathbf{p}_b||\mathbf{p}_a|}{2^{\delta_a} 2^{\delta_b} s}}\, \hat{a}_J^{ba}$ and constraint on eigenvalues:

$$\mathrm{Im}(a_J^i) \geqslant |a_J^i|^2$$

- When $s \to \infty$ get $\hat{a}_J = a_J$

Unitarity diagrams

In the limit of large s, can neglect the cubic couplings:



$$\frac{\lambda}{8\pi}\sqrt{1-\frac{4m^2}{s}} \qquad \frac{\kappa^2\sqrt{1-\frac{4m^2}{s}}}{8\pi(M^2-s)} \to 0 \qquad \frac{\kappa^2 \log\left(\frac{M^2}{-4m^2+M^2+s}\right)}{8\pi\sqrt{s(s-4m^2)}} \to 0$$

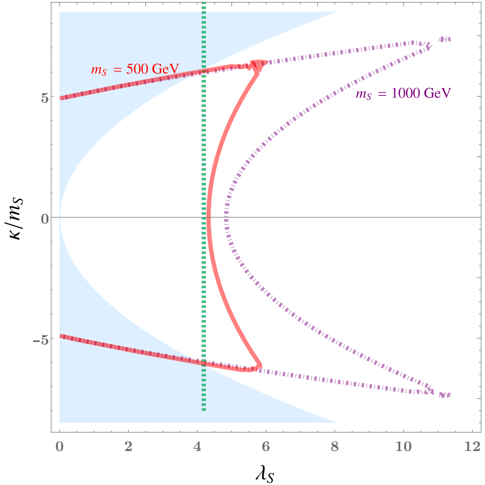
- The constraint is a quantification of perturbativity in the quartic coupling
- But it throws away a lot of information! And implies that cubic couplings are not bounded by unitarity!
- Indeed, if we consider the limit $s \sim \kappa^2 \gg M_W^2$, then the Goldstone boson equivalence theorem is still valid (so only compute scalar diagrams) but now get genuine constraints!

Simplest example

Take a trivial example of

$$\mathcal{L} \supset \frac{1}{2}M_S^2 S^2 + \frac{1}{3}\kappa S^3 + \frac{1}{2}\lambda_S S^4$$

We shouldn't be able to take κ arbitrarily large!! And indeed we have limits:



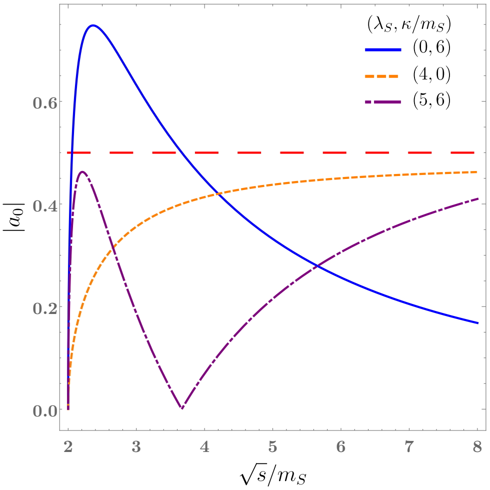
New information corresponds to:

- Vacuum stability
- Perturbativity of cubic coupling

Simple to derive a_0 for this case:

$$a_0 = -\frac{1}{32\pi}\left[\sqrt{1-\frac{4m_S^2}{s}}\left(12\lambda_S + \frac{4\kappa^2}{s-m_S^2}\right) + \frac{8\kappa^2}{\sqrt{s(s-4m_S^2)}}\log\frac{m_S^2}{s-3m_S^2}\right].$$

For $\lambda_S \lesssim \kappa/m_S$ has a maximum near $s \sim 6m_S^2$:



Stronger and weaker constraints

Constraints can strengthen or weaken:

- Typically maximum a_0 for s just above threshold: can be large enhancement of constraint
- But: maximum $s \leftrightarrow$ cutoff Λ: if a_0 is increasing with s then can weaken constraints
- Also: trilinear coupling contribution can negatively interfere with quartic.

Implementation

We implemented this calculation for all uncoloured scalars into `SARAH`:

- Can calculate scattering diagrams, and output `Fortran` code linked to `SPheno` library for spectrum generation and numerical evaluation of unitarity
- Choose the best value of s in range given (e.g. if define theory with a cutoff then constraints can be weaker)
- Compute eigenvalues of scattering matrix
- Cut out irreducible submatrices if we are near a pole (perturbation expansion effectively breaks down there)

Example: THDM

These constraints even improve bounds on quartic couplings in theories with a vev! E.g. in the THDM:

$$
\begin{aligned}
V_{\text{Tree}} =& \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_2^\dagger H_1|^2 \\
&+ m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + \left(M_{12}^2 H_1^\dagger H_2 + \frac{1}{2} \lambda_5 (H_2^\dagger H_1)^2 + \text{h.c.} \right)
\end{aligned}
$$

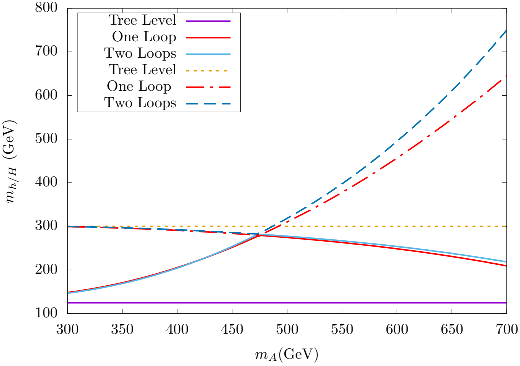
- With CP, have 8 parameters plus two expectation values minus 2 vacuum conditions and the weak vev $\rightarrow$ 7 free parameters.
- Can trade these for $m_h, m_H, m_A, m_{H^\pm}, \tan\beta, \tan\alpha$ and M_{12}^2:

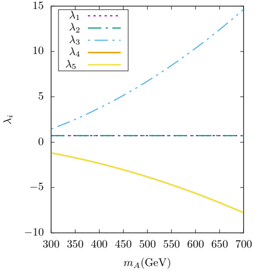
$$
\begin{aligned}
\lambda_1 =& \frac{1 + t_\beta^2}{2(1 + t_\alpha^2) v^2} \left(m_H^2 - M_{12}^2 t_\beta + t_\alpha^2 (m_h^2 - M_{12}^2 t_\beta) \right) \\
\lambda_2 =& \frac{1 + t_\beta^2}{2(1 + t_\alpha^2) t_\beta^3 v^2} \left(-M_{12}^2 - M_{12}^2 t_\alpha^2 + t_\beta (m_h^2 + m_H^2 t_\alpha^2) \right) \\
\lambda_3 =& \frac{1}{(1 + t_\alpha^2) t_\beta v^2} \Big[m_h^2 t_\alpha + 2 m_{H^+}^2 (1 + t_\alpha^2) t_\beta \\
&+ m_h^2 t_\alpha t_\beta^2 - m_H^2 t_\alpha (1 + t_\beta^2) - M_{12}^2 (1 + t_\alpha^2)(1 + t_\beta^2) \Big] \\
\lambda_4 =& \frac{1}{t_\beta v^2} \left(M_{12}^2 + m_A^2 t_\beta - 2 m_{H^+}^2 t_\beta + M_{12}^2 t_\beta^2 \right) \\
\lambda_5 =& \frac{1}{t_\beta v^2} \left(M_{12}^2 - m_A^2 t_\beta + M_{12}^2 t_\beta^2 \right)
\end{aligned}
$$

Unphysical couplings

- Main problem with this: it is very easy to have huge underlying unphysical couplings!

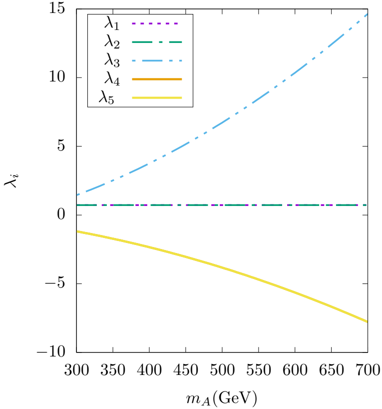
E.g. enforce the alignment limit of $\tan\alpha = -1/\tan\beta$, we can scan over the other parameters. If we take the Heavy Higgs mass to be 300 GeV and scan only over e.g. $m_A = m_{H^+}$ we find for loop corrections to masses:

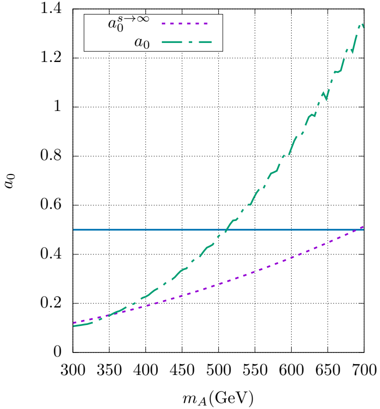




Unitarity

- Better check of perturbativity: use unitarity
- Naively $s \to \infty$ limit is enough because only quartic couplings





- For the above point $t_\beta = -1/t_\alpha = 1, m_A = m_{H+}, \rightarrow \lambda_1 = \lambda_2$ and $\lambda_4 = \lambda_5$:

$$\mathcal{L} \supset -\frac{2}{v}[4m_{12}^2 + 2m_H^2 + m_h^2](h + \frac{h^2}{2v})H^2 \\ -\frac{2}{v}[4m_{12}^2 + 2m_A^2 + m_h^2](h + \frac{h^2}{2v})A^2,$$

- For the extreme case $m_H = m_h \sim \sqrt{s} \ll m_A = m_{H^+} = |M_{12}|$ (nb not the case on previous slide, where $|M_{12}| = 300$ GeV), we find the hH^2 coupling dominates:

$$\frac{a_0^{\max}}{a_0^{s\rightarrow\infty}} = \frac{-2M_{12}^2 \log\left(\frac{m_h^2}{s-3m_h^2}\right)}{\sqrt{s\left(s-4m_h^2\right)}} \sim \frac{1}{2}\frac{m_A^2}{m_h^2}$$

- This is dominated by the near-threshold behaviour we saw before, and can become very large!

Comparison with loop corrections: MSSM limit

The loop corrections to THDM were calculated in $s \to \infty$ limit by [Grinstein, Murphy, Uttayarat, 1512.04567]

Results clearly seen in the "MSSM-like" limit:

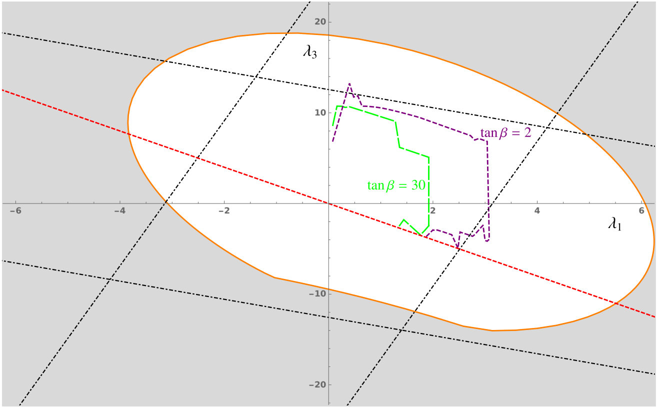
$$\lambda_1 = \lambda_2, \lambda_5 = 0, \lambda_4 = -\lambda_3 - 2\lambda_1$$

Classic constraints are then just

$$|8\lambda_1 - \lambda_3| \leqslant 8\pi, \qquad |2\lambda_1 + 2\lambda_3| \leqslant 8\pi.$$

Constraints from finite s vs. loop corrections

But: we generate cubic interactions from vevs of $H_1, H_2 \to$ improved constraints at finite s!



Summary

- Unitarity constraints give a quantitative definition of perturbativity
- ... and vacuum stability $\rightarrow$
- Can now get all of this information automatically: can explore all sorts of models
- Currently testing implementation of the constraints from colourful states