Symmetry, Confinement, and the Higgs Phase

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8th Intl Conference on
New Frontiers in Physics

Kolymbari, Greece

August 2019

PRD 2017, 2018 and in progress

What is "confinement"?

Suppose we have an SU(N) gauge theory with matter fields in the fundamental representation, e.g. QCD. Wilson loops have perimeter-law falloff asymptotically, Polyakov lines have a non-zero VEV, what does it mean to say such theories (QCD in particular) are confining?

Most people take it to mean “color confinement” or

C-confinement

There are only color neutral particles in the asymptotic spectrum.

The problem with C-confinement is that it also holds true for gauge-Higgs theories, deep in the Higgs regime, where there are

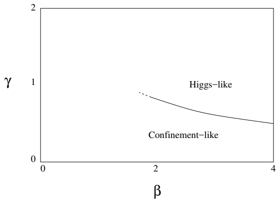
- only Yukawa forces,
- no linearly rising Regge trajectories,
- no color electric flux tubes.

If C-confinement is “confinement,” then the Higgs phase is also confining.

C-confinement in gauge-Higgs theories

How we know this:

1. **Elitzur's** Theorem: No such thing as spontaneous symmetry breaking of a local gauge symmetry.
2. The **Fradkin-Shenker-Osterwalder-Seiler** (FSOS) Theorem: There is no transition in coupling-constant space which isolates the Higgs phase from a confinement-like phase.
3. **Frölich-Morchio-Strocchi** (FMS) and also **'t Hooft** (1980): physical particles (e.g. W's) in the spectrum are created by gauge-invariant operators in the Higgs region.



FMS show how to recover the usual results of perturbation theory, starting from gauge-invariant composite operators.

Conclusion: If the confinement-like (QCD-like) region has a color neutral spectrum, then so does the Higgs-like region.

Higgs and Confinement: what's the difference?

The Higgs and confinement regions are both massive. Yet QCD and the weak interactions seem physically so different.

Questions:

- Can that difference be formulated precisely? Is there some variety of confinement other than C confinement?
- Are the confinement-like and Higgs-like regions of a gauge-Higgs theory differentiated by the breaking of a symmetry?
- If so, then what symmetry? And how is symmetry breaking related to a transition in the type of confinement?

Beyond C-confinement?

In a pure SU(N) gauge theory there is a different and stronger meaning that can be assigned to the word "confinement," which goes beyond C-confinement.

Of course the spectrum consists only of color neutral objects: glueballs.

But such theories *also* have the property that the static quark potential rises linearly or, equivalently, that large planar Wilson loops have an area-law falloff.

Is there any way to generalize this property to gauge theories with matter in the fundamental representation?

Separation-of-charge (“S_c”) confinement

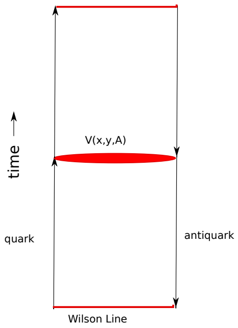
The Wilson area-law criterion for pure gauge theories is equivalent to “S_c-confinement.”

A static $q\overline{q}$ pair, connected by a Wilson line, evolves in Euclidean time to some state

$$\Psi_V \equiv \overline{q}^a(\mathbf{x})V^{ab}(\mathbf{x},\mathbf{y};A)q^b(\mathbf{y})\Psi_0$$

where $V(\mathbf{x},\mathbf{y};A)$ is a gauge bi-covariant operator transforming as

$$V^{ab}(\mathbf{x},\mathbf{y};A) \rightarrow g^{ac}(\mathbf{x},t)V^{cd}(\mathbf{x},\mathbf{y};A)g^{\dagger db}(\mathbf{y},t)$$



The energy above the vacuum energy $\mathcal{E}_{vac}$ is

$$E_V(R) = \langle \Psi_V | H | \Psi_V \rangle - \mathcal{E}_{vac}$$

S_c-confinement

$$\lim_{R\to\infty} E_V(R) = \infty$$

for ***ANY*** choice of bi-covariant $V(\mathbf{x}, \mathbf{y}; A)$.

For an SU(N) pure gauge theory, $E_V(R) \geq E_0(R)$, where $E_0(R) \sim \sigma R$ is the ground state energy of a static quark-antiquark pair.

Our proposal: S_c-confinement should also be regarded as the confinement criterion in gauge+matter theories. The crucial element is that the bi-covariant operators $V^{ab}(\mathbf{x}, \mathbf{y}; A)$ *must depend only on the gauge field A* at a fixed time, and not on the matter fields.

The idea is to study the energy $E_V(R)$ of physical states with large separations R of static color charges, *unscreened by matter fields*.

If $V^{ab}(\mathbf{x}, \mathbf{y}; A)$ would also depend on the matter field(s), then it is easy to violate the S_c-confinement criterion, e.g. let ϕ be a matter field in the fundamental representation, and

$$V^{ab}(\mathbf{x}, \mathbf{y}, \phi) = \phi^a(\mathbf{x})\phi^{\dagger b}(\mathbf{y})$$

Then

$$\Psi_V = \{\overline{q}^a(\mathbf{x})\phi^a(\mathbf{x})\} \times \{\phi^{\dagger b}(\mathbf{y})q^b(\mathbf{y})\}\Psi_0$$

corresponds to two color singlet (static quark + Higgs) states, only weakly interacting at large separations. Operators V of this kind, which depend on the matter fields, are excluded.

This also means that the lower bound $E_0(R)$, unlike in pure gauge theories, is *not* the lowest energy of a state containing a static quark-antiquark pair.

It is the lowest energy of such states when color screening by matter is excluded.

SU(2) gauge-Higgs theory

Most of our numerical work is one in this model, with a unimodular $|\phi| = 1$ Higgs field. In SU(2) the doublet can be mapped to an SU(2) group element

$$\vec{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \Longrightarrow \phi = \begin{bmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{bmatrix}$$

and the corresponding action is

$$S = \beta \sum_{plaq} \frac{1}{2} \text{Tr}[UUU^\dagger U^\dagger] + \gamma \sum_{x,\mu} \frac{1}{2} \text{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x+\widehat{\mu})]$$

Existence of S_c-confinement

1. Does S_c-confinement exist *anywhere* in the $\beta - \gamma$ phase diagram, apart from pure gauge theory ($\gamma = 0$)?

   Yes. We can show that gauge-Higgs theory is S_c-confining at least in the region

$$\gamma \ll \beta \ll 1 \quad \text{and} \quad \gamma \ll \frac{1}{10}$$

   This is based on strong-coupling expansions and a theorem (Gershgorim) in linear algebra.

2. Then does S_c-confinement hold *everywhere* in the $\beta - \gamma$ phase diagram?

   No. We can construct V operators which violate the S_c-confinement criterion when γ is large enough.

So there must exist a transition between S_c and C confinement.

Loss of S_c-confinement

Away from strong coupling, there is no guarantee of S_c-confinement.

If we can find *even one* V at some β, γ such that E_V does not grow linearly with R, then S_c-confinement is lost at that β, γ.

For $V =$ Wilson line, $E_V(R) \propto R$ even for non-confining theories. Not useful!
Instead we consider

1. **The Dirac state**
   generalization of the lowest energy state with static charges in an abelian theory.

2. **Pseudomatter**
   Introduce fields built from the gauge field which transform like matter fields. See if these induce string-breaking.

3. **"Fat link" states**
   Wilson lines built from smoothed links.

Dirac, Pseudomatter, Fat Link states

- **Dirac state:** A generalization of the ground state in the abelian theory with static charges. Let $G_C(\mathbf{x}, A)$ be the gauge transformation $A \to$ Coulomb gauge. Then

$$V^{ab}(x, y; A) = G_C^{\dagger ac}(\mathbf{x}; A) G_C^{cb}(\mathbf{y}; A)$$

- **Pseudomatter:** Let φ_n be the eigenstates of the covariant Laplacian

$$(-D_i D_i)^{ab}_{\mathbf{xy}} \varphi_n^b(\mathbf{y}) \quad = \quad \lambda_n \varphi_n^a(\mathbf{x})$$

Then

$$V^{ab}(\mathbf{x}, \mathbf{y}; A) \quad = \quad \varphi_1^a(\mathbf{x}) \varphi_1^{\dagger b}(\mathbf{y})$$

- **Fat Links:** $V(\mathbf{x}, \mathbf{y}; A)$ on the lattice is just a Wilson line constructed from “fat” links, obtained by the usual iterative procedure.

The Calculation

In general

$$E_V(R) = -\lim_{t\to 0} \frac{d}{dt} \log \left[\frac{\langle \Psi_V | e^{-Ht} | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \right] - \mathcal{E}_{vac}$$

on the lattice

$$E_V(R) = -\log \left[\frac{\left\langle \mathrm{Tr}\left[U_0(x,t) V(x,y,t+1) U_0^\dagger(y,t) V(y,x,t) \right] \right\rangle}{\langle \mathrm{Tr}\,[V(x,y,t) V(y,x,t)] \rangle} \right]$$

This is what we calculate numerically.

Results

Each of these states defines a V operator, and we can calculate $E_V(R)$ by lattice Monte Carlo.

- We find a line of S_c to C-confinement transition in the $\beta - \gamma$ plane for the V operator corresponding to the Dirac state.
- We find an S_c to C-confinement transition for the V operator constructed from pseudomatter fields. The transition line is close to (but a little below) the transition line for the Dirac state.
- The fat link state seems to be everywhere S_c-confining. This doesn't mean that the gauge-Higgs theory is everywhere S_c-confining. It means instead that not every operator can detect the transition to C-confinement.

See *arXiv:1708.08979* for details.

Gauge choice to V operator

Given an $F[U] = 0$ gauge, let $g_F(x; U)$ be the transformation to the gauge. Then define

$$V_F(\mathbf{x}, \mathbf{y}; U) = g_F(\mathbf{x}; U) g_F^\dagger(\mathbf{y}; U)$$

In an F-gauge, $V_F = \mathbb{1}$. Then in this gauge

$$E_V(R) = -\log \left[\frac{1}{N} \langle \text{Tr} \left[U_0(\mathbf{x}, t) U_0^\dagger(\mathbf{x}_0, t) \right] \rangle_F \right]$$

gauge choice, continued

In any F gauge, there is a remnant global subgroup of the gauge symmetry of (at least) $g(\mathbf{x}, t) = z(t) \in Z_N$. Let $\langle \phi(x) \rangle_F$ be the VEV of ϕ in this gauge.

If $\langle \phi(x) \rangle_F \neq 0$ in the thermodynamic limit, then this remnant global symmetry is broken.

U_0 is sensitive to this symmetry, and will also pick up a finite VEV if the symmetry is broken. Then

$$\begin{aligned} \lim_{R\to\infty} E_V(R) &= \lim_{R\to\infty} -\log\left[\frac{1}{N}\mathrm{Tr}\left[\langle U_0(\mathbf{x}, t)\rangle \langle U_0^\dagger(\mathbf{y}, t)\rangle\right]\right] \\ &= \text{finite} \end{aligned}$$

So breaking remnant symmetry in an F gauge implies C confinement. But the breaking happens in different places in different gauges.

Where does C confinement begin? Is there a necessary condition of some kind?

Custodial Symmetry

Define a *custodial symmetry* to be a symmetry of the matter fields such that any operator which transforms under that symmetry also transforms under the gauge symmetry.

Example: SU(2) gauge-Higgs theory

$$S_H = \gamma \sum_{x,\mu} \frac{1}{2} \text{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x+\widehat{\mu})]$$

is invariant under $\text{SU(2)}_{gauge} \times \text{SU(2)}_{global}$:

$$\begin{aligned} U_\mu(x) &\rightarrow L(x) U_\mu(x) L^\dagger(x+\hat{\mu}) \\ \phi(x) &\rightarrow L(x)\phi(x) R \end{aligned}$$

$R \in \text{SU(2)}_{global}$ is a custodial symmetry transformation.

Custodial symmetry breaking

Define a partition function for spacelike links and the scalar field at fixed time $t = 0$

$$
\begin{aligned}
\mathbf{Z}(U,\phi) &= \int DU_{i,t\neq 0} D\phi_{t\neq 0} DU_0 \; e^{-S} \\
\mathbf{Z}(U) &= \int D\phi_{t=0} \, \mathbf{Z}(U,\phi)
\end{aligned}
$$

and probability distribution

$$
\langle Q \rangle = \int DU \; Q(U) P(U) \;\; , \;\; P(U) = \frac{\mathbf{Z}(U)}{Z}
$$

At *fixed* U, $\mathbf{Z}(U,\phi)$ has no local gauge symmetry, but it does have custodial symmetry, which can break spontaneously:

$$
\overline{\phi}(\mathbf{x}; U) = \frac{1}{\mathbf{Z}(U)} \int D\phi \; \phi(\mathbf{x}) \mathbf{Z}(U,\phi)
$$

Of course this symmetry breaking depends on U, and $\overline{\phi}(\mathbf{x})$ depends on $\mathbf{x}$.

The question is whether the symmetry is broken for configurations U selected from probability distribution $P(U)$.

Gauge-invariant order parameter

The gauge-invariant order parameter for custodial symmetry breaking is

$$\langle\Omega\rangle = \int DU \frac{1}{V} \sum_{\mathbf{x}} |\overline{\phi}(\mathbf{x}; U)| \frac{\mathbf{Z}(U)}{Z}$$

which is non-zero in the thermodynamic limit if custodial symmetry is broken.

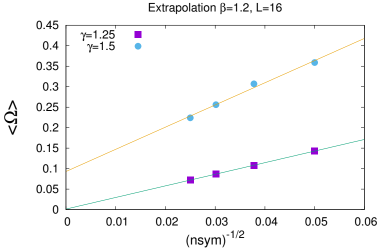
In practice compute $\langle\Omega\rangle$ by a Monte-Carlo-in-a-Monte-Carlo procedure. Update U, ϕ as usual, but at data-taking, keep spacelike links fixed on a $t = 0$ timeslice. Update everything else, and compute

$$\frac{1}{V_3} \sum_{\mathbf{x}} |\overline{\phi}(\mathbf{x}, t = 0; U)|$$

at $t = 0$. Average over data taking sweeps, and extrapolate to $V_3 \to \infty$.

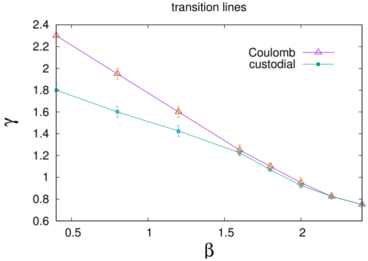
Note that there is no gauge-fixing of any sort in this computation.

Transition lines



Custodial symmetry breaking is determined from extrapolation of $\langle \Omega \rangle$ to $n_{sym} \to \infty$. In this case $\gamma_c \approx 1.4$

Remnant symmetry breaking in Coulomb gauge is determined from a peak in the susceptability of $\langle \phi \rangle$.



Transition lines for SU(2) gauge theory in the $\beta - \gamma$ plane.

Note that custodial symmetry breaking occurs before the breaking in Coulomb gauge.

Custodial and gauge symmetry breaking

Theorem

$$\langle \Omega \rangle \geq \langle \phi \rangle_F$$

for all physical gauges $F(U) = 0$. And there exists at least one physical gauge such that

$$\langle \Omega \rangle = \langle \phi \rangle_{F_\Omega}$$

Custodial symmetry breaking is therefore a

- necessary and sufficient, and
- gauge invariant

condition for the existence of spontaneous breaking of a global subgroup of the gauge symmetry, via $\langle \phi \rangle > 0$ in a physical F gauge.

Broken custodial symmetry

- implies spontaneous breaking of the subgroup of gauge transformations $g(\mathbf{x}, t) = z(t) \in Z_N$ in a physical gauge;
- which implies $\langle U_0 \rangle \neq 0$ in that gauge;
- which implies C confinement.

Let us define

$$
\begin{aligned}
|\text{charged}\rangle &= \overline{q}^a(\mathbf{x}) V^{ab}(\mathbf{x}, \mathbf{y}; U) q^b(\mathbf{y}) |\text{vac}\rangle \\
|\text{neutral}\rangle &= (\overline{q}^a \phi^a)_{\mathbf{x}} (\phi^{\dagger b} q^b)_{\mathbf{y}} |\text{vac}\rangle
\end{aligned}
$$

Take $\mathbf{x} \rightarrow \infty$. This defines charged and uncharged quark states at site $\mathbf{y}$, distinguished by a long-range color electric field of the charged quark.

Charged and uncharged states

Custodial broken phase: Evaluate overlap with $V(\mathbf{x},\mathbf{y};U) = g_F^\dagger(\mathbf{x};U)g_F(\mathbf{y};U)$, $\langle\phi\rangle_F > 0$. Then in the $R = |\mathbf{x}-\mathbf{y}| \to \infty$ limit

$$\langle \text{charged}|\text{neutral}\rangle \propto \langle \phi^{\dagger a}(\mathbf{x})\phi^a(\mathbf{y})\rangle \neq 0$$

Custodial unbroken phase: $\langle\phi\rangle_F = 0$, and there is no obvious Higgs mechanism in any physical $F(U) = 0$ gauge. Moreover, at $R \to \infty$,

$$\langle \text{charged}|\text{neutral}\rangle = 0$$

for all charged states. Orthogonality due to a long-range electric field in the charged state.

- If finite energy electric field $\Longrightarrow$ massless phase.
  This has been ruled out numerically in SU(2) gauge-Higgs.
- The alternative is an infinite energy electric field $\Longrightarrow$ S_c confinement.

What about Goldstone's Theorem?

Custodial symmetry is not a gauge symmetry. Why are there no massless excitations in the broken phase?

- Goldstone implies long-range correlations in $\mathbf{Z}(U)$, at fixed U

$$\overline{\phi(x;U)\phi(y;U)} - \overline{\phi(x;U)} \times \overline{\phi(y;U)}$$

  Such a correlator however, being locally gauge non-invariant, would necessarily vanish in the full theory, i.e.

$$\left\langle \overline{\phi(x;U)\phi(y;U)} \right\rangle - \left\langle \overline{\phi(x;U)} \times \overline{\phi(y;U)} \right\rangle = 0$$

- Follow *Guralnik, Hagen, Kibble (circa 1964)*: To see if there are *physical* long range correlations, fix to a physical gauge. But then the Hamiltonian acquires a non-local term. This invalidates one of the assumptions that goes into the Goldstone Theorem.

Conclusions

We have suggested two gauge-invariant criteria for distinguishing between the confinement and Higgs sectors of a gauge-Higgs theory:

1. S_c vs. C confinement
2. unbroken vs. spontaneously broken custodial symmetry

Broken custodial symmetry $\Longrightarrow \langle\phi\rangle_F > 0$ in a physical gauge $\Longrightarrow$ C confinement.
In the unbroken phase there is no obvious route to the Higgs mechanism, and we have argued that S_c confinement is implied.

According to our arguments...

the custodial symmetry transition line and the S_c to C transition line coincide.

EXTRA

SLIDES

proof, part 1

Introduce a breaking term

$$S_\eta = -J \sum_{\mathbf{x}} \mathrm{Tr}[\eta^\dagger(\mathbf{x})\phi(\mathbf{x})]$$

and

$$\begin{aligned} \overline{\phi}_{JV}(\mathbf{x};\eta) &= \frac{1}{\mathbf{Z}(U)} \int D\phi \phi(\mathbf{x}) \mathbf{Z}(U,\phi) e^{-S_\eta} \\ \Omega_{JV}(U) &= \max_\eta \frac{1}{V} \sum_{\mathbf{x}} |\overline{\phi}_{JV}(\mathbf{x};\eta)| \end{aligned}$$

Then

$$\begin{aligned} |\langle\phi\rangle_F| &= \lim_{J\to 0} \lim_{V\to\infty} \frac{1}{Z} \left| \int DU \delta[F(U)] \Delta_F[U] \right. \\ & \quad \left. \times \frac{1}{V} \sum_{\mathbf{x}} \int D\phi \phi(x) \mathbf{Z}(U,\phi) e^{J\sum_{\mathbf{x}} \mathrm{Tr}\phi(\mathbf{x})} \right| \\ &\leq \lim_{J\to 0} \lim_{V\to\infty} \frac{1}{Z} \int DU \delta[F(U)] \Delta_F[U] \\ & \quad \times \frac{1}{V} \sum_{\mathbf{x}} \left| \int D\phi \phi(x) \mathbf{Z}(U,\phi) e^{J\sum_{\mathbf{x}} \mathrm{Tr}\phi(\mathbf{x})} \right| \end{aligned}$$

proof, part 2

$$
\begin{aligned}
|\langle\phi\rangle_F| &\leq \lim_{J\to 0}\lim_{V\to\infty}\frac{1}{Z}\int DU\delta[F(U)]\Delta_F[U] \\
&\quad \times\frac{1}{V}\max_{\eta}\sum_{\mathbf{x}}\left|\int D\phi\phi(x)\mathbf{Z}(U,\phi)e^{-S_\eta}\right| \\
&\leq \langle\Omega\rangle
\end{aligned}
$$

which is the first part of the the theorem. Next, define the gauge

$$
\widehat{F}(U) = \frac{\overline{\phi}(\mathbf{x};U)}{|\overline{\phi}(\mathbf{x};U)|} - \mathbb{1} = 0
$$

Since $\Omega(U)$ is gauge invariant, it can be evaluated in this $\widehat{F}(U) = 0$ gauge in particular.

proof, part 3

Then

$$
\begin{aligned}
\langle \Omega \rangle &= \lim_{J\to 0} \lim_{V\to\infty} \frac{1}{Z} \int DU \delta[\widehat{F}(U)] \Delta_{\widehat{F}}[U] \\
&\quad \times \frac{1}{V} \max_{\eta} \sum_{\mathbf{x}} \left| \int D\phi \phi(x) \mathbf{Z}(U,\phi) e^{-S_\eta} \right| \\
&= \lim_{J\to 0} \lim_{V\to\infty} \frac{1}{Z} \left| \int DU \delta[\widehat{F}(U)] \Delta_{\widehat{F}}[U] \right. \\
&\quad \left. \times \frac{1}{V} \max_{\eta} \sum_{\mathbf{x}} \int D\phi \phi(x) \mathbf{Z}(U,\phi) e^{-S_\eta} \right| \\
&= \langle \phi \rangle_{\widehat{F}}
\end{aligned}
$$

which establishes the second part of the theorem.

Orthogonality

Integrating out the static quark fields, it is easy to see that

$$\langle \text{charged}_{\mathbf{xy}} | \text{neutral}_{\mathbf{xy}} \rangle \quad \propto \quad \langle \phi^{\dagger a}(\mathbf{x}) V^{ab}(\mathbf{x}, \mathbf{y}; U) \phi^b(\mathbf{y}) \rangle$$

Then we integrate out the scalar field to obtain

$$\lim_{R\to\infty} \langle \text{charged}_{\mathbf{xy}} | \text{neutral}_{\mathbf{xy}} \rangle$$
$$= \lim_{R\to\infty} \int DU \, \text{Tr}[V(\mathbf{x}, \mathbf{y}; U) G(\mathbf{y}, \mathbf{x}; U)] \frac{\mathbf{Z}(U)}{Z}$$

where

$$G(\mathbf{y}, \mathbf{x}; U) \quad = \quad \frac{1}{\mathbf{Z}(U)} \int D\phi \phi^\dagger(\mathbf{x}) \phi(\mathbf{y}) e^{-S}$$

Unbroken custodial symmetry in $\mathbf{Z}(U, \phi)$, for fixed U configurations drawn from the probability distribution $\mathbf{Z}(U)/Z$, implies $G(\mathbf{x}, \mathbf{y}, U) \to 0$ as $R \to \infty$, even if the background U field is fixed to some F-gauge. Then, since $V(\mathbf{x}, \mathbf{y}; U)$ is normalized and therefore bounded in the $R \to \infty$ limit, we conclude that

$$\lim_{R\to\infty} \langle \text{charged}_{\mathbf{xy}} | \text{neutral}_{\mathbf{xy}} \rangle \quad = \quad 0$$