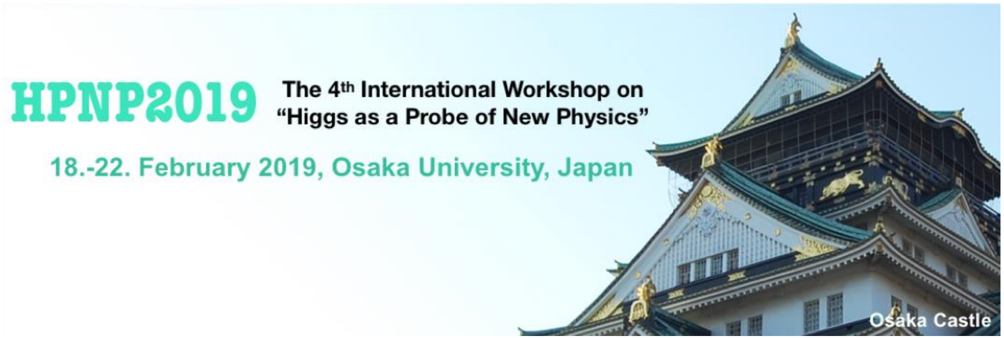
Topological Aspects of Two Higgs Doublet Models



Feb 21, 2019

Topological Materials Science

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1858
CALAMVS
GLADIO
FORTIOR

Collaborators

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Chandrasekhar Chatterjee (Keio U.), **Yu Hamada** (Kyoto U.)

References

2HDM

Eto, Kurachi, MN, Phys.Lett. B785 (2018) 447-453 [arXiv:1803.04662 [hep-ph]] ← Focusing on domain walls
Eto, Kurachi, MN, JHEP 1808 (2018) 195 [arXiv:1805.07015 [hep-ph]] ← Focusing on vortices
Eto, Hamada, Kurachi, MN, in preparation ← Focusing on monopoles

Georgi-Machacek model

Chatterjee, Kurachi, MN, Phys.Rev. D97 (2018) 115010 [arXiv:1801.10469 [hep-ph]]

SM: topologically trivial
BSM: topologically nontrivial

Cosmic strings, domain walls, monopoles …

Plan of My Talk

Plan of My Talk

§ 1 Introduction: SM

§ 2 Domain walls and membranes in 2HDM

§ 3 Vortices (cosmic strings) in 2HDM

§ 4 Monopoles in 2HDM

§ 5 Summary

Symmetry breaking: $G \rightarrow H$
Either gauge or global symmetries

↓

Nambu-Goldstone modes
Vacuum manifold or Order parameter space(OPS): G/H

↓

Topology of OPS: $\pi_n(G/H)$

↓

Topological solitons, defects/textures

N.D.Mermin
Rev.Mod.Phys.('79),
G.E.Volovik
Universe in a helium droplet

They (especially vortices) determine
the dynamics of the system!

Classification of topological objects

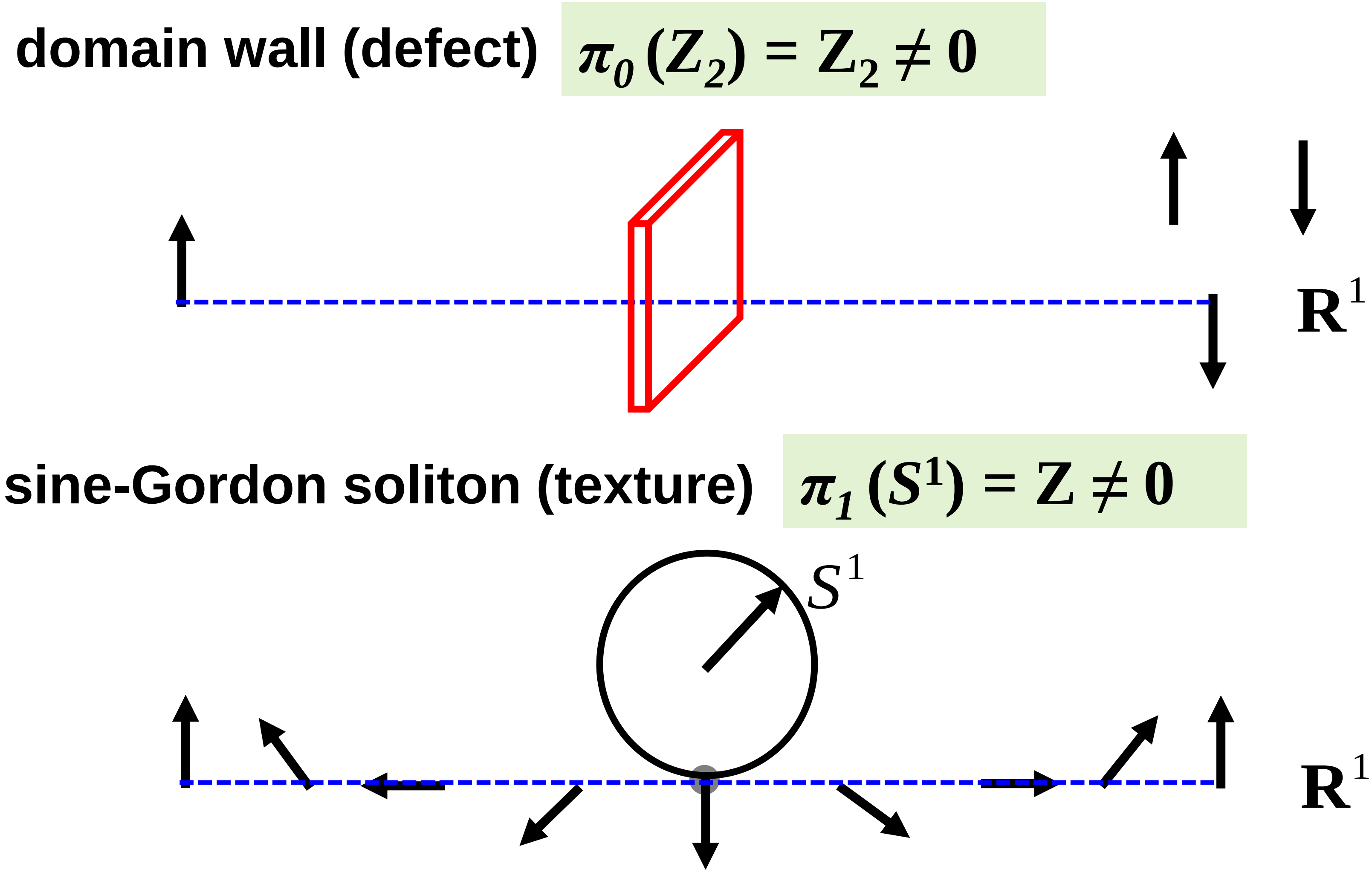
dim	Topological defects	Topological textures
***d*=1**	**Domain wall** π_0	**Sine-Gordon soliton(kink)** π_1
***d*=2**	**Vortex, cosmic string** π_1	**Lumps, baby Skyrmion** π_2
***d*=3**	**Monopole** π_2	**Skyrmion** π_3

$$\partial \mathbf{R}^d \cong S^{d-1} \to \boldsymbol{G/H}$$

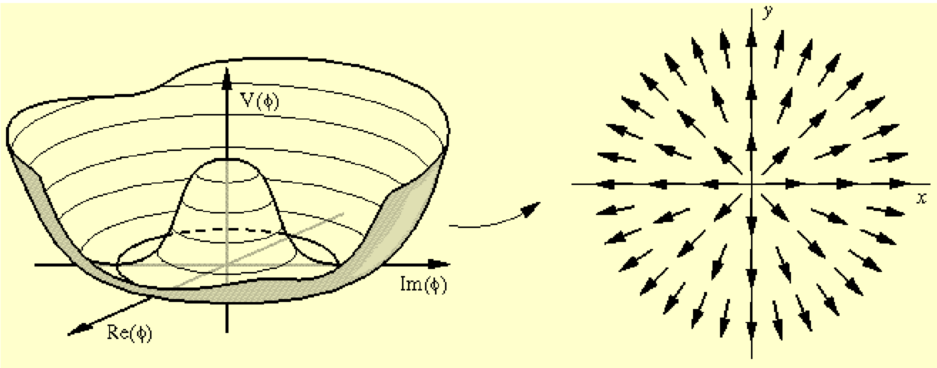
$$\boldsymbol{\pi_{d\text{-}1}(G/H) \neq 0}$$

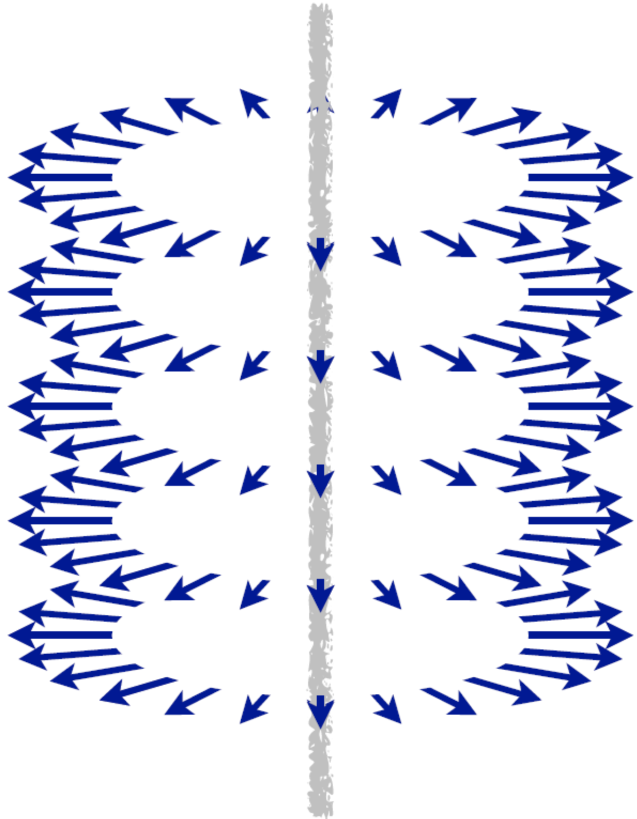
$$\mathbf{R}^d + \{\infty\} = S^d \to \boldsymbol{G/H}$$

$$\boldsymbol{\pi_{d}(G/H) \neq 0}$$

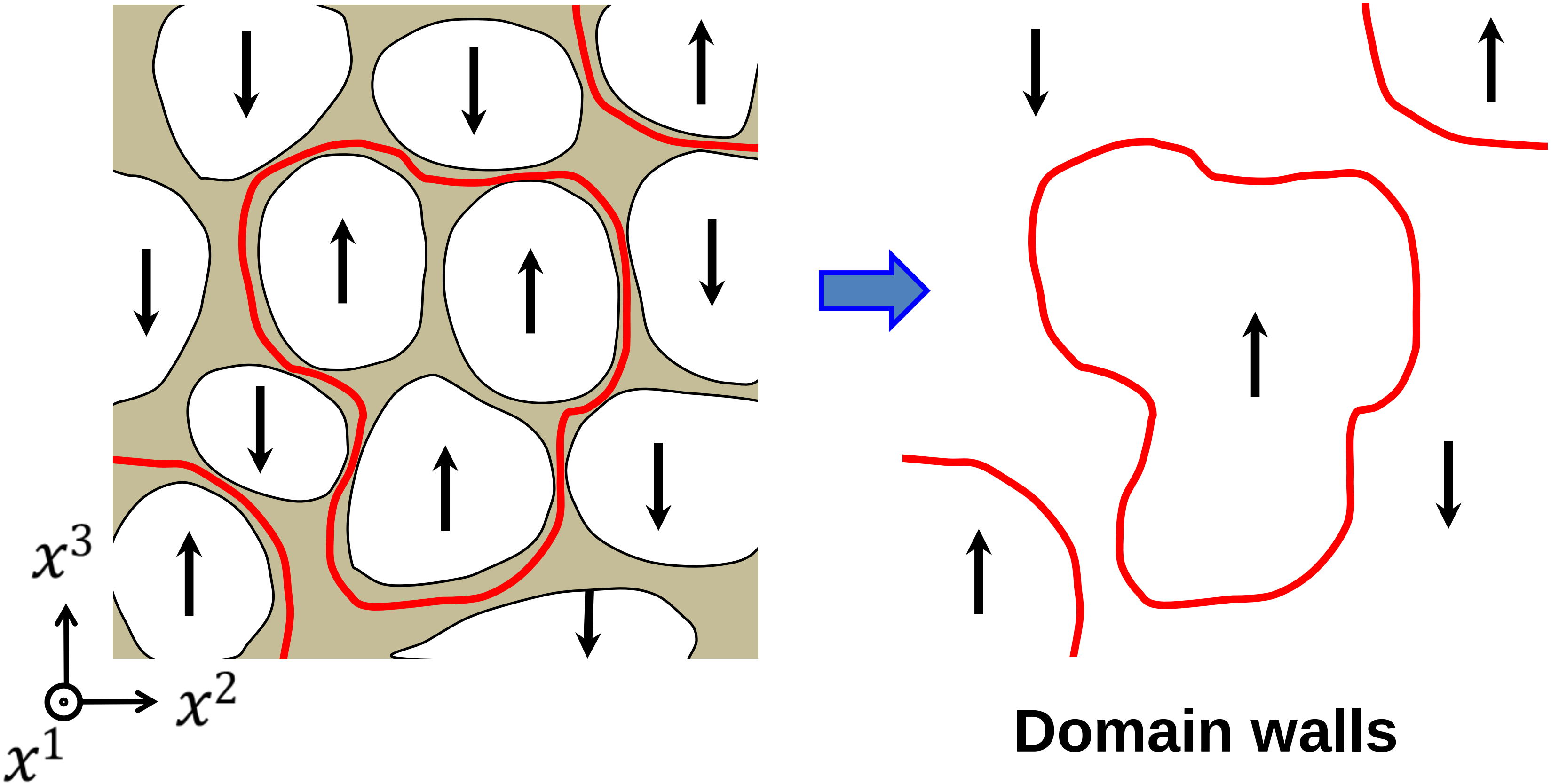

domain wall (defect)
$\pi_0(Z_2) = Z_2 \neq 0$
$\mathbf{R}^1$
sine-Gordon soliton (texture)
$\pi_1(S^1) = Z \neq 0$
S^1
$\mathbf{R}^1$

vortex, cosmic string (defect) $\pi_1(S^1) = Z \neq 0$





How are they created?
Kibble-Zurek mechanism @ phase transition



Standard model (SM)

$$G = SU(2)_W \times U(1)_Y \rightarrow H = U(1)_{\mathrm{em}}$$

Vacuum manifold of SM $\quad G/H = SU(2) = S^3$

$\pi_0(S^3) = 0$ No wall

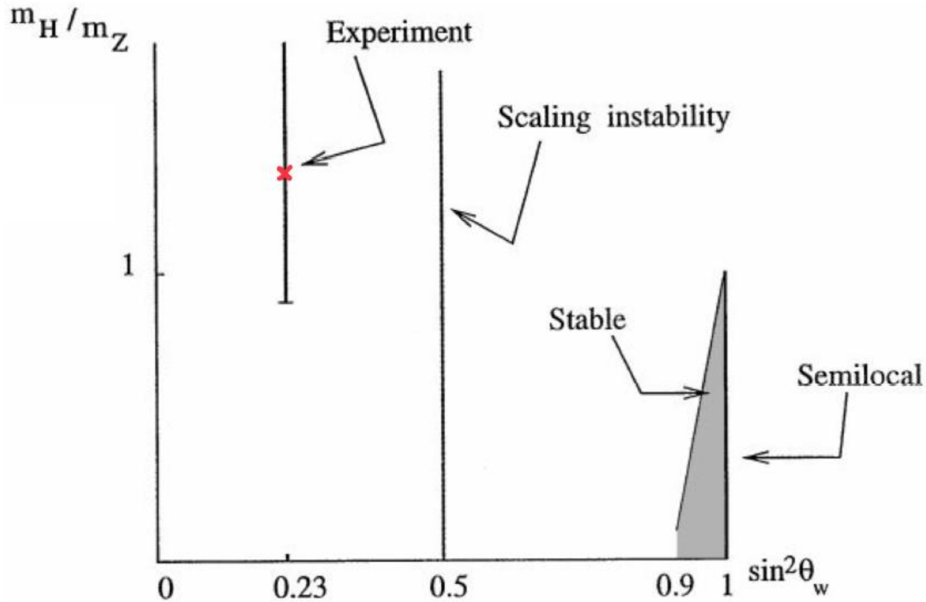
$\pi_1(S^3) = 0$ No cosmic string, No sine-Gordon

$\pi_2(S^3) = 0$ No monopole, No baby Skyrmion

$\pi_3(S^3) = Z$ Skyrmion? (unstable)

Electro-weak (EW) string in SM

Z-string
Nambu ('77)
Vachaspati ('92)



Achucarro & Vachaspati, Phys.Rep ('00)

EW monopole in SM

Monopole of E&M

Z-string

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Lagrangian of 2HDM

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^{a}_{\mu\nu}W^{a\mu\nu} + \sum_{i=1,2}\left(D_\mu\Phi_i^\dagger D^\mu\Phi_i\right) - V.$$

$$\begin{aligned} V =\ & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right) + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 \\ & + \beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right\}, \\ & + \left\{\left[\beta_6\Phi_1^\dagger\Phi_1 + \beta_7\Phi_2^\dagger\Phi_2\right]\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right\} \end{aligned}$$

Lagrangian of 2HDM

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} + \sum_{i=1,2}\left(D_\mu\Phi_i^\dagger D^\mu\Phi_i\right) - V.$$

$$V = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right) + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+\beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \mathrm{h.c.}\right\},$$
$$+\left\{\left[\beta_6\Phi_1^\dagger\Phi_1 + \beta_7\Phi_2^\dagger\Phi_2\right]\left(\Phi_1^\dagger\Phi_2\right) + \mathrm{h.c.}\right\}$$

(softly broken) Z_2 : $\Phi_1 \to +\Phi_1$ and $\Phi_2 \to -\Phi_2$

Lagrangian of 2HDM

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} + \sum_{i=1,2}\left(D_\mu\Phi_i^\dagger D^\mu\Phi_i\right) - V.$$

$$V = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right) + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+\beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \mathrm{h.c.}\right\},$$

CP invariance: $\Phi_i \to i\sigma_2\Phi_i^*$ **$m_{12}{}^2$, β_5: real ($m_{12}{}^2 \geqq 0$)**

VEVs: $\Phi_1 = e^{-i\alpha}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \Phi_2 = e^{i\alpha}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

$v_{\mathrm{EW}}^2/2 = (v_1^2 + v_2^2)$

$v_{\mathrm{EW}}\ (\simeq 246\ \mathrm{GeV})$

α=0 (mod π/2): CP preserving
$\alpha\neq$0 (mod π/2): SSB of CP

Matrix notation: $H = (i\sigma_2\Phi_1^*,\ \Phi_2) = \begin{pmatrix} \phi_{1,2}^* & \phi_{2,1} \\ -\phi_{1,1}^* & \phi_{2,2} \end{pmatrix}$

$$V = \frac{m_{11}^2 + m_{22}^2}{2}\mathrm{Tr}(H^\dagger H) - \frac{m_{11}^2 - m_{22}^2}{2}\mathrm{Tr}\left(H^\dagger H\sigma_3\right) - m_{12}^2\,(\det H + \mathrm{h.c.})$$
$$+\frac{2(\beta_1+\beta_2)+3\beta_3}{12}\mathrm{Tr}\left(H^\dagger H H^\dagger H\right) + \frac{2(\beta_1+\beta_2)-3\beta_3}{12}\mathrm{Tr}\left(H^\dagger H\sigma_3 H^\dagger H\sigma_3\right)$$
$$-\frac{\beta_1-\beta_2}{3}\mathrm{Tr}\left(H^\dagger H\sigma_3 H^\dagger H\right) + (\beta_3+\beta_4)\det(H^\dagger H) + \left(\frac{\beta_5}{2}\det H^2 + \mathrm{h.c.}\right),$$

$SU(2)_W$ x $U(1)_Y$ gauge transformation

$$H \to \exp\left(\frac{i}{2}\alpha_a(x)\sigma_a\right) H \exp\left(-\frac{i}{2}\beta(x)\sigma_3\right) \quad D_\mu H = \partial_\mu H - g\frac{i}{2}\sigma_a W_\mu^a H + g'\frac{i}{2}H\sigma_3 B_\mu$$

Alignment v_1=v_2 (tanβ=1) when $m_{11} = m_{22}, \quad \beta_1 = \beta_2$

Custodial symmetry $H \to U^\dagger H U, \quad U \in \mathrm{SU}(2)_C$ **exact when** $\beta_1 = \frac{3}{4}\beta_3$

$m_{11} = m_{22}, \quad \beta_1 = \beta_2$

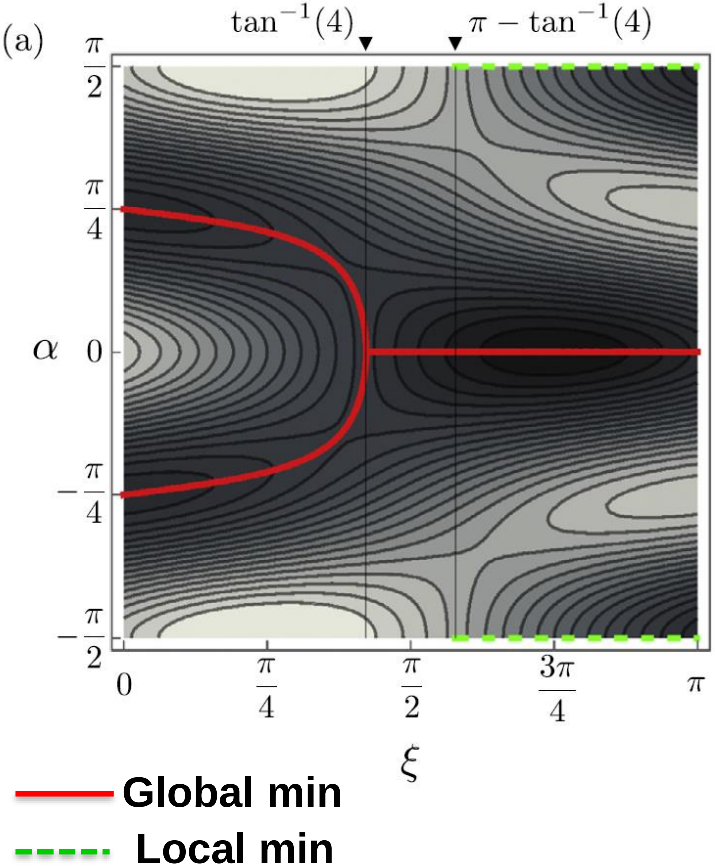
$$\Phi_1 = e^{-i\alpha}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \Phi_2 = e^{i\alpha}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

Double sine-Gordon potential

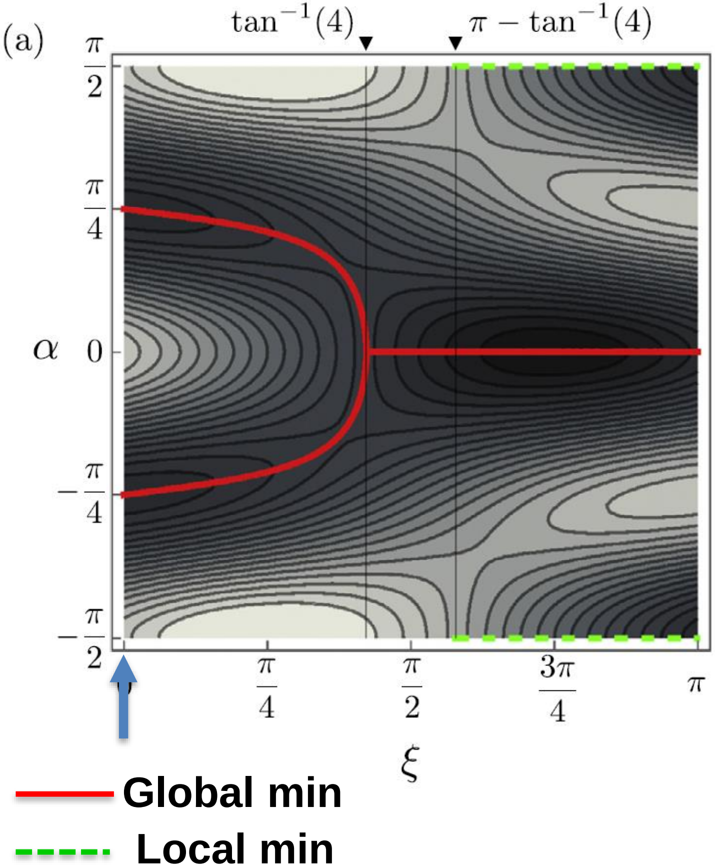
$$\begin{aligned} V_\xi(\alpha) &= -2m_{12}^2 v_1 v_2 \cos 2\alpha + \beta_5 v_1^2 v_2^2 \cos 4\alpha \\ &= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2/v_1 v_2\right)^2 + \beta_5^2}\,(-\sin\xi \cos 2\alpha + \cos\xi \cos 4\alpha) \end{aligned}$$

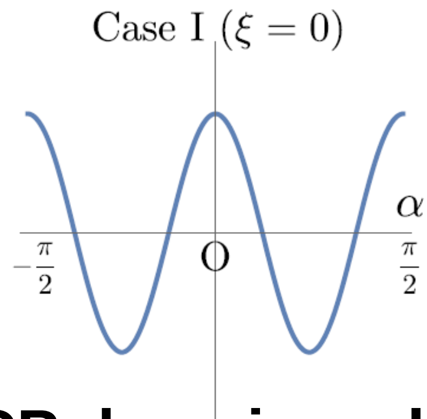
$$\sin\xi = \frac{2(m_{12}^2/v_1 v_2)}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}} \quad \cos\xi = \frac{\beta_5}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}}$$

$$V_\xi(\alpha) = -2m_{12}^2 v_1 v_2 \cos 2\alpha + \beta_5 v_1^2 v_2^2 \cos 4\alpha$$
$$= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2/v_1 v_2\right)^2 + \beta_5^2}\,(-\sin\xi \cos 2\alpha + \cos\xi \cos 4\alpha)$$



$$
\begin{aligned}
V_\xi(\alpha) &= -2m_{12}^2 v_1 v_2 \cos 2\alpha + \beta_5 v_1^2 v_2^2 \cos 4\alpha \\
&= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2/v_1 v_2\right)^2 + \beta_5^2}\,(-\sin\xi \cos 2\alpha + \cos\xi \cos 4\alpha)
\end{aligned}
$$

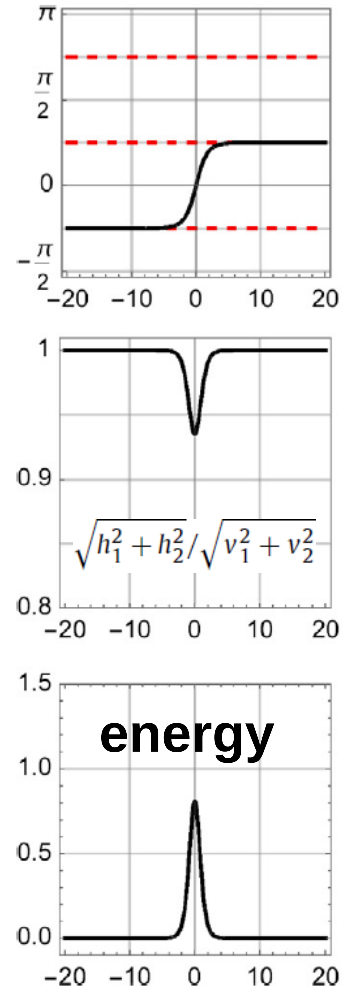




CP domain wall

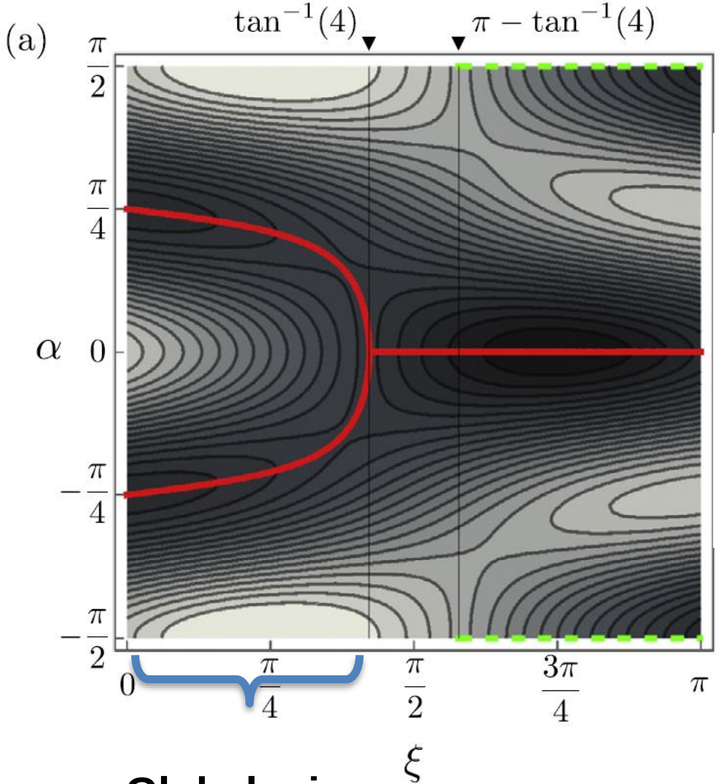
CP recovered on the wall

Battye, Brawn & Pilaftsis ('11)



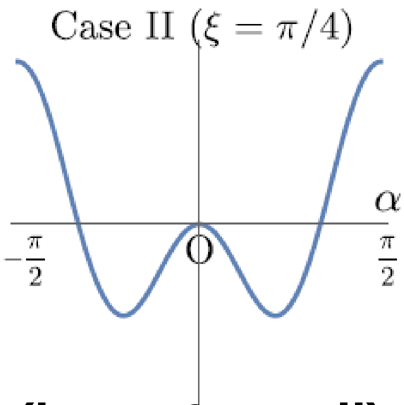
$$V_\xi(\alpha) = -2m_{12}^2 v_1 v_2 \cos 2\alpha + \beta_5 v_1^2 v_2^2 \cos 4\alpha$$

$$= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2/v_1 v_2\right)^2 + \beta_5^2}\,(-\sin\xi \cos 2\alpha + \cos$$



Global min

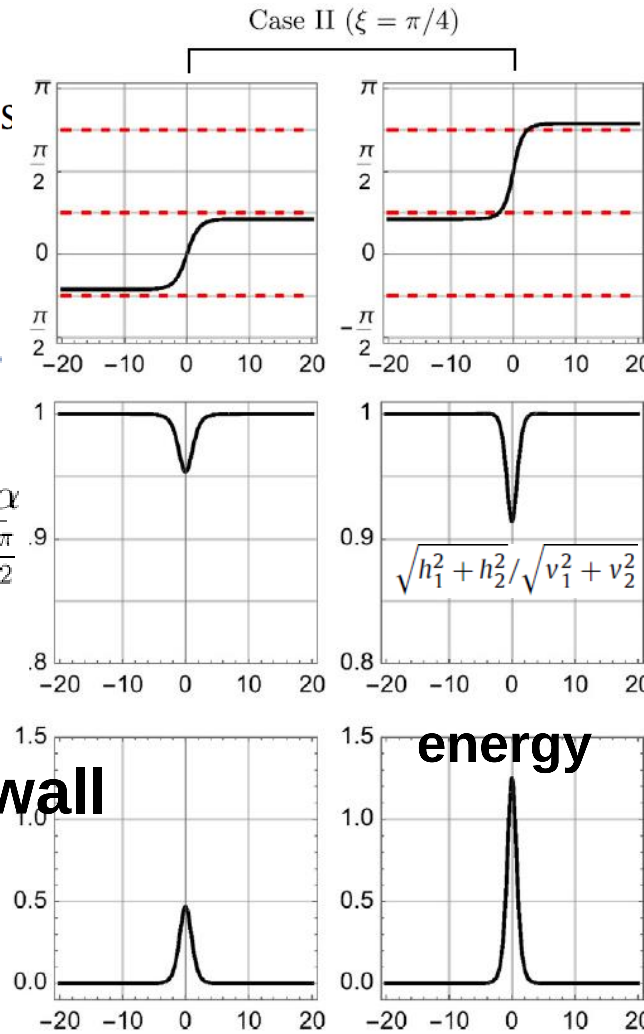
Local min



(large & small)

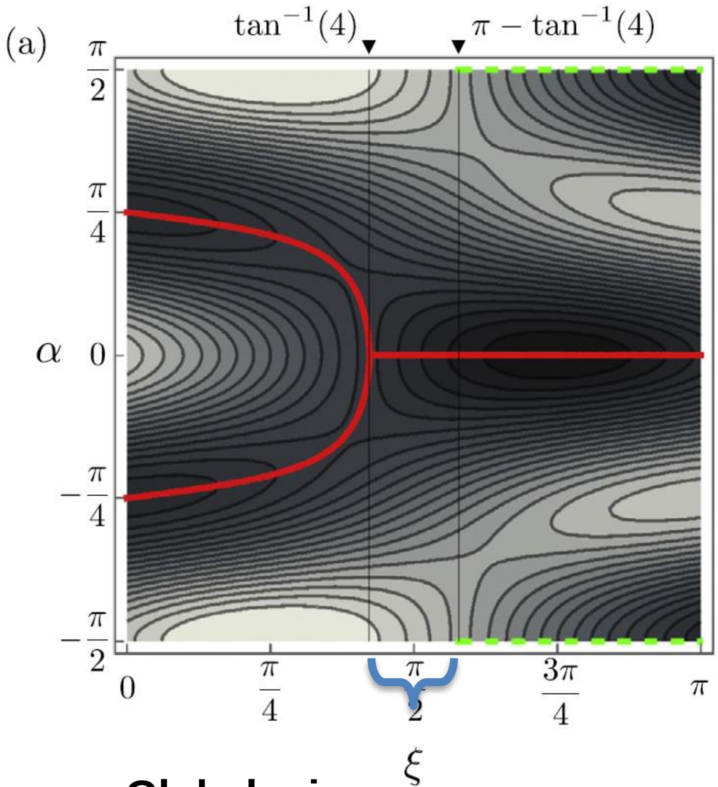
CP domain wall

CP recovered on the wall



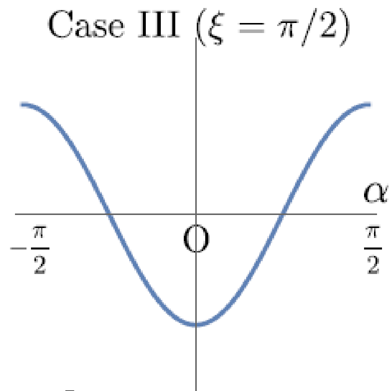
Battye, Brawn & Pilaftsis ('11)

$$V_\xi(\alpha) = -2m_{12}^2 v_1 v_2 \cos 2\alpha + \beta_5 v_1^2 v_2^2 \cos 4\alpha$$
$$= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2/v_1 v_2\right)^2 + \beta_5^2}\,(-\sin\xi \cos 2\alpha + \cos\xi \cos 4\alpha)$$



Global min

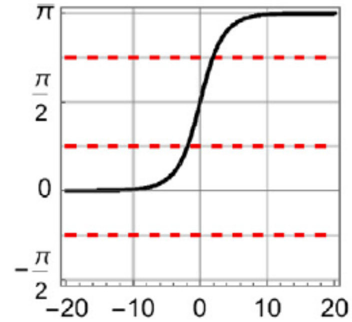
Local min

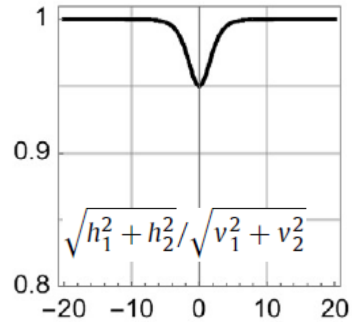


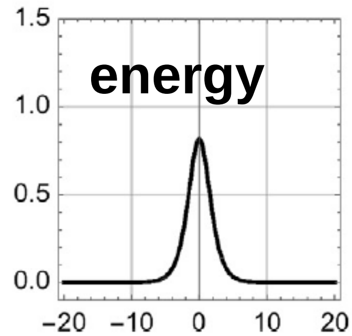
Membrane (sine-Gordon kink)

CP broken around the wall (recovered on the wall)

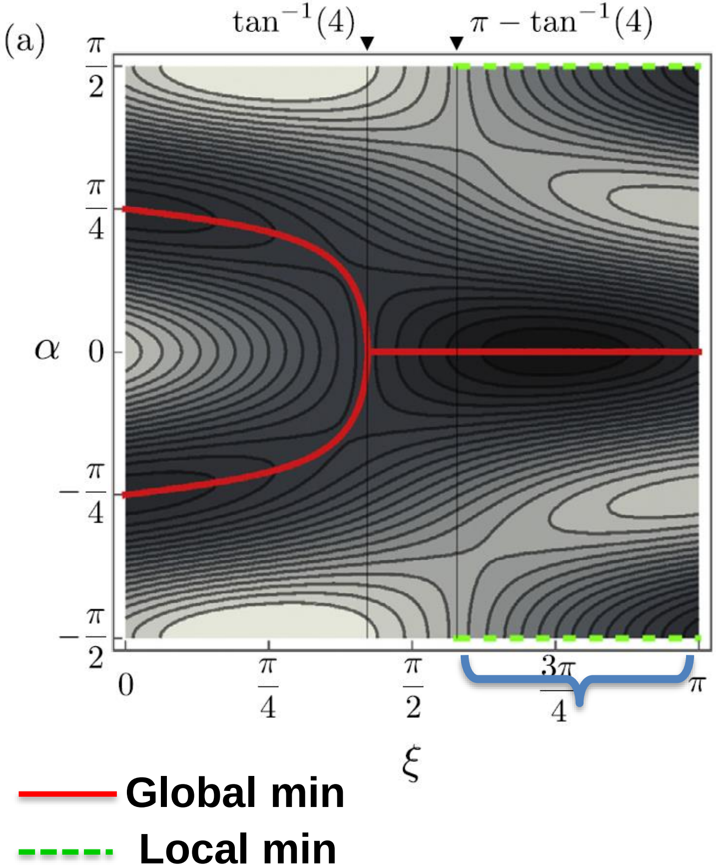
Bachas & Tomaras ('95)

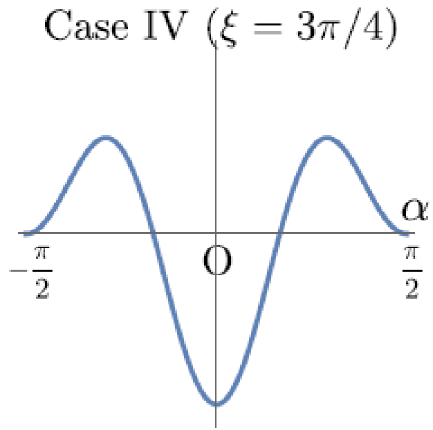


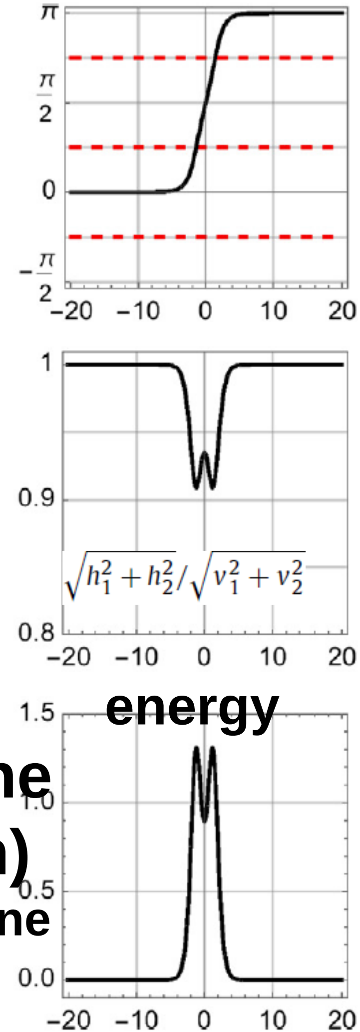




$$V_\xi(\alpha) = -2m_{12}^2 v_1 v_2 \cos 2\alpha + \beta_5 v_1^2 v_2^2 \cos 4\alpha$$
$$= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2/v_1 v_2\right)^2 + \beta_5^2}\,(-\sin\xi \cos 2\alpha + \cos\xi \cos 4\alpha)$$





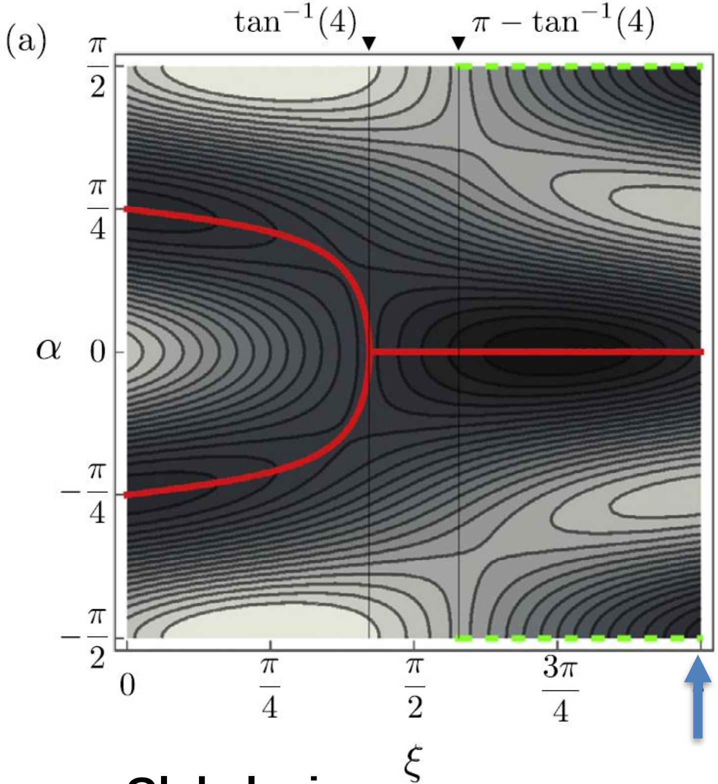


Composite membrane
(double sine-Gordon)

CP broken around membrane
(recovered on membrane)

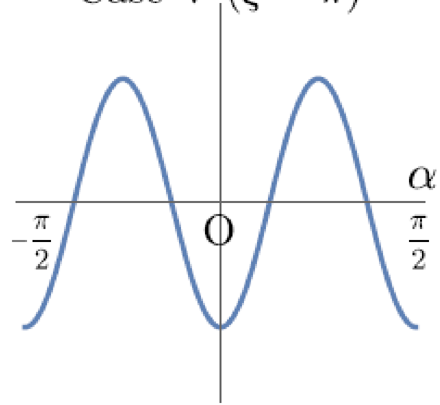
new

$$V_\xi(\alpha) = -2m_{12}^2 v_1 v_2 \cos 2\alpha + \beta_5 v_1^2 v_2^2 \cos 4\alpha$$
$$= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2/v_1 v_2\right)^2 + \beta_5^2}\,(-\sin\xi \cos 2\alpha + \cos\xi \cos 4\alpha)$$



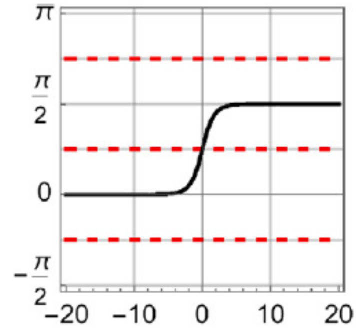
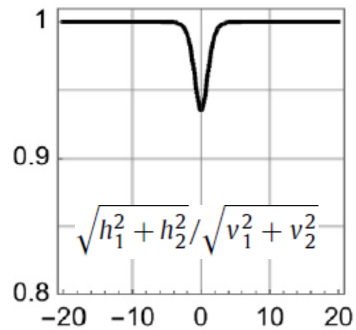
Global min

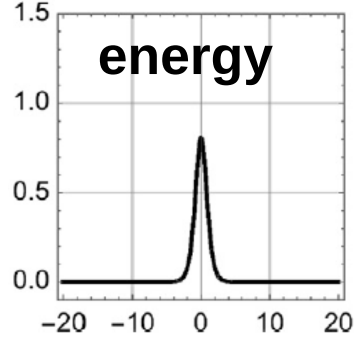
Local min



Z_2 domain wall

CP broken on the wall



new

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Simplification of parameters

(1) Alignment v_1=v_2 (tanβ=1) when $m_{11} = m_{22}, \quad \beta_1 = \beta_2$

$$V_{v_1=v_2} = -m^2\mathrm{Tr}(H^\dagger H) + \lambda_1 \mathrm{Tr}\left((H^\dagger H)^2\right) + \lambda_2(\mathrm{Tr}(H^\dagger H))^2 + \lambda_4 \mathrm{Tr}\left(H^\dagger H \sigma_3 H^\dagger H \sigma_3\right)$$
$$-m_{12}^2\left(\det H + \mathrm{h.c.}\right) + \left(\frac{\beta_5}{2}\det H^2 + \mathrm{h.c.}\right),$$

(2) Exact custodial symmetry $H \to U^\dagger H U, \quad U \in \mathrm{SU}(2)_C$ **when** $\beta_1 = \frac{3}{4}\beta_3$

(3) Exact $\mathrm{U}(1)_a: \ H \to e^{i\alpha} H$ **when** $m_{12} = \beta_5 = 0$ $\quad m_{11} = m_{22}, \quad \beta_1 = \beta_2$

The most symmetric Higgs sector

$$m_{11} = m_{22}, \quad \beta_1 = \beta_2 = \frac{3}{4}\beta_3, \quad m_{12} = \beta_5 = 0,$$

$$V = -m^2\mathrm{Tr}[H^\dagger H] + \lambda_1 \mathrm{Tr}\left[(H^\dagger H)^2\right] + \lambda_2\left(\mathrm{Tr}[H^\dagger H]\right)^2$$

(4) Gauge sector: sin θ_W=0 (g'=0)

Consider the simplest case and then relax the conditions gradually

Stable non-Abelian string

U(1)$_a$: global string (log div tension)

$$H_0 = v \begin{pmatrix} h(r) & 0 \\ 0 & f(r)e^{i\theta} \end{pmatrix} = v e^{i\frac{\theta}{2}} e^{-i\frac{\theta}{2}\sigma_3} \begin{pmatrix} h(r) & 0 \\ 0 & f(r) \end{pmatrix}$$

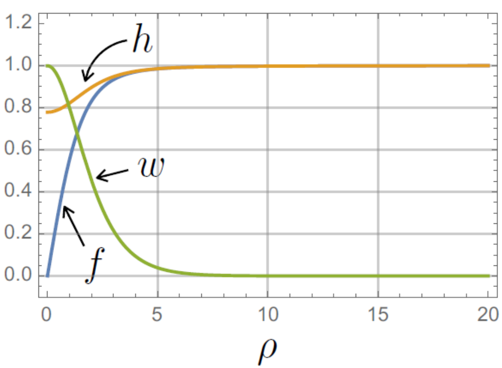
SU(2)$_W$: flux tube

$$W^a_{i,0} = \delta^{a3}\frac{1}{g}\epsilon_{ij}\frac{x^j}{r^2}(1 - w(r)), \quad W^a_{3,0} = 0,$$

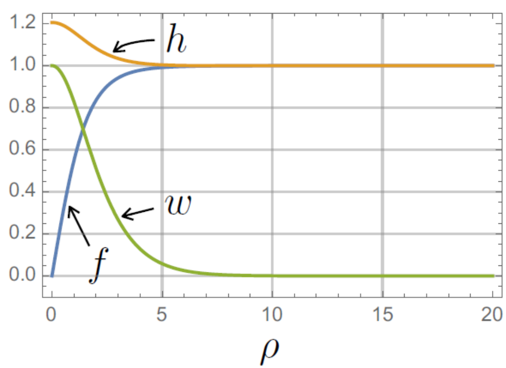
$$h'(0) = 0,\ f(0) = 0,\ w(0) = 1, \quad h(\infty) = f(\infty) = 1,\ w(\infty) = 0.$$

$$\Phi^3_{12} = \int d^2x\, W^3_{12} = -\frac{2\pi}{g}$$

$\gamma_1 < \gamma_3$



$\gamma_3 < \gamma_1$



$$\gamma_1 = \frac{\sqrt{2}m_1}{m_W}, \quad \gamma_3 = \frac{\sqrt{2}m_3}{m_W}$$

$$m_1^2 = 2m^2$$

$$m_3^2 = \frac{\lambda_1}{\lambda_1 + 2\lambda_2}2m^2 = 4\lambda_1 v^2$$

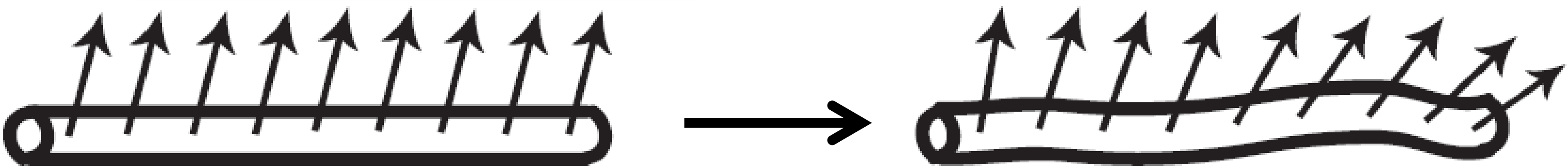
SU(2)$_C$ is recovered at $r \to \infty$

& spontaneously broken at $r \to 0$ (vortex core)

$$H\big|_{\text{NA string}} \to v e^{i\frac{\theta}{2}} \begin{pmatrix} h(0) & 0 \\ 0 & 0 \end{pmatrix}$$

Nambu-Goldstone modes localized around a vortex

$$\frac{\mathrm{SU}(2)_C}{\mathrm{U}(1)_c} \simeq \mathbb{C}P^1 \simeq S^2$$ → **Moduli of a vortex**

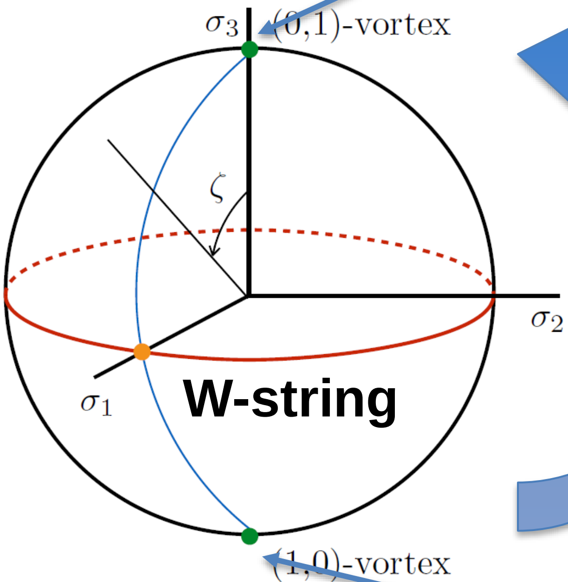


"ground state" **1+1 dim effective theory** **fluctuations**

$CP^1 \Leftrightarrow$ SU(2) magnetic flux

Z-string

$$H_\pi = v \begin{pmatrix} f(r)e^{i\theta} & 0 \\ 0 & h(r) \end{pmatrix} \qquad \Phi^3_{12} = \int d^2x\, W^3_{12} = \frac{2\pi}{g}$$



SU(2)$_C$ custodial symmetry

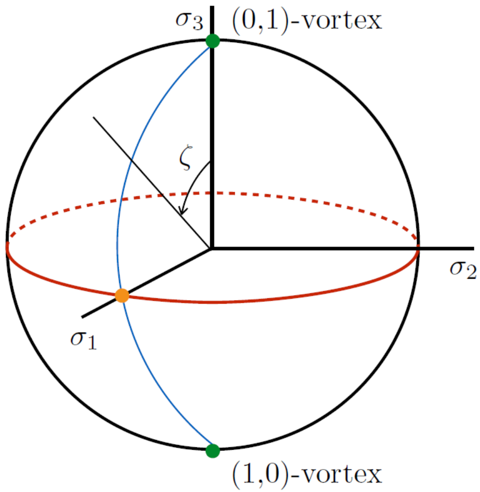
opposite flux

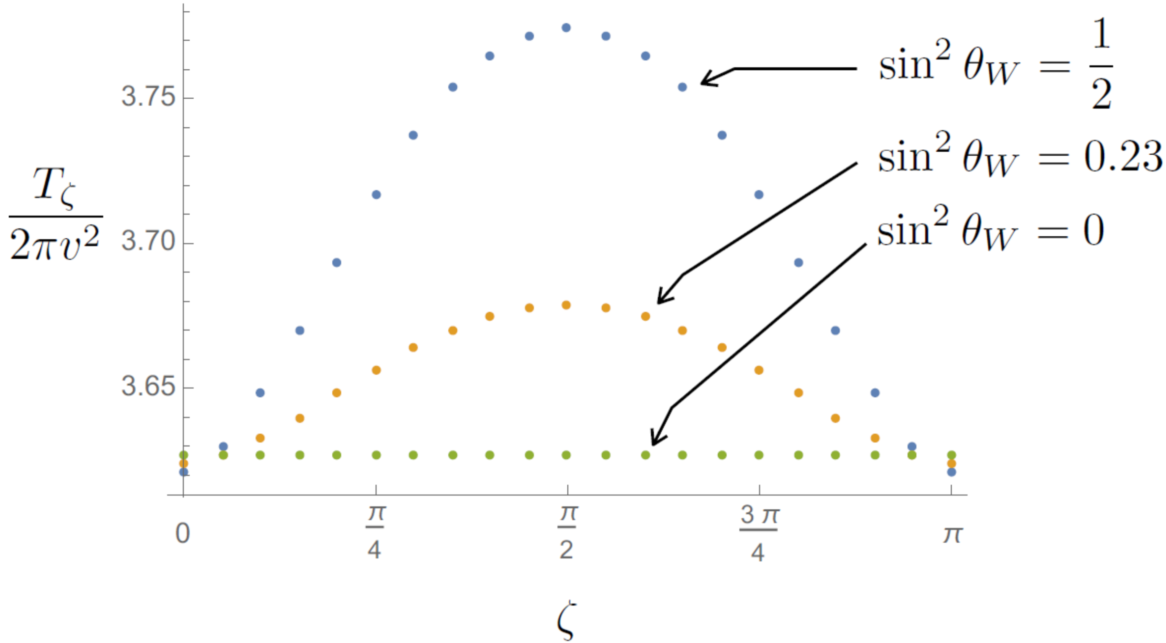
Z-string

$$H_0 = v \begin{pmatrix} h(r) & 0 \\ 0 & f(r)e^{i\theta} \end{pmatrix} \qquad \Phi^3_{12} = \int d^2x\, W^3_{12} = -\frac{2\pi}{g}$$

Topological Z-string @ ~~$\sin\theta_W=0$ ($g'=0$)~~, $v_1=v_2$ ($\tan\beta=1$), $SU(2)_C$

$U(1)_Y$ lifts up all NA vortices minimizing Z-strings (N & S poles)

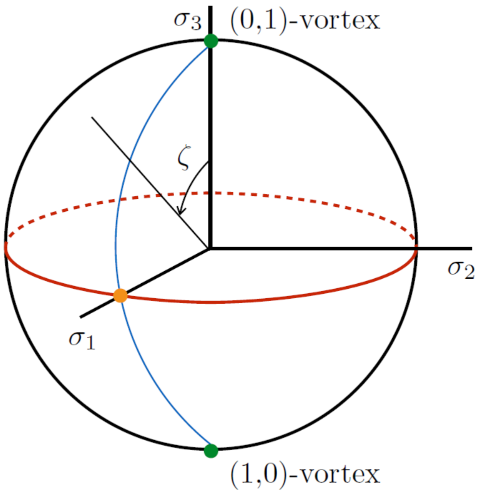


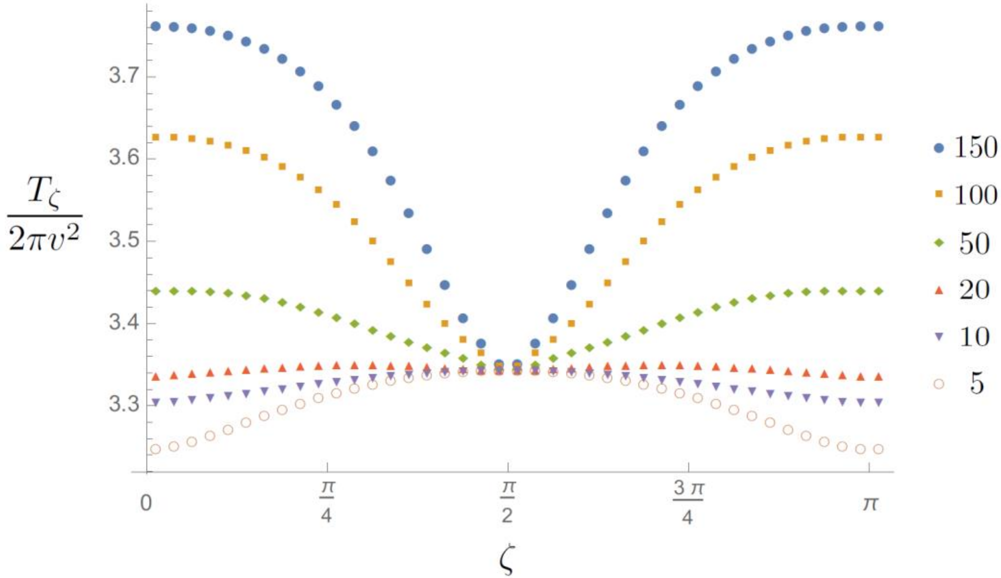


$$\Phi_Z = \pm\frac{2\pi\cos\theta_W}{g}$$

Topological Z/W-string @ ~~sin θ_W=0(g'=0)~~, v_1=v_2 (tanβ=1), ~~SU(2)$_C$~~

~~SU(2)$_C$~~ introduces a potential minimizing either Z or W-strings





NEW: There is a parameter region a W-string is stable, unlike SM.

Topological Z/W-string @ ~~sin θ_W=0 (g'=0)~~, ~~v_1=v_2 (tanβ=1)~~, ~~SU(2)~~$_C$

$$H^{(0,1)} = \begin{pmatrix} v_1 h(r) & 0 \\ 0 & v_2 f(r) e^{i\theta} \end{pmatrix}, \quad Z_i^{(0,1)} = \frac{2\sin^2\beta\cos\theta_W}{g}\epsilon_{ij}\frac{x^j}{r^2}(1-w(r)),$$

$$\Phi_Z^{(0,1)} = 2\pi\frac{\sin^2\beta\cos\theta_W}{g}$$

$$\sin^2\beta = \frac{v_1^2}{v_1^2+v_2^2}$$

$$H^{(1,0)} = \begin{pmatrix} v_1 f(r) e^{i\theta} & 0 \\ 0 & v_2 h(r) \end{pmatrix}, \quad Z_i^{(1,0)} = -\frac{2\cos^2\beta\cos\theta_W}{g}\epsilon_{ij}\frac{x^j}{r^2}(1-w(r)),$$

$$\Phi_Z^{(1,0)} = -2\pi\frac{\cos^2\beta\cos\theta_W}{g}$$

$$\cos^2\beta = \frac{v_2^2}{v_1^2+v_2^2}$$

Fractionally quantized magnetic fluxes

Dvali & Senjanovic ('94)

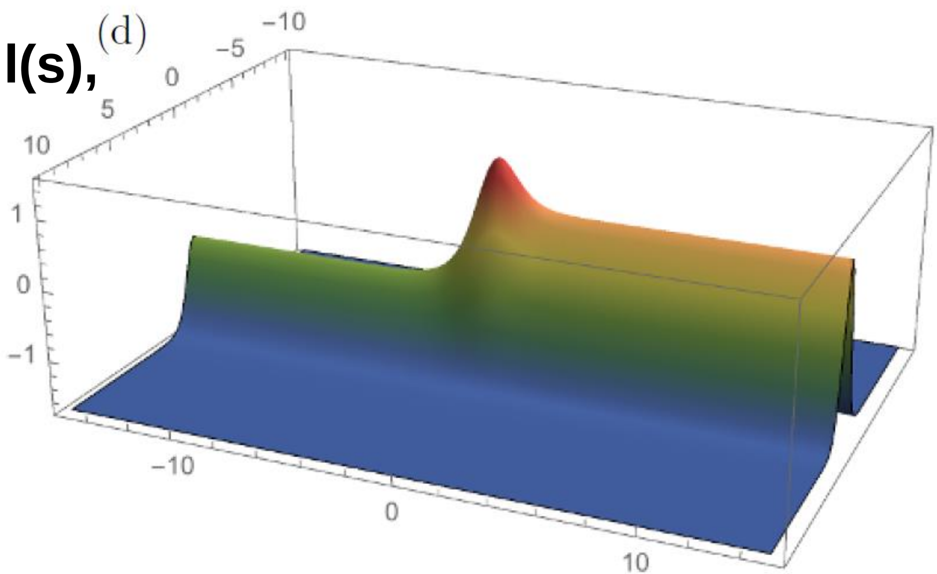
(3) Exact ~~$U(1)_a: H \to e^{i\alpha}H$~~ when ~~$m_{12} = \beta_5 = 0$~~

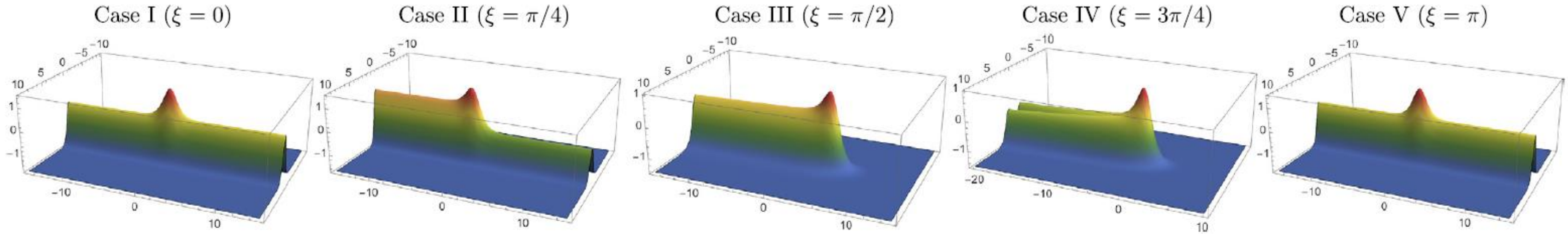
$U(1)_a$ must be explicitly broken to remove Nambu-Goldstone boson.

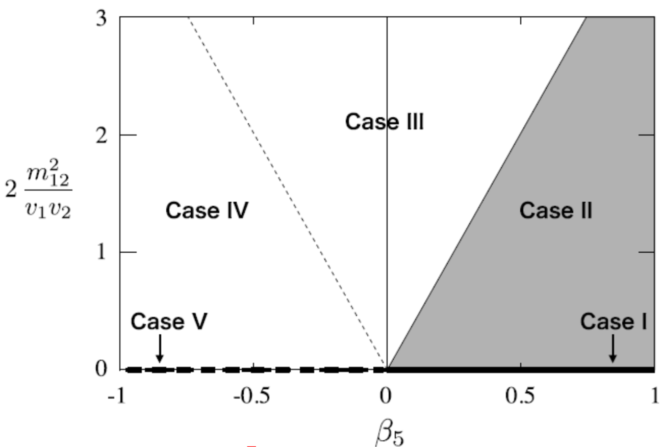
$$V = -m^2 \mathrm{Tr}[H^\dagger H] - \mu^2 \mathrm{Tr}[H^\dagger H \sigma_3] + \lambda_1 \mathrm{Tr}\left[(H^\dagger H)^2\right] + \lambda_2 \left(\mathrm{Tr}[H^\dagger H]\right)^2$$
$$+ \lambda_3 \mathrm{Tr}[H^\dagger H \sigma_3 H^\dagger H] + \lambda_4 \mathrm{Tr}[H^\dagger H \sigma_3 H^\dagger H \sigma_3]$$
$$- m_{12}^2 (\det H + \mathrm{h.c.}) + \left(\frac{\beta_5}{2} \det H^2 + \mathrm{h.c.}\right).$$

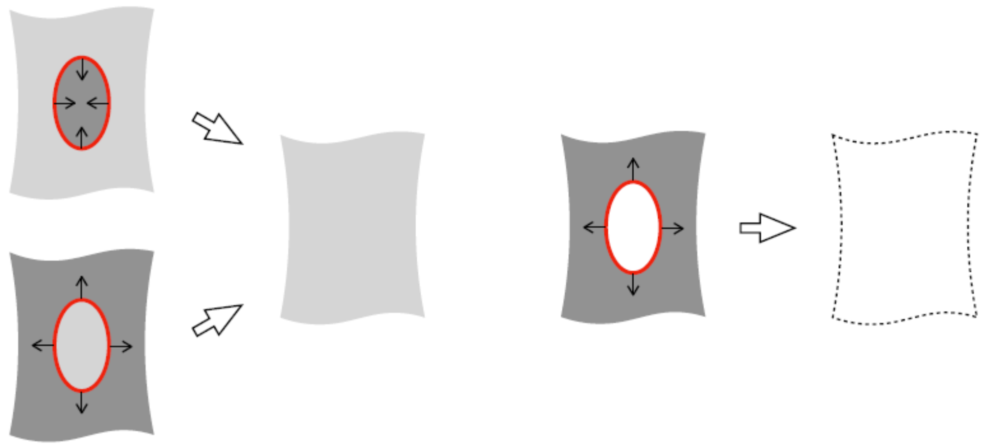
The case in which we discussed domain walls.

A string is attached by wall(s), like axion strings.









(a) region I

(b) region II

Constraints on m_{12} & β_5

Cosmological domain wall problem

Cases I, II, V: Cosmologically forbidden

Cases III, IV: Cosmologically safe

Plan of My Talk

§ 1 Introduction: SM

§ 2 Domain walls and membranes in 2HDM

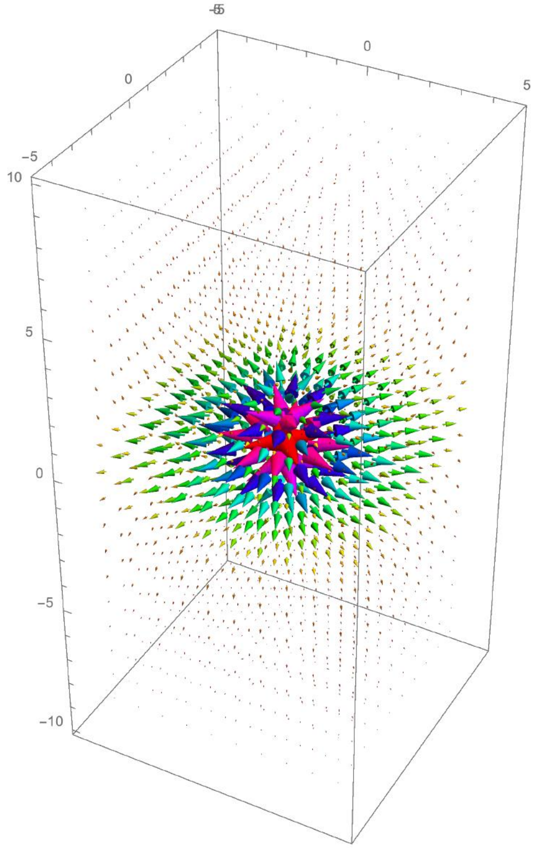
§ 3 Vortices (cosmic strings) in 2HDM

§ 4 Monopoles in 2HDM

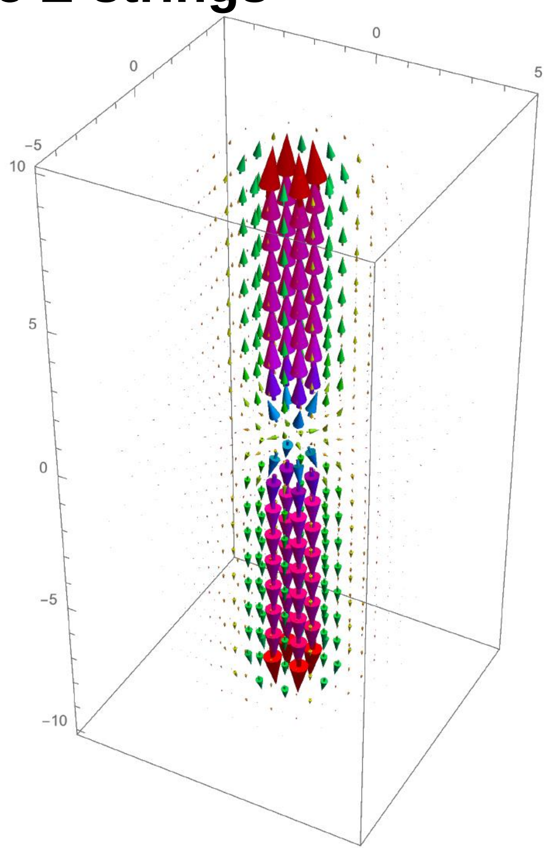
§ 5 Summary

Stable Nambu monopole!! (preliminary)

Magnetic flux of E&M is hedgehog



Magnetic Z-flux is confined to Z-strings



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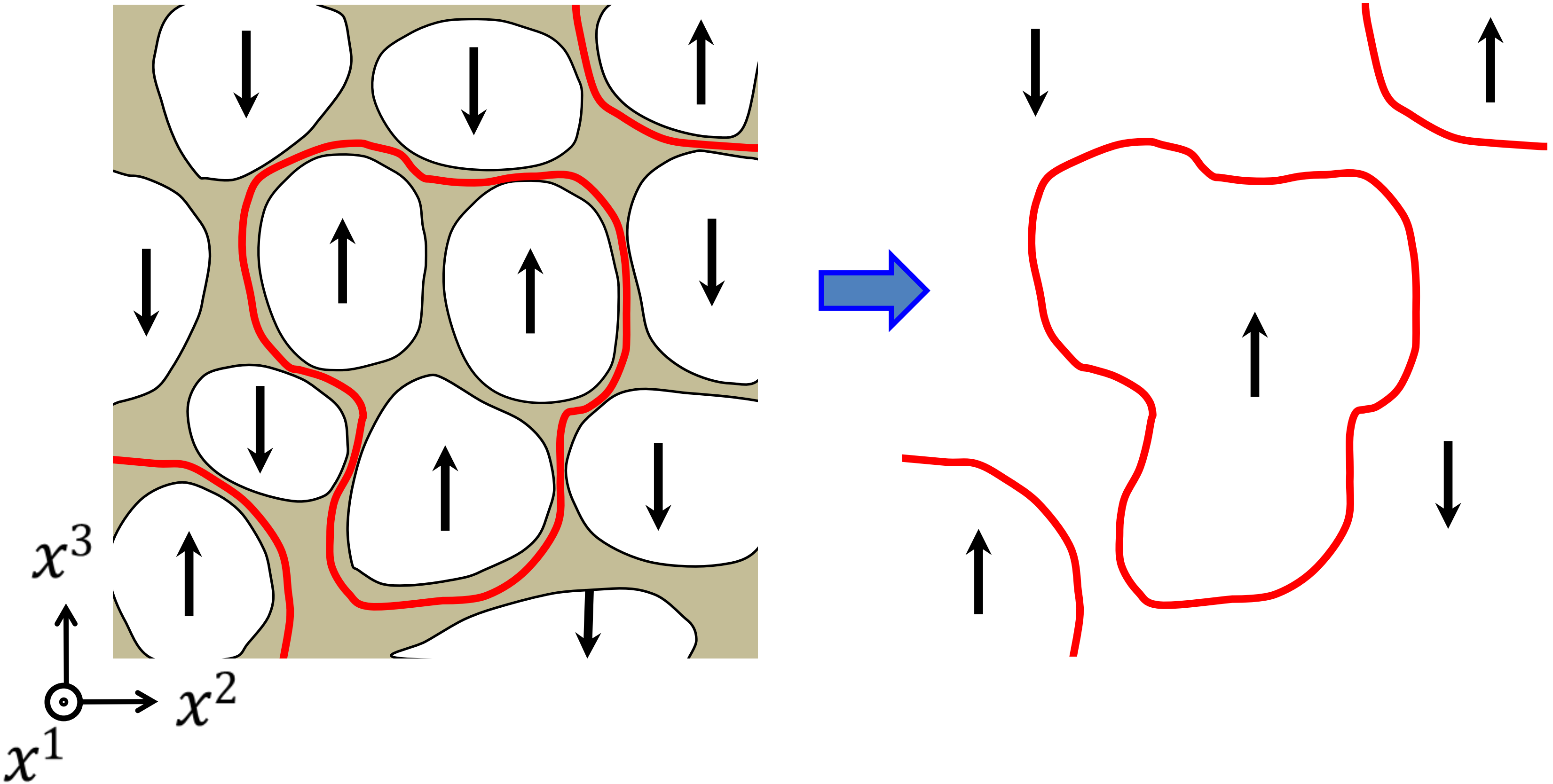
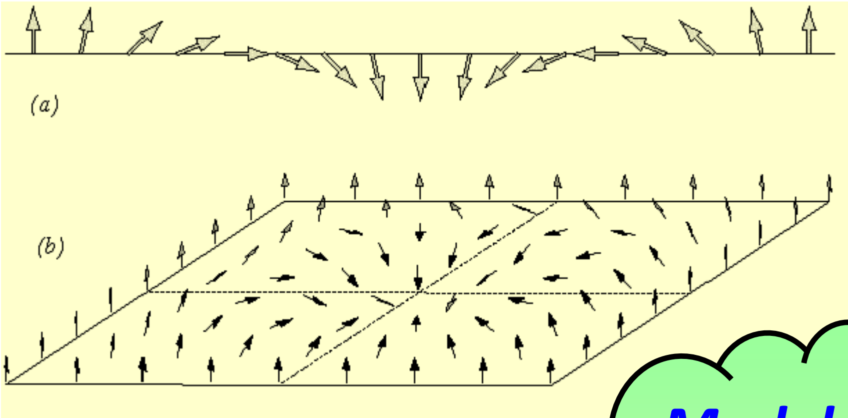
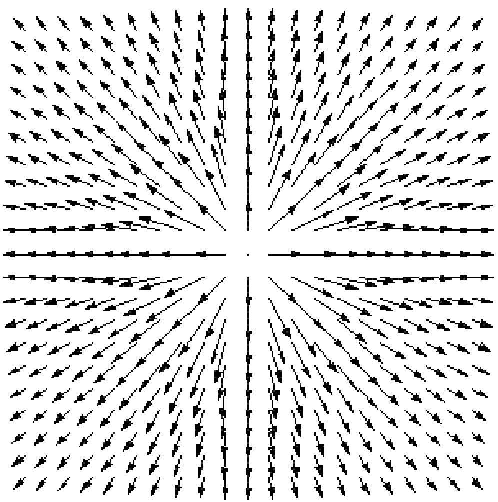


x^3
x^2
x^1

What is a Skyrmion?





3dim hedgehog

Model of nucleon in HEP

1D Skyrmion

=Sine-Gordon kink

$$\pi_1(S^1) = \mathbf{Z}$$

2D Skyrmion

$$\pi_2(S^2) = \mathbf{Z}$$

3D Skyrmion

$$\pi_3(S^3) = \mathbf{Z}$$

T.H.R. Skyrme

A Nonlinear theory of strong interactions
Proc.Roy.Soc.Lond. A247 (1958) 260-278

A Unified Field Theory of Mesons and Baryons
Nucl.Phys. 31 (1962) 556-569

Lump, baby Skyrmion

$$\pi_2(S^2) \simeq \mathbf{Z}$$