Generalised CP Symmetry in Modular-Invariant Models of Flavour

in collaboration with J. T. Penedo, S. T. Petcov and A. V. Titov

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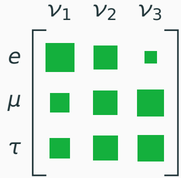
Modular invariance for flavour

Why modular invariance?

- Naturally leads to a discrete flavour symmetry
- Discrete flavour symmetry can explain 2 large + 1 small lepton mixing angles
- More predictive than conventional discrete symmetry approach: avoids multiple new scalar fields, baroque potentials and shaping symmetries

$|U_{PMNS}| =$

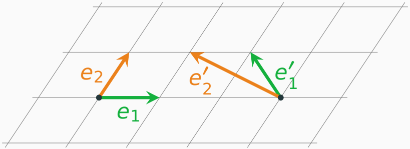


Modular transformations

modulus

$$\tau = \frac{e_2}{e_1}$$

$$\tau' = \frac{e'_2}{e'_1}$$



$$\tau' = \frac{a\tau + b}{c\tau + d}$$

modular group $\Gamma = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \begin{matrix} a, b, c, d \in \mathbb{Z}, \\ ad - bc = 1 \end{matrix} \right\}$

Modular invariance for flavour

Transformation of fields

$$\phi_i' = (c\tau + d)^{-k} \; \rho\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]_{ij} \; \phi_j$$

weight ("charge") — $(c\tau + d)^{-k}$

unitary representation of Γ_N — $\rho\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]_{ij}$

Γ_N is a quotient of Γ

N	2	3	4	5
Γ_N	S_3	A_4	S_4	A_5

discrete symmetry group

Modular-invariant interactions

$Y(\tau)$ → $(c\tau + d)^k \rho$

$\phi_1 \, \phi_2 \, \ldots \, \phi_n$ → $(c\tau + d)^{-(k_1+k_2+\ldots+k_n)} \rho_1 \otimes \rho_2 \otimes \ldots \otimes \rho_n$ × itself

modular form of weight k and level N

$$k = k_1 + k_2 + .. + k_n$$
$$\rho \otimes \rho_1 \otimes \rho_2 \otimes \ldots \otimes \rho_n \supset \mathbf{1}$$

N \ k	0	2	4	6
2 (S_3)	1	2	3	4
3 (A_4)	1	3	5	7
4 (S_4)	1	5	9	13
5 (A_5)	1	11	21	31

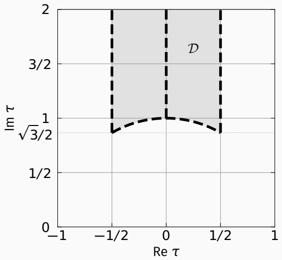
CP + modular invariance

Why combine modular invariance with CP invariance?

- Observed CP violation is related to the flavour structure (3 generations)
- CP symmetry arises together with modular symmetry in models with extra dimensions
- Further increase of predictive power

Consistency condition $CP^{-1} \circ \Gamma \circ CP = \Gamma$ yields:

- $\tau \xrightarrow{CP} -\tau^*$ (without loss of generality)
- $\phi_i(t, \vec{x}) \xrightarrow{CP} \phi_i^\dagger(t, -\vec{x})$ (in some basis of Γ_N)
- $Y_i(\tau) \xrightarrow{CP} Y_i(-\tau^*) = Y_i(\tau)^*$
- CP invariance $\Leftrightarrow g = g^*$ (less parameters)
- CP is conserved if $\tau \in \partial\mathcal{D}$, or $\mathrm{Im}\tau = 0$
- Otherwise, both CP and flavour symmetry are broken by the VEV of τ



Example: CP-invariant S_4 model

$$L \sim (\mathbf{3}, 2)\,, \qquad E^c \sim (\mathbf{1}', 0) \oplus (\mathbf{1}, 2) \oplus (\mathbf{1}', 2)\,, \qquad N^c \sim (\mathbf{3}, 0)$$

$$\begin{aligned} W = {} & \alpha \left(E_1^c L\, Y_{\mathbf{3}'}^{(2)}\right)_{\mathbf{1}} H_d + \beta \left(E_2^c L\, Y_{\mathbf{3}}^{(4)}\right)_{\mathbf{1}} H_d + \gamma \left(E_3^c L\, Y_{\mathbf{3}'}^{(4)}\right)_{\mathbf{1}} H_d \\ & + \boxed{g} \left(N^c L\, Y_{\mathbf{2}}^{(2)}\right)_{\mathbf{1}} H_u + \boxed{g'} \left(N^c L\, Y_{\mathbf{3}'}^{(2)}\right)_{\mathbf{1}} H_u + \Lambda \left(N^c N^c\right)_{\mathbf{1}} \end{aligned}$$

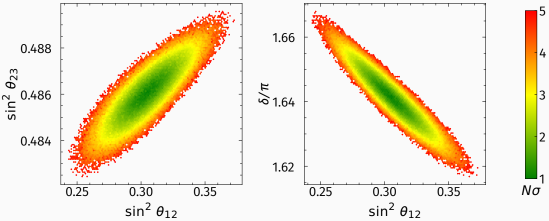
- CP invariance condition: $\boxed{\mathrm{Im}\left(g'/g\right) = 0}$
- 7 parameters to fit 8 observables, $N\sigma = 1.012$
- Predictions at a few percent level:

$$m_1 = 0.012 \text{ eV}, \quad m_2 = 0.015 \text{ eV}, \quad m_3 = 0.051 \text{ eV},$$
$$\Sigma m = 0.078 \text{ eV}, \quad |\langle m \rangle| = 0.012 \text{ eV},$$
$$\delta/\pi = \pm 1.64, \quad \alpha_{21}/\pi = \pm 0.35, \quad \alpha_{31}/\pi = \pm 1.25$$

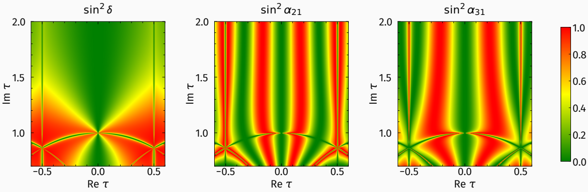
- $\tau \approx 0.099 + i\,1.016$ breaks both CP and flavour symmetry

Example: CP-invariant S_4 model

Correlations



CP-conserving values of τ



Summary

- Modular symmetry can be combined with CP in a unique way
- τ is the only source of CPV and flavour symmetry breaking
- Modular + CP allows to construct predictive models of flavour

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Thank you!