Hierarchy problem
- from bottom-up and/or top-down -

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"and" = a hope to connect these two approaches
"or" = they are not yet connected.

Hierarchy "problems" are ubiquitous in nature.
Particle physics, Cosmology
Condensed matter physics,
Fluid dynamics, Astrophysics, .....
Hierarchy of scales in fluid: "effective theory" approach

Karman vortices in various scales

Behind Cheju island

Jupiter (NASA Voyager2)

Navier-Stokes eq. is described by a single dimensionless parameter $= R$.

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} \quad \Rightarrow \quad \frac{D\tilde{u}}{Dt} = -\tilde{\nabla} \tilde{p} + \frac{1}{R} \tilde{\Delta} \tilde{u}$$

$$R = \frac{L U}{\nu}$$

Reynolds number

But Cheju Karman vortices cannot be explained if we use the molecular viscosity $\nu$.

$$\nu(\text{Cheju}) \sim 10^8 \, \nu(\text{micro})$$

Eddy viscosity molecular viscosity
Viscosity is scale-dependent.

The framework of LEET can be determined by the IR physics, e.g., symmetry (Navier-Stokes eq., Chiral Lagrangian) renormalizability (Standard Model)

But, the parameters in the EFT are obtained by integrating out UV physics.

This is nothing but the idea of Renormalization Group.

⇒ UV determines IR parameters through RG.
Suppose that we were always faced to the sun. How could we understand why the distance is fixed at $r=10^7$ km? why it is different from $r$ on Pluto? why the sun is not falling down to the earth?

In this setup, a classical solution (configuration) is first determined by the initial condition first, and then we can calculate its effective potential on each planet. Thus, effective potentials are different on Earth and on Pluto. Phenomena on planets, e.g., Coriolis force, is solution-dependent. Solution comes first, Effective potential next!

"IR" geometry determines effective potential in a top-down way.
Naturalness problem = Hierarchy of various scales in nature
In this talk, I will focus on the hierarchy of $M_{EW}$ against $M_{Planck}$ from bottom-up (part 1) and from top-down (part 2).

Especially I will pay particular attention to the vev of Higgs $<H>$ itself.

Note that $<H>$ is a solution to the EOM of Higgs field.
Part 1
Bottom-up approach to the Higgs hierarchy problem

- LHC and Classical conformal models –

  N. Okada, Y. Orikasa, SI

  Y. Orikasa, SI
  PTEP(2012)

  M. Hashimoto, Y. Orikasa, SI

- QCD induced EW phase transition -

  K. Kohri & K. Shimada, SI

  P. Serpico, K. Shimada, SI
  PRL (2017)
We have now many experimental data about the Higgs sector including Higgs mass, Higgs VEV, quartic coupling, Yukawa couplings → Rough picture of the Higgs potential: (at least) 3 important points

1. Higgs VEV \( v = 246 \text{ GeV} \) → SM (present universe)
2. UV scale \( M_{UV} > 10^{10} \text{ GeV} \) or \( M_{\text{Planck}} \) → gravity, string theory & origin of Higgs
3. \( h=0 \) (origin) → history of the early universe

The behaviors around 2 and 3 control the early universe, but they are only indirectly accessible by using RG and theoretical biases.
2 important lessons from LHC for the Higgs potential

(1) mass = 125 GeV

still controversial ...

Degrassi et.al.(12)
Elias-Miro et.al.(12)
Alkhin, Djouadi, Moch (12)

"EW" physics may be directly related to Planck scale physics without intermediate scales in between.

(2) No deviations from SM / no TeV SUSY? \(\Rightarrow\) Naturalness

If \(\mu=0\) at UV scale, it will be never radiatively generated in the IR if there are no intermediate scales strongly coupled to the SM.

\(\Rightarrow\) Classically conformal models (scalar potential is radiatively generated.)

Flat potential \(V(H)=0\) at \(M_{Pl}\)

Scalar fields often appear as moduli with flat potential in string theory.

\[ V = -\mu^2 |H|^2 + \lambda(|H|^2)^2 \]
From UV to IR: Naturalness problem = how can we interpolate?

"UV physics" either Quantum scale invariance in QG
or String theory (flat moduli in non-susy vacua)

Start from a flat potential $V(H)=0$ at $M_{Pl}$, or $M_{string}$

No intermediate scales between UV and IR

Coleman-Weinberg mechanism
Radiatively generate

"IR Physics": SM + (inevitably) an additional sector
because CW mechanism requires that
the beta-function of the quartic scalar coupling must be positive $\beta_\phi > 0$
(Within the SM, $\beta_{Higgs} < 0$ and CW mechanism does not work.)

Several possibilities of extensions:

1. SM + scalars
   Foot, Kobakhidze, Volkas (07),
   Meissner, Nicolai (07)

2. SM + gauge sector
   B-L: SI, Okada, Orikasa (09)
   L-R symmetric: Holthausen, Lindner, Schmidt (10)
   $SU(2)_X$: Hambye Strumia (13), ...

3. SM + strongly couple sector
   Hur, P. Ko (11),
   Holthausen, Kubo, Kim, Kindner (13) .......
In the following, we focus on a specific model "classically conformal (B-L)-extension of SM"

IR physics
- Standard Model
- B-L sector
  - $U(1)_{B-L}$ gauge
  - SM singlet scalar
  - Right-handed $\nu$

UV physics
- Planck scale
  - $V(h)=0$
  - No intermediate scales

(1) Free from naturalness problem (no intermediate scales)
   B-L breaking at 10 TeV by CW mechanism $\rightarrow$ triggers EWSB at 100 GeV
   Everything occurs radiatively.

(2) Minimal extension of SM & Phenomenologically viable
   $\nu$ oscillation, (resonant) leptogenesis,
   DM candidate (e.g. right-handed $\nu$ with $Z_2$)

IR physics
- Standard Model
- $U(1)_{B-L}$ gauge
- SM singlet scalar
- Right-handed $\nu$
- $Z'$ necessary for $\beta_\phi > 0$
- $\phi$ CW mechanism
- $v_R^i$ $i=1$~$3$ necessary for anomaly cancellation
Scalar potential:

\[ V(\phi, h) = \frac{\lambda_h}{4} h^4 + \frac{\lambda_{\text{mix}}}{4} \phi^2 h^2 + \frac{\lambda_{\phi}(\phi)}{4} \phi^4 \]

Minimum if \( \beta_\lambda > 0 \)

\[
\langle H \rangle = \sqrt{-\frac{\lambda_{\text{mix}}}{\lambda_H}} M_{B-L}
\]

Some properties of the potential

1. Negative small value of the scalar mixing \( \lambda_{\text{mix}} \) can be radiatively induced from the flat potential \( \lambda_{\text{mix}} = 0 @ M_{\text{Pl}} \).

\[
\frac{d\lambda_{\text{mix}}}{dt} = \frac{1}{16\pi^2} \left( 6g_{B-L}^2 g_{\text{mix}}^2 + \lambda_{\text{mix}}(\cdots) \right) \quad \rightarrow \quad \lambda_{\text{mix}} \sim -g_{B-L}^2 g_{\text{mix}}^2
\]

Orikasa, SI ('12)
② The condition $\beta_\phi > 0$ for CW mechanism in B-L sector

$$B \approx \frac{c_0^4}{8\pi^2} \left( 3(2g)^4 - 2\text{Tr} \left( \frac{Y}{\sqrt{2}} \right)^4 - D\lambda_{\text{mix}}^2 \right)$$

$B= 2\lambda \quad c_0 \sim 1$

$$m_{Z'} > \frac{2\text{Tr} m_N^4}{3} + \frac{D m_h^4}{3}$$

Right-handed neutrinos are lighter than $M_{Z'}$, typically TeV or lighter.

③ B-L scalar (pNG boson) is very light.

$$m_\phi < 470 \text{ GeV}$$

$$m_\phi = \sqrt{B M}$$

④ Vacuum energy at the origin is given by $Z'$ mass

$$V_0 = B M^4 / 16 < \left( \frac{3}{128 \pi^2} \right) m_{Z'}^4 \ll M^4$$

$V_0$ becomes smaller if $m_\phi$ becomes lighter.

⑤ $M_{Z'}$ is typically 5-10 TeV.

$$\langle H \rangle = \sqrt{-\frac{\lambda_{\text{mix}}}{\lambda_H}} M_{B-L}$$
"Classically conformal B-L extension of SM"
is a minimal & phenomenologically viable model

In order to verify (or falsify) the model
Look for $Z'$: 5-10 TeV but $\alpha_{B-L}$ is relatively small
$\phi$: difficult to find, too light and weakly coupled
$\nu_R$: TeV scale (resonant) leptogenesis
But it is difficult to distinguish this kind of models from others.

Is there any clearer evidence for the classical conformality?

Yes

Supercooling of the B-L and EW symmetry breaking in the early universe

Witten (80)
Serpico, Shimada, SI (17)
see also, Konstandin, Servant (11)
After primordial inflation, universe is preheated up to high $T >> T_c$.
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(h=0, $\phi=0$)

Trapped in symmetric phase for a long time

$V_0 \approx m_{Z'}$

Thermal barrier never disappears (classical conformal)

High- $T$

$T = T_c \sim m_{Z'}$

2nd inflation starts

$T \ll T_c$

valley
Supercooling of (B-L)+EW $\rightarrow$ PT much below $T_c$

(h=0, $\phi=0$)

Trapped in symmetric phase for a long time

$V_0 \approx m_{Z'}$

Thermal barrier never disappears (classical conformal)

Bubble of true vacuum is created by tunneling.
Percolation of true vacuum

Bubbles of true vacuum are created by tunneling

\[ \Gamma \approx T^4 e^{-S_3/T} \]

\( T_p \) : percolation temperature

Universe is occupied with true vacuum bubbles
$T_P = \text{percolation temperature can be calculated by tunneling rate.}$

If $g < 0.2$, universe is never occupied with true vacuum until $T$ decreases down to 100 MeV or less.  

Note that de Sitter fluctuation $\sim T_{\text{GH}} = \mathcal{H}/2\pi$ is negligible at 100 MeV.

$V_0^{1/4} \sim m_{Z'}$
When temperature decreases down to 100 MeV at \((\phi=0, h=0)\)

\[
\langle \bar{q}_i q_i \rangle \sim \Lambda_{\text{QCD}}^3
\]

In the standard scenario with \(N_f=(2+1)\) → crossover

In the present case, since \(h=0\) all \(N_f=6\) quarks are massless!

→ 1\textsuperscript{st} order phase transition for \(N_f \geq 3\)

Pisarski Wilczek (1983)
$\langle \bar{q}_i q_i \rangle \sim \Lambda_{QCD}^3$  \hspace{1cm} SU(2)$_L$ doublet with U(1)$_Y$ charge

Electroweak symmetry is broken (EWSB)

$SU(2)_L \times U(1)_Y \rightarrow U(1)$

Higgs linear term is generated via Yukawa interaction

$y_i h \langle \bar{q}_i q_i \rangle \sim (y_i \Lambda_{QCD}^3) h$

Quigg Shrock (09)

Witten (81)
Buchmuller et al (90)
Kuzmin et.al.(92)

new minimum

$(\phi = 0, h = v_{QCD})$

$v_{QCD} = \left( \frac{y_t \langle \bar{t}t \rangle}{\sqrt{2} \lambda_h} \right)^{1/3} \sim \Lambda_{QCD}$

$(\phi, h) = (M, 246 \text{GeV})$
tunnels before QCDPT

$g > 0.2$

$T_i \sim 10 \text{ TeV}$

$V_0^{1/4} \sim m_Z$

temperature at which de Sitter expansion starts at the origin

$g < 0.2$

QCDPT

$(\phi = 0, h = v_{QCD})$

$V(\phi) \sim \left(g^2 T_{QCD}^2 - |\lambda_{\text{mix}}| v_{QCD}^2 / 2\right)\phi^2$

Interplay of [QCD $\chi$SB, confinement, B-L, EW]: similar to the 2-step EWSB by M.J. Ramsey-Musolf
Towards the true minimum directly rolling down

\[ T_i \sim 10 \text{ TeV} \quad V_0^{1/4} \sim m_Z, \]

temperature at which de Sitter expansion starts at the origin

\[ g < 0.2 \]

QCDPT

\[ (\phi = 0, h = v_{QCD}) \]

Towards the true minimum

\[ V(\phi) \sim (g^2 T_{QCD}^2 - |\lambda_{mix}| v_{QCD}^2 / 2) \phi^2 \]

Interplay of [QCD χSB, confinement, B-L, EW]:

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Towards the true minimum

Interplay of [QCD $\chi$SB, confinement, B-L, EW]:
Super-cooling generates the second inflation with $N \sim 10$

$$N = \ln \frac{T_{i} \cdot T_{QCD}^N}{T_{QCD}^N} \sim \ln \frac{m_{Z'}}{T_{QCD}^N}$$

$N \sim 10$ in region (I)(II) for $m_{Z'} = 10$ TeV

$N < 10$ for $g > 0.2$
Cosmological consequences

In scenario (I) \[ \text{QCD + (B-L) + EW} \] strong \text{1st order phase transition expected}

Bubble collisions $\rightarrow$ Sizable Stochastic \text{Gravitational Waves} from \text{1st order PT}

Many other implications:
- \text{dilution factor} $10^6$ dilutes relics from high energy (high temperature)
  - WIMP DM, baryon asymmetry $\rightarrow$ supercool DM
  - Hambye Strumia Teresi (18)
- \text{low reheat temperature} after 2\text{nd mini-inflation} ($T_R < 100 \text{ GeV}$)
  - Baryon asymmetry at low T: e.g. Cold EWBG
  - Konstandin, Servant (11) ....
- \text{Enhanced scalar fluctuations}
- \text{PBH} by 1st order QCD PT or Ultra-compact mini halo
  - Jedamzik (97), Ricotti et(09)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{GW.png}
\caption{eLISA sensitivities from C.Caprini et al. (2016)
Serpico, Shimada, SI ('17)
Jinno, Takimoto ('17)
Hashino et.al. ('18) ....}
\end{figure}
Summary of part 1

In classically conformal models motivated by naturalness, the early universe is drastically different.

Extreme SUPERCOOLING and the second inflation with $N \sim 10$

$$N = \ln \frac{T_i}{T_{QCD}} \sim \ln \frac{m_{Z'}}{T_{QCD}}$$

Interesting cosmological consequences
- Stochastic GW, PBH, Cold EWBH, axion abundance, supercool DM

Supercooling is also expected in other models, e.g. Randall-Sundrum models, Harling, Servant (18)
Part 2
Top-down approach to the Higgs hierarchy problem
- Revolving D-branes –

N. Kitazawa, SI PTEP (15)
N. Kitazawa, SI arXiv:1812.08912
In a field theoretic approach, there are many different proposals to solve the hierarchy problem. But there is one common basic assumption: "Calculate the Higgs potential first!"

And then obtain a solution = minimum of the potential. one solution to one Higgs potential  
→ Then we are often faced with the naturalness problem
Question:
Can we first obtain a solution of $\langle H \rangle$ in a geometric way and then calculate low-energy EFT in the SM.

Analogy with

Higgs potential itself may be a function of $\langle H \rangle$.

Any radius of orbits is a solution to the effective potential.
Initial condition (angular momentum) gives a different orbit.

The mechanism looks as if any value of $\langle H \rangle$ is a minimum of $V(H; \langle H \rangle)$.
Stringy view of our "universe" and "Higgs" sector:

Geometry in string theory = Dynamics in QFT

[1] (3+1)-dimensional space-time is embedded in d=9+1. Either compactification or brane-world scenario

[2] Higgs (scalar) field is a geometrical "moduli" field
e.g. distance between D-branes
volume / shape of extra-dimension etc.

[3] VEVs of moduli fields are proportional to the geometrical size.

The natural scale of M should be the string scale, not the EW scale.

Hierarchy problem in string theory
An example of D-brane configurations for SM

Attractive force between D3s and anti-D7 due to open string 1-loop amplitudes

\[ \mu^2 \sum_a |Z_3^{(a)}|^2 \]

1-loop suppressed

Repulsive centrifugal force by revolution of D3s

Solution: Hierarchy of EW scale

High angular frequency \( \omega = \mu \sim \frac{g}{4\pi} M_s \)

Low velocity \( v = \omega d \sim \frac{v_0}{M_s} \ll 1 \)

N. Kitazawa SI PTEP, 2015
It is possible to make a classically stable state with a short distance; 
\[ d \ll l_{\text{string}}. \]
Such short distance region is described by low energy modes of open strings (D-brane EFT) instead of closed strings (supergravity).

\[ \rightarrow \] Hierarchy problem can be avoided by using the geometrical approach in string theory

But the large angular frequency causes two serious problems
- Lorentz symmetry is violated in the dispersion relation of Higgs field (Coriolis force).
- closed string emission \( \rightarrow \) unstable

\[ \omega = \mu \sim \frac{g}{4\pi} M_s \]
Is it possible to make a "sufficiently stable resonant state" of D-branes with $d \ll l_{\text{string}}$ and $\omega \ll m_{\text{string}}$?

The first step is to calculate potential between revolving branes.

H. Ohta, T. Suyama SI (18)

Next step: supersymmetry breaking.

To avoid large angular frequency, we need weaker attractive force

$\Rightarrow$ BPS configuration of D-branes

no-force (flat potential) at rest,
but attract each other when they are moving.
Experimental test of the geometric scenario

Lorentz violation in the Higgs sector

(***Coriolis force*** for Higgs field since it is geometrical.)

\[ \omega^2 = M^2 + \left(1 + \frac{4\omega_0^2}{M^2}\right)p^2 + 16\frac{\omega_0^4}{M^6}p^4 + \ldots \]

\[ \omega_0 < 0.1 \text{ GeV} \]

N. Kitazawa, SI ('18)

Message in Part 2 (top-down approach)

In string theory, Higgs has a **geometrical meaning**. And if it is stabilized by **stationary motion**, it will be tested (or falsified) by looking at a tiny violation of Lorentz invariance in the Higgs sector.
Summary

Higgs physics @ LHC + ILC + ...

early universe

String theory space-time physics

- Naturalness
- Stability
- Yukawa couplings

- Dark matter
- Baryogenesis
- Inflation, PBH, ...

- Moduli = geometry
- SUSY breaking
- Dark energy

- From the bottom-up
  - Classically conformal = no dimensionful parameter in Lagrangian
  - \( \rightarrow \) CW mechanism \( \rightarrow \) \( <H> \) given radiatively
  - Intriguing phenomenology & cosmology (supercooling)

- From the top-down
  - Flat moduli = no dimensionful parameter in Lagrangian
  - \( \rightarrow \) revolution of D3 breaks SUSY \( \rightarrow \) \( <H> \) given geometrically
  - Lorentz violation in Higgs sector
In both approaches, No dimensionful parameters in Lagrangian but its dimensionful value $<H>$ is governed by "dimensional transmutation".

First determine Higgs vev $<H>$, then calculate the Higgs potential.

I hope to connect bottom-up and top-down in near future.

Higgs is a probe for new physics = HPNP