Quark-diquark models and fluctuations in QCD

Eugenio Megías[1]*
E. Ruiz Arriola[1] and L.L. Salcedo[1]

[1] **Department of Atomic, Molecular and Nuclear Physics, and Carlos I Institute of Theoretical and Computational Physics, University of Granada, Spain.**
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Other references: E. Megías et al. PRD74 (2006) 065005, 114014; PRL109 (2012) 151601; PRD89 (2014) 076006; PRD94 (2016) 096010; PoS Hadron2017 (2018) 232; Acta Phys. Polon. Proc 11 (2018) 563; Nucl. Part. Phys. Proc. 300 (2018) 196.

Issues

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References

- **QCD at finite T:** Phys.Lett. B563 (2003) 173-178; Phys.Rev. D69 (2004). 116003.
- **Polyakov-Nambu–Jona-Lasinio:** hep-ph/0410053; AIP Conf.Proc. 756 (2005) 436-438; Phys.Rev. D74 (2006) 065005; Rom.Rep.Phys. 58 (2006) 081-086; PoS JHW2005 (2006) 025; AIP Conf.Proc. 892 (2007) 444-447; Eur.Phys.J. A31 (2007).
- **Dim-2 Condensates:** JHEP 0601 (2006) 073; Phys.Rev. D75 (2007) 105019; Nucl.Phys.Proc.Suppl. 186 (2009) 256-259; Phys.Rev. D81 (2010) 096009.
- **Hadron Resonance Gas for Polyakov loop:** Phys.Rev.Lett. 109 (2012) 151601; Nucl. Phys. Proc. Suppl. 234 (2013) 313-316; Acta Phys. Polon. B 45, 2407 (2014).
- **Polyakov loop Spectroscopy:** Phys.Rev. D89 (2014) 076006; Nucl. Part. Phys. Proc. 258-259 (2015) 109.
- **Heavy Quark Physics:** Nucl. Part. Phys. Proc. 270-272 (2016) 170-174; Phys.Rev. D94 (2016) 096010.
- **Fluctuations:** 1711.09837; 1805.05214; PRD99 (2019) 074020.

Issues

QCD Thermodynamics

- The partition function of QCD

$$Z_{\rm QCD} = {\rm Tr}\, e^{-H_{\rm QCD}/T} = \sum_n e^{-E_n/T}\,, \qquad H_{\rm QCD}\psi_n = E_n\psi_n\,.$$

- Spectrum of QCD ⟶ Thermodynamics
  Hadron Resonance Gas Model
- In the confined phase: Colour singlet states (hadrons + $\cdots$???)
  ⟶ Low temperature partonic expansion **[EM, E. Ruiz Arriola, L.L. Salcedo, Acta Phys. Pol. B45 '14]**:

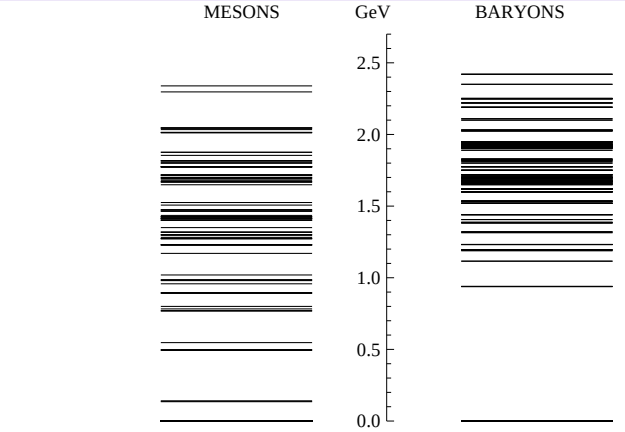
$$Z = \underbrace{Z_0}_{\text{Vacuum}} \cdot \underbrace{Z_{[q\bar{q}]}}_{\text{Mesons}} \cdot \underbrace{Z_{[qqq]} \cdot Z_{[\bar{q}\bar{q}\bar{q}]}}_{\text{Baryons}} \cdot \underbrace{Z_{[q\bar{q}g]}}_{\text{Hybrids}} \cdot \underbrace{Z_{[q\bar{q}q\bar{q}]}}_{\text{Tetraquarks}} \cdot \cdots$$

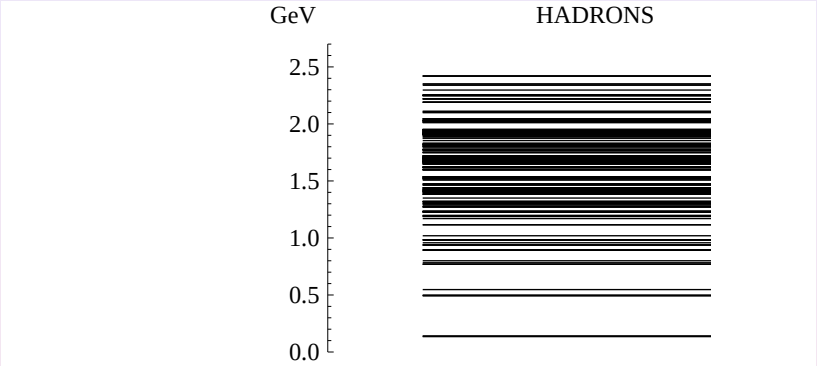
- In the deconfined phase: quarks and gluons ⟶ quark-gluon plasma.
- Phase transition is a crossover ⟶ Do we see quark-gluon substructure BELOW the "*phase transition*"?

Issues

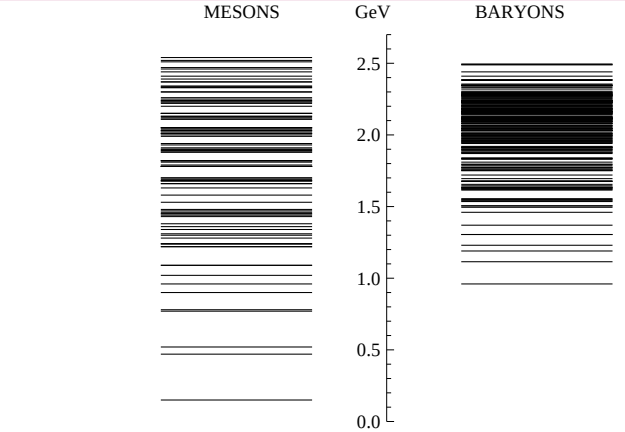
Hadron spectrum (u,d,s)

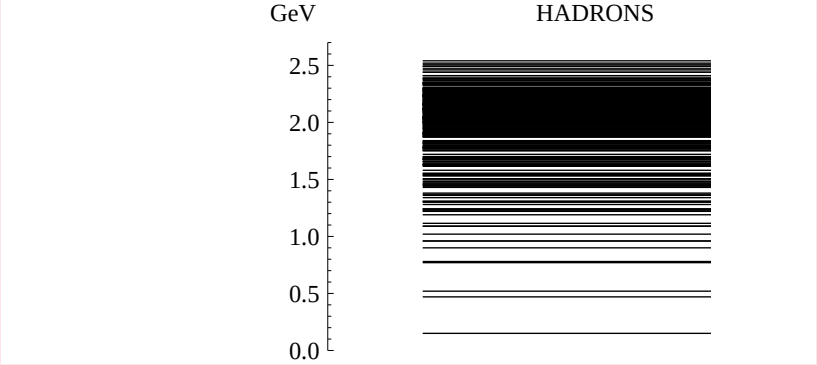
- Particle Data Group (PDG) compilation 2016





- Relativized Quark Model (RQM), Isgur, Godfrey, Capstik 1985





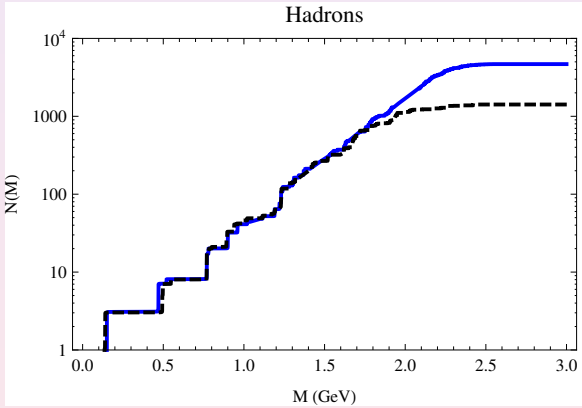
Cumulative number of states

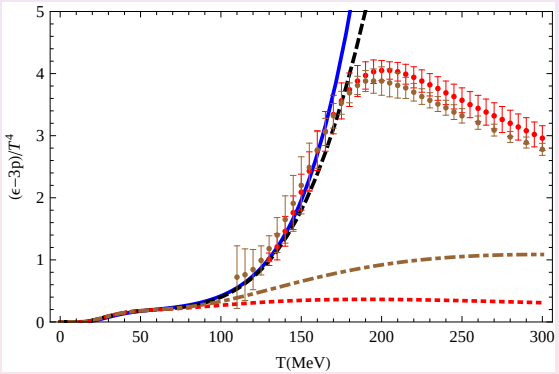
- Cumulative number $\equiv$ number of bound states below M.

$$N(M) = \sum_n \Theta(M - M_n)\,,$$

- Which states count?

$$N(M) = N_{[q\bar{q}]}(M) + N_{[qqq]}(M) + N_{[\bar{q}\bar{q}\bar{q}]}(M) + \cdots\,,$$



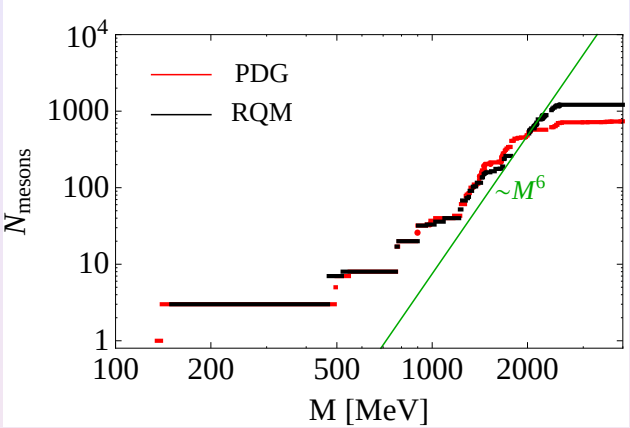


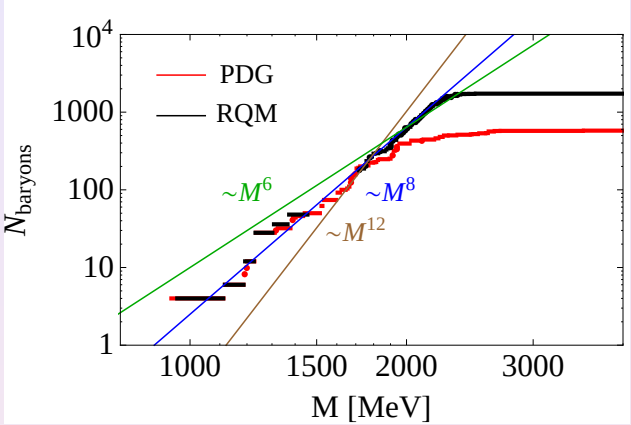
Assumption: $N_{\text{hadrons}} \sim e^{M/T_H}$

$$Z_{\text{HRG}} = \text{Tr}\, e^{-H_{\text{HRG}}/T} \underset{T \to T_H^-}{\longrightarrow} \frac{A}{T_H - T}\,, \qquad T_H \sim 150\,\text{MeV} \equiv \text{Hagedorn temperature}$$

- Non-interacting Hadron-Resonance Gas works for $T \lesssim 0.8 T_c$.

Cumulative number of states





- n-parton Hamiltonian:

$$H_n = \sum_{i=1}^{n} \sqrt{p_i^2 + m^2} + \sum_{i<j}^{n} v_{ij}(r_{ij}) \,, \qquad n = 2 \text{ (mesons)}, \; n = 3 \text{ (baryons)} \,.$$

- Within a semiclassical approximation:

$$N_n(M) \sim g_n \int \prod_{i=1}^{n} \frac{d^3x_i d^3p_i}{(2\pi)^3} \, \delta\!\left(\textstyle\sum_{i=1}^{n} \mathbf{x}_i\right) \delta\!\left(\textstyle\sum_{i=1}^{n} \mathbf{p}_i\right) \theta(M - H_n(p,x)) \sim \left(\frac{M^2}{\sigma}\right)^{3n-3}$$

- Large mass expansion: $N_{[q\bar{q}]} \sim M^6$, $N_{[qqq]} \sim M^{12}$, $N_{[q\bar{q}q\bar{q}]} \sim M^{18}$.
- Data indicate $N_{\rm baryons}(M) \sim M^6 - M^8 \ll M^{12}$ ⟶ qqq excited spectrum effectively behaves as a 2-body system.

Issues

Fluctuations of conserved charges

- Conserved charges ⟶ $[Q_a, H] = 0.$
- In the *uds* sector the only conserved charges are:

$$Q \equiv \text{Electric charge}, \quad B \equiv \text{Baryon number}, \quad S \equiv \text{Strangeness}.$$

- In the hot vacuum (no chemical potential):

$$\langle Q \rangle_T = 0\,, \qquad \langle B \rangle_T = 0\,, \qquad \langle S \rangle_T = 0\,.$$

- Fluctuations ⟶ Susceptibilities:

$$\boxed{\chi_{ab}(T) \equiv \frac{1}{VT^3}\langle \Delta Q_a \Delta Q_b \rangle_T\,, \qquad \Delta Q_a = Q_a - \langle Q_a \rangle_T\,.}$$

- At high temperature ($N_c = 3$ and flavors u, d, s):

$$\chi_{BB}(T) \to 1/3\,, \quad \chi_{BQ}(T) \to 0\,, \qquad \chi_{BS}(T) \to -1/3\,,$$
$$\chi_{SS}(T) \to 1\,, \qquad \chi_{QS}(T) \to 1/3\,, \quad \chi_{QQ}(T) \to 2/3\,.$$

Fluctuations of conserved charges

[M.Asakawa, M.Kitazawa, Prog. Part. Nucl. Phys. 90 (2016)].

- Fluctuations of conserved charges can be computed from the grand-canonical partition function:

$$Z = \mathrm{Tr}\exp\left[-\left(H - \sum_a \mu_a Q_a\right)/T\right], \qquad \Omega = -T\log Z\,,$$

by differentiation

$$\boxed{-\frac{\partial\Omega}{\partial\mu_a}\bigg|_{\mu_a=0} = \langle Q_a\rangle_T\,, \quad -T\frac{\partial^2\Omega}{\partial\mu_a\partial\mu_b}\bigg|_{\mu_a=0=\mu_b} = \langle\Delta Q_a\Delta Q_b\rangle_T \equiv VT^3\chi_{ab}(T)}$$

- $Q_a \in \{Q, B, S\}$, or, in the quark-flavor basis, $Q_a \in \{u, d, s\}$ where

$$B = \frac{1}{3}(u+d+s)\,, \quad Q = \frac{1}{3}(2u-d-s)\,, \quad S = -s\,.$$

Issues

Fluctuations within the HRG approach

- Fluctuations within the HRG approach:

$$Q_a = \sum_{i \in \text{Hadrons}} q_i^a N_i \,, \quad \chi_{ab}(T) = \sum_{i,j \in \text{Hadrons}} q_i^a q_j^b \langle \Delta N_i \Delta N_j \rangle_T \,, \quad a, b \in \{Q, B, S\}$$

where $q_i^a \in \{Q_i, B_i, S_i\} \equiv$ charge with symmetry a of ith-hadron.

- Averaged number of hadrons of type i is

$$\langle N_i \rangle_T = V \int \frac{d^3k}{(2\pi)^3} \frac{g_i}{e^{E_{k,i}/T} - \xi_i} \,,$$

with $E_{k,i} = \sqrt{k^2 + M_i^2}$, and $\xi = \pm 1$ for bosons/fermions.

- Susceptibilities:

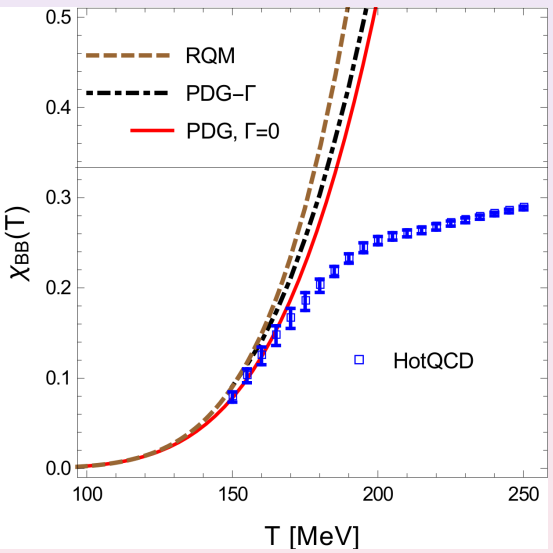
$$\chi_{ab}(T) = \frac{1}{2\pi^2} \sum_{i \in \text{Hadrons}} g_i \, q_i^a \, q_i^b \sum_{n=1}^{\infty} \zeta_i^{n+1} \frac{M_i^2}{T^2} K_2\left(\frac{nM_i}{T}\right) \,,$$

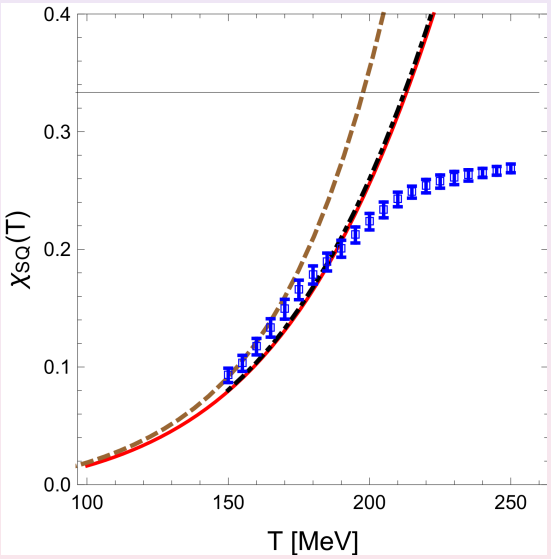
with $a, b = B, Q, S$, and $K_2(z)$ is the Bessel function of 2nd kind.

Fluctuations and missing states

- Fluctuations of Conserved Charges ➞ Good description of lattice data for $T \lesssim 160$ MeV.

Lattice data of Fluctuations [A.Bazavov et al., PRD86 (2012)]

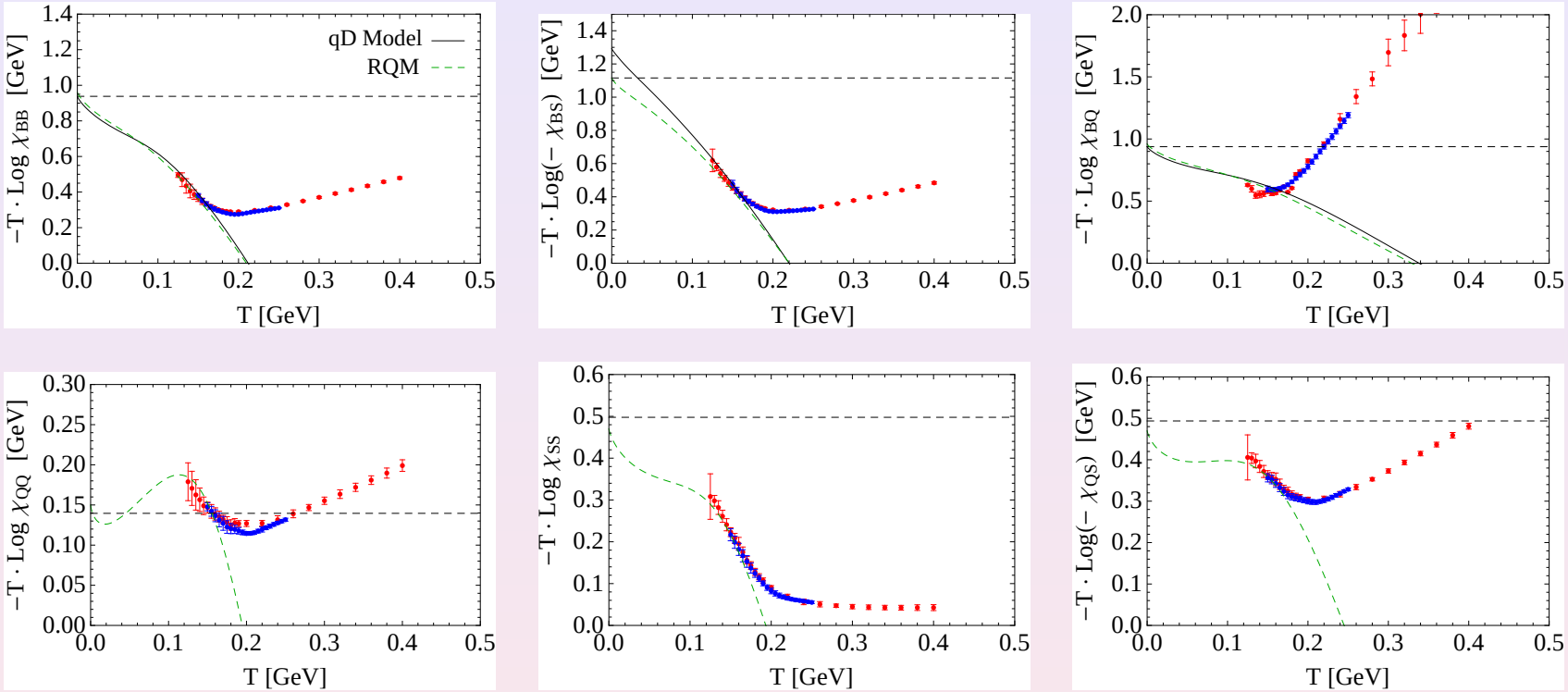




➞ *Fluctuations as a diagnostic tool to study missing states*.
Example: RQM seems to have too many baryonic states, but not too many charged states.

[E.Ruiz Arriola, W.Broniowski, E.M., L.L.Salcedo, 1612.07091].

Susceptibilities within the HRG approach



Lattice data: [A. Bazavov et al. PRD86 '12], [S. Borsanyi et al. JHEP 01 '12].

- Low temperature behavior ➡ dominated by lowest-lying state

$$\chi_{BB}(T) \underset{T\to 0}{\sim} e^{-M_p/T}, \quad \chi_{BS}(T) \underset{T\to 0}{\sim} e^{-M_{\Lambda^0}/T}, \quad \chi_{BQ}(T) \underset{T\to 0}{\sim} e^{-M_p/T},$$

$$\chi_{QQ}(T) \underset{T\to 0}{\sim} e^{-M_{\pi^\pm}/T}, \quad \chi_{SS}(T) \underset{T\to 0}{\sim} e^{-M_{K^\pm}/T}, \quad \chi_{QS}(T) \underset{T\to 0}{\sim} e^{-M_{K^\pm}/T}.$$

Issues

Quark-diquark model for baryons

- Relativistic quark-diquark model for baryons $B \equiv [\underbrace{(qq)}_{D}\, q]$
  [E. Santopinto, J. Ferretti, PRC92 '15]:

$$H_{qD} = \sqrt{\vec{p}^2 + m_q^2} + \sqrt{\vec{p}^2 + m_D^2} + V_{qD}(r)$$

- It can be proved that $V_{qD}(r) \simeq V_{q\bar{q}}(r)$ (up to an additive constant)

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = (\mathbf{3} \otimes \bar{\mathbf{3}}) \oplus (\mathbf{3} \otimes \mathbf{6}),$$

$$e^{-F_{qD}(\mathbf{x}_1,\mathbf{x}_2,T)/T} \equiv e^{-F_{q\bar{q}}(\mathbf{x}_1,\mathbf{x}_2,T)/T} + e^{-F_{3\otimes 6}(\mathbf{x}_1,\mathbf{x}_2,T)/T} \simeq e^{-F_{q\bar{q}}(\mathbf{x}_1,\mathbf{x}_2,T)/T}.$$

  [EM, E. Ruiz Arriola, L.L. Salcedo PRD99 '19].
- $V_{qD}(r) = -\frac{\tau}{r} + \sigma r + \mu\,, \qquad \text{with} \qquad \tau = \pi/12\,, \quad \sigma = (0.42\,\text{GeV})^2.$
- Parameters of the model are controlled by:
  i) $m_{\text{cons}} \equiv$ constituent quark mass
  ii) $\hat{m}_s \equiv$ current quark mass for the strange quark

$$m_{u,d} = m_{\text{cons}}\,, \qquad m_s = m_{\text{cons}} + \hat{m}_s\,.$$

Quark-diquark model for baryons

- Distinguish between two kinds of diquarks:
  - scalar D: $[q_1 q_2] \equiv$ quark content notation
  - axial vector D_{AV}: $\{q_1 q_2\}$
  - mass difference: $\Delta m_D := m_{D_{AV}} - m_D \simeq 0.21\,\mathrm{GeV}$ **[R.L.Jaffe, Phys. Rep. 409 '05].**
- Take into account the breaking of flavor $\mathrm{SU}(3)$ for diquarks:

$$m_D = \underbrace{m_{D,\mathrm{ns}}}_{\text{diquark with no s-quarks}} + n_s \hat{m}_s \,.$$

- Natural choice:

$$m_{D,\mathrm{ns}} = 2 m_{\mathrm{cons}}$$

- Free parameters of the model:

$$m_{\mathrm{cons}}, \quad \hat{m}_s, \quad \mu \quad \text{and} \quad m_{D,\mathrm{ns}} \,.$$

Quark-diquark model for baryons

Baryon	Total deg.	$Q=-1$	$Q=0$	$Q=1$	$Q=2$
$[nn]n$	4	-	2	2	-
$\{nn\}n$	36	6	12	12	6
$[nn]s$	2	-	2	-	-
$\{nn\}s$	18	6	6	6	-
$[ns]n$	8	2	4	2	-
$\{ns\}n$	24	6	12	6	-
$[ns]s$	4	2	2	-	-
$\{ns\}s$	12	6	6	-	-
$\{ss\}n$	12	6	6	-	-
$\{ss\}s$	6	6	-	-	-

Table : Spin-isospin degeneracies of the baryonic states within the quark-diquark model. The columns from $Q=-1$ to $Q=2$ contain the degeneracies by distinguishing between the electric charges of the states.

$n \equiv$ light flavors u, d $\qquad$ $s \equiv$ strange quark.

- Quantum numbers of these states: Q , $B=1$, $S=-n_s$.

Issues

Baryon spectrum

- Variational method ⟶ Diagonalize the Hamiltonian:

$$H_{qD} = \sqrt{\vec{p}^2 + m_q^2} + \sqrt{\vec{p}^2 + m_D^2} + V_{qD}(r)\,.$$

- For the diagonalization process, we use as basis the isotropic harmonic oscillator

$$R_{n\ell}(r) = \frac{e^{-\frac{r^2}{2b^2}}}{\sqrt[4]{\pi}} \left(\frac{r}{b}\right)^{\ell} \sqrt{\frac{(n-1)!\,2^{\ell+n+1}}{b^3(2\ell+2(n-1)+1)!!}}\, L_{n-1}^{\ell+\frac{1}{2}}(r^2/b^2)\,.$$

- Matrix elements

$$\begin{aligned}\langle n\ell|H_{qD}|n'\ell\rangle &= \int_0^\infty dp\, \hat{u}^*_{n\ell}(p)\hat{u}_{n'\ell}(p)\left[\sqrt{p^2+m_q^2}+\sqrt{p^2+m_D^2}\right] \\ &\quad + \int_0^\infty dr\, u^*_{n\ell}(r)u_{n'\ell}(r)V_{qD}(r)\,,\end{aligned}$$

with $u_{n\ell}(r) = r\,R_{n\ell}(r)$.

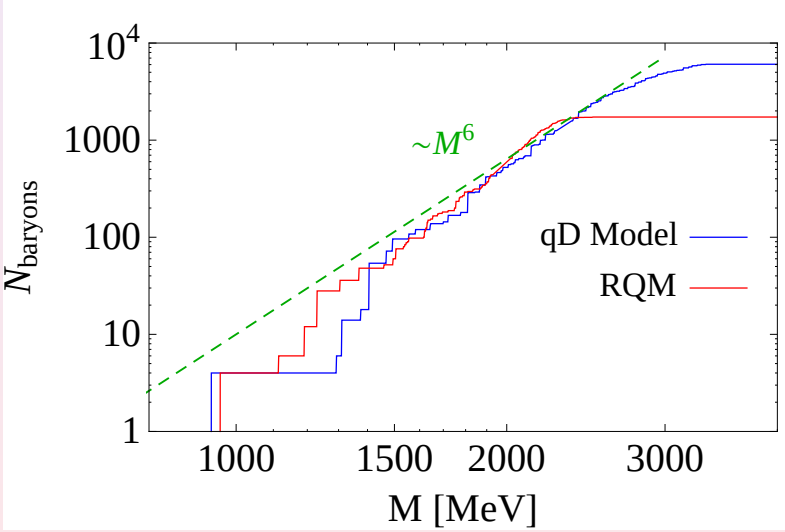
- $b \equiv$ minimize energy levels. Typically: $0.55\,\mathrm{fm} \lesssim b \lesssim 0.65\,\mathrm{fm}$.

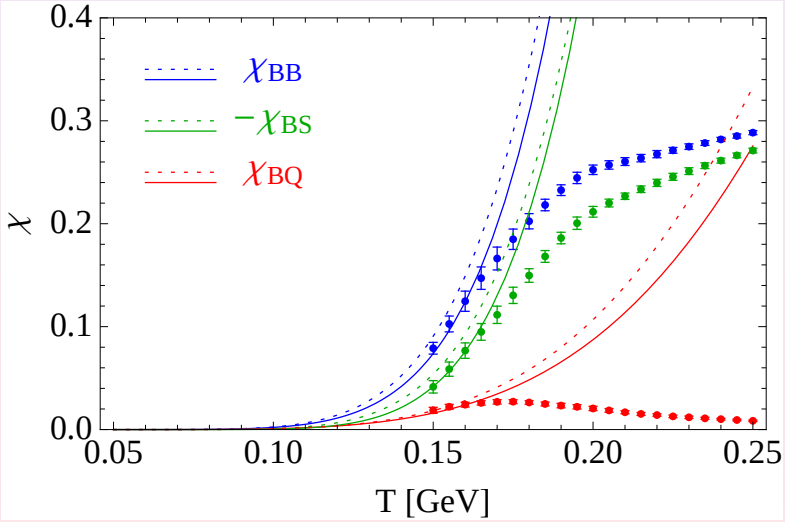
Baryon spectrum

- Typical choice of parameters:

$$m_{D,\mathrm{ns}} = 0.6\,\mathrm{GeV}\,, \quad m_{u,d} = 0.3\,\mathrm{GeV}\,,$$
$$\hat{m}_s = 0.10\,\mathrm{GeV}\,, \quad \mu = -0.459\,\mathrm{GeV}\,.$$

- Spectrum of baryons → Baryonic susceptibilities:





Solid lines: qD model.

Dotted lines: RQM spectrum.

[EM, E.Ruiz Arriola, L.L.Salcedo, PRD99 (2019) 074020].

Issues

Baryonic susceptibilities

- Spectrum of baryons within the quark-diquark model → baryonic susceptibilities:

$$\chi_{BB}(T), \quad \chi_{BQ}(T), \quad \chi_{BS}(T),$$

$$\text{with} \qquad \chi_{ab}(T) = \frac{1}{2\pi^2} \sum_{i \in \text{Hadrons}} g_i \, q_i^a \, q_i^b \sum_{n=1}^{\infty} \zeta_i^{n+1} \frac{M_i^2}{T^2} K_2\left(\frac{nM_i}{T}\right).$$

- 4 free parameters in the model → perform the best fit to the lattice data → $\bar{\chi}^2$ minimization:

$$\bar{\chi}^2 = \bar{\chi}^2_{BB} + \bar{\chi}^2_{BQ} + \bar{\chi}^2_{BS}, \quad \text{where} \quad \bar{\chi}^2_{ab} = \sum_{j=1}^{j_{\max}} \frac{\left(\chi^{\text{lat}}_{ab}(T_j) - \chi^{\text{HRG}}_{ab}(T_j)\right)^2}{(\Delta\chi^{\text{lat}}_{ab}(T_j))^2}.$$

- Best fit:

$$T \lesssim 125\,\text{MeV} - 150\,\text{MeV}.$$

Baryonic susceptibilities

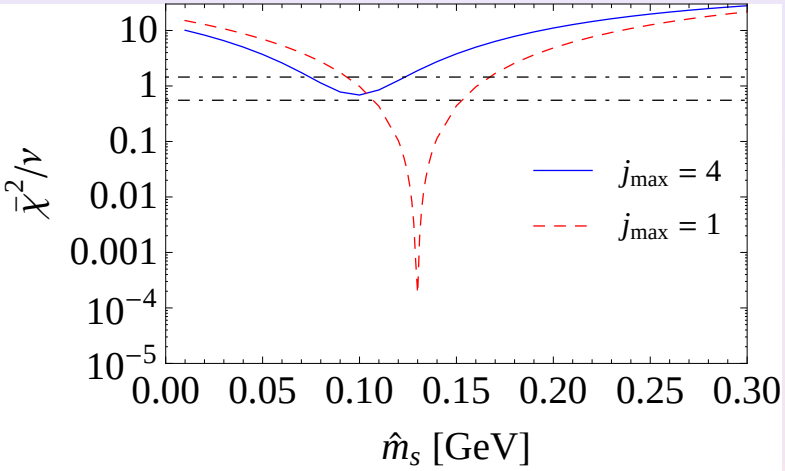


Figure : $\bar{\chi}^2/\nu$ as a function $\hat{m}_s$. We have fixed:

$$m_{\rm cons} = 0.3\,{\rm GeV} \qquad \text{and} \qquad m_{D,{\rm ns}} = 2m_{\rm cons}.$$

Lattice data of **[A.Bazavov et al. PRD86 '12]**.

- Best fit: $\hat{m}_s = 0.10\,{\rm GeV}$ and $\mu = -0.459\,{\rm GeV}$.

Baryonic susceptibilities

- General fits to study the full validity of the model:

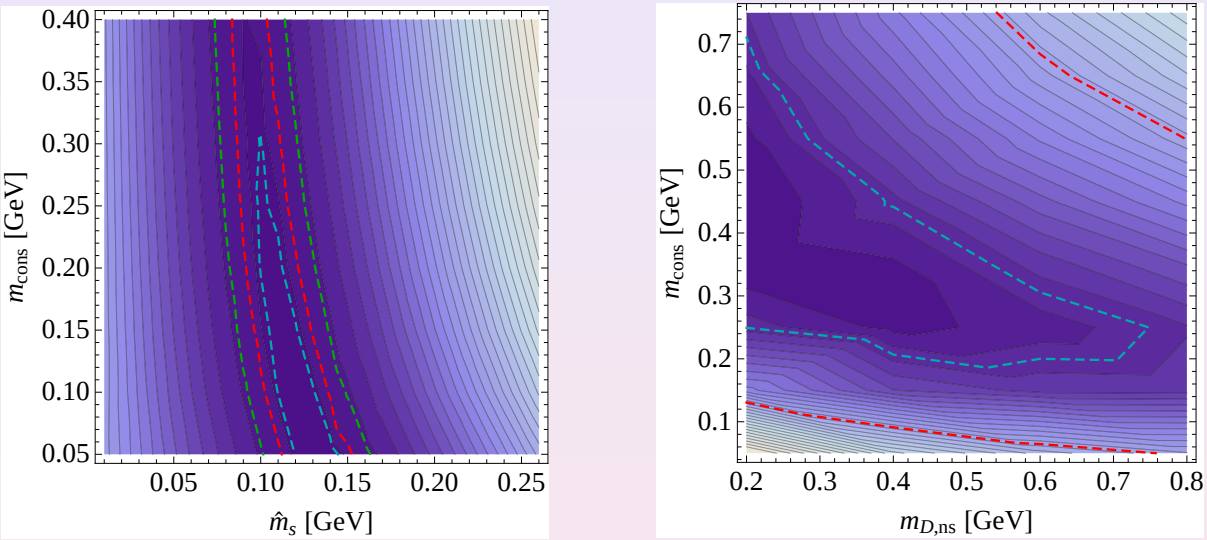
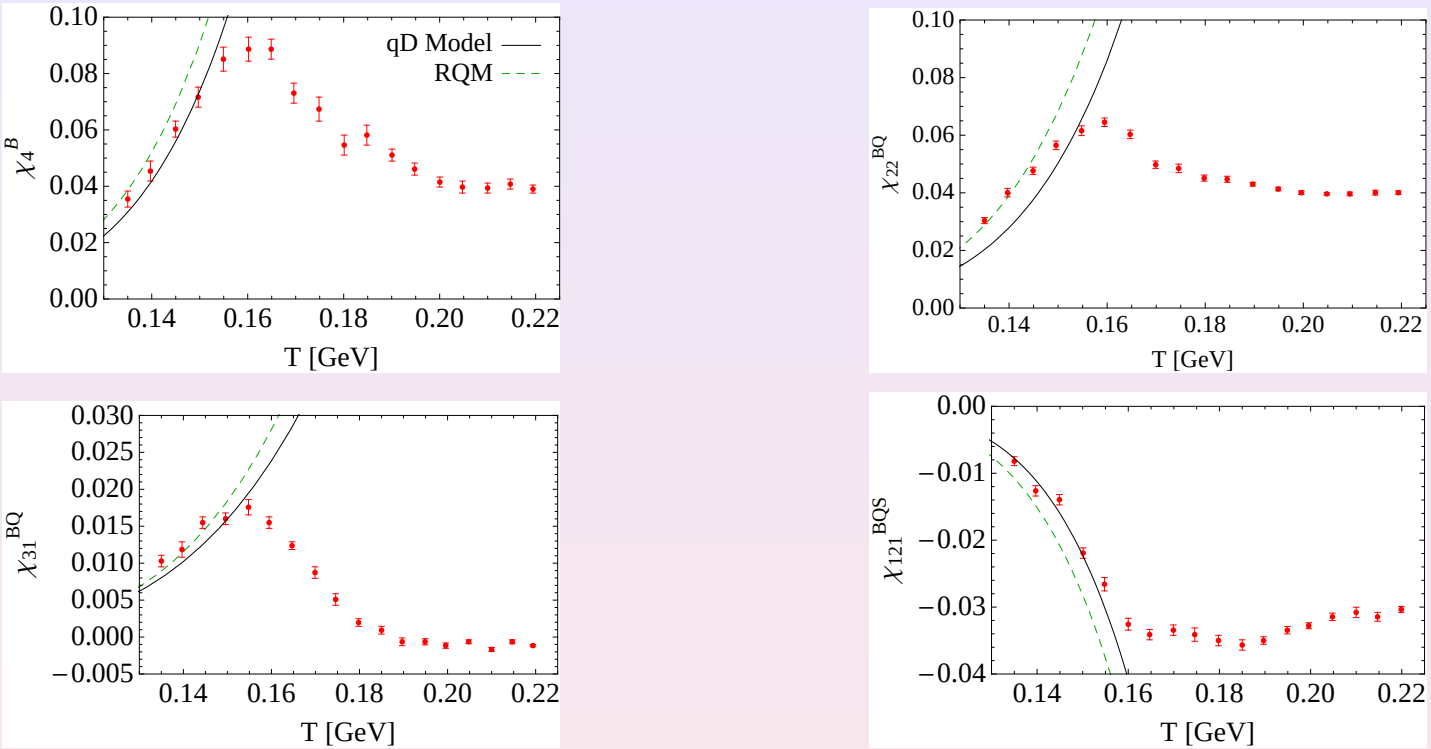


Figure : $\bar{\chi}^2/\nu$ from a fit to the lattice data of the baryonic fluctuations from **[A.Bazavov et al. PRD86 '12]** with $T \leq 150\,\mathrm{MeV}$. The dashed lines correspond to $\bar{\chi}^2/\nu = 0.77$ (blue), $\bar{\chi}^2/\nu = 1$ (red) and $1 + \sqrt{2/\nu}$ (green).

- Left panel: plane $(\hat{m}_s, m_{\rm cons})$ as free parameters, with $m_{D,\rm ns} = 2m_{\rm cons}$.
- Right panel: plane $(m_{D,\rm ns}, m_{\rm cons})$ with $\hat{m}_s = 0.10\,\mathrm{GeV}$ $\longrightarrow$ $m_{D,\rm ns} = 2m_{\rm cons}$ not assumed.

Baryonic susceptibilities of 4th order



- Baryonic susceptibilities of 4th order, χ_4^B, χ_{22}^{BQ}, χ_{31}^{BQ} and χ_{121}^{BQS}, from the quark-diquark model (solid black).

Dots → lattice data of **[S.Borsanyi et al. JHEP 10 (2018) 205]**.
Dashed green lines → result from the spectrum of the RQM.

Issues

Semiclassical estimates of the susceptibilities

- Cumulative number

$$N(M) = \mathrm{Tr}(\Theta(M - \hat{H})) \simeq \int \frac{d^3x d^3p}{(2\pi)^3} \Theta(M - H) + \cdots ,$$

with

$$H = \sqrt{\vec{p}^2 + m_q^2} + \sqrt{\vec{p}^2 + m_D^2} + \sigma r - \frac{4\alpha_S}{3r} + \mu .$$

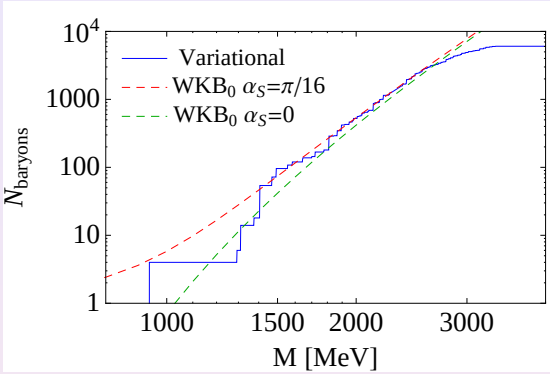
- The integral can be performed analytically as an expansion in $M^2/\sigma \gg 1$. When $m_q = 0 = m_D$:

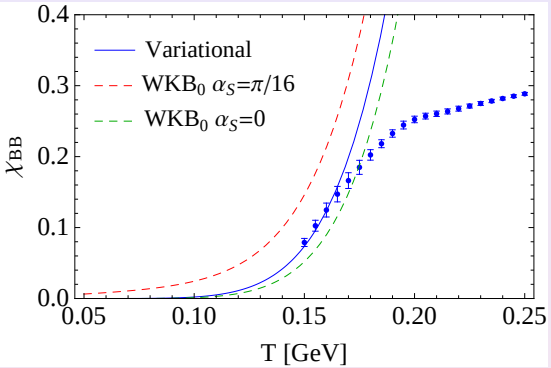
$$N_{\mathrm{WKB}}(M) = \frac{M^6}{720\pi\sigma^3} + \alpha_S \frac{M^4}{36\pi\sigma^2} + \alpha_S^2 \frac{2M^2}{9\pi\sigma} - \frac{M^2}{9\pi\sigma} + \cdots .$$

- This leads to the baryonic susceptibility:

$$\chi_{BB}^{\mathrm{WKB}}(T) = \frac{127\pi^5}{94500}\left(\frac{T^2}{\sigma}\right)^3 + \frac{31\pi^3}{5670}\alpha_S\left(\frac{T^2}{\sigma}\right)^2 + \frac{7\pi}{405}\alpha_S^2\frac{T^2}{\sigma} - \frac{7\pi}{810}\frac{T^2}{\sigma} + \cdots .$$

Semiclassical estimates of the susceptibilities





- When considering finite mass effects:

$$N_{\rm WKB}(M) = \frac{64\sqrt{2}}{945\pi\sigma^3}\left(\frac{m_q m_D}{m_q+m_D}\right)^{3/2}(M-m_q-m_D)^{9/2}+\cdots,$$

and

$$\chi_{BB}^{\rm WKB}(T) = \frac{(m_q m_D)^{3/2}T^3}{\pi^2\sigma^3}e^{-(m_q+m_D)/T}+\cdots, \quad \text{for} \quad T \ll m_q+m_D\,.$$

Conclusions

- At low temperatures hadrons can be considered as a complete basis of states in terms of a Hadron Resonance Gas (HRG) approach. The HRG works at $T \lesssim 0.8\,T_c$.
- Close T_c many hadrons are needed to saturate the sum rule $\Longrightarrow$ previous studies suggest that there are in the QCD spectrum:
  i) conventional missing states ($q\bar{q}$ and qqq),
  ii) hybrid states ($q\bar{q}g$ and $qqqg$),
  iii) tetraquarks, pentaquarks, etc ($q\bar{q}q\bar{q}$, $q\bar{q}qqq, \cdots$).
- Cumulative number of baryons suggests that qqq excited spectrum effectively behaves as a 2-body system $\longrightarrow$ quark-diquark picture of baryons.
- Fluctuations of conserved charges in confined phase of QCD allow to study hadron spectrum and missing states in 3 sectors:
  i) electric charge, ii) baryon number, and iii) strangeness.
- We have studied the spectrum of baryons within a quark-diquark model, and used it to compute the baryonic susceptibilities $\longrightarrow$ good agreement with lattice data of susceptibilities.

Thank You!