The impact of fluctuations on the QCD Phase Structure

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Germany
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QC$_3$D phase structure

vacuum/nuclear matter/transition & only corners of the phase diagram are known from “first principles”
conjectured QC$_3$D phase structure

knowledge so far
mostly based on model calculations

assumptions:
equilibrium, homogeneous phases,
infinite volume, ....
conjectured QC$_3$D phase structure

**Open/unclear questions:**

- **CEP:** existence/location/number
- relation between chiral & deconfinement?
  - chiral $\leftrightarrow$ deconfinement CEP?
- Quarkyonic phase: coincidence of both transitions at $\mu=0$ & $\mu>0$?
- inhomogeneous phases? $\rightarrow$ more favored?
- axial anomaly restoration around chiral transition?
- finite volume effects? $\rightarrow$ lattice comparison/
  influence boundary conditions
- role of fluctuations? so far mostly Mean-Field results
  $\rightarrow$ effects of fluctuations important
  - examples: size of crit reg. around CEP
- What are good experimental signatures?
  $\rightarrow$ cumulants?

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equilibrium, homogeneous phases,
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conjectured QC$_3$D phase structure

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infinite volume, ....

Theory:

→ Lattice: but simulations restricted to small $\mu$

→ Models: effective theories parameter dependency

→ Functional QFT methods: FRG, DSE, nPI

Theoretical aim:
deeper understanding & more realistic HIC description

→ existence of critical end point(s)?
Agenda

- **Role of (quantum and thermal) Fluctuations for QCD phases**
  - from mean-field approximations (MFA)
  - to the Functional Renormalization Group (FRG)

- **Effect of different FRG truncations**

- **Columbia plot**
Functional Renormalization Group

\[ \Gamma_k[\phi] \] scale dependent effective action

\[ t = \ln (k/\Lambda) \quad R_k \text{ regulators} \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi} \]

**FRG (average effective action)**

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \partial_t R_k \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right) \]

\[ k \partial_k \Gamma_k[\phi] \sim \frac{1}{2} \]

**Ansatz for \( \Gamma_k \): Example: Leading order derivative expansion**

\[ \Gamma_k = \int d^4 x \bar{q} \left[ i \gamma_\mu \partial^\mu - g (\sigma + i \bar{\pi} \gamma_5) \right] q + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \bar{\pi})^2 + V_k(\phi^2) \]

\[ V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4} (\sigma^2 + \bar{\pi}^2 - v^2)^2 - c \sigma \]

22.08.2019 | B.-J. Schaefer | Giessen University |
Example: grid technique

Scale evolution of meson potential:

phase transition: First order

phase transition: Second Order
FRG and QCD

- full dynamical QCD FRG flow:

  fluctuations of **gluon**, **ghost**, **quark** and (via hadronization) **meson**

  \[
  \frac{\partial_t \Gamma_k[\phi]}{} = \frac{1}{2} - - + \frac{1}{2}
  \]

  in presence of **dynamical quarks**: **gluon propagator** is modified

  pure Yang Mills flow + matter back-coupling
chiral phase transition:

flow for quark-meson model truncation:

neglect YM contributions and bosonic fluctuations

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} \]

without bosonic fluctuations:
extended Mean-field approximation
Phase diagram $N_f=2$

$O(4) \sim SU(2) \times SU(2)$

chiral limit

no spinodal lines!

two phase diagrams

$O(4)$ universality class

$\sigma \neq 0$

$\sigma = 0$

tricritical point

$[BJS, Wambach 2005]$
Phase diagram $N_f=2$

- 2nd order, $O(4)$
- 2nd order, $Z(2)$
- 1st order
- crossover

$O(4) \sim SU(2) \times SU(2)$

Chiral limit

No spinodal lines!

O(4) universality class

$\sigma \neq 0$

$\sigma = 0$

Tricritical point

$T_c$ vs $\mu$

$T$ vs $\mu$

$\mu_B$, $\mu^\text{tri}_B$, $\mu^\text{CEP}_B$, $m_B$, $m_\text{phys}$

$T_c$, $T$, $m_\pi = 0$ MeV, $m_\pi = 138$ MeV

Two phase diagrams

[BJS, Wambach 2005]
FRG: Quark-Meson with Polyakov

\[ N_f = 2 \text{ quark flavor} \]

without back reaction \( (T_0(\mu) = \text{const}) \)

with back reaction \( (T_0(\mu)) \)

\[ m_\pi = 138 \text{ MeV} \]

\[ \chi \text{ crossover} \]
\[ \Phi \text{ crossover} \]
\[ \bar{\Phi} \text{ crossover} \]
\[ \chi \text{ 1st order} \]
\[ \sigma(T=0)/2 \]

[Herbst, Pawlowski, BJS 2010,2013]
Critical Endpoint?

Exact location of CEP not (yet) accessible with lattice, FRG & DSE

so far:

we can exclude CEP for small densities: $\mu_B/T < 2 \ldots 3$

Higher densities: dynamical baryons needed!
Higher cumulants

infinite volume: influence of fluctuations

findings:

- (quark) fluctuations pushes CEP to smaller $T$ and bigger $\mu$
- Fluctuations wash out phase transition $\rightarrow$ broader negative regions

[S. Resch, BJS to be published]
Higher cumulants

findings:

(quark) fluctuations pushes CEP to smaller \( T \) and bigger \( \mu \)

Fluctuations wash out phase transition → broader negative regions

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[S. Resch, BJS to be published]
Role of (quantum and thermal) Fluctuations for QCD phases from mean-field approximations (MFA) to the Functional Renormalization Group (FRG)

Effect of different FRG truncations

Columbia plot
FRG analysis for $N_f = 2+1$ quark flavor

\[
\Gamma_k = \int_x \left\{ \bar{q} Z_{q,k} (\gamma_\mu \partial_\mu + \gamma_0 \mu) q + \bar{q} h_k \cdot \Sigma_5 q + \text{tr} (Z_{\Sigma,k} \partial_\mu \Sigma \cdot \partial_\mu \Sigma^\dagger) + \tilde{U}_k (\Sigma) \right\}
\]

\[
q^T = (l, l, s) \quad \quad \Sigma_5 = T_a (\sigma_a + i \gamma_5 \pi_a)
\]

- effective potential
  \[
  \tilde{U}_k = U_k (\rho_1, \tilde{\rho}_2) - j_l \sigma_l - j_s \sigma_s - c_k \xi \quad \xi = \det \Sigma + \det \Sigma^\dagger
  \]

- wave function renormalizations
  \[
p^2 + m^2 \rightarrow Z_k p^2 + m^2
  \]

- Yukawa couplings
  \[
h_k = \begin{pmatrix}
h_{l,k} & h_{l,k} & h_{ls,k} \\
h_{l,k} & h_{l,k} & h_{ls,k} \\
h_{sl,k} & h_{sl,k} & h_{s,k}
\end{pmatrix}
\]

\[\text{U(3) x U(3) sym. potential} \]
\[\text{(two chiral invariants)} \]
\[\rho_i = \text{tr} (\Sigma \cdot \Sigma^\dagger)^i \]
\[\text{explicit chiral symmetry breaking:} \]
\[\text{finite light & strange current quark masses} \]
\[\text{anomalous U(1)A breaking} \]
\[\text{via 't Hooft determinant} \]

- FRG analysis for $N_f = 2+1$ quark flavor

\[\text{[Rennecke, BJS 2018]}\]
Different FRG truncations

- different truncations:

$$\Gamma_k = \int_x \left\{ \bar{q} Z_{q,k} (\gamma_\mu \partial_\mu + \gamma_0 \mu) q + \bar{q} h_k \cdot \Sigma_5 q + \text{tr} \left( Z_{\Sigma,k} \partial_\mu \Sigma \cdot \partial_\mu \Sigma^\dagger \right) + \tilde{U}_k (\Sigma) \right\}$$

<table>
<thead>
<tr>
<th>truncation</th>
<th>running couplings</th>
</tr>
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<tbody>
<tr>
<td>LPA'+Y</td>
<td>$\bar{U}<em>k, \bar{h}</em>{l,k}, \bar{h}<em>{s,k}, Z</em>{l,k}, Z_{s,k}, Z_{\phi,k}$</td>
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<td>LPA</td>
<td>$\bar{U}_k$</td>
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LPA = local potential approximation = leading order derivative expansion

Y = Yukawa coupling running

LPA' = beyond local potential approximation include wave function renormalization
Chiral Phase Diagram

- Critical Endpoint for different truncations:

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<th>running couplings</th>
<th>( (T_{\text{CEP}}, \mu_{\text{CEP}}) ) [MeV]</th>
</tr>
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<tbody>
<tr>
<td>LPA' + Y</td>
<td>( \tilde{U}<em>k, \tilde{h}</em>{l,k}, \tilde{h}<em>{s,k}, Z</em>{l,k}, Z_{s,k}, Z_{\phi,k} )</td>
<td>(61,235)</td>
</tr>
<tr>
<td>LPA + Y</td>
<td>( \tilde{U}<em>k, \tilde{h}</em>{l,k}, \tilde{h}_{s,k} )</td>
<td>(46,255)</td>
</tr>
<tr>
<td>LPA</td>
<td>( \tilde{U}_k )</td>
<td>(44,265)</td>
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[Rennecke, BJS 2018]
Masses

• quark masses

• meson masses

driven by the mesonic wave-function renormalization

mesons decouple more rapidly beyond LPA
Masses

- Quark masses
- Meson masses

Driven by the mesonic wave-function renormalization:

Mesons decouple more rapidly beyond LPA

[Threshold behavior]

[Renonecke, BJS 2018]
Mixing angles

- Mixing angles determine light and strange quark content of $\sigma$, $f_0$, $\eta$, $\eta'$ mesons

\[
\begin{pmatrix}
  f_0 \\
  \sigma \\
  \eta \\
  \eta'
\end{pmatrix} =
\begin{pmatrix}
  \cos \varphi_s & -\sin \varphi_s \\
  \sin \varphi_s & \cos \varphi_s \\
  \cos \varphi_p & -\sin \varphi_p \\
  \sin \varphi_p & \cos \varphi_p
\end{pmatrix}
\begin{pmatrix}
  \sigma_t \\
  \sigma_s \\
  \eta_t \\
  \eta_s
\end{pmatrix}
\]

Significant effects on pseudoscalar mixing beyond LPA!

Consequence: Chiral partners of $\eta$ and $\eta'$ change!

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<td>$(\eta, f_0)$</td>
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- Columbia plot
Quark mass sensitivity: Columbia plot

For physical quark masses: smooth phase transitions $\rightarrow$ deconfinement: analytic change of d.o.f.
$\rightarrow$ associated global QCD symmetries only exact in two mass limits

1. infinite quark masses (center symmetry)  
   Order parameter: VEV of traced Polyakov loop(s)

2. massless quarks (chiral symmetry)  
   Order parameter: chiral condensate(s)

for finite quark masses: both symmetries explicitly broken

[de Forcrand D’Elia 2017]
Role of axial anomaly?

For physical quark masses: smooth phase transitions → deconfinement: analytic change of d.o.f.
→ associated global QCD symmetries only exact in two mass limits

1. infinite quark masses (center symmetry)
   Order parameter: VEV of traced Polyakov loop(s)

2. massless quarks (chiral symmetry)
   Order parameter: chiral condensate(s)

still conflicting lattice results!

open issue: Nf=2: O(4)? U(2) × U(1)/U(2)?
→ similar crit. exponents
or even 1st order?

→ dep. on strength of axial anomaly!

• example:

  if U_A(1) broken @T_c → O(4) universality

  → there exist tricritical m_tri

[de Forcrand D’Elia 2017]
location of tri-critical point still an open question (maybe shifts to infinite strange quark mass)

below physical point

above physical point

1st order $Z(2)$ crossover

$m_{\pi} = 0$

$m_{\pi,c} = \infty$

$H = \frac{m_l}{m_{\text{phys}}} - \frac{m_c}{m_{\text{phys}}}$

Role of axial anomaly?
location of tri-critical point still an open question (maybe shifts to infinite strange quark mass)

below physical point

above physical point

$m_\pi = 0$

$m_\pi, c$

$m_\pi = \infty$

$1^{\text{st}}$ order $Z(2)$ crossover

$2^{\text{nd}}$ order $2^{\text{nd}}$ order $Z(2)$

$N_f = 2$

$N_f = 3$

$N_f = 1$

physical point

$H = \frac{m_l}{m_{\text{phy}}}$

$m_{u,d} = m_l$

$\mu/T = 0$

$[\text{HotQCD 2019}]$

Role of axial anomaly?
Mean-field analysis

chiral critical surface
standard scenario

finite chemical potential

Nf=2+1 quark-meson model
Mean-field approximation

with $U_A(1)$-anomaly

[BJS, M. Wagner 2009]

without $U_A(1)$-anomaly
FRG analysis

Nf=2+1 FRG QM truncation:
initial action in the UV: 7 parameters, (4 couplings, 2 explicit symmetry breaking, 1 ‘t Hooft determinant)
axial $U_A(1)$ symmetry: on or off

How to fix initial action in the UV away from the physical mass point?

1.) fixed UV -scenario:
vary only explicit SB
remaining parameter
adjusted @ physical
point

$\alpha=1$ physical mass point

$\alpha=0$ chiral limit
FRG analysis

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initial action in the UV: 7 parameters, (4 couplings, 2 explicit symmetry breaking, 1 ‘t Hooft determinant)
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How to fix initial action in the UV away from the physical mass point?

1.) fixed UV -scenario:
vary only explicit SB
remaining parameter adjusted @ physical point

No symmetry breaking in the chiral limit!

[Resch, Rennecke, BJS 2019]
FRG analysis

Nf=2+1 FRG QM truncation:

initial action in the UV: 7 parameters, (4 couplings, 2 explicit symmetry breaking, 1 ‘t Hooft determinant)
axial $U_A(1)$ symmetry: on or off

How to fix initial action in the UV away from the physical mass point?

2.) fixed $f_\pi$-scenario: (motivated by chiral perturbation theory)
fix $f_\pi$ in the infrared when explicit SB is varied
trick: vary only UV cutoff while initial action is fixed

[Resch, Rennecke, BJS 2019]
Columbia plots

FRG
[Resch, Rennecke, BJS 2019]

all fluctuations quarks & mesons

with $U_A(1)$-anomaly

without $U_A(1)$-anomaly
Columbia plots

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[Resch, Rennecke, BJS 2019]

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Extended Mean-field analysis
[Resch, Rennecke, BJS 2019]

influence of vacuum fluctuations of quarks

without $U_A(1)$-anomaly

Extended Mean-field analysis
[Resch, Rennecke, BJS 2019]

influence of vacuum fluctuations of quarks
Columbia plots

findings:
- conventional bending of chiral critical surface ➔ critical endpoint @ physical mass point
- tricritical strange quark mass far away from light chiral limit
- First-order region around chiral limit very small ➔ in agreement with lattice
  [Enrödi, et al. 2007, de Forcrane et al. 2017]
- Nf=2+1 ➔ Nf=2 analytically connected two-flavor chiral limit
- influence of axial anomaly on chiral critical line

all fluctuations quarks & mesons

with $U_A(1)$-anomaly

without $U_A(1)$-anomaly
quantum and thermal fluctuations on QCD phase diagram via FRG investigation with different truncations: LPA, LPA’, LPA’+Y

- fluctuations are important (beyond LPA)
- mass sensitivity of the chiral phase structure (Columbia plot)