# NLO and FSR NNLO radiative corrections for Drell–Yan processes at LHC

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20th Annual RDMS CMS Collaboration Conference
Tashkent, September 12 – 15, 2018

# Outline

## NLO

- Notations
- EWK Boson Self Energies. EWK/QCD Vertices
- EWK Boxes. Asymptotic Approach
- Real photon/gluon and inverse gluon bremsstrahlung
- $M$- and $G/N$- methods
- Rebuilding to fully differential cross section. Code READY
- Independence from unphysical parameters
- Comparison with existing results on hadronic level
- Numerical estimations at CMS setup

## NNLO

- Approaches to NNLO radiative corrections
- Estimation of FSR NNLO RC: Soft (naive) Approach
- Hard (real) Approach

# Introduction

Despite the fact that the Standard Model (SM) keeps for oneself the status of consistent and experimentally confirmed theory, the search of New Physics (NP) manifestations is continued. The possible traces of NP can be

- the supersymmetry,
- extra spatial dimensions,
- extra neutral gauge bosons, etc.

One of powerful tool in the modern experiments at LHC from this point of view is the investigation of **Drell–Yan lepton-pair production**:,

$$\mathbf{pp} \to \gamma, \mathbf{Z} \to \mathbf{l^+l^-X} \tag{1}$$

at **large invariant mass** of lepton-pair: $\mathbf{M} \geq \mathbf{1}$ TeV.

# Drell-Yan process (1970, BNL)

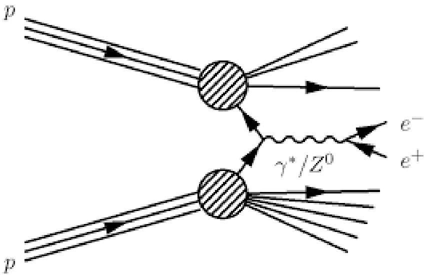


Рис. 1: **Drell-Yan process with neutral current ($\gamma$/Z)**

- $\sqrt{\mathbf{S}}$ is total energy in c.m.s. of hadrons
- **M** is dilepton $\mathbf{l}^+\mathbf{l}^-$ invariant mass ($\mathbf{l} = \mathbf{e}, \mu$)
- **y** is dilepton rapidity

# Pioneer papers on RC to DY process

QED RC:

- **V. Mosolov, N. Shumeiko**, Nucl. Phys. B **186**, 394 (1981);
- **A. Soroko, N. Shumeiko**, Yad. Fiz. **52**, 514 (1990).

Рис. 2: *Nikolai Maximovich Shumeiko (1942–2016)*

# Pioneer papers on EWK and QCD RC to DY process

- EWK RC:
  **U. Baur, et al. (ZGRAD)**, Phys.Rev.D **65**: 033007, (2002).
- QCD NLO RC:
  **H. Baer, et al.**, Phys.Rev.D 40, 2844 (1989); Phys.Rev.D **42**, 61 (1990).
- QCD NNLO RC:
  **R. Hambert, W.L. van Neerven, T. Matsuura**, Nucl.Phys.B **359**, 343 (1991).

# Some modern codes for NLO and NNLO RC for DY process at hadronic colliders (in the ABC order)

- DYNNLO (S. Catani, L. Cieri, G. Ferrera et. al)
- FEWZ (R. Gavin, Y. Li, F. Petriello, S. Quackenbush et. al)
- HORACE (C.Carloni Calame, G.Montagna, O.Nicrosini et. al)
- LPPG (E. Dydyshka, V. Yermolchyk)
- MC@NLO (S. Frixione, F. Stoeckli, P. Torrielli et. al)
- PHOTOS (N. Davidson, T. Przedzinski, Z. Was et al.)
- POWHEG (L. Barze, G. Montagna, P. Nason et. al)
- RADY (S. Dittmaier, A. Huss, C. Schwinn et. al)
- READY (V. Zykunov, RDMS CMS)
- SANC (Dubna group: A.Andonov, A.Arbuzov, D.Bardin et.al)
- WINHAC (W. Placzek, S. Jadach, M.W. Krasny et. al)
- WZGRAD (U. Baur, W. Hollik, D. Wackeroth et al.)

# Current experimental situation at CMS LHC

- The measured Drell–Yan cross sections and forward-backward asymmetries **are consistent with the SM predictions** at

$$\sqrt{\mathbf{S}} = 8 \text{ TeV } (\mathbf{19.7\ fb^{-1}}) \text{ for } \mathbf{M} \leq \mathbf{2} \text{ TeV},$$

$$\sqrt{\mathbf{S}} = 13 \text{ TeV } (\mathbf{85\ fb^{-1}}) \text{ for } \mathbf{M} \leq \mathbf{3} \text{ TeV}$$

  - differential $\frac{\mathbf{d}\sigma}{\mathbf{dM}}$ cross sections,
  - double-differential $\frac{\mathbf{d}^2\sigma}{\mathbf{dMdy}}$ cross sections,
  - $\mathbf{A_{FB}}$ asymmetries.

- The latest published results can be found in

  CMS PAS-SMP-16-009, CMS PAS-SMP-17-001

  (PAS = Physics Analysis Summaries)

- NNLO RC are taken into account by using of **FEWZ 3.1**
- NNLO PDFs are **CT10 NNLO** and **NNPDF2.1**.

# Mathematical Content

At the edges of kinematical region (extra large $\sqrt{\mathbf{S}}$, **M**) the important task is make the correction procedure of background both accurate and fast. For the latter it is desirable to obtain the set of **compact** formulas for the EWK and QCD RC.

To get leading effect of **Weak RC** in the region of large invariant dilepton mass we used the Sudakov Logarithms (**SL**):

$$\mathbf{l_{i,x}} = \ln \frac{\mathbf{m_i^2}}{|\mathbf{x}|} \quad (\mathbf{i} = \mathbf{Z}, \mathbf{W}; \quad \mathbf{x} = \mathbf{s}, \mathbf{t}, \mathbf{u}), \tag{2}$$

**V. Sudakov, Sov. Phys. JETP 3, 65 (1956)**.

Collinear Logarithms (**CL**) play leading role in **QED RC and QCD RC**

$$\ln \frac{\mathbf{m_f^2}}{|\mathbf{x}|} \quad (\mathbf{f} = \mathbf{e}, \mu, \mathbf{q}; \quad \mathbf{x} = \mathbf{s}, \mathbf{t}, \mathbf{u}). \tag{3}$$

# Notations and Born amplitude

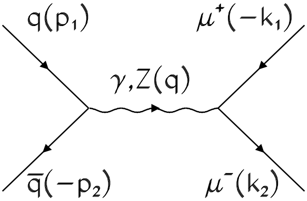


Рис. 3: **The lowest order graph giving contribution to the DY scattering at parton level**

The standard set of Mandelstam invariants for the partonic elastic scattering:

$$\mathbf{s} = (\mathbf{p_1} + \mathbf{p_2})^2,\ \mathbf{t} = (\mathbf{p_1} - \mathbf{k_1})^2,\ \mathbf{u} = (\mathbf{k_1} - \mathbf{p_2})^2. \qquad (4)$$

# **Convolution formula** for Born and $V$-contribution

$$\sigma_V^H = \frac{1}{3}\int d^3\Gamma \sum_{q=u,d,s,c,b} \theta_K\theta_M\theta_D [f_q^A(x_1,Q^2) f_{\bar q}^B(x_2,Q^2)\sigma_V^{q\bar q}(t) +$$

$$+f_{\bar q}^A(x_1,Q^2) f_q^B(x_2,Q^2)\sigma_V^{\bar q q}(t)], \quad \int d^3\Gamma[...] = \int\limits_0^1 dx_1 \int\limits_0^1 dx_2 \int\limits_{-S}^0 dt[...],$$

where $\mathbf{V} = \{\mathbf{0}, \mathrm{BSE}, \mathrm{LV}, \ \mathrm{HV}, \mathbf{b}, \mathrm{fin}\}$, $\ \mathbf{b} = \{\gamma\gamma, \gamma \mathbf{Z}, \mathbf{ZZ}, \mathbf{WW}\}$.

$\theta_K = \theta(s+t)$, $\ \theta_M$, $\ \theta_D$ are kinematical factors.

The propagator for $\mathbf{j}$-boson depends on its mass and width:

$$D^{js} = \frac{1}{s - m_j^2 + i m_j \Gamma_j}. \qquad (5)$$

# Born cross section and coupling constants

Born cross section looks like

$$\sigma_0^{q\bar{q}} = \frac{2\pi\alpha^2}{s^2} \sum_{i,j=\gamma,Z} D^i D^{j*} (b_+^{i,j} t^2 + b_-^{i,j} u^2), \tag{6}$$

where

$$b_\pm^{n,k} = \lambda_{q}{}_+^{n,k} \lambda_{l}{}_+^{n,k} \pm \lambda_{q}{}_-^{n,k} \lambda_{l}{}_-^{n,k}, \tag{7}$$

$$\lambda_{f}{}_+^{i,j} = v_f^i v_f^j + a_f^i a_f^j,\ \ \lambda_{f}{}_-^{i,j} = v_f^i a_f^j + a_f^i v_f^j, \tag{8}$$

$$v_f^\gamma = -Q_f,\ \ a_f^\gamma = 0,\ \ v_f^Z = \frac{I_f^3 - 2s_W^2 Q_f}{2s_W c_W},\ \ a_f^Z = \frac{I_f^3}{2s_W c_W}. \tag{9}$$

The Feynman rules from paper **M. Böhm, H. Spiesberger, W. Hollik, Forschr.Phys. 34 (1986) 687–751** were used.

# Main features of QCD RC and EWK RC calculation

- the t'Hooft–Feynman gauge,
- on-mass renormalization scheme ($\alpha, \alpha_s, \mathbf{m_W}, \mathbf{m_Z}, \mathbf{m_H}$ and the **fermion masses** as independent parameters),
- ultrarelativistic limit.

QCD result can be obtained from QED case by substitution:

$$\mathbf{Q_q^2}\alpha \to \sum_{\mathbf{a=1}}^{\mathbf{N^2-1}} \mathbf{t^a t^a} \alpha_\mathbf{s} = \frac{\mathbf{N^2-1}}{\mathbf{2N}} \mathbf{I} \alpha_\mathbf{s} \to \frac{\mathbf{4}}{\mathbf{3}} \alpha_\mathbf{s}, \qquad (10)$$

here $\mathbf{2t^a}$ – Gell-Man matrices, $\mathbf{N = 3}$.

# EWK Boson Self Energies

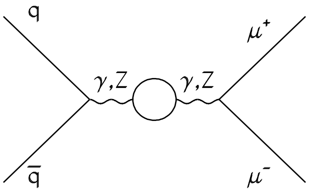


Рис. 4: $\gamma\gamma$-, $\gamma Z$- and $ZZ$-Self Energy diagrams

$$\begin{aligned}\sigma^{\mathbf{q\bar{q}}}_{\mathrm{BSE}} \quad = \quad & -\frac{\mathbf{4}\alpha^{\mathbf{2}}\pi}{\mathbf{s^2}}\Big[\sum_{\mathbf{i,j}=\gamma,\mathbf{Z}} \mathbf{\Pi^i D^i D^{j}}^{*} \sum_{\chi=+,-} \lambda_{\mathbf{q}\chi}^{\mathbf{i,j}}\lambda_{\mathbf{l}\chi}^{\mathbf{i,j}}(\mathbf{t^2}+\chi\mathbf{u^2})+ \\ & +\mathbf{\Pi}^{\gamma\mathbf{Z}}\mathbf{D^Z}\sum_{\mathbf{i}=\gamma,\mathbf{Z}} \mathbf{D^{j}}^{*} \sum_{\chi=+,-}(\lambda_{\mathbf{q}\chi}^{\gamma,\mathbf{j}}\lambda_{\mathbf{l}\chi}^{\mathbf{Z,j}}+\lambda_{\mathbf{q}\chi}^{\mathbf{Z,j}}\lambda_{\mathbf{l}\chi}^{\gamma,\mathbf{j}})(\mathbf{t^2}+\chi\mathbf{u^2})\Big]\end{aligned}$$

is connected with the renormalized $\gamma$–, $Z$– and $\gamma Z$–self energies as

$$\mathbf{\Pi}^{\gamma}=\frac{\hat{\mathbf{\Sigma}}^{\gamma}}{\mathbf{s}},\ \mathbf{\Pi^Z}=\frac{\hat{\mathbf{\Sigma}}^{\mathbf{Z}}}{\mathbf{s}-\mathbf{m_Z^2}},\ \mathbf{\Pi}^{\gamma\mathbf{Z}}=\frac{\hat{\mathbf{\Sigma}}^{\gamma\mathbf{Z}}}{\mathbf{s}}.$$

# Light and Heavy Vertices (EWK RC)

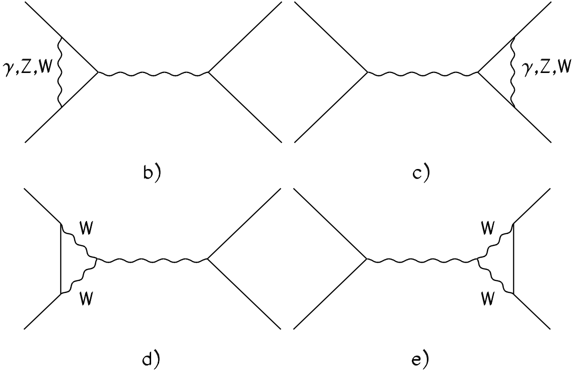


Рис. 5: **Feynman graphs for Vertices diagrams. Unsigned helix lines mean $\gamma$ or $Z$.**

# EW Form Factors

The results are presented as the **Form Factor set** to the Born vertices (as, for example, in **M. Böhm** *et al.*, **Fortschr. Phys. 34, 687 (1986)**, so we can easily use they to construct the cross section: all that we need is to replace the coupling constants in Born vertex to the corresponding form factors:

$$\mathbf{v_f^j} \to \delta \mathbf{F_V^{jf}}, \mathbf{a_f^j} \to \delta \mathbf{F_A^{jf}}. \qquad (11)$$

Electroweak **form factors** $\delta \mathbf{F^{if}_{V,A}}$ in ultrarelativistic limit depend on the Sudakov logarithms by means of functions $\mathbf{\Lambda_{2,3}(m_i)}$ as:

$$\mathbf{\Lambda_2(m_i)} = \frac{\pi^2}{3} - \frac{\mathbf{7}}{\mathbf{2}} - \mathbf{3l_{i,s}} - \mathbf{l^2_{i,s}}, \quad \mathbf{\Lambda_3(m_i)} = \frac{\mathbf{5}}{\mathbf{6}} - \frac{\mathbf{1}}{\mathbf{3}}\mathbf{l_{i,s}}. \qquad (12)$$

Then Ver={HV, LV} contribution to cross section looks like

$$\sigma^{\mathbf{q\bar{q}}}_{\rm Ver} = \frac{\mathbf{4}\pi\alpha^{\mathbf{2}}}{\mathbf{s^2}} {\rm Re} \sum_{\mathbf{i,j}=\gamma,\mathbf{Z}} \mathbf{D^i D^j}^* \sum_{\chi=+,-} (\lambda^{\rm F}{}^{\mathbf{i,j}}_{\mathbf{q}\,\chi} \lambda_{\mathbf{l}}{}^{\mathbf{i,j}}_{\chi} + \lambda_{\mathbf{q}}{}^{\mathbf{i,j}}_{\chi} \lambda^{\rm F}{}^{\mathbf{i,j}}_{\mathbf{l}\,\chi})(\mathbf{t^2} + \chi \mathbf{u^2}).$$

# EWK Light and Heavy Boxes

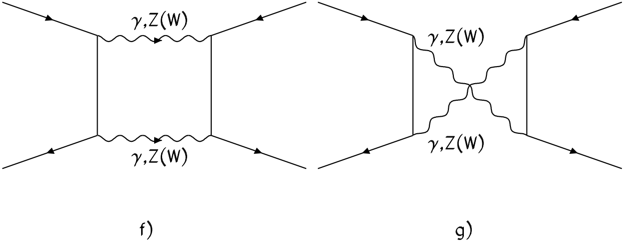


Рис. 6: **Feynman graphs for Boxes**

The calculation of two heavy boson contribution is more complicate procedure.

Fortunately there is a way to avoid many of troubles with the integration of box contribution – **Asymptotic Approach** (V.Z., PRD **75**, 073019, 2007 [hep-ph/0509315]).

# $\gamma Z$, $ZZ$-boxes vs. $WW$-box

To obtain the $WW$-**box contribution** one should:

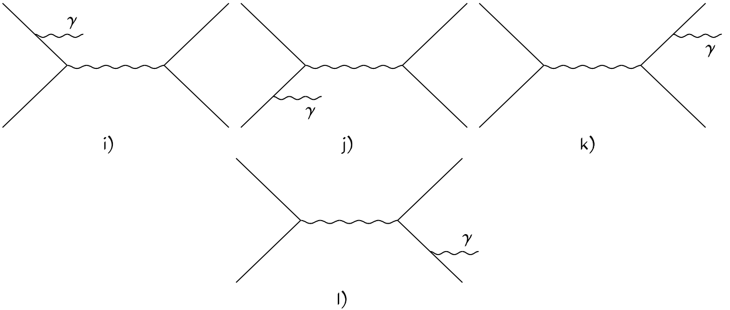
1. to do the trivial substitution $\mathbf{Z} \to \mathbf{W}$,
2. to take into consideration the charge conservation law (some parton $WW$-box diagrams are forbidden).

The second feature of $WW$-boxes explains the **fact of domination of** $WW$-**box** in comparison with $ZZ$ (and $\gamma Z$)-boxes.
The $ZZ$, $\gamma Z$-contributions are proportional to difference

$$\mathbf{I^2_{Z,t}} - \mathbf{I^2_{Z,u}} = \ln \frac{\mathbf{u}}{\mathbf{t}} (\mathbf{I^1_{Z,t}} + \mathbf{I^1_{Z,u}}), \tag{13}$$

whereas the $WW$–box does not contain the difference (13) and are proportional to $\mathbf{I^2_{W,x}}$.

# Photon/gluon bremsstrahlung



i) j) k) l)

Рис. 7: $\gamma$ **bremsstrahlung diagrams. Unsigned helix lines – $\gamma$ or $Z$.**

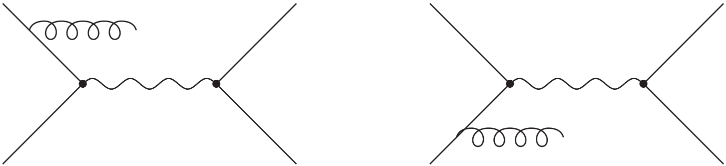


Рис. 8: **Gluon bremsstrahlung diagrams.**

# Inverse gluon bremsstrahlung

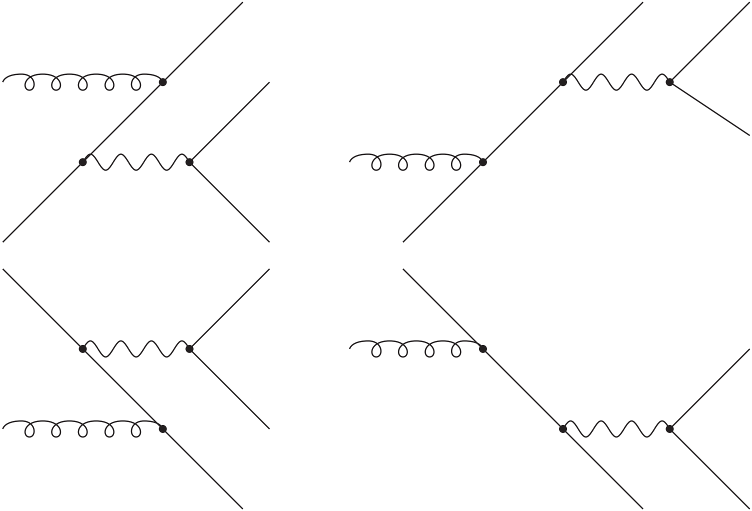


Рис. 9: **Inverse gluon bremsstrahlung diagrams**

# Phase space via 4 invariants (*M*-method)

Phase space looks like

$$\mathbf{I}^{6}_{\Omega}[\mathbf{A}] = \int\limits_{0}^{1} \mathbf{dx_1} \int\limits_{0}^{1} \mathbf{dx_2}\ \frac{\mathbf{4s}}{\pi^2} \int \mathbf{d\Phi}\ \theta^{\mathbf{R}}_{\mathbf{M}} \theta^{\mathbf{R}}_{\mathbf{D}}\ \mathbf{A},$$

with phase space of 3-particle final state

$$\int \mathbf{d\Phi} = \frac{\pi}{\mathbf{4s}} \iiiint\limits_{\Omega} \mathbf{dtdvdzdu_1} \frac{\mathbf{1}}{\pi\sqrt{\mathbf{R_{u_1}}}}$$

with Gram determinant $\mathbf{R_{u_1}}$, radiative invariants based on 4-momenta of real photon/gluon, $\mathbf{p}$:

$$\mathbf{z_1} = \mathbf{2p_1p},\ \mathbf{u_1} = \mathbf{2p_2p},\ \mathbf{z} = \mathbf{2k_1p},\ \mathbf{v} = \mathbf{2k_2p}.$$

For numerical integration we used Monte Carlo routine based on the **VEGAS** algorithm: **G. Peter Lepage'1978.**

# Phase space in new $G/N$-method

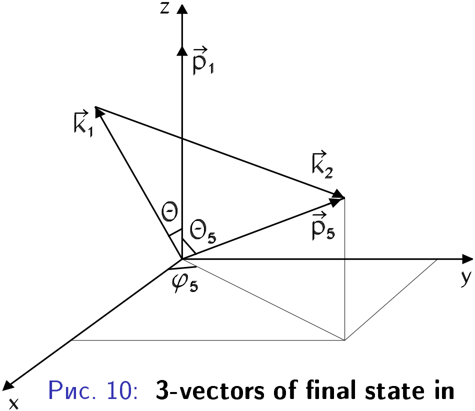


Рис. 10: **3-vectors of final state in c.m.s. of quarks**

It is suitible to use

- c.m.s. of quarks,
- reverse vector

$$\vec{\mathbf{p}}_5 = -\vec{\mathbf{p}}$$

- with

$$\theta_{\mathbf{p}} = \pi - \theta_{\mathbf{5}},\ \varphi_{\mathbf{p}} = \pi + \varphi_{\mathbf{5}}.$$

$$\int \mathbf{d\Phi} \ldots = \int\limits_{\omega}^{\mathbf{\Omega}} \mathbf{p_0 dp_0} \int\limits_{-1}^{1} \mathbf{d} \cos\theta \int\limits_{-1}^{1} \mathbf{d} \cos\theta_{\mathbf{p}} \int\limits_{0}^{2\pi} \mathbf{d}\varphi_{\mathbf{p}} \frac{\pi |\vec{\mathbf{k}}_1| \ldots}{\mathbf{4k_{20} K_A(k_{10})}}.$$

# Some detailes of $G/N$-method

Factor in phase space is

$$\mathbf{K_A}(\mathbf{x}) = \mathbf{1} + \frac{\mathbf{x}(\mathbf{1} - \mathbf{p_0}\mathbf{A}/\sqrt{\mathbf{x}^2 - \mathbf{m}^2})}{\sqrt{\mathbf{x}^2 - \mathbf{2}\mathbf{p_0}\mathbf{A}\sqrt{\mathbf{x}^2 - \mathbf{m}^2} + \mathbf{p}_0^2}},$$

with $\mathbf{A}$ – cosine between $\vec{\mathbf{k}}_1$ and $\vec{\mathbf{p}}_5$:

$$\mathbf{A} = \sin\theta \sin\theta_{\mathbf{5}} \cos\varphi_{\mathbf{5}} + \cos\theta \cos\theta_{\mathbf{5}}.$$

Lepton energy depends on sign of $\mathbf{A}$:

$$\mathbf{k_{10}} = \frac{\mathbf{BC} \pm \sqrt{\mathbf{C}^2 + \mathbf{m}^2(\mathbf{1} - \mathbf{B}^2)}}{\mathbf{1} - \mathbf{B}^2}, \qquad (14)$$

where

$$\mathbf{B} = \frac{\sqrt{\mathbf{s}} - \mathbf{p_0}}{\mathbf{A}\mathbf{p_0}}, \ \mathbf{C} = \frac{(\mathbf{2}\mathbf{p_0} - \sqrt{\mathbf{s}})\sqrt{\mathbf{s}}}{\mathbf{2}\mathbf{A}\mathbf{p_0}}. \qquad (15)$$

# One usefull possibility of $G/N$-method

Using $G/N$-metod we can combain soft and hard photon/gluon parts to avoid of $\omega$-dependance:

$$\mathbf{soft} + \mathbf{hard} \;\; = \int\limits_{\lambda}^{\omega} \mathbf{dp_0} \ldots + \int\limits_{\omega}^{\boldsymbol{\Omega}} \mathbf{dp_0} \ldots = \int\limits_{\lambda}^{\boldsymbol{\Omega}} \mathbf{dp_0} \ldots \; .$$

# Rebuilding to fully differential cross section

Here we rebuild all of the cross sections to completely differential form

$$\sigma_{\mathbf{C}} \to \sigma_{\mathbf{C}}^{(3)} \equiv \frac{\mathbf{d}^3\sigma_{\mathbf{C}}}{\mathbf{dMdyd}\psi},$$

where

$\mathbf{y} \equiv |\mathbf{y}(\mathbf{l}^-\mathbf{l}^+)|$ – dilepton rapidity,

$\psi$ – cosine of angle between $\vec{\mathbf{P}}_{\mathbf{A}}$ and $\vec{\mathbf{k}}_1$.

For non-radiative part the translation to differential form simply to do using the Jackobian $\mathbf{J_N}$:

$$\mathbf{J_N} = \frac{\mathbf{D}(\mathbf{x_1},\mathbf{x_2},\mathbf{t})}{\mathbf{D}(\mathbf{M},\mathbf{y},\psi)} = \frac{\mathbf{4M^3e^{2y}}}{\mathbf{S}[\mathbf{1}+\psi+(\mathbf{1}-\psi)\mathbf{e^{2y}}]^2}.$$

The radiative Jackobian can introduce in the following way

$$\mathbf{J_R^{(3)}} = \frac{\mathbf{4Me^{2y}}}{\mathbf{S}} \frac{(\mathbf{v}+\mathbf{M}^2)(\mathbf{z_1}+\mathbf{M}^2)(\mathbf{u_1}+\mathbf{M}^2)}{[(\mathbf{1}+\psi)(\mathbf{z_1}+\mathbf{M}^2)+(\mathbf{1}-\psi)\mathbf{e^{2y}}(\mathbf{u_1}+\mathbf{M}^2)]^2}.$$

# Leading Logs for **EWK** bremsstrahlung, **QCD** gluon bremsstrahlung, and Inverse Gluon Emission (**IGE**)

Common features of formulas:

- Collinear $\mathbf{u_1}$- and $\mathbf{z_1}$-peaks for ISR, $\mathbf{p} = (\mathbf{1} - \eta)\mathbf{p_{1,2}}$, $\gamma/g$ is collinear to $\mathbf{q}, \mathbf{\bar{q}}$
- Collinear $\mathbf{z}$- and $\mathbf{v}$-peaks for FSR, $\mathbf{p} = \frac{\mathbf{1}-\eta}{\eta}\mathbf{k_{1,2}}$, $\gamma/g$ is collinear to $\mu^+, \mu^-$
- Proportional to the Born expressions: $\mathbf{J_N}$ and $\mathbf{t_B^2} + \chi \mathbf{u_B^2}$
- PDFs grouped into combinations $\mathbf{f_q^A}(\mathbf{x_1^B})\mathbf{f_{\bar{q}}^B}(\frac{\mathbf{x_2^B}}{\eta})$
- EWK/QCD and IGE splitting functions

$$\frac{\mathbf{1} + \eta^2}{\eta} \text{ and } \frac{(\mathbf{1} - \eta)^2 + \eta^2}{\eta}$$

are factorized at Collinear Logs

# Quark Mass Singularity in QED- and QCD-corrections

To solve Quark Mass Singularity (QS) problem in $\overline{\mathrm{MS}}$-scheme, then **CL-terms** are adsorbing into PDFs depending on the factorization scale, $M_{sc}$. The part to be subtracted is

$$\sigma_{\rm QS} = \frac{1}{3}\int d^3\Gamma \int\limits_0^{1-2\omega/M} d\eta \sum_{q=u,d,s,c,b} \Big[ \Big( f_q(x_1,Q^2)\Delta\bar{q}(x_2,\eta) +$$

$$+\Delta q(x_1,\eta) f_{\bar{q}}(x_2,Q^2)\Big)\sigma_0^{q\bar{q}} + (q \leftrightarrow \bar{q}) \Big]\theta_K\theta_M\theta_D,$$

$$\Delta q(x,\eta) = C_{\rm RC}\left[\frac{1}{\eta}f_q(\frac{x}{\eta},M^2_{\rm sc})\theta(\eta-x) - f_q(x,M^2_{\rm sc})\right]\frac{1+\eta^2}{1-\eta}\times$$

$$\times\left(\ln\frac{M^2_{\rm sc}}{m_q^2(1-\eta)^2} - 1\right),\ C_{\rm QED} = \frac{\alpha}{2\pi}Q_q^2,\ C_{\rm QCD} = \frac{4}{3}\frac{\alpha_s}{2\pi}.$$

For IGE the result of QS-term substraction is trivial:

$$\sigma_{\rm IGE} - \sigma_{\rm IGE,QS} = \sigma_{\rm IGE}(m_q \to M_{\rm sc}).$$

# Discussion of numerical results. Code READY

In the following the scale of radiative corrections and their effect on the observables of Drell–Yan processes will be discussed using FORTRAN program **READY**: (**R**adiative corr**E**ctions to l**A**rge invariant mass **D**rell-**Y**an process).

We used the following set of prescriptions:

- the standard PDG set of SM input electroweak parameters:
- the light quark "effective" masses provide $\mathbf{\Delta}\alpha^{(\mathbf{5})}_{\mathbf{had}}(\mathbf{m^2_Z}) = \mathbf{0.0276}$,
- 5 active flavors of quarks in proton, their masses as regulators of the collinear singularity,
- CTEQ, MRST 2004QED, and MSTW8 sets of PDFs (with the choice $\mathbf{Q} = \mathbf{M_{sc}} = \mathbf{M}$).

# CMS detector setup

We impose the experimental restriction conditions

- on the detected lepton angle $-\zeta^* \leq \zeta \leq \zeta^*$ and on the rapidity $|\mathbf{y(l)}| \leq \mathbf{y(l)}^*$; for CMS detector the cut values of $\zeta^*$ and $\mathbf{y(l)}^*$ are determined as

$$\mathbf{y(l)}^* = -\ln\ \tan\frac{\theta^*}{\mathbf{2}} = \mathbf{2.5}\ (\text{or} = \mathbf{2.4}),$$

- the second standard CMS restriction $\mathbf{p_T(l)} \geq \mathbf{20\ GeV}$,
- the "bare" setup for muons identification requirements (no smearing, no recombination of muon and photon).

# Independence of EWK RC from $\omega$ (GeV) and quark masses at $\mathbf{l} = \mu$, $\sqrt{\mathbf{S}} = \mathbf{14}$ TeV, $\mathbf{M} = \mathbf{2}$ TeV, $\mathbf{y} = \mathbf{0}$, $\psi = \mathbf{0}$

| $\omega$ | $m_q/m_u$ | $\delta_{fin}$ | $\delta^{hard}$ | $\delta_{fin} - \delta^{soft}_{QS}$ | $\delta^{hard} - \delta^{hard}_{QS}$ | $\delta_{tot}$ |
|---|---|---|---|---|---|---|
| 10 | 10.0 | −0.4555 | 0.3294 | −0.3292 | 0.2250 | −0.1042 |
|  | 1.0 | −0.4846 | 0.3527 | −0.3291 | 0.2250 | −0.1042 |
|  | 0.1 | −0.5136 | 0.3759 | −0.3291 | 0.2250 | −0.1041 |
| 1 | 10.0 | −0.7117 | 0.5831 | −0.4862 | 0.3799 | −0.1064 |
|  | 1.0 | −0.7581 | 0.6235 | −0.4862 | 0.3799 | −0.1064 |
|  | 0.1 | −0.8045 | 0.6639 | −0.4862 | 0.3799 | −0.1064 |
| 0.1 | 10.0 | −0.9679 | 0.8390 | −0.6256 | 0.5190 | −0.1066 |
|  | 1.0 | −1.0316 | 0.8967 | −0.6256 | 0.5190 | −0.1066 |
|  | 0.1 | −1.0953 | 0.9545 | −0.6256 | 0.5190 | −0.1066 |
| 0.01 | 10.0 | −1.2241 | 1.0951 | −0.7476 | 0.6410 | −0.1066 |
|  | 1.0 | −1.3052 | 1.1702 | −0.7476 | 0.6410 | −0.1066 |
|  | 0.1 | −1.3862 | 1.2454 | −0.7476 | 0.6410 | −0.1066 |
| 0.001 | 10.0 | −1.4803 | 1.3513 | −0.8522 | 0.7456 | −0.1066 |
|  | 1.0 | −1.5787 | 1.4438 | −0.8522 | 0.7456 | −0.1066 |
|  | 0.1 | −1.6771 | 1.5362 | −0.8522 | 0.7456 | −0.1066 |

# Independence of QCD RC from $\omega$ (GeV) and quark masses at $\mathbf{l} = \mu$, $\sqrt{\mathbf{S}} = \mathbf{14}$ TeV, $\mathbf{M} = \mathbf{2}$ TeV, $\mathbf{y} = \mathbf{0}$, $\psi = \mathbf{0}$

| $\omega$ | $\mathbf{m_q/m_u}$ | $\delta_{\mathbf{fin}}$ | $\delta^{\mathbf{hard}}$ | $\delta_{\mathbf{fin}} - \delta^{\mathbf{soft}}_{\mathbf{QS}}$ | $\delta^{\mathbf{hard}} - \delta^{\mathbf{hard}}_{\mathbf{QS}}$ | $\delta_{\mathbf{tot}}$ |
|---|---|---|---|---|---|---|
| 10 | 10.0 | −5.6024 | 4.5893 | 2.1076 | −1.7306 | 0.3770 |
| | 1.0 | −7.3746 | 5.9937 | 2.1122 | −1.7306 | 0.3815 |
| | 0.1 | −9.1469 | 7.3980 | 2.1167 | −1.7306 | 0.3861 |
| 1 | 10.0 | −9.0318 | 7.9905 | 4.7309 | −4.3551 | 0.3758 |
| | 1.0 | −11.8625 | 10.4443 | 4.7313 | −4.3551 | 0.3762 |
| | 0.1 | −14.6932 | 12.8982 | 4.7318 | −4.3551 | 0.3767 |
| 0.1 | 10.0 | −12.4611 | 11.4170 | 8.4329 | −8.0567 | 0.3762 |
| | 1.0 | −16.3503 | 14.9284 | 8.4329 | −8.0567 | 0.3762 |
| | 0.1 | −20.2395 | 18.4399 | 8.4330 | −8.0567 | 0.3763 |
| 0.01 | 10.0 | −15.8905 | 14.8461 | 13.1958 | −12.8196 | 0.3763 |
| | 1.0 | −20.8382 | 19.4159 | 13.1958 | −12.8196 | 0.3763 |
| | 0.1 | −25.7858 | 23.9858 | 13.1959 | −12.8196 | 0.3763 |
| 0.001 | 10.0 | −19.3198 | 18.2754 | 19.0175 | −18.6412 | 0.3763 |
| | 1.0 | −25.3260 | 23.9037 | 19.0175 | −18.6412 | 0.3763 |
| | 0.1 | −31.3322 | 29.5321 | 19.0175 | −18.6412 | 0.3763 |

# Comparison at Hadronic Level

$$\frac{d\sigma}{dMdy} = \int\limits_{-\zeta^*}^{\zeta^*} d\psi \sigma^{(3)} \theta_D; \quad \frac{d\sigma}{dM} = \int\limits_{-\zeta^*}^{\zeta^*} d\psi \int\limits_{-\ln \frac{\sqrt{S}}{M}}^{+\ln \frac{\sqrt{S}}{M}} dy\ \sigma^{(3)} \theta_D.$$

Comparing the relative EWK RC to $\mathbf{d}\sigma/\mathbf{dM}$ with the results of

- **HORACE** (C. M. Carloni Calame, G. Montagna, O. Nicrosini, A. Vicini // JHEP. 2007. Vol. 10. P. 109, arXiv:0710.1722)
- **SANC** (A. Andonov *et al.* Comput. Phys. Commun. 2006. Vol. 174. P. 481 [hep-ph/0411186])
- **ZGRAD2** (U. Baur *et al.* Phys. Rev. D. 2002. Vol. 65, 033007, P. 1–19. [hep-ph/0108274])

published in Proc. of Les Houches 2007, Physics at TeV colliders, arXiv:0803.0678 [hep-ph] we have a good agreement at $M \geq 0.5$ TeV.

# Comparison of $M$-distribution

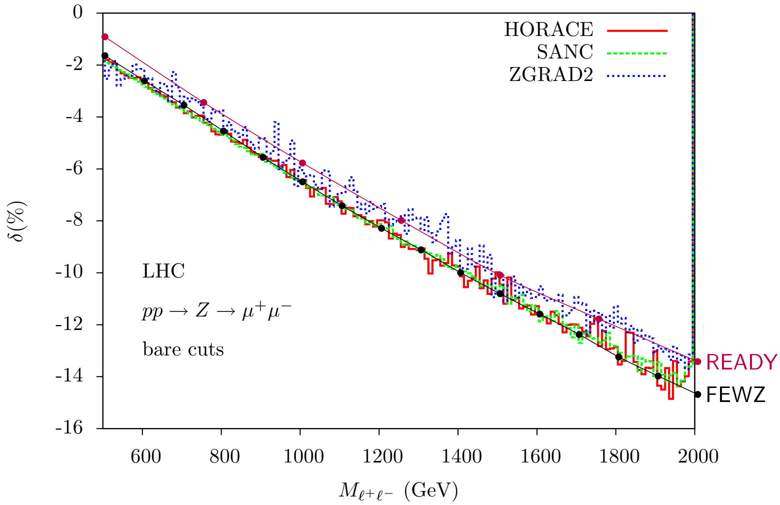


Рис. 11: **Relative electroweak corrections $\delta(\%)$ to $d\sigma/dM$ vs $M$. READY accuracy is $< 0.1\%$, a time per dot is $\sim 1200$ s.**

# Comparison of forward-backward asymmetry

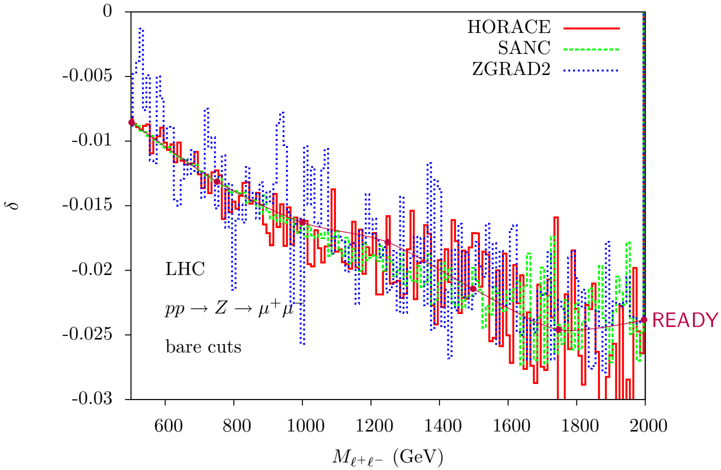


Рис. 12: **The difference between the NLO and LO predictions for $A_{FB}$ due to electroweak corrections.**

# READY5.7 settings

Form of in57.dat input-file:

1 ! 1=PDG'08, 2=arXiv:1606.02330
2 ! 1=d$\sigma$/dM/dy/d$\psi$, 2=d$\sigma$/dM, 4=$A_{\mathrm{FB}}$
1 ! 1=EWK, 2=QCD, 3=EWK+QCD, 4=NNLO QED FSR
1 ! 1=muon, 2=electron
4 ! 1,2,3-CTEQ (MSbar,DIS,LO), 4-MRST4, 5-MSTW8, ...
10000 ! base number of iterations by VEGAS
13000. ! LHC energy, GeV
0.986614 ! y limitation of CMS (y=2.5, cos(a)=0.986614)
20.0 ! $p_T$ limitation of CMS (20 GeV)

# Approaches to NNLO corrections for Drell–Yan

Contributions needed to calculate (in order of difficulty increasing):

- FSR radiation (EWK)
- ISR radiation (EWK and QCD)
- Inverse contributions
- Their interference and/or interplay

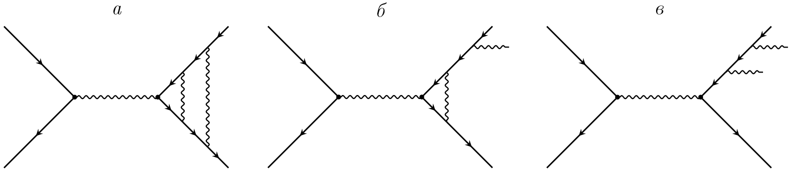


Рис. 13: *а* – *T*-part, *б* – *O*-part, *в* – *D*-part.

# NNLO QED FSR radiative corrections

Simplest (but principal) part of NNLO radiative correction is NNLO QED FSR contribution.
To get it we need to calculate

- **Q-part**:
  Quadratic NLOs, or square of one-loop NLO FSR corrections
- **T-part**:
  all Two-loop FSR diagrams with photon
- **O-part**:
  One-photon emission with NLO V-contributions (soft and hard)
- **D-part**:
  Double-photon emission (soft and hard)

# NNLO QED FSR with **soft real photons**

Summing up $Q$-, $T$-, $O$-, $D$-parts (and subtracting $K$-part) on partonic level we get:

$$
\begin{aligned}
\sigma_{\mathrm{NNLO}} &= \sigma_Q + \sigma_T + \sigma_O + \sigma_D - \sigma_K = \\
&= \sigma_0 \Big[ |\mathbf{F}^{(1)}(\mathbf{s})|^2 + 2\mathrm{Re}\mathbf{F}^{(2)}(\mathbf{s}) + \delta_1^{\mathbf{S}} \cdot 2\mathrm{Re}\mathbf{F}^{(1)}(\mathbf{s}) + \frac{1}{2}(\delta_1^{\mathbf{S}})^2 - \\
&\qquad - \frac{1}{2}\left(\frac{\alpha}{\pi}\right)^2 \frac{2}{3}\pi^2(\mathbf{L}-1)^2 \Big].
\end{aligned}
$$

All important form factorts $\mathbf{F}^{(1)}(\mathbf{s})$, $\mathbf{F}^{(2)}(\mathbf{s})$, and $\delta_1^{\mathbf{S}}$ expressed via three logarithms – **collinear**, **infrared**, and **soft** ones:

$$
\mathbf{L} = \log\frac{\mathbf{s}}{\mathbf{m}^2},\ \ \mathbf{L}_\lambda = \log\frac{\lambda^2}{\mathbf{m}^2},\ \ \mathbf{L}_\omega = \log\frac{2\omega}{\sqrt{\mathbf{s}}},
$$

where $\lambda$ is **mass** of internal virtual photon, $\omega$ is **maximal energy** of soft real photon.

# One-loop form factors via logs

$$\begin{aligned}
\mathbf{F}^{(1)}(\mathbf{s}) &= \frac{\alpha}{\pi}\Big[-\frac{1}{4}\mathbf{L}^2 + \frac{1}{2}\mathbf{L}_\lambda\mathbf{L} + \frac{3}{4}\mathbf{L} - \frac{1}{2}\mathbf{L}_\lambda - 1 + \frac{\pi^2}{3} + \\
&\quad + \mathbf{i}\pi\Big(\frac{1}{2}\mathbf{L} - \frac{1}{2}\mathbf{L}_\lambda - \frac{3}{4}\Big)\Big], \\
\delta_1^{\mathbf{S}} &= \frac{\alpha}{\pi}\Big[\frac{1}{2}\mathbf{L}^2 - \mathbf{L}_\lambda\mathbf{L} + 2\mathbf{L}_\omega\mathbf{L} + \mathbf{L}_\lambda - 2\mathbf{L}_\omega - \frac{\pi^2}{3}\Big],
\end{aligned}$$

where

$$\mathbf{L} = \log\frac{\mathbf{s}}{\mathbf{m}^2},\ \ \mathbf{L}_\lambda = \log\frac{\lambda^2}{\mathbf{m}^2},\ \ \mathbf{L}_\omega = \log\frac{2\omega}{\sqrt{\mathbf{s}}}.$$

# Two-loop form factor via logs

$$\begin{aligned}
\mathrm{Re}\mathbf{F}^{(2)}(\mathbf{s}) \quad = \quad & \Big(\frac{\alpha}{\pi}\Big)^2 \Big[\frac{1}{32}\mathbf{L}^4 - \frac{3}{16}\mathbf{L}^3 + \Big(\frac{17}{32} - \frac{5}{4}\zeta_2\Big)\mathbf{L}^2 \\
& + \Big(-\frac{21}{32} + 3\zeta_2 + \frac{3}{2}\zeta_3\Big)\mathbf{L} + \frac{2}{5}\zeta_2^2 - \frac{9}{4}\zeta_3 - 3\zeta_2 \log 2 \\
& - \frac{1}{2}\zeta_2 + \frac{405}{216} + \mathbf{L}_\lambda^2\Big(\frac{1}{8}\mathbf{L}^2 - \frac{1}{4}\mathbf{L} + \frac{1}{8} - \frac{3}{4}\zeta_2\Big) \\
& + \mathbf{L}_\lambda\Big(-\frac{1}{8}\mathbf{L}^3 + \frac{1}{2}\mathbf{L}^2 + \Big(-\frac{7}{8} + \frac{5}{2}\zeta_2\Big)\mathbf{L} + \frac{1}{2} - \frac{13}{4}\zeta_2\Big)\Big].
\end{aligned}$$

This is result of **F.A. Berends, W.L. Van Neerven, G.J.H. Burgers (Nucl. Phys. B., 1988, Vol. 297, 429)**.

# The $\lambda$-independance of FSR NNLO result

The relative corrections to the cross section $\mathbf{d}\sigma/\mathbf{dM}$ for $\mu$-case at $\mathbf{M} = \mathbf{2}$ TeV and $\omega = \omega_{\text{eff}}$ inducing different contributions to FSR NNLO correction

$$\mathrm{NNLO} = Q + T + O + D - K$$

depending on $\lambda$:

| $\lambda$, GeV | $Q$ | $T$ | $O$ | $D$ | $K$ | NNLO |
|---|---|---|---|---|---|---|
| $10^{-1}$ | 0.0398 | 0.0310 | $-0.1037$ | 0.0380 | 0.0062 | $-0.0011$ |
| $10^{-2}$ | 0.0899 | 0.0761 | $-0.2740$ | 0.1131 | 0.0062 | $-0.0011$ |
| $10^{-3}$ | 0.1606 | 0.1406 | $-0.5242$ | 0.2282 | 0.0062 | $-0.0011$ |
| $10^{-4}$ | 0.2519 | 0.2246 | $-0.8547$ | 0.3834 | 0.0062 | $-0.0011$ |

# Total Analytical NNLO Result

We control the cancellation of collinear logs of highest orders – NNLO result contains only $\mathbf{L}^2$, $\mathbf{L}^1$, and $\mathbf{L}^0$:

$$\sigma_{\rm NNLO} = \left(\frac{\alpha}{\pi}\right)^2 \Big[ \mathbf{c}_2 \mathbf{L}^2 + \mathbf{c}_1 \mathbf{L} + \mathbf{c}_0 \Big] \sigma_0,$$

where

$$\begin{aligned}
\mathbf{c}_2 &= 2\mathbf{L}_\omega^2 + 3\mathbf{L}_\omega - 2\zeta_2 + \frac{9}{8}, \\
\mathbf{c}_1 &= -4\mathbf{L}_\omega^2 + \mathbf{L}_\omega(4\zeta_2 - 7) + \frac{11\zeta_2}{2} + 3\zeta_3 - \frac{45}{16}, \\
\mathbf{c}_0 &= 2\mathbf{L}_\omega^2 + 4\mathbf{L}_\omega(1 - \zeta_2) - \frac{6\zeta_2^2}{5} + \frac{3\zeta_2}{8} - 6\zeta_2 \ln 2 - \frac{9\zeta_3}{2} + \frac{19}{4}.
\end{aligned}$$

# **Naive Choice** of maximal energy of soft real photon

How we can choice of $\omega$ to correspond to experimental situation of CMS LHC detector?

**NAIVE POSSIBILITY**: to use effective values at certain kinematical point wich reproduce exact NLO RC.

| $M$, TeV | $\delta_{\rm NLO,FSR}$ | $\omega_{\rm eff}/M$ | $\delta_{\rm NNLO,FSR}$ |
|---|---|---|---|
| 0.5 | $-0.0628$ | 0.093 | $-0.0025$ |
| 1.0 | $-0.0773$ | 0.083 | $-0.0022$ |
| 1.5 | $-0.0895$ | 0.076 | $-0.0018$ |
| 2.0 | $-0.1017$ | 0.069 | $-0.0011$ |
| 2.5 | $-0.1104$ | 0.063 | $-0.0002$ |
| 3.0 | $-0.1222$ | 0.057 | $+0.0008$ |

# REAL (NOT NAIVE!) HARD APPROACH

Now we use notations of contributions to NNLO cross section of **F.Berends, et.al., NPB, 1988**:

- **V** × **V** – two virtual photons **(Q-part plus T-part)**
- **V** × **S** – one virtual and one soft photon **(soft O-part)**
- **V** × **H** – one virtual and one hard photon **(hard O-part)**
- **S** × **S** – two soft photons **(soft D-part minus K-part)**
- **H** × **H** – two hard photons **(hard D-part)**
- **S** × **H** – one soft and one hard photon

For hard part calculation were used simple Leading Logs Approximation

# $\omega$-independance (at fixing $\lambda$)

Using fixing $\lambda = 10^{-4}$ GeV we get:

| $\omega$, GeV | V × V | V × S | V × H | sum |
|---|---|---|---|---|
| $10^{-2}$ | 0.4764 | −0.0466 | −0.8014 | −0.3716 |
| $10^{-1}$ | 0.4765 | −0.2418 | −0.6062 | −0.3715 |
| $10^{0}$ | 0.4765 | −0.4371 | −0.4112 | −0.3718 |
| $10^{1}$ | 0.4766 | −0.6322 | −0.2190 | −0.3748 |
| $10^{2}$ | 0.4766 | −0.8275 | −0.0516 | −0.4026 |

| $\omega$, GeV | S × S | S × H | H × H | sum | NNLO |
|---|---|---|---|---|---|
| $10^{-2}$ | −0.0051 | 0.0388 | 0.3436 | 0.3773 | 0.0058 |
| $10^{-1}$ | 0.0245 | 0.1534 | 0.1994 | 0.3773 | 0.0058 |
| $10^{0}$ | 0.0940 | 0.1882 | 0.0951 | 0.3774 | 0.0056 |
| $10^{1}$ | 0.2036 | 0.1450 | 0.0306 | 0.3792 | 0.0044 |
| $10^{2}$ | 0.3532 | 0.0446 | 0.0034 | 0.4012 | −0.0015 |

The last line reproduce “naive” soft approach number.

# $\lambda$-independance (at fixing $\omega$ )

Using fixing $\omega = 10^0$ GeV we get:

| $\lambda$, GeV | S × S | V × V | V × S | sum |
|---|---|---|---|---|
| $10^{-6}$ | 0.3530 | 0.9467 | −1.1663 | 0.1335 |
| $10^{-5}$ | 0.2036 | 0.6917 | −0.7618 | 0.1335 |
| $10^{-4}$ | 0.0940 | 0.4765 | −0.4370 | 0.1335 |
| $10^{-3}$ | 0.0245 | 0.3013 | −0.1923 | 0.1335 |
| $10^{-2}$ | −0.0051 | 0.1661 | −0.0275 | 0.1335 |

| $\lambda$, GeV | S × H | V × H | H × H | sum | NNLO |
|---|---|---|---|---|---|
| $10^{-6}$ | 0.3568 | −0.5798 | 0.0951 | −0.1279 | 0.0056 |
| $10^{-5}$ | 0.2725 | −0.4955 | 0.0951 | −0.1279 | 0.0057 |
| $10^{-4}$ | 0.1882 | −0.4112 | 0.0951 | −0.1279 | 0.0057 |
| $10^{-3}$ | 0.1040 | −0.3270 | 0.0951 | −0.1279 | 0.0057 |
| $10^{-2}$ | 0.0197 | −0.2427 | 0.0951 | −0.1279 | 0.0057 |

# **Real** FSR NNLO relative corrections $\delta_{\mathrm{NNLO}}$ for CMS LHC

| $M$, TeV | $(\mathrm{V+S})^2$ | $(\mathrm{V+S+H}) \times \mathrm{H}$ | total NNLO |
|---|---|---|---|
| 0.50 | 0.0556 | −0.0533 | 0.0023 |
| 0.75 | 0.0735 | −0.0706 | 0.0030 |
| 1.00 | 0.0885 | −0.0850 | 0.0035 |
| 1.25 | 0.1015 | −0.0975 | 0.0041 |
| 1.50 | 0.1132 | −0.1086 | 0.0046 |
| 1.75 | 0.1237 | −0.1186 | 0.0051 |
| 2.00 | 0.1334 | −0.1278 | 0.0056 |
| 2.25 | 0.1425 | −0.1364 | 0.0062 |
| 2.50 | 0.1509 | −0.1443 | 0.0067 |
| 2.75 | 0.1589 | −0.1517 | 0.0072 |
| 3.00 | 0.1664 | −0.1586 | 0.0078 |

For $(\mathrm{V+S})^2$ and $(\mathrm{V+S+H}) \times \mathrm{H}$ was used the fixing $\omega = 10^0$ GeV.

# Conclusions

- The NLO EWK+QCD and FSR NNLO RC to Drell–Yan process at **extra large M** in **fully differential form** have been studied.
- The results are **the compact expressions (both for NLO and NNLO)**, they expand in Sudakov and collinear logarithms.
- The new $G/N$-method of taking into account of radiative events without any approximations is demonstrated.
- At the parton/hadron level FORTRAN **code READY** gives a good coincidence for cross section and $A_{\mathrm{FB}}$ with other groups at $M > 0.5$ TeV and **fast convergence**.
- We have the new result on NNLO FSR RC **with hard photons** to Drell–Yan process at CMS LHC. Our next steps are taking into account ISR QED and QCD modes, their interplay, etc.

# ACKNOWLEDGMENTS

- With great thanks to our teacher Nikolai Maximovich Shumeiko.
- I thank organizers of this Conference.
- Thanks the RDMS CMS members, also A. Arbuzov, Yu. Bystritsky, and Ya. Dyadyshka for the stimulating discussions, to group SANC and D. Wackeroth for a detailed comparison of the results.
- I thank CERN (CMS Group) and Memorial University (Canada, NFL), where part of this work was carried out, for warm hospitality during my visits.