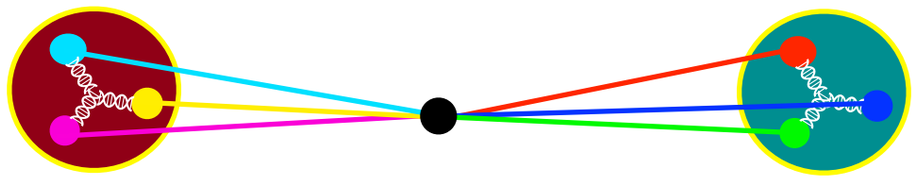
Lattice QCD updates on *B* violating matrix elements

Michael Wagman



BLV2019

IFT, Madrid

Oct 22, 2019

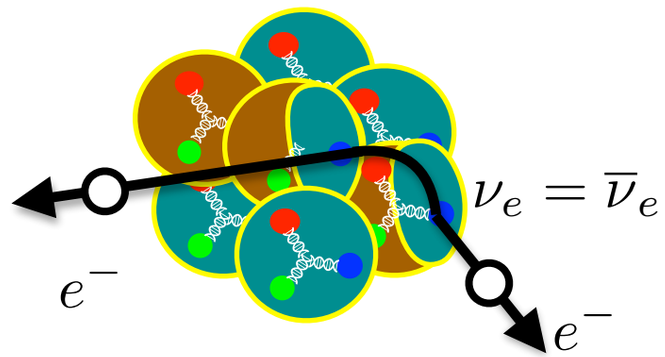
Fermilab

B-L Violation

B-L is an "accidental" symmetry of Standard Model operators with dim ≤ 4

Dim 5: ***B-L* violating**, *L* violating
Majorana neutrino mass

$$\mathcal{L}_5 \sim \left(\frac{1}{\Lambda_{BSM}}\right)(H^T \ell^*)(\bar{\ell} H)$$



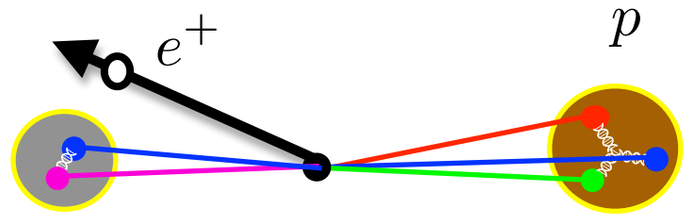
Double-β decay

$\Lambda_{BSM} \gtrsim 10^{10}$ GeV

✓ Leptogenesis

Dim 6: ***B-L* conserving**, *B* violating
proton decay operators

$$\mathcal{L}_6 \sim \left(\frac{1}{\Lambda^2_{BSM}}\right) uude + \ldots$$



Proton decay

$\Lambda_{BSM} \gtrsim 10^{16}$ GeV

✗ Washed out by sphalerons?

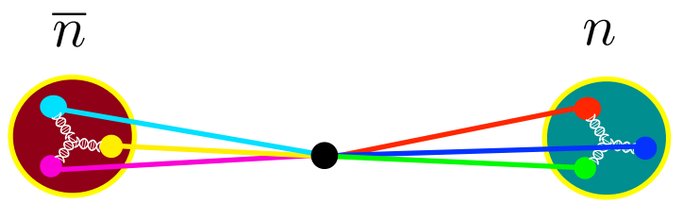
Lattice QCD: Aoki, Izubuchi, Shintani, Soni, PRD96 (2017)

Yoo, Aoki, Izubuchi, Syritsyn, Lattice2018

Dim 7: see talk by Martin Hirsch

Dim 9: ***B-L* violating**, *B* violating
Majorana neutron mass

$$\mathcal{L}_9 \sim \left(\frac{1}{\Lambda^5_{BSM}}\right) uddudd + \ldots$$



Neutron-antineutron oscillations

$\Lambda_{BSM} \gtrsim 10^5$ GeV

✓ Post-sphaleron baryogengesis

Neutron-Antineutron Oscillations

$n\bar{n}$ oscillation phenomenology similar to meson, neutrino oscillations

$$\mathcal{P}_{n\overline{n}} = \sin^2(t/\tau_{n\overline{n}})e^{-\Gamma_n t} \qquad \tau_{n\overline{n}}^{-1} = < \overline{n}|H_{n\overline{n}}|n >$$

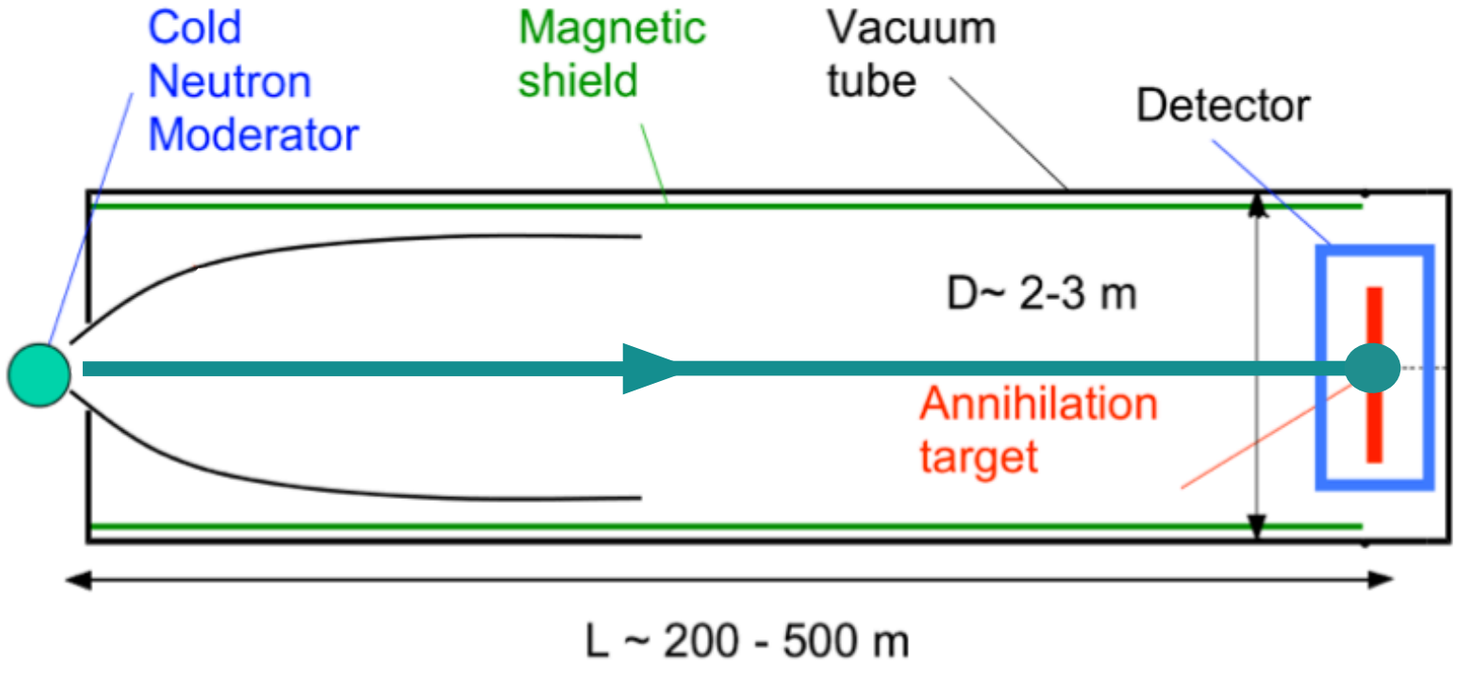
See talk by Yuri Kamyshkov

In order to turn experimental constraints into BSM physics constraints, we need theory predictions of $\tau_{n\overline{n}}^{-1}$ including QCD strong interaction effects

Institut Laue-Langevin (ILL)

$$\tau_{n\overline{n}} > 0.89 \times 10^8 \text{ s}$$

Baldo-Ceolin et al, Zeitschrift für Physik C Particles and Fields (1994)



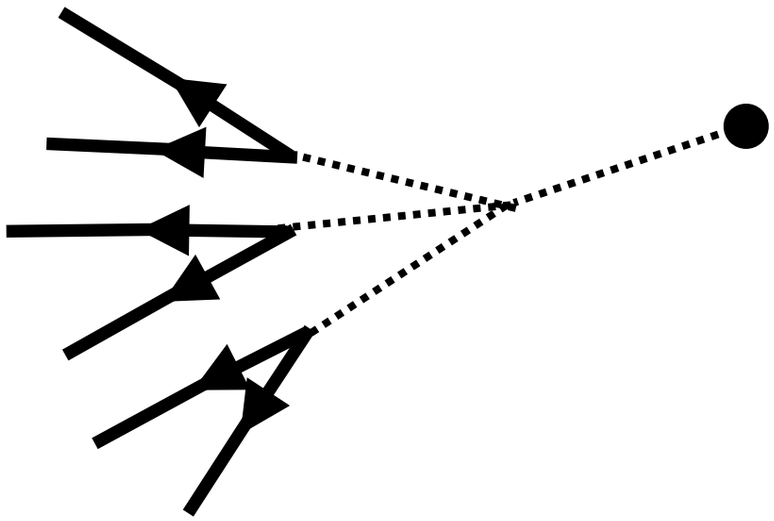
Mohapatra, J. Phys. G 36 (2009)

$n\bar{n}$ and Baryogenesis

BSM theories describe quark-level amplitudes for $|\Delta B| = 2$ processes

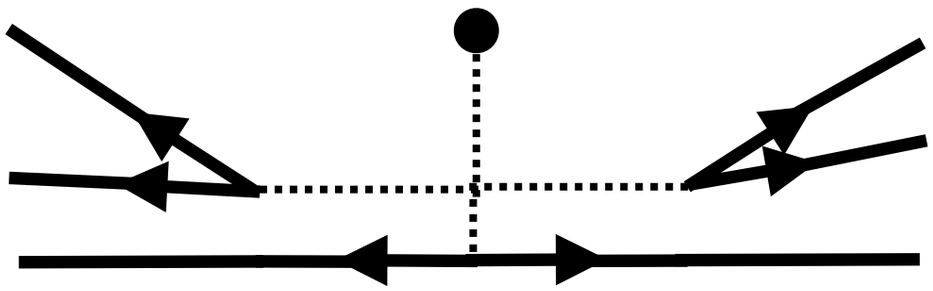
Example: Left-Right symmetric $SU(2)_L \times SU(2)_R \times SU(4)_C$ gauge theory

Baryon asymmetry from decay of BSM Higgs boson with *B-L* violating VEV



Mohapatra, Marshak PRL 44 (1980)

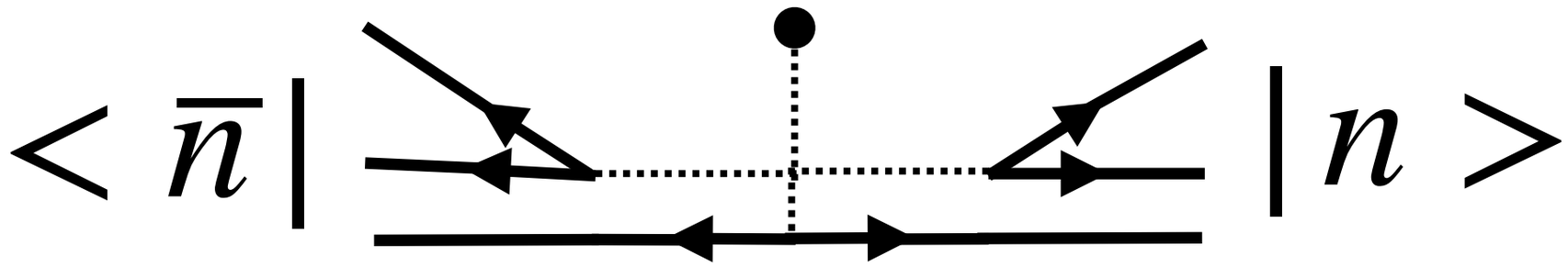
Rearranging diagram gives quark-level amplitude for $n\bar{n}$ oscillations



η_B and $\tau_{n\bar{n}}^{-1}$ related, although free parameters enter for *CP*-violation

QCD for $n\overline{n}$

Predicting $\tau_{n\overline{n}}^{-1}$ requires strong interaction effects in (anti)neutron states

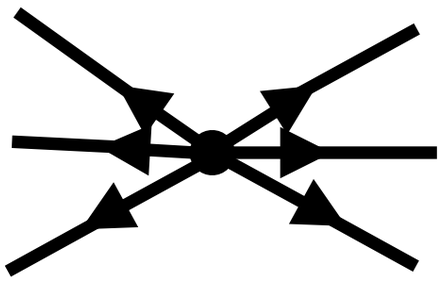


Lattice QCD allows path integrals describing (anti)neutron states to be defined and numerically computed, even at strong coupling

Problem: UV cutoff set by lattice spacing. Resource limits $a^{-1} \ll \Lambda_{BSM}$

QCD for $n\overline{n}$

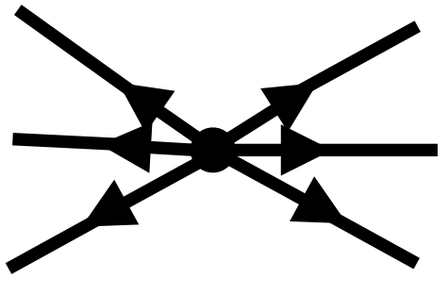
Predicting $\tau_{n\overline{n}}^{-1}$ requires strong interaction effects in (anti)neutron states

$$< \overline{n} \Big|$$  $$\Big| n >$$

Lattice QCD allows path integrals describing (anti)neutron states to be defined and numerically computed, even at strong coupling

Problem: UV cutoff set by lattice spacing. Resource limits $a^{-1} \ll \Lambda_{BSM}$

Solution: Exploit separation of QCD and BSM scales with EFT

 $$= \left(\frac{\lambda f^3 \tilde{v}_{B-L}}{\Lambda_{BSM}^5} \right) \left[(u_i C P_R d_j)(u_k C P_R d_l)(u_m C P_R d_n) T^{AAS}_{ijklmn} \right]$$

Six-Quark Operators

Arbitrary BSM theory predictions possible once QCD matrix elements calculated for complete basis of dim-9 six-quark operators

$$\mathcal{L}_9 = \frac{1}{\Lambda_{BSM}^5} \sum_I C_I^{\overline{\rm MS}}(\Lambda_{BSM}) Q_I^{\overline{\rm MS}}(\Lambda_{BSM})$$

Basis of 18 six-quark operators

Kuo, Love, PRL 45 (1980) Chang, Chang, Phys. Lett. B 92 (1980)

Complete basis of 14 operators (spin-color-flavor Fierz), renormalization

Caswell, Milutinovic, Phys. Lett. 122B (1983)

MIT bag model matrix element calculations

Rao, Shrock, Nucl. Phys. B 232 (1984)

14x14 renormalization matrix relates regularized and renormalized

$$Q_I^{\overline{\rm MS}}(\mu) = \sum_J Z_{IJ}^{\overline{\rm MS}}(\mu, a) Q_J^{\rm lattice}(a)$$

Chiral Operators

Chiral symmetry can be used to build an operator basis with no mixing under renormalization in perturbative QCD: $Q_1, ..., Q_7, Q_1^P, ..., Q_7^P$

Buchoff, MW et al, PRD 96 (2016)

Building blocks: **Singlet diquark** **Vector diquark**

$$\mathcal{D}_{L,R} = qCP_{L,R}i\tau^2 q \qquad \mathcal{D}^a_{L,R} = qCP_{L,R}i\tau^2\tau^a q$$

---

4 $SU(2)_L$ **singlet operators:**

$$Q_1 = \mathcal{D}_R\mathcal{D}_R\mathcal{D}_R^+T^{AAS} \qquad Q_2 = \mathcal{D}_L\mathcal{D}_R\mathcal{D}_R^+T^{AAS} \qquad Q_3 = \mathcal{D}_L\mathcal{D}_L\mathcal{D}_R^+T^{AAS}$$

$$Q_4 = \mathcal{D}_R^3\mathcal{D}_R^3\mathcal{D}_R^+T^{SSS} - \text{traces}$$

---

10 $SU(2)_L$ **non-singlet operators:**

$$Q_5 = \mathcal{D}_R^-\mathcal{D}_L^+\mathcal{D}_L^+T^{SSS} \qquad Q_6 = \mathcal{D}_R^3\mathcal{D}_L^3\mathcal{D}_L^+T^{SSS} \qquad Q_7 = \mathcal{D}_R^+\mathcal{D}_L^3\mathcal{D}_L^3T^{SSS} - \text{traces}$$

$$Q_I^P = -Q_I(L \leftrightarrow R)$$

Symmetry Constraints

$n\overline{n}$ matrix elements $\mathcal{M}_I = < \overline{n}|Q_I|n >$ further constrained by symmetry

Parity: $\mathcal{M}_I^P = -\mathcal{M}_I$ **Isospin:** $\mathcal{M}_4 = 0$ $\mathcal{M}_5 = \mathcal{M}_6 = -\frac{3}{2}\mathcal{M}_7$

$SU(2)_L$ non-singlet operators suppressed in EFT by $v = \text{Higgs VEV}$

$$\mathcal{M}_{n-\overline{n}} = \frac{1}{\Lambda_{BSM}^5}\left[\sum_{I=1,2,3}\widetilde{C}_I\mathcal{M}_I + \frac{v^2}{\Lambda_{BSM}^2}\sum_{I=1,2,3,5}\widetilde{C}_I^P\mathcal{M}_I^P + \frac{v^4}{\Lambda_{BSM}^4}\widetilde{C}_5\mathcal{M}_5\right]$$

Rinaldi, Sryitsyn, MW et al, PRD 99 (2019)

In isospin limit:

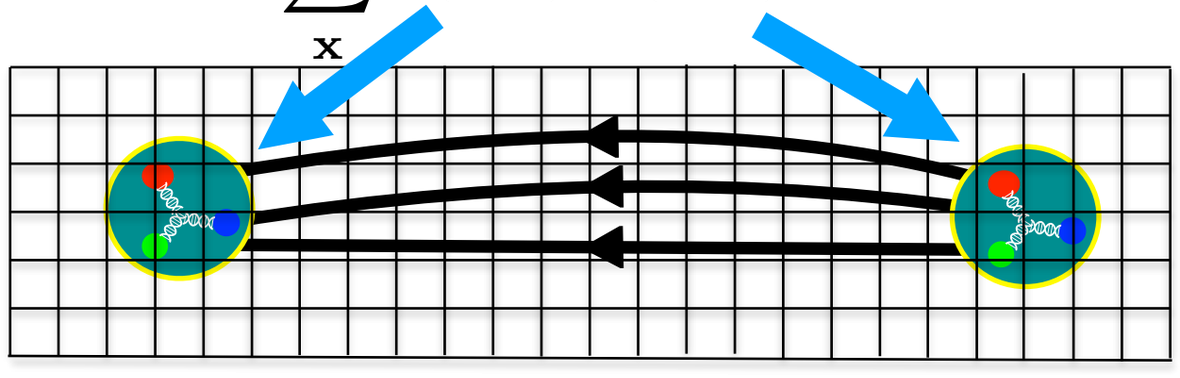
3 dominant matrix elements $\mathcal{M}_1,\ \mathcal{M}_2,\ \mathcal{M}_3$

1 subdominant matrix element $\mathcal{M}_5$

The Neutron

Neutron correlation function: path integral with neutron source/sink separated in Euclidean (imaginary) time

$$G_n(t) = \int \mathcal{D}\overline{q}\mathcal{D}q\mathcal{D}U \; e^{-S_{QCD}} \sum_{\mathbf{x}} n(\mathbf{x},t) \; \overline{n}(\mathbf{0},0)$$



Path integrals evaluated by Monte Carlo sampling field configurations

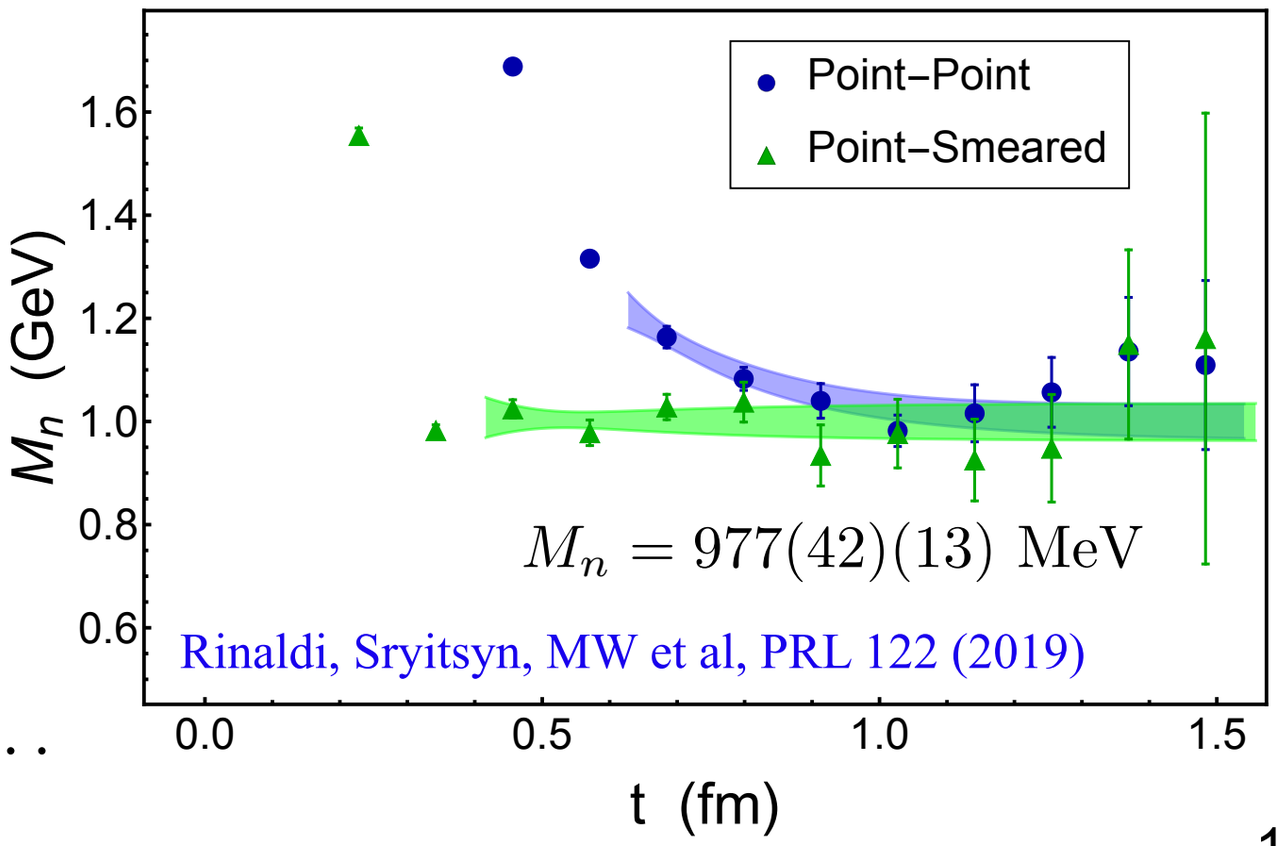
Physical quark mass configurations

Blum et al (RBC, UKQCD), PRD 93 (2016)

Neutron mass extracted by fitting to spectral representation

$$G_n(t) = Z_0 e^{-M_n t} + Z_1 e^{-(M_n+\delta_n)t} + \ldots$$

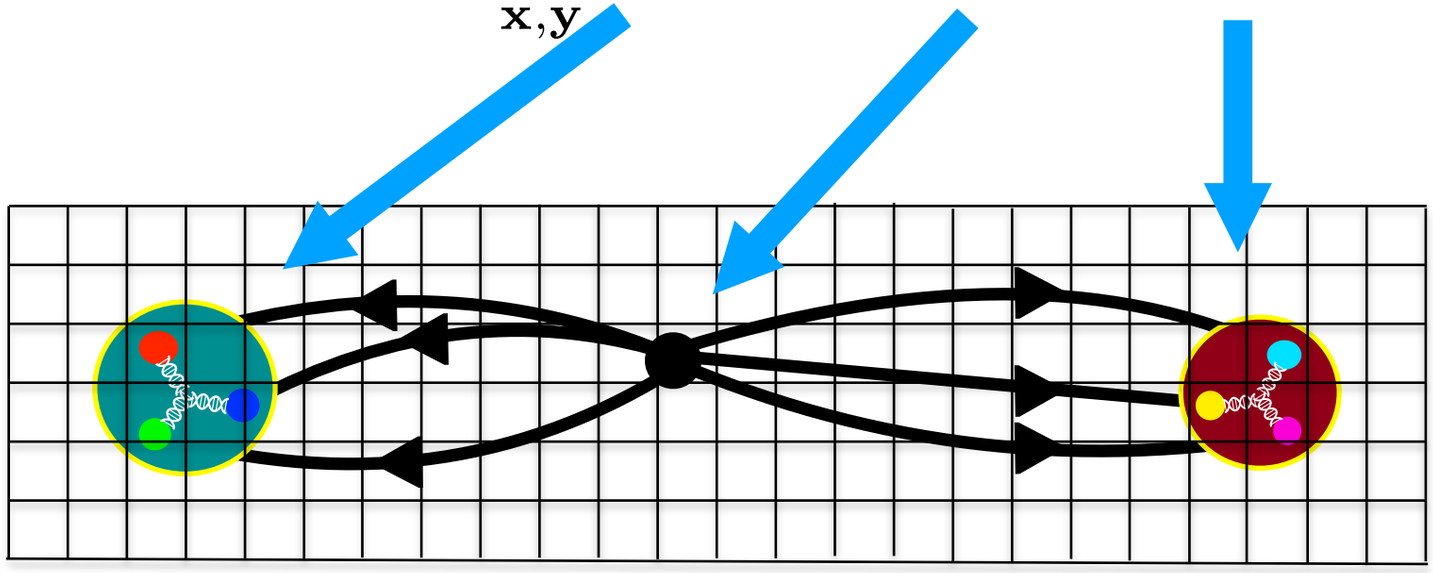
Effective mass: $M_n(t) = -\partial_t \ln G_n(t)$



$n\bar{n}$ Matrix Elements

$n\bar{n}$ correlation function includes Q_I between (anti)neutron source and sink

$$G_I^{n\bar{n}}(t,\tau) = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}U e^{-S_{QCD}} \sum_{\mathbf{x},\mathbf{y}} n(\mathbf{x}, t-\tau)\; Q_I^\dagger(0)\; n(\mathbf{y}, -\tau)$$



Rinaldi, Sryitsyn, MW et al, PRL 122 (2019) Rinaldi, Sryitsyn, MW et al, PRD 99 (2019)

Ratio of $n\bar{n}$ and neutron correlation functions gives matrix elements plus excited state effects

$$\frac{G_I^{n\bar{n}}(t,\tau)}{G_n(t)} = \mathcal{M}_I + \mathcal{A}_I e^{-\delta_n t} + \mathcal{B}_I e^{-\delta_n \tau} + \mathcal{C}_I e^{-\delta_n (t-\tau)}$$

Non-Perturbative Renormalization

Lattice regularized operators require renormalization

RI/MOM scheme convenient for LQCD matrix elements

Martinelli et al, Nucl Phys B445 (1995)

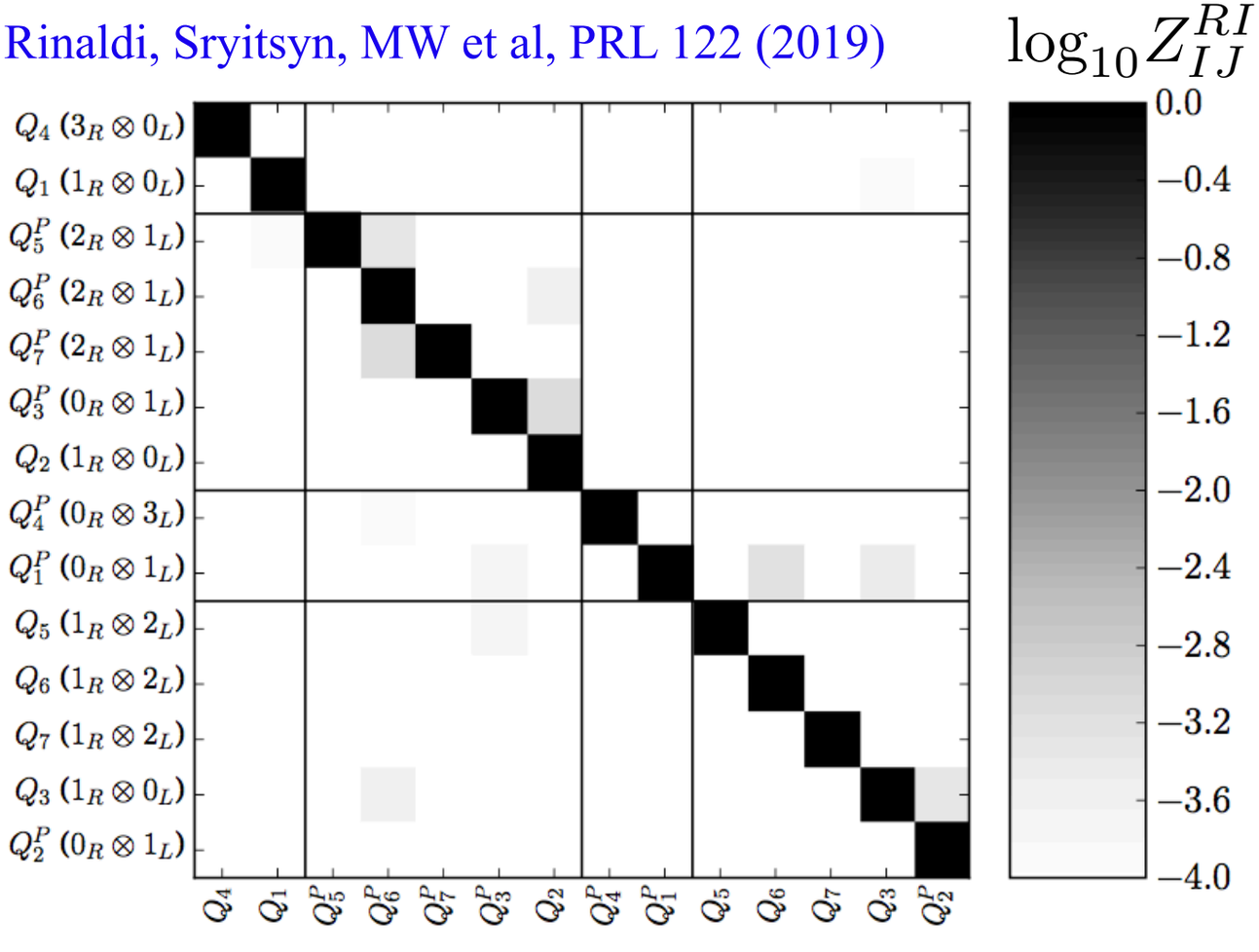
$$Z_q^{RI}(\mu)^3 Z_{IJ}^{RI}(\mu)\Lambda_J(\mu) = \Lambda_I^{tree}(\mu)$$

Quark field renormalization

Blum et al (RBC, UKQCD), PRD 93 (2016)

Vertex function

Rinaldi, Sryitsyn, MW et al, PRL 122 (2019)



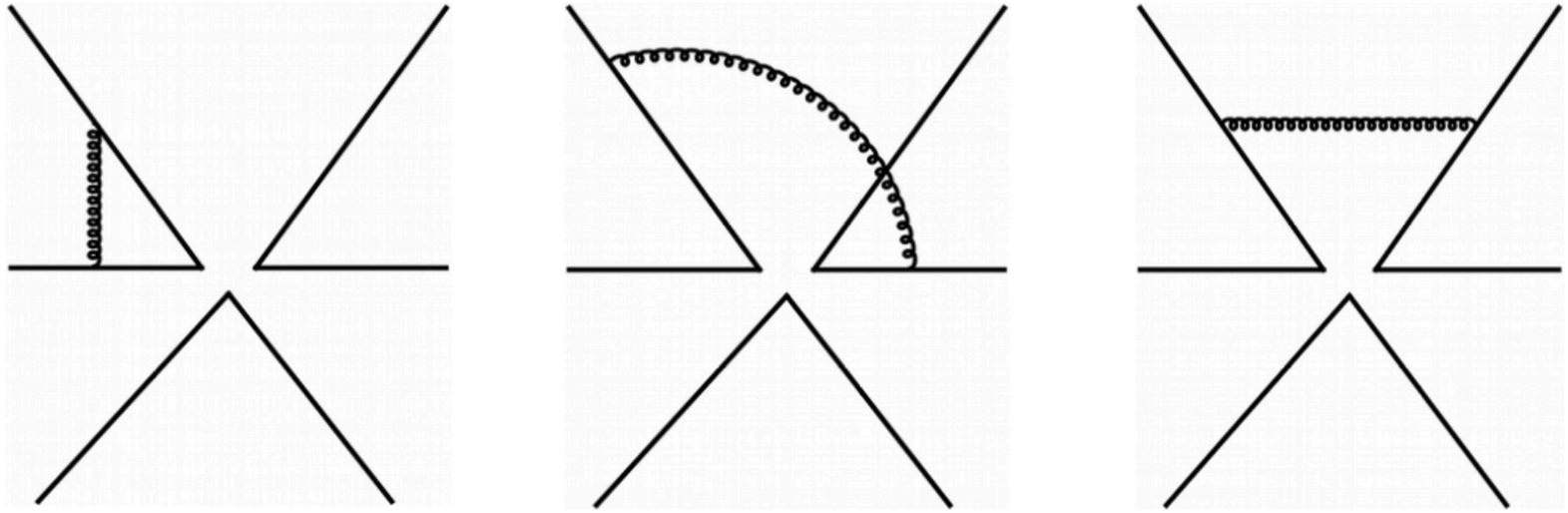
Matrix of renormalization factors diagonal up to 10^{-3}

Negligible mixing from quark mass effects, lattice artifacts, non-perturbative $U_1(A)$ violation

One-Loop Matching

One-loop running (leading log): Caswell and Milutinovic, Phys. Lett. 122B (1983)

One-loop matching (next-to-leading-log) from finite parts of computed using same RI/MOM scheme as non-perturbative renormalization



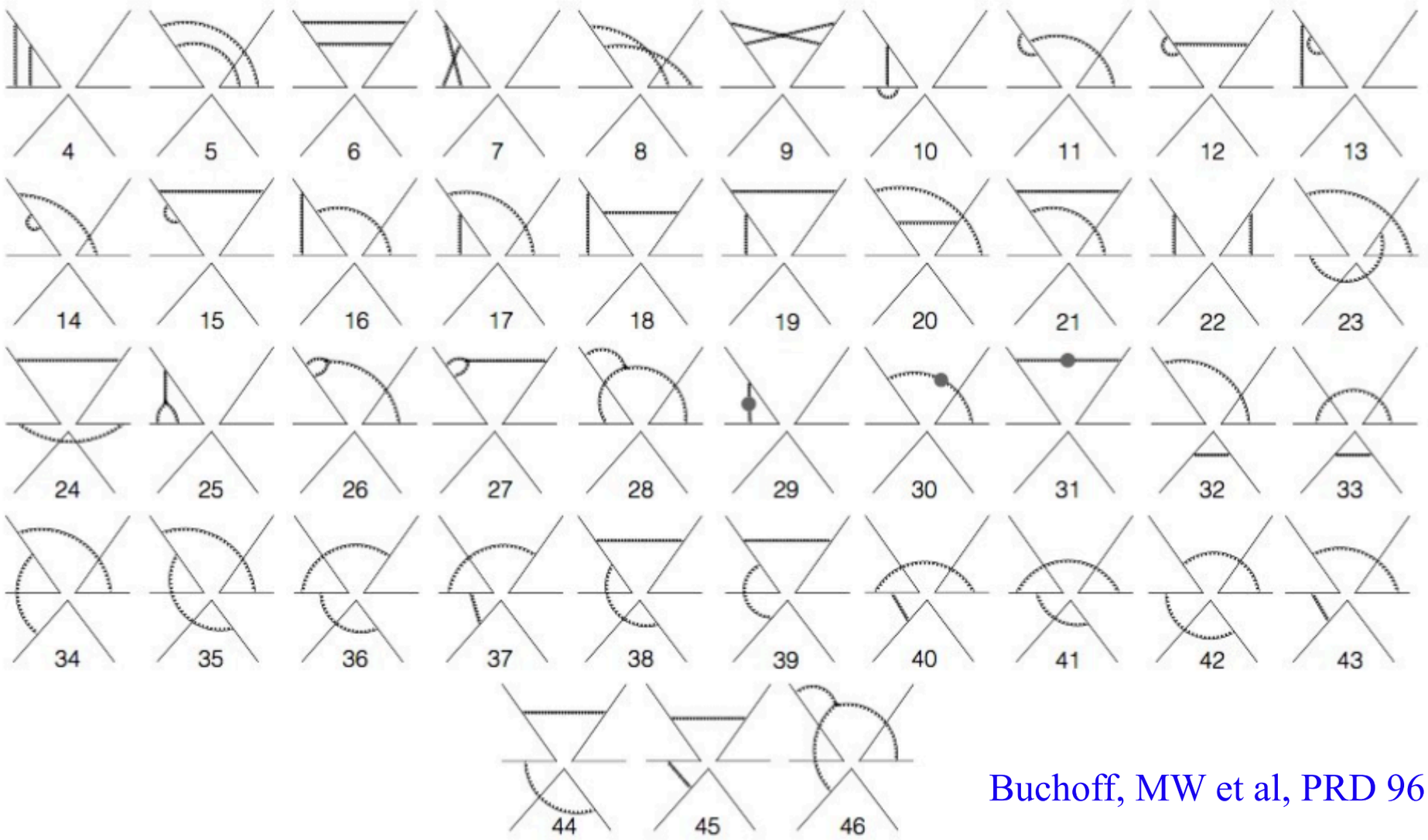
15 one-loop diagrams in 3 topologies Buchoff, MW et al, PRD 96 (2016)

All topologies identical to four-quark weak matrix element diagrams with a pair of spectator quarks, integrals can be checked against literature

Two-Loop Running

Two-loop running (next-to-leading-log)

350 two-loop diagrams in 46 topologies. Several new to literature



Buchoff, MW et al, PRD 96 (2016)

Evanescent operators (zero in D=4 but present in dimensional regularization) lead to non-flavor-blind counterterms unusual in QCD

$n\bar{n}$ Error Budget (Q_1)

• Statistics Rinaldi, Sryitsyn, MW et al, PRL 122 (2019)	• 27% uncertainty
• Excited states Rinaldi, Sryitsyn, MW et al, PRL 122 (2019)	• 4% uncertainty
• Discretization effects	• 3% estimated uncertainty
• Finite volume Bijnens and Kofoed, Eur Phys J C 77 (2017)	• 1% estimated uncertainty
• Quark mass dependence Rinaldi, Sryitsyn, MW et al, PRL 122 (2019)	• 1% estimated uncertainty (isospin breaking)
• Non-perturbative renormalization Rinaldi, Sryitsyn, MW et al, PRL 122 (2019)	• 57% effect
• 1-loop RG evolution Caswell and Milutinovic, Phys. Lett. 122B (1983)	• 36% effect $\mu = 2\ \mathrm{GeV} \rightarrow \Lambda_{BSM} = 700\ \mathrm{TeV}$
• 2-loop RG evolution Buchoff, MW et al, PRD 96 (2016)	• 9% effect $\mu = 2\ \mathrm{GeV} \rightarrow \Lambda_{BSM} = 700\ \mathrm{TeV}$
• 1-loop renorm scheme matching Buchoff, MW et al, PRD 96 (2016)	• 8% effect

QCD $n\bar{n}$ Results

Rinaldi, Sryitsyn, MW et al, PRL 122 (2019)

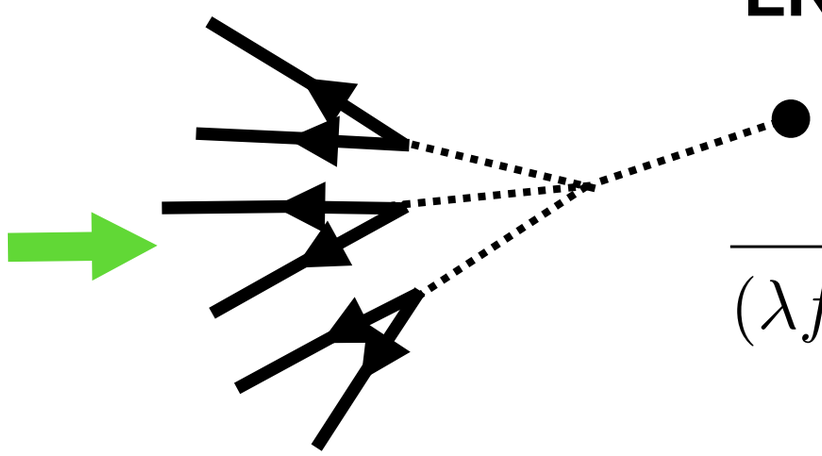
	$\mathcal{M}_I^{\overline{\rm MS}}(700\ {\rm TeV})\ [10^{-5}\ {\rm GeV}^6]$
Q_1	$-26(7)$
Q_2	$144(26)$
Q_3	$-47(11)$
Q_5	$-0.23(10)$

Standard Model EFT:

$$\tau_{n\text{-}\bar{n}}^{-1} = \frac{10^{-9}\ {\rm s}^{-1}}{(700\ {\rm TeV})^{-5}} \left| 4.2(1.1)\widehat{C}_1^{\overline{\rm MS}}(\mu) - 8.6(1.5)\widehat{C}_2^{\overline{\rm MS}}(\mu) + 4.5(1.1)\widehat{C}_3^{\overline{\rm MS}}(\mu) + 0.096(43)\widehat{C}_5^{\overline{\rm MS}}(\mu) \right|_{\mu=2\ {\rm GeV}}$$

ILL:

$$\tau_{n\bar{n}} > 0.89 \times 10^8\ {\rm s}$$



LR-symmetric example:

$$\frac{\Lambda_{BSM}}{\left(\lambda f^3 \widetilde{v}_{B-L}\right)^{1/5}} > 390 \pm 22\ {\rm TeV}$$

Experimental Implications

Rinaldi, Sryitsyn, MW et al, PRL 122 (2019) Rao, Shrock, Nucl. Phys. B 232 (1984)

	$\mathcal{M}_I^{\overline{\rm MS}}(700\ {\rm TeV})\ [10^{-5}\ {\rm GeV}^6]$	MIT Bag $\times$ RG $[10^{-5}\ {\rm GeV}^6]$
Q_1	$-26(7)$	$-6.4,\ -5.2$
Q_2	$144(26)$	$16,\ 19$
Q_3	$-47(11)$	$-9.1,\ -7.6$
Q_5	$-0.23(10)$	$-0.28,\ 0.15$

For fixed BSM parameters, QCD predicts experimental sensitivity is **25 - 64 times higher** than predicted using MIT bag model

$$N_{events} \propto \tau_{n\overline{n}}^{-2} \approx \left(\sum_{I=1}^{3} \widehat{C}_I^{\overline{\rm MS}}(\Lambda_{BSM}) \mathcal{M}_I^{\overline{\rm MS}}(\Lambda_{BSM}) \right)^2$$

For $SU(2)_L \times SU(2)_R \times SU(4)_C$ BSM Higgs mass/coupling lower bound from ILL **390 TeV** instead of **290 TeV**

Deuteron decay searches

Large underground volume detectors can constrain $\tau_{n\overline{n}}$

— Given nuclear suppression factor R_A

$$\Gamma_A^{-1} = R_A \tau_{n\overline{n}}^2$$

Deuteron suppression computed in chiral EFT to NLO

$R_d = (1.1 \pm 0.3) \times 10^{22} s^{-1}$ Oosterhof, Long, de Vries, Timmermans, van Kolck, PRL 122 (2019)

SNO constraint: $\Gamma_d^{-1} > 1.18 \times 10^{31}$ years

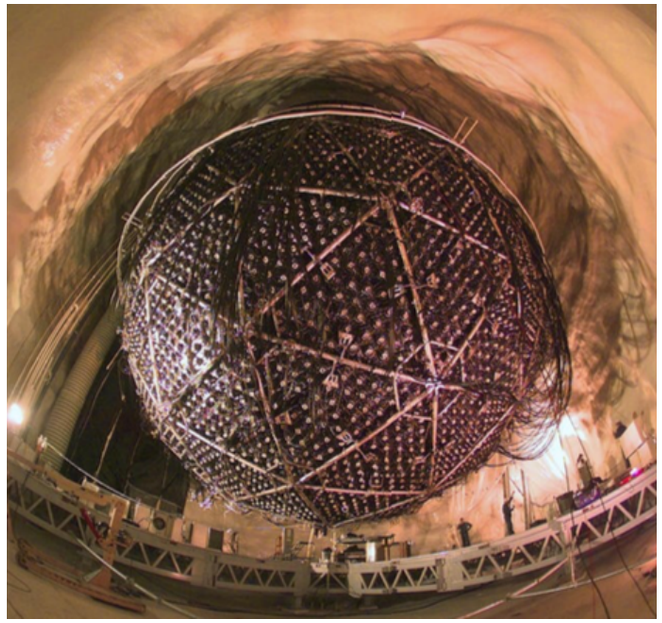
Aharmin et al (SNO), PRD 96 (2017)

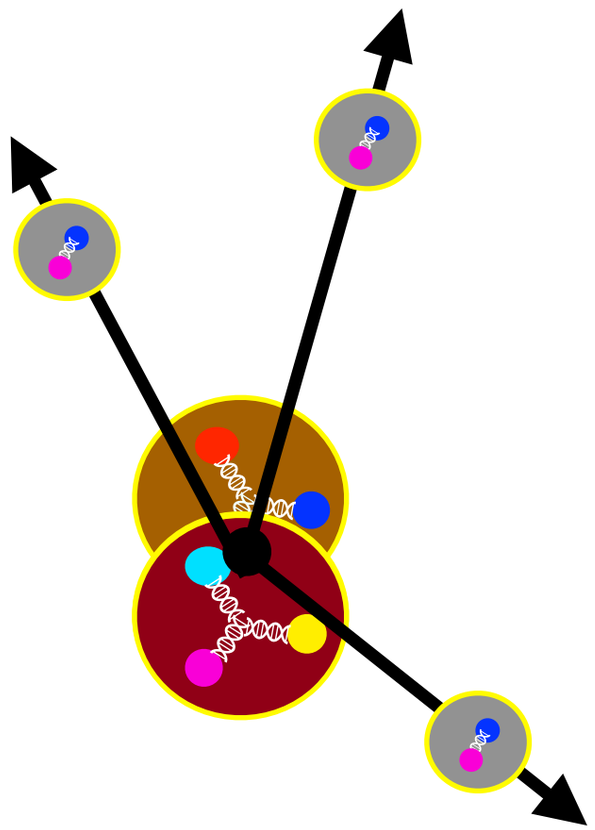
SNO + ChEFT: $\tau_{n\overline{n}} > 1.6 \times 10^8$ s

SNO + ChEFT + QCD: $\dfrac{\Lambda_{BSM}}{(\lambda f^3 \widetilde{v}_{B-L})^{1/5}} > 439$ TeV

Rinaldi, Sryitsyn, MW et al, PRL 122 (2019)

SNO





Oxygen decay searches

Super K extracts stronger but more theoretically uncertain bounds from oxygen decay

Nuclear optical potential model

$$R_O \approx 5.17 \times 10^{22}\ \mathrm{s}^{-1}$$ Friedman, Gal, PRD 78 (2008)

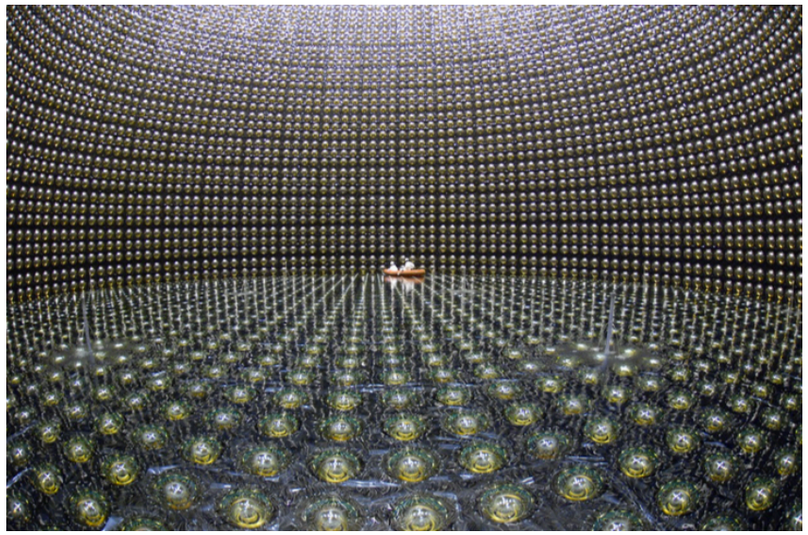
Super K constraint $\Gamma^{-1}_{O^{16}} > 19 \times 10^{31}\ \mathrm{years}$

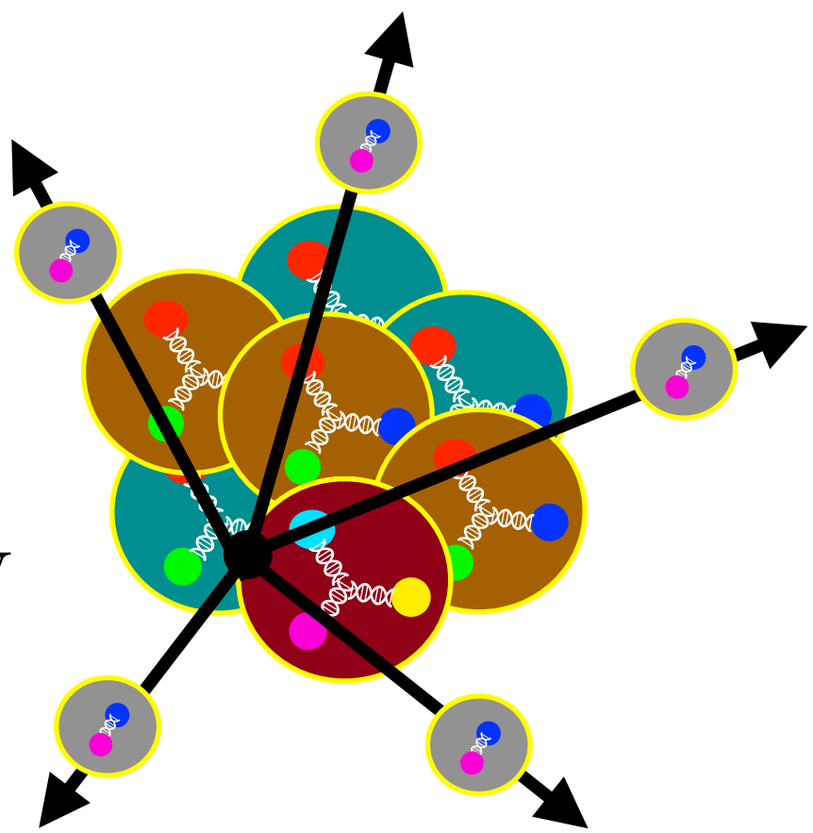
Abe et al (Super K), PRD 91 (2015)

Super K & optical potential $\tau_{n\bar{n}} \gtrsim 2.7 \times 10^{8}\ \mathrm{s}$

Super K & optical potential & QCD: $\dfrac{\Lambda_{BSM}}{(\lambda f^3 \widetilde{v}_{B-L})^{1/5}} \gtrsim 480\ \mathrm{TeV}$

Rinaldi, Sryitsyn, MW et al, PRL 122 (2019)





Experimental Outlook

Future experiments at ESS could improve direct bounds on $\tau_{n\bar{n}}$ by 10-100

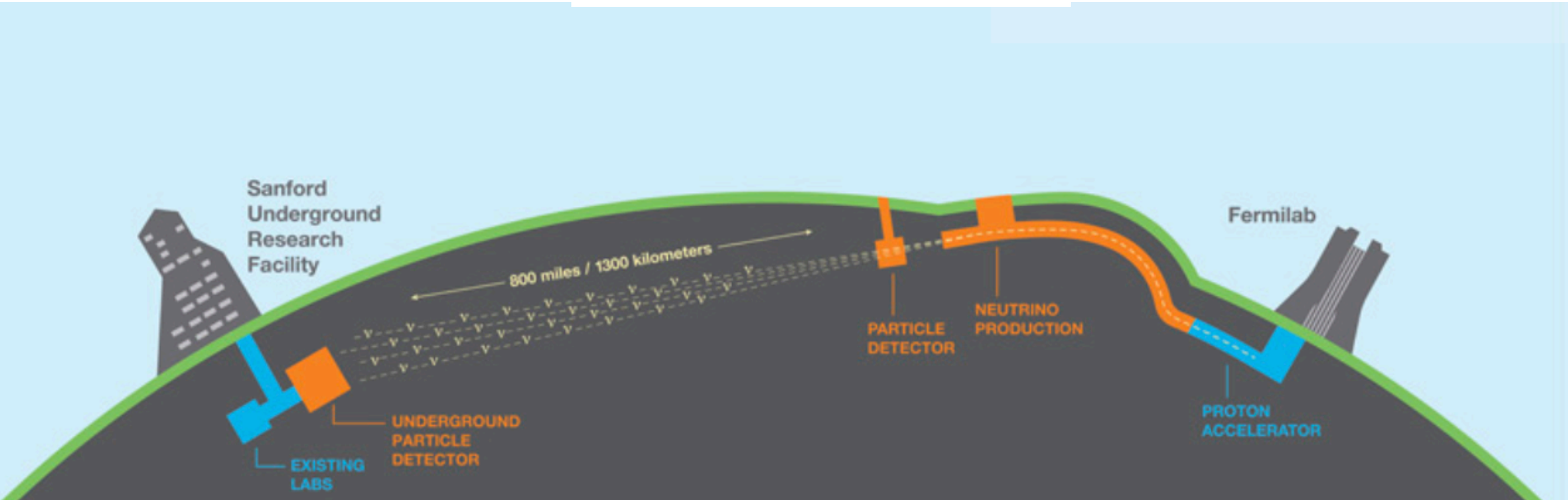
DUNE and Hyper-K will probe *B* violation in nuclei at even higher BSM scales

Argon nuclear suppression factor computed in optical potential model

Barrow et al, arXiv:1906.02833

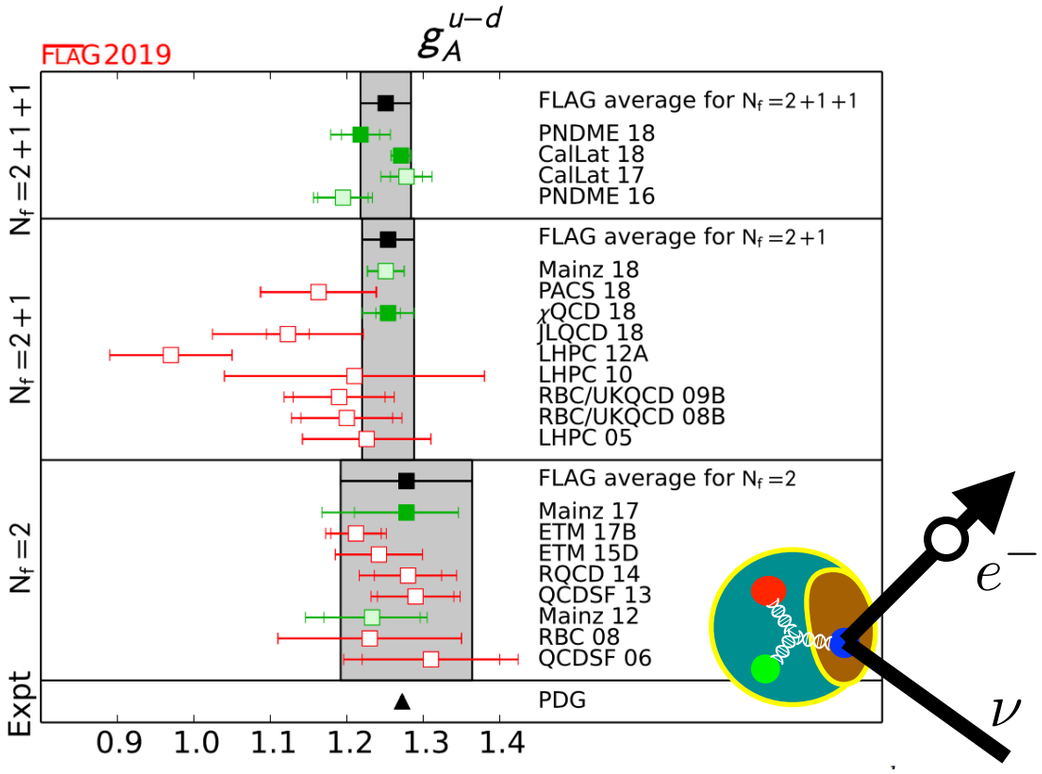
$R_{Ar} \sim 5.6 \times 10^{22}\ \mathrm{s}^{-1}$

See talk/poster by Joshua Barrow



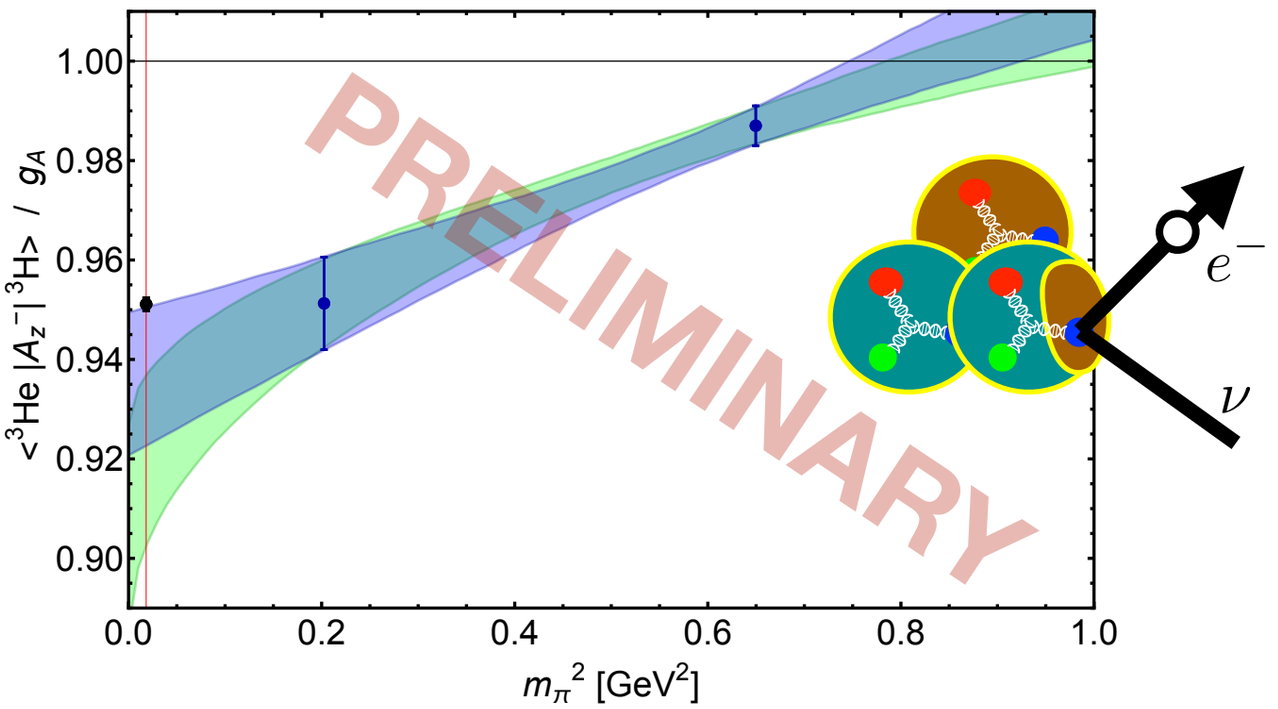
Towards *B-L* Violation in Nuclei

$$n \to p\, e^-\, \overline{\nu}_e$$



Aoki et al [FLAG], arXiv:1902.08191

$$^3\mathrm{H} \to {}^3\mathrm{He}\, e^-\, \overline{\nu}_e$$

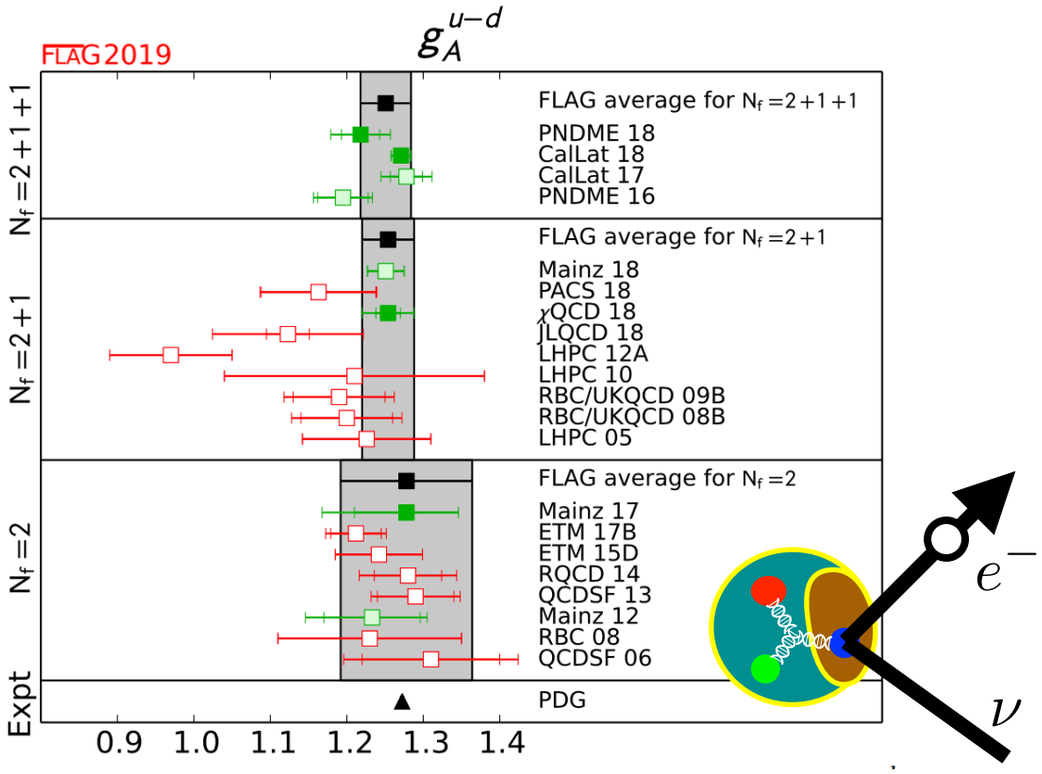


Savage et al [NPLQCD], PRL 119 (2017)

NPLQCD, *in preparation*

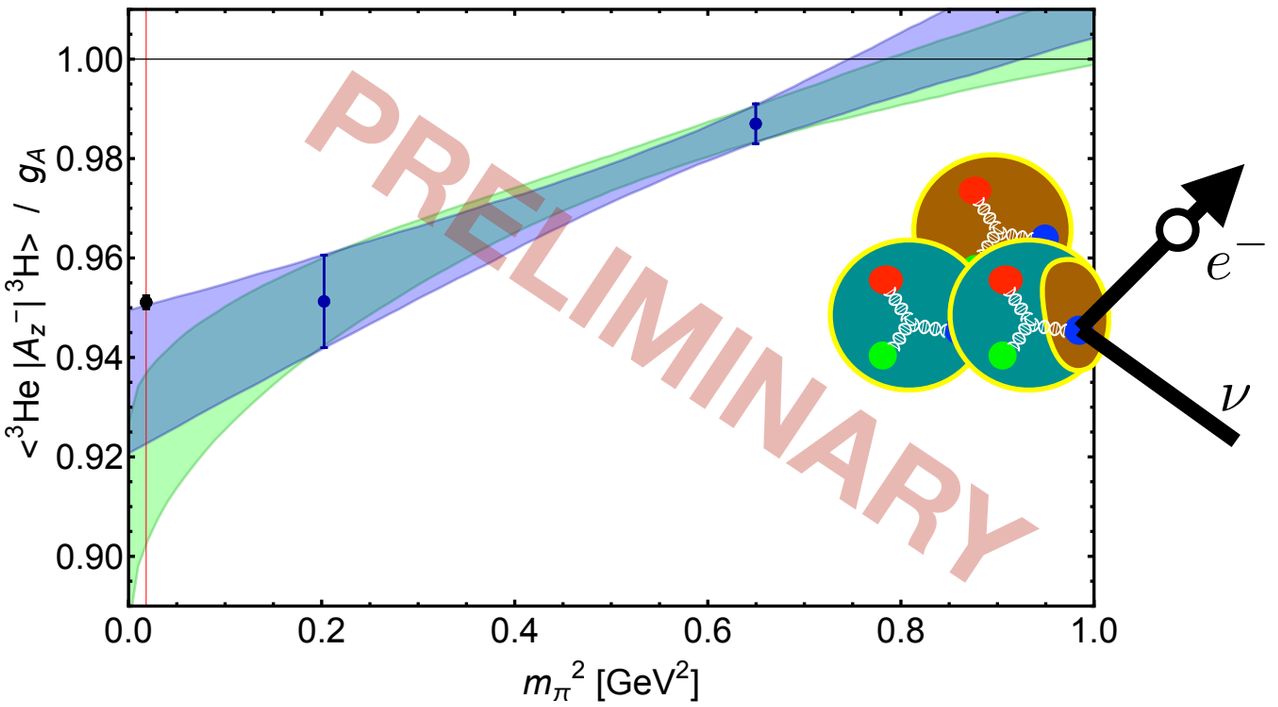
Towards *B-L* Violation in Nuclei

$$n \to p\, e^-\, \overline{\nu}_e$$



Aoki et al [FLAG], arXiv:1902.08191

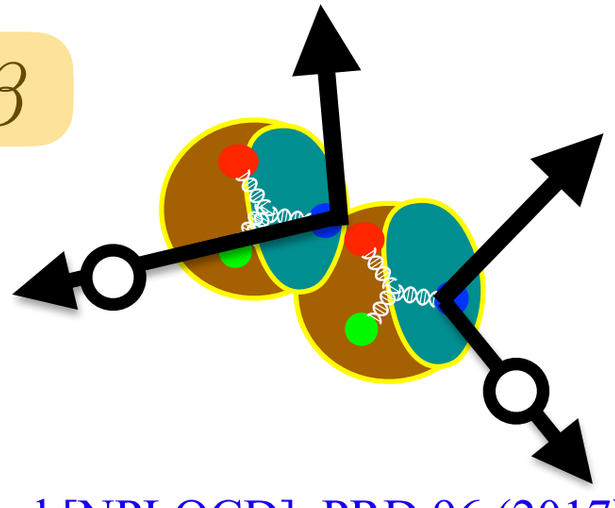
$$^3\mathrm{H} \to {}^3\mathrm{He}\, e^-\, \overline{\nu}_e$$



Savage et al [NPLQCD], PRL 119 (2017)

NPLQCD, *in preparation*

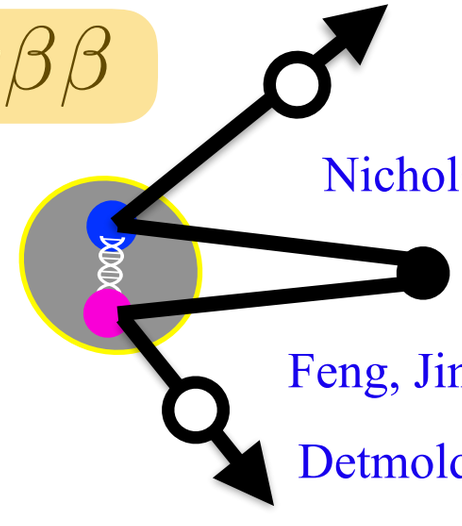
$$2\nu\beta\beta$$



Tiburzi et al [NPLQCD], PRD 96 (2017)

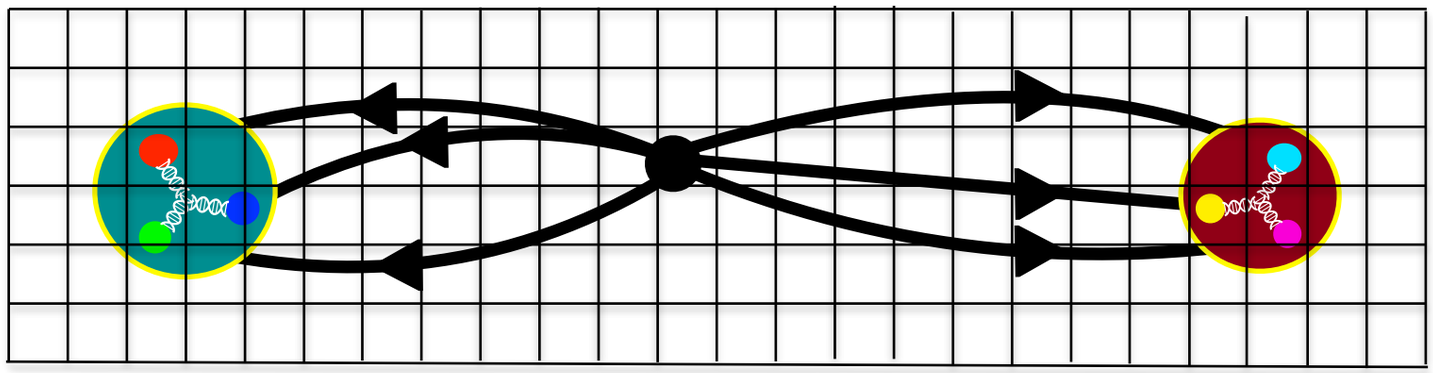
Shanahan et al [NPLQCD], PRL 119 (2017)

$$0\nu\beta\beta$$



Nicholson et al, PRL 121 (2018)

Feng, Jin, Tuo, Xia, PRL 122 (2019)

Detmold, Murphy, LATTICE 2018

Questions