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QCD in the heavy dense regime for general N_c: Quarkyonic matter?

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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

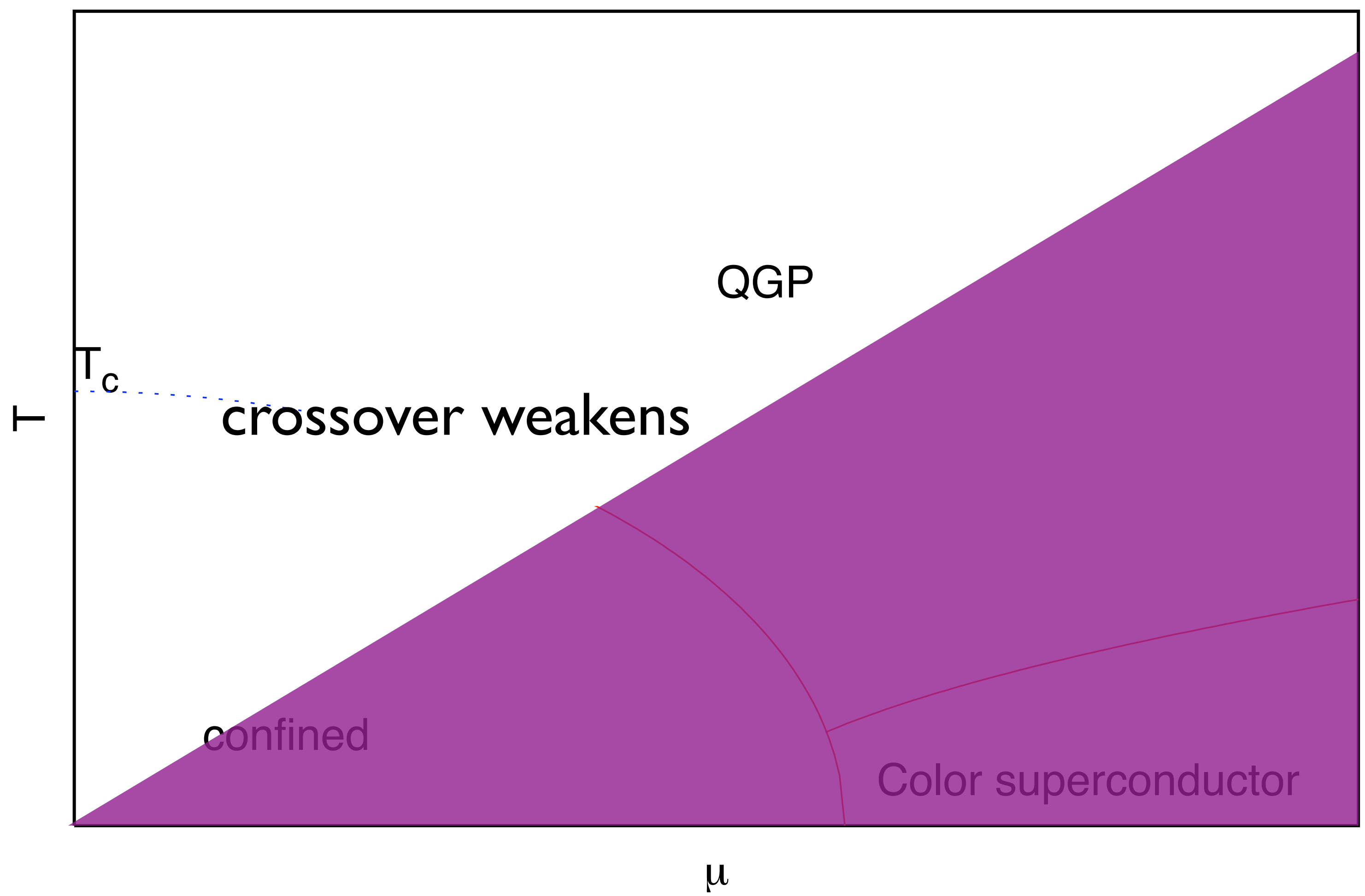
- Review: effective lattice theory for finite density QCD
- The nuclear liquid gas transition
- What happens at large N_c

HIC for FAIR Helmholtz International Center

NIC

CRC-TR 211 Strong-interaction matter under extreme conditions

The lattice-calculable region of the phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region, some signals beyond

Effective lattice theory for heavy dense QCD

O.P. with Fromm, Langelage, Lottini, Neuman, Glesaaen

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \; \det Q \; e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \; e^{-S_{eff}[L]}$$

  Spatial integration after analytic strong coupling and hopping expansion $\sim \frac{1}{g^2}, \frac{1}{m_q}$

- Step II.: mild sign problem of effective theory

- Analytic solution by linked cluster expansion

Effective theory: start from Wilson's lattice action

Pure gauge part: character expansion

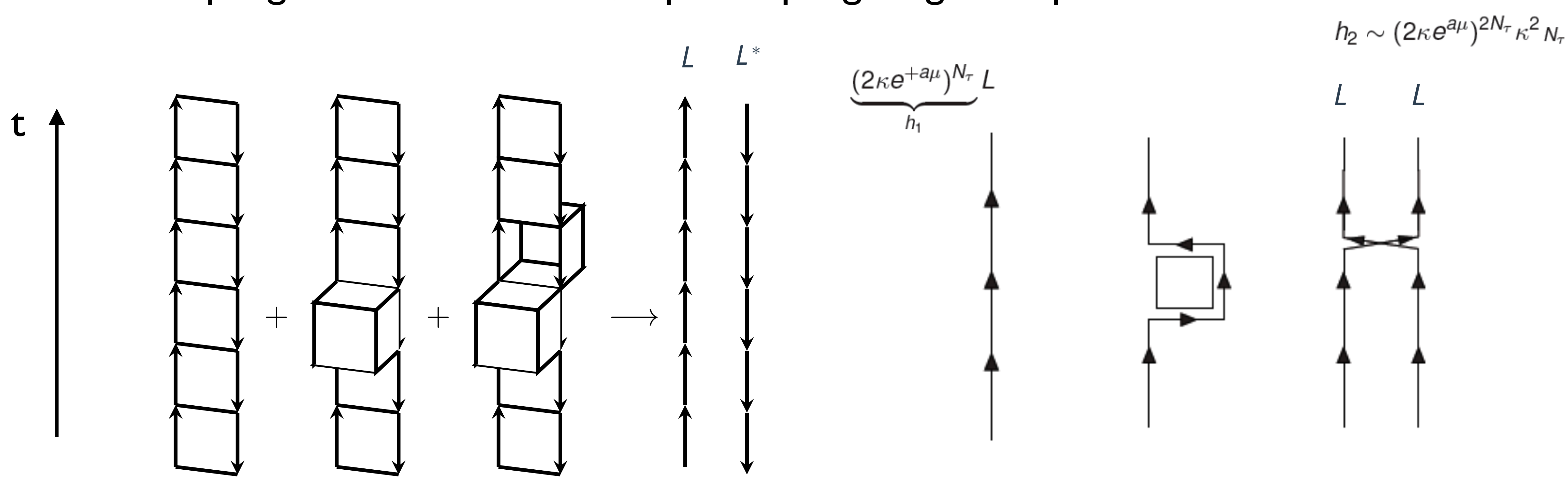
$$u(\beta) = \frac{\beta}{18} + \frac{\beta^2}{216} + \ldots < 1$$

$$\beta = \frac{2N_c}{g^2}$$

Fermion determinant: hopping expansion

$$\kappa = \frac{1}{2am + 8}$$

Generates couplings over all distances, n-pt. couplings, higher reps…. :



$$\lambda(u, N_\tau \geq 5) \;=\; u^{N_\tau} \exp\left[N_\tau \left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]$$

The effective 3d theory

$$-S_{\text{eff}} = \sum_i \lambda_i(u,\kappa,N_\tau) S_i^{\text{S}} - 2N_f \sum_i \left[h_i(u,\kappa,\mu,N_\tau) S_i^{\text{A}} + \bar{h}_i(u,\kappa,\mu,N_\tau) S_i^{\dagger\text{A}} \right]$$

effective couplings

$$S_i^{A,S} = S_i^{A,S}[L, L^*]$$

This is a 3d continuous spin model!

cf. Svetitsky-Yaffe conjecture for universality of SU(N) Yang-Mills

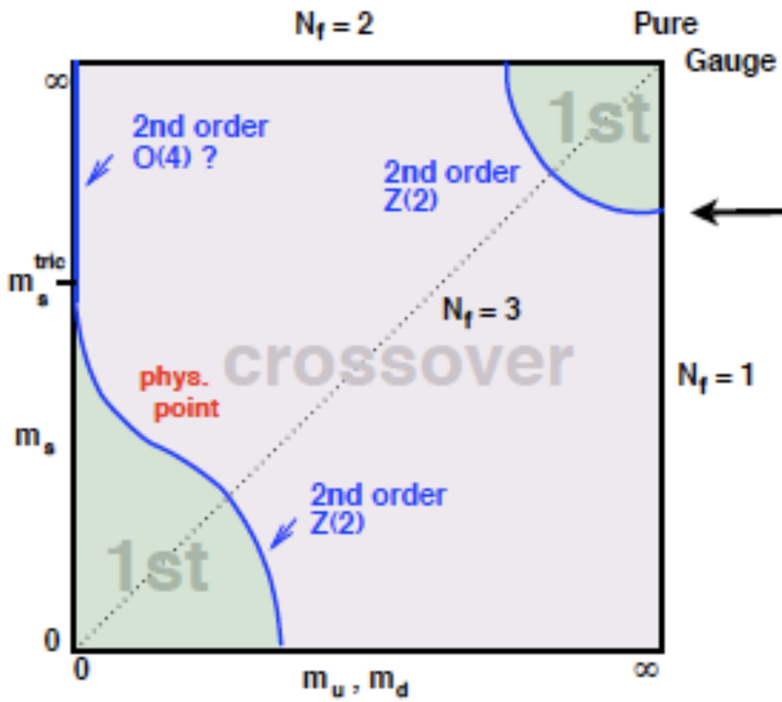
$$L = \text{Tr}W$$

$$Z = \int DW \prod_{<\mathbf{x},\mathbf{y}>} \left[1 + \lambda(L_\mathbf{x} L_\mathbf{y}^* + L_\mathbf{x}^* L_\mathbf{y})\right]$$

$$\times \prod_\mathbf{x} [1 + h_1 L_\mathbf{x} + h_1^2 L_\mathbf{x}^* + h_1^3]^{2N_f} [1 + \bar{h}_1 L_\mathbf{x}^* + \bar{h}_1^2 L_\mathbf{x} + \bar{h}_1^3]^{2N_f}$$

$$\times \prod_{<\mathbf{x},\mathbf{y}>} \left(1 - h_2 \text{Tr} \frac{h_1 W_\mathbf{x}}{1 + h_1 W_\mathbf{x}} \text{Tr} \frac{h_1 W_\mathbf{y}}{1 + h_1 W_\mathbf{y}}\right) \left(1 - h_2 \text{Tr} \frac{\bar{h}_1 W_\mathbf{x}^\dagger}{1 + \bar{h}_1 W_\mathbf{x}^\dagger} \text{Tr} \frac{\bar{h}_1 W_\mathbf{y}^\dagger}{1 + \bar{h}_1 W_\mathbf{y}^\dagger}\right) \quad \ldots.$$

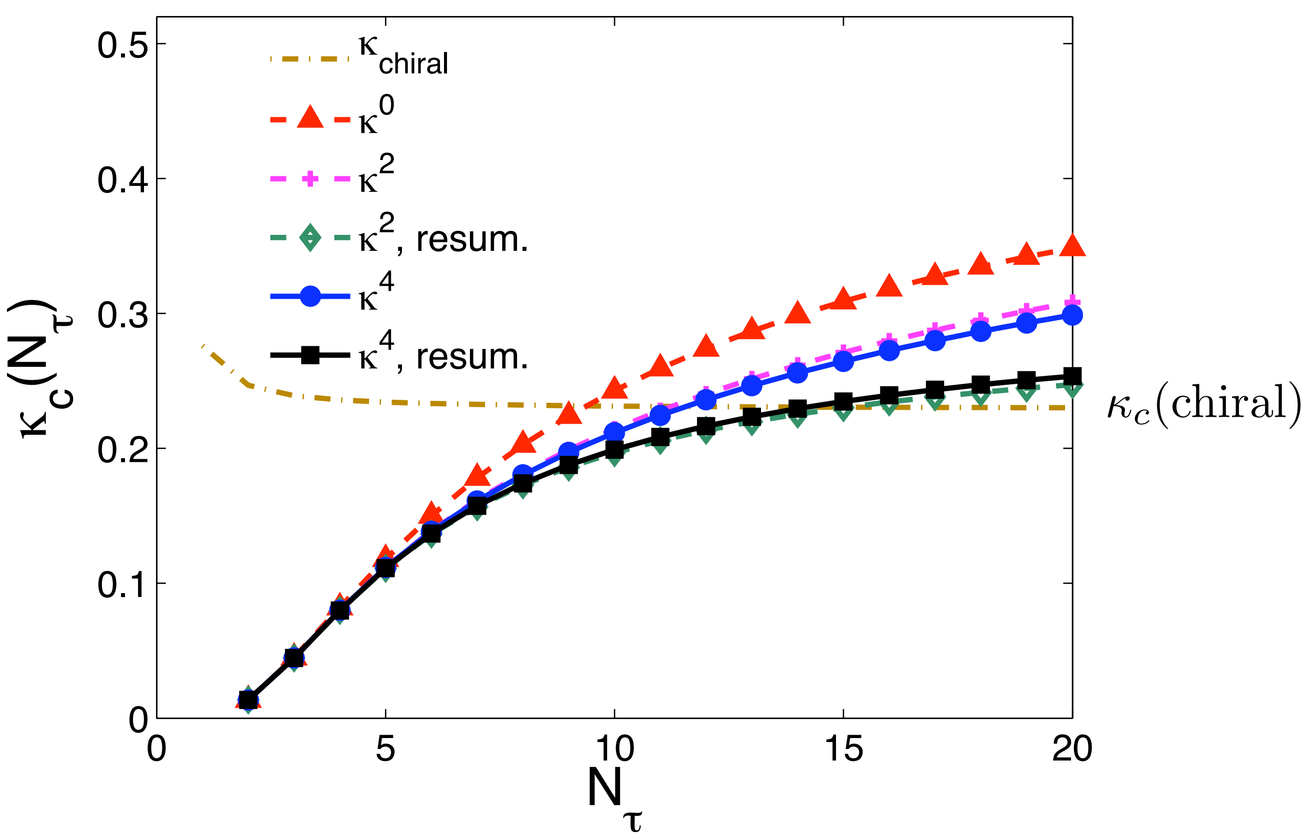
The deconfinement transition for heavy quarks



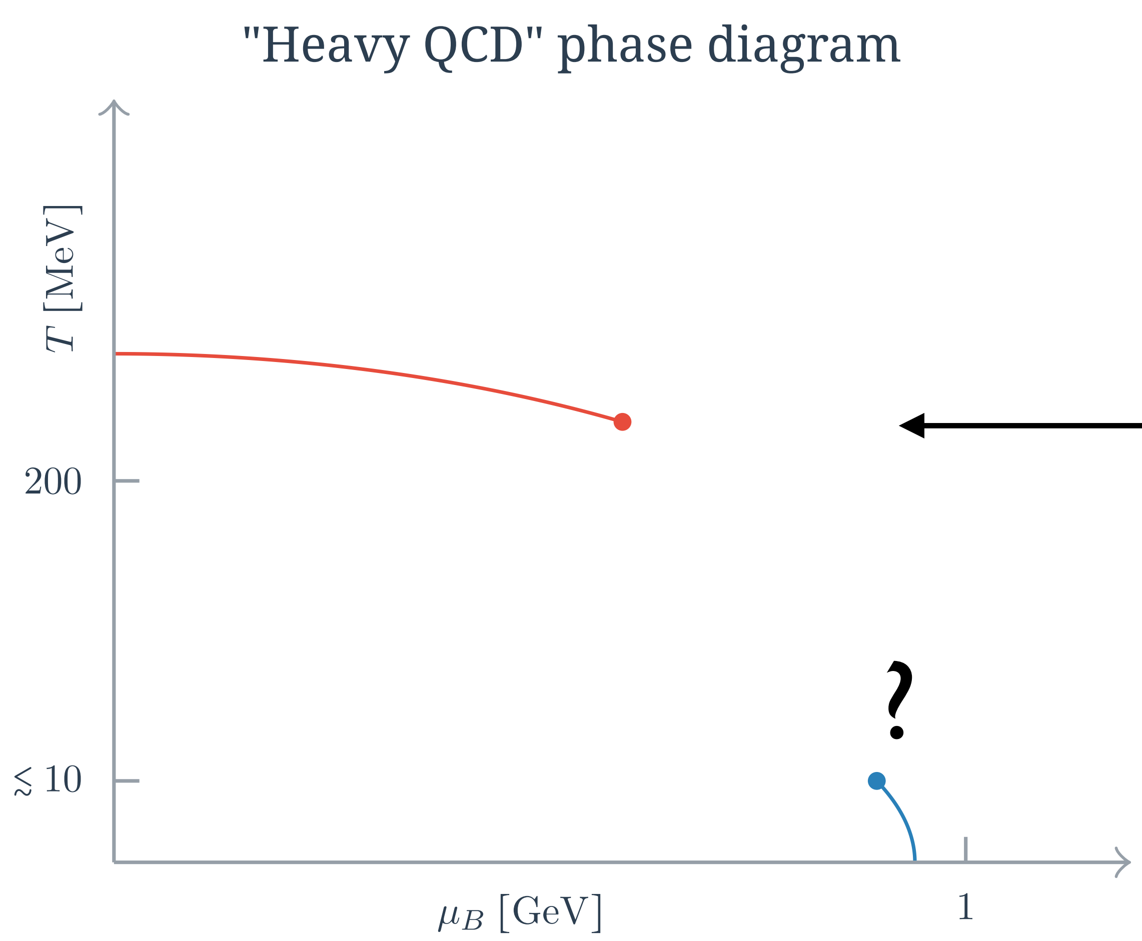
eff. theory — 4d MC, WHOT — 4d MC, de Forcrand et al

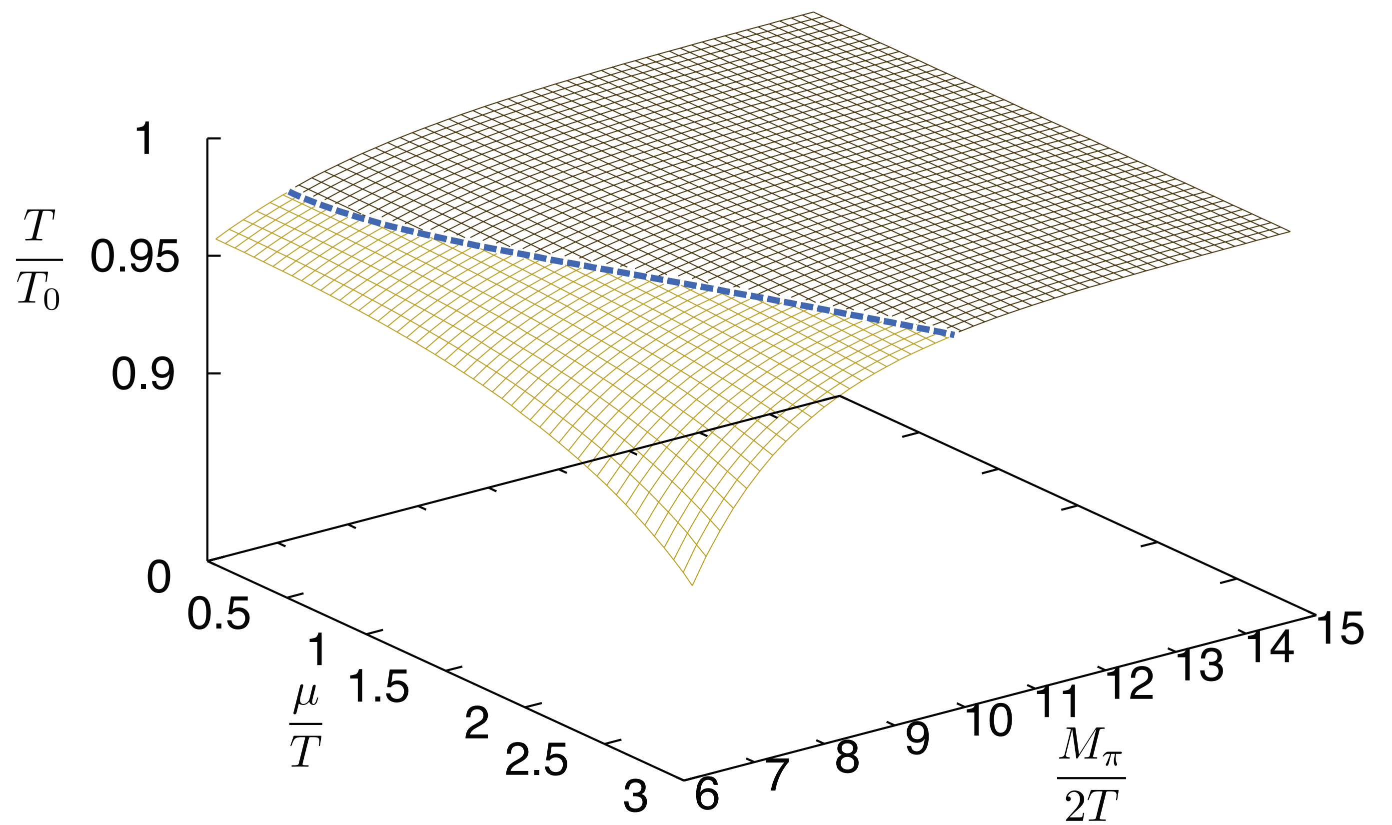
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

Accuracy ~5%, predictions for Nt=6,8,... available!



The fully calculated deconfinement transition





Continuum, functional methods:

Fischer, Lücker, Pawlowski 15

Fromm, Langelage, Lottini, O.P. 11

Cold and dense: static strong coupling limit

Fromm, Langelage, Lottini, Neuman, O.P. , PRL 13

T=0: anti-fermions decouple:

$$\begin{aligned} h_1 &= (2\kappa e^{a\mu})^{N_\tau} = e^{\frac{\mu-m}{T}} \\ \bar{h}_1 &= (2\kappa e^{-a\mu})^{N_\tau} = e^{\frac{-\mu-m}{T}} \end{aligned}$$

$$Z(\beta=0) \xrightarrow{T\rightarrow 0} \left[\prod_f \int dW (1 + h_1 L + h_1^2 L^* + h_1^3)^2\right]^V = z_0^V$$

$N_f = 1$: $\quad z_0 = 1 + 4h_1^3 + h_1^6$ free baryon gas

spin 3/2, 0

Silver blaze phenomenon + Pauli principle: $\lim_{T\rightarrow 0} a^3 n = \begin{cases} 0, & \mu < m \\ 2N_c, & \mu > m \end{cases}$

1st order phase transition from vacuum to saturated baryon crystal

$N_f = 2:$

Δ^- $\quad$ $2n + 4\Delta^0$ $\quad$ $2p + 4\Delta^+$ $\quad$ Δ^{++}

$$z_0 = (1 + 4h_d^3 + h_d^6) + (6h_d^2 + 4h_d^5)h_u + (6h_d + 10h_d^4)h_u^2 + (4 + 20h_d^3 + 4h_d^6)h_u^3$$
$$+(10h_d^2 + 6h_d^5)h_u^4 + (4h_d + 6h_d^4)h_u^5 + (1 + 4h_d^3 + h_d^6)h_u^6$$

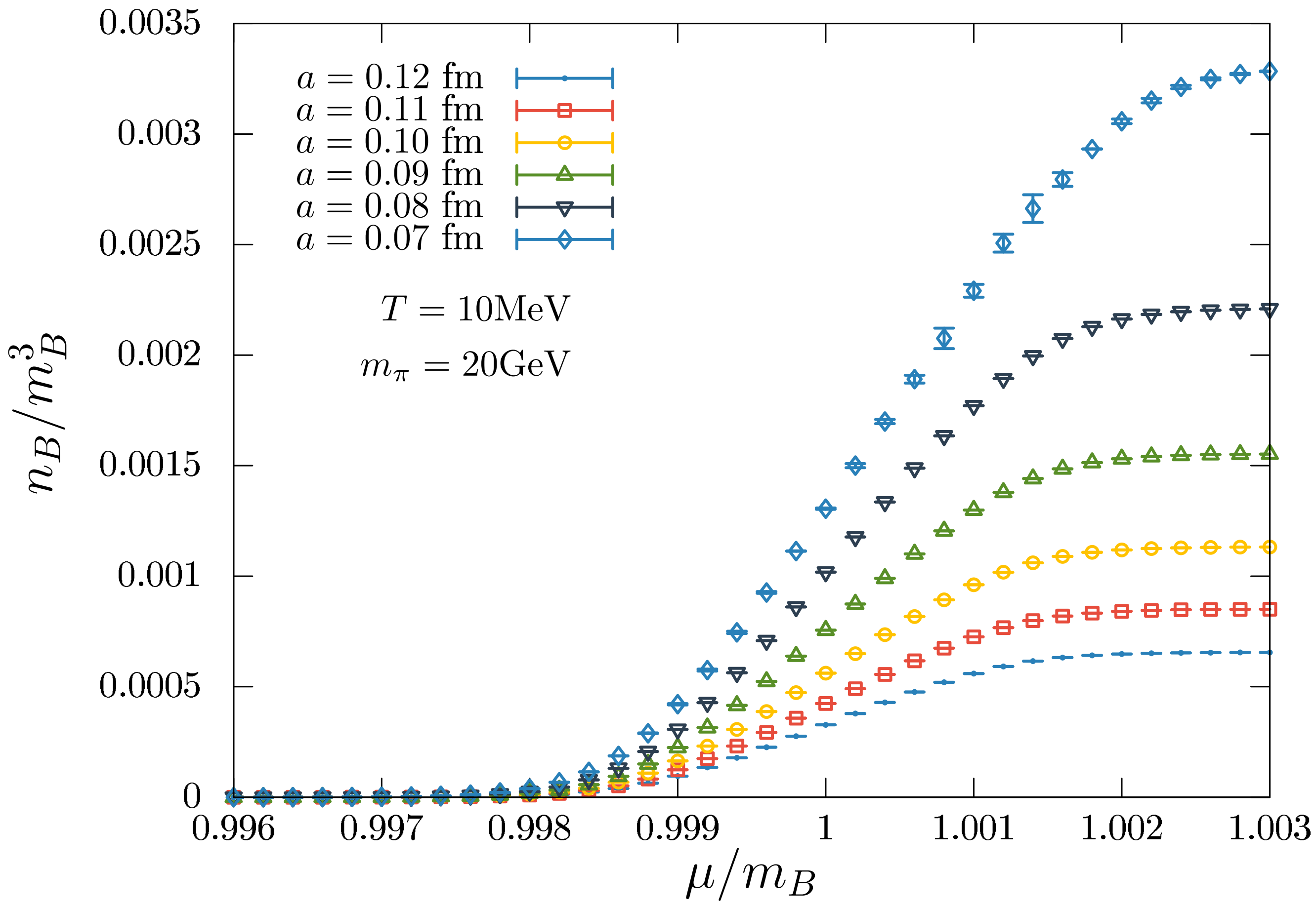
"Di-baryons": 3 spin 1 triplets, 1 spin 0 singlet, $\Delta^{++}\Delta^0$, $\quad pp$

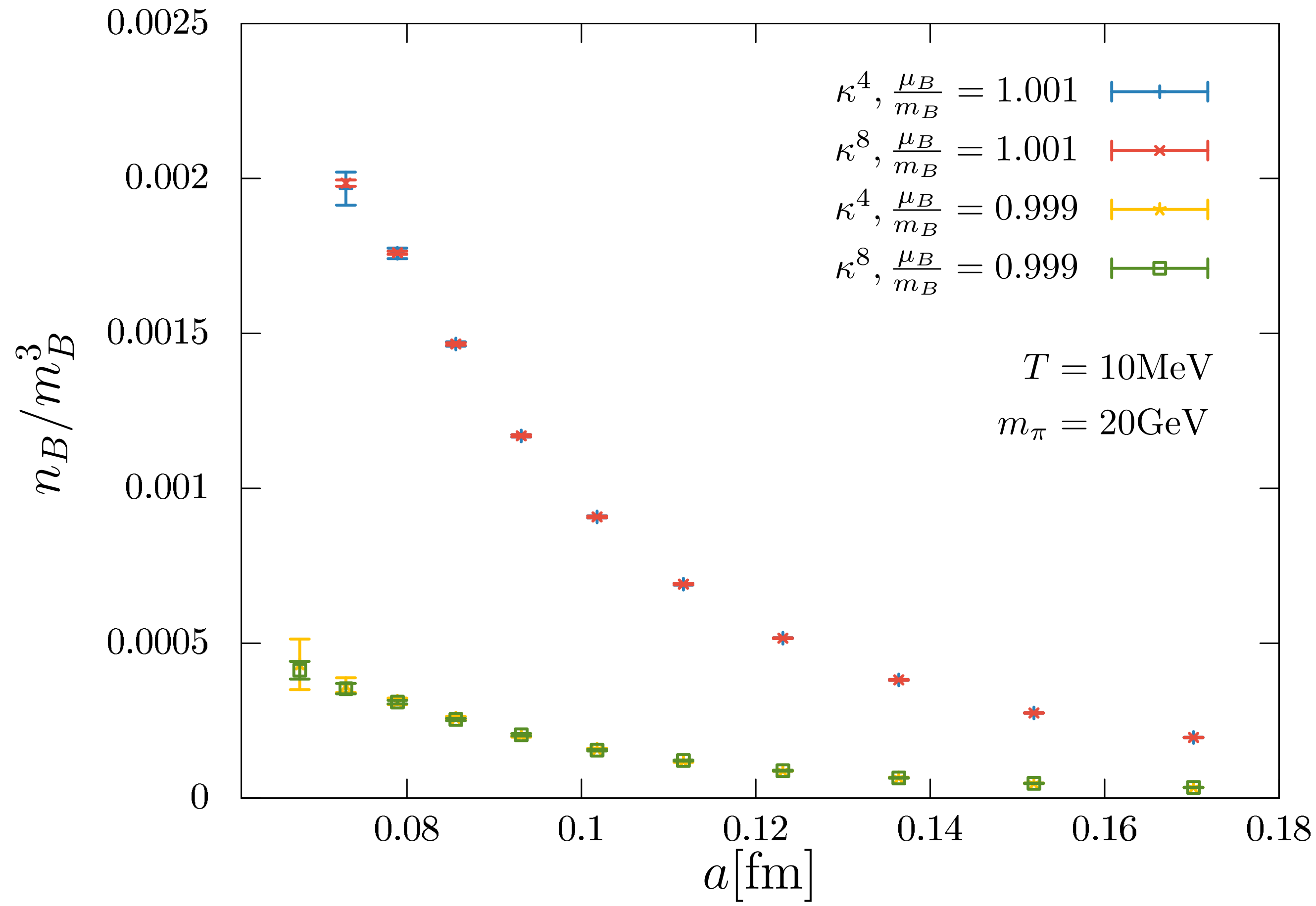
Complete spin-flavour structure of baryons (mesons for isospin chemical potential)

Gauge and Lorentz symmetries!

Cold and dense regime

Glesaaen, Neuman, O.P., JHEP 15

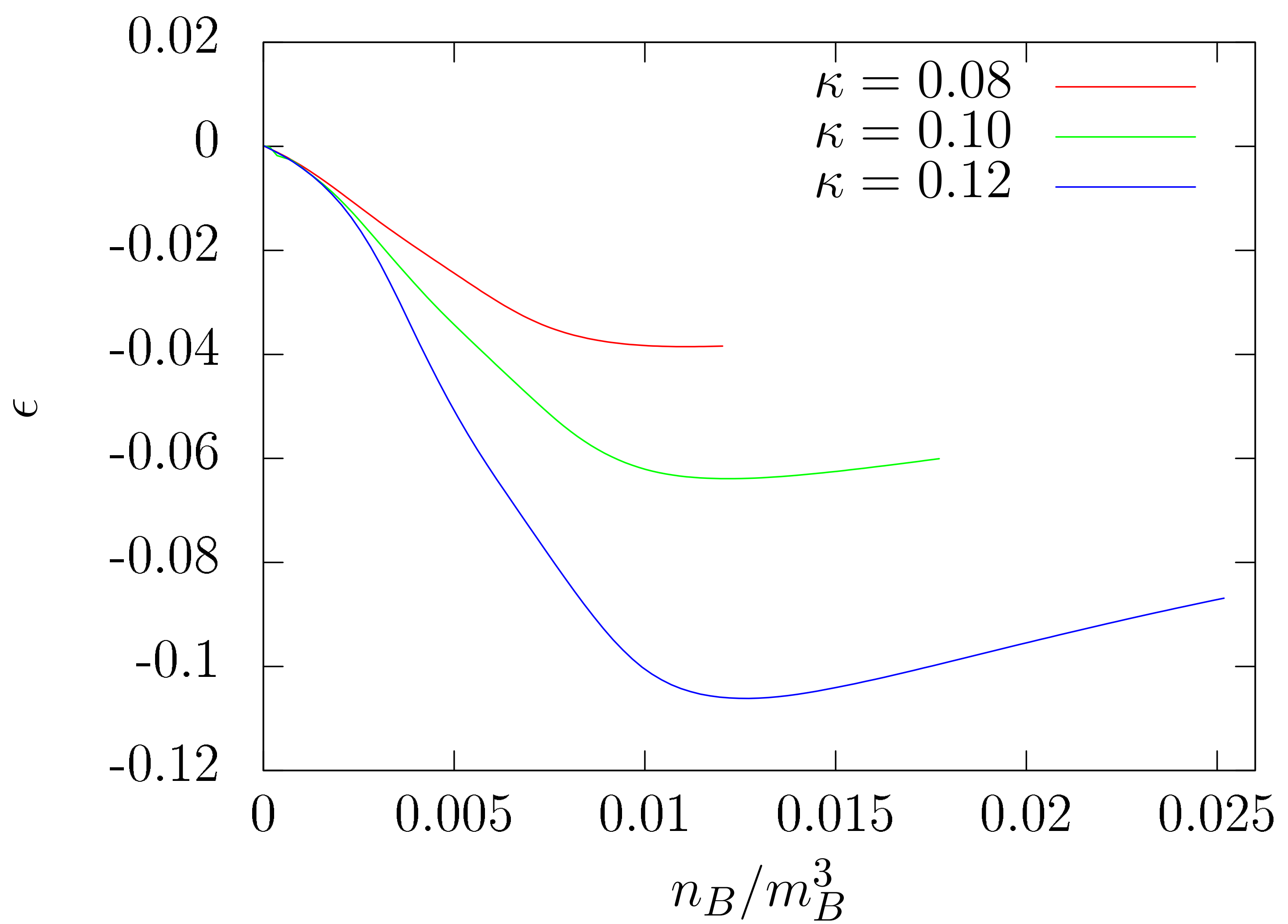




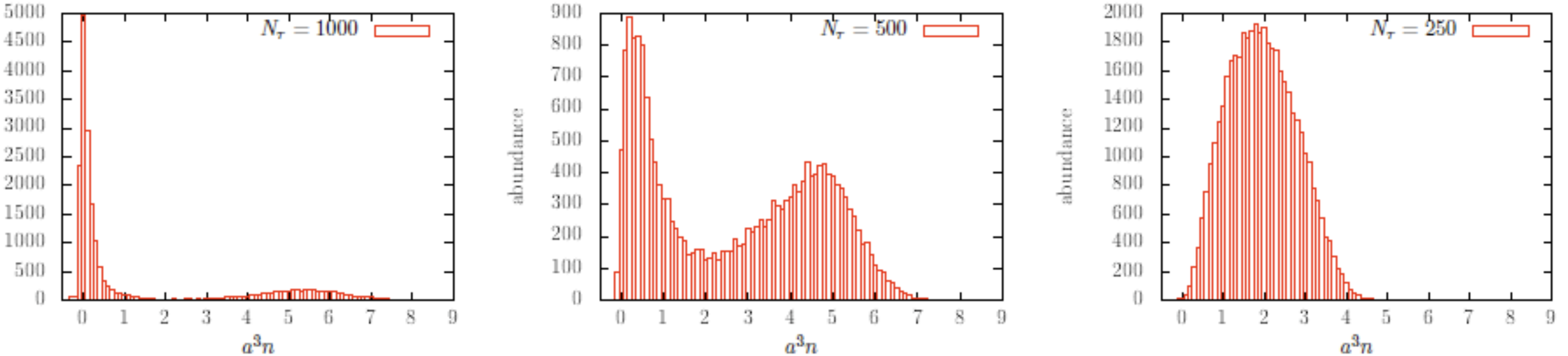
- Continuum approach ~a as expected for Wilson fermions
- Cut-off effects grow rapidly beyond onset transition: lattice saturation!
- Finer lattice necessary for larger density!

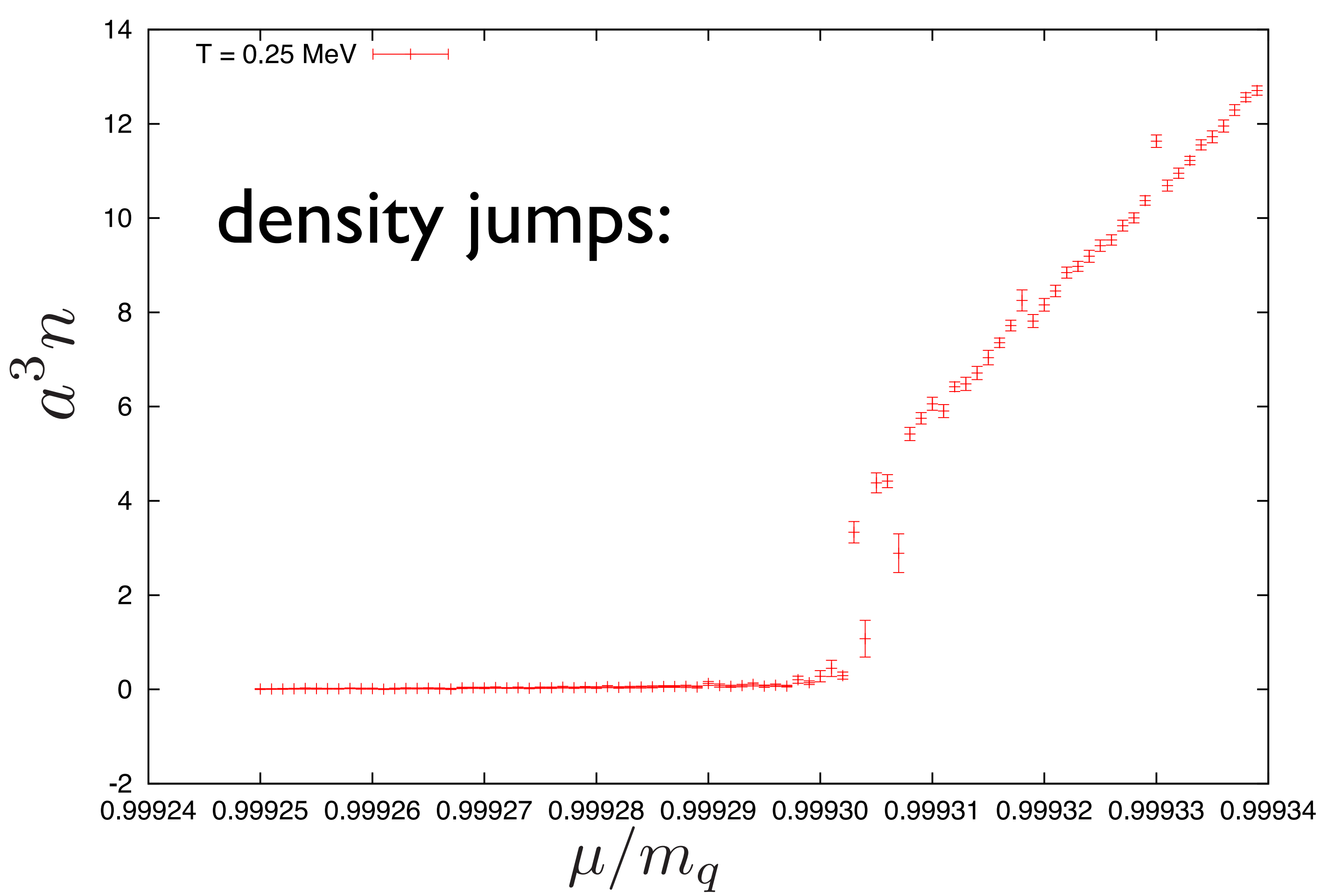
Binding energy per nucleon

$$\epsilon \equiv \frac{e - n_B m_B}{n_B m_B} \overset{LO}{=} -\frac{4}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 \kappa^2 = -\frac{1}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 e^{-am_M}$$



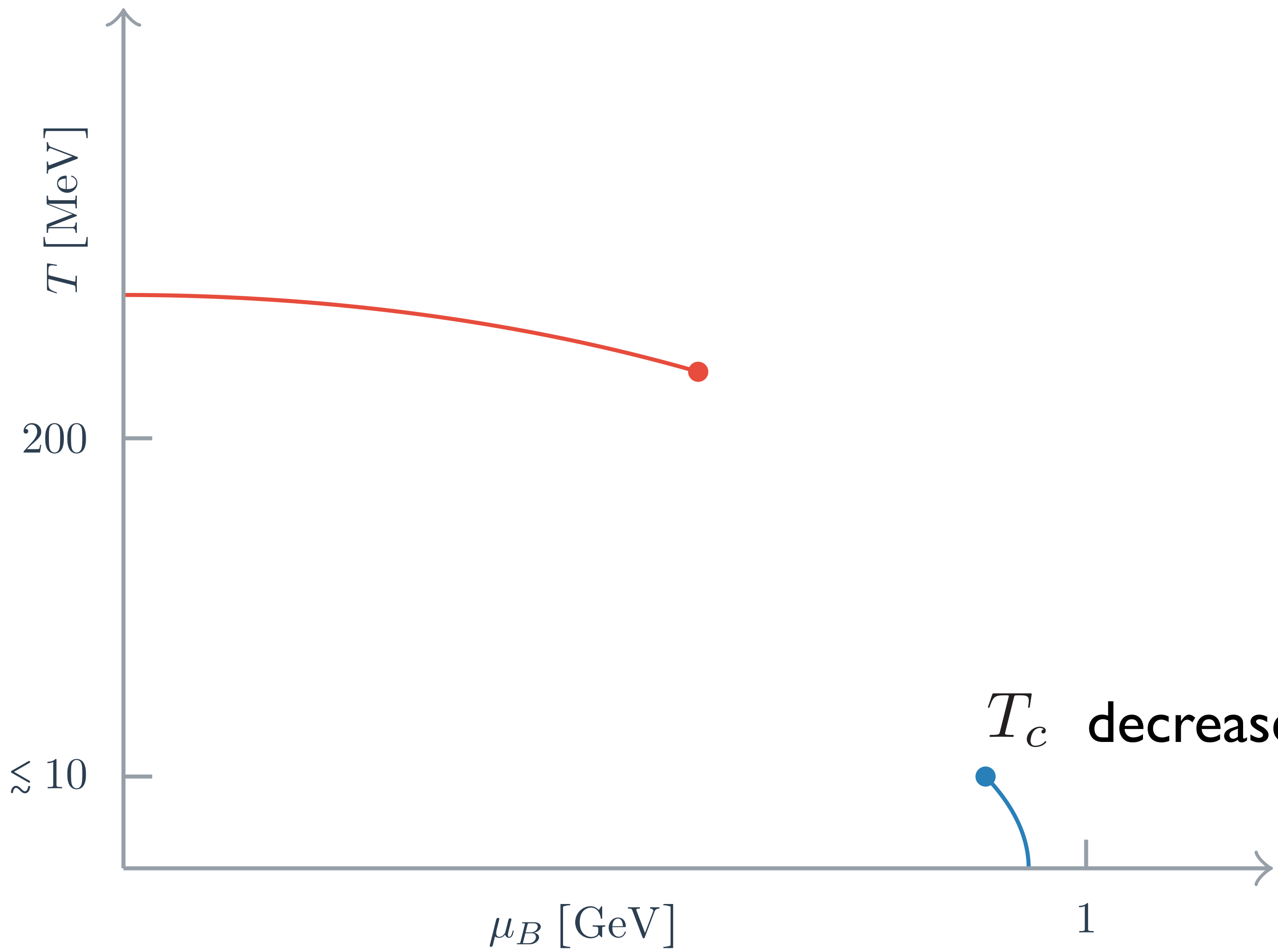
Light quarks: first order transition + endpoint





- phase coexistence: first order
- for higher $T = \frac{1}{aN_\tau}$ crossover
- nuclear liquid gas transition!

"Heavy QCD" phase diagram



T_c decreases with growing quark mass

QCD at large N_c

Definition, 't Hooft 1974 : $N_c \longrightarrow \infty, \quad g^2 N_c = const.$

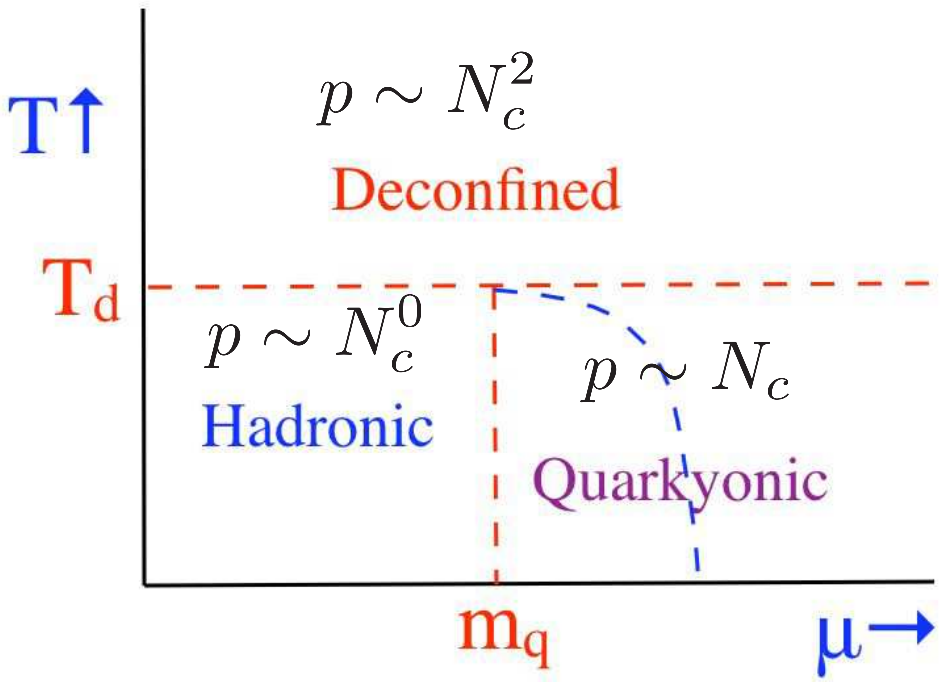
- suppresses quark loops in Feynman diagrams
- mesons are free;
  corrections: cubic interactions $\sim 1/\sqrt{N_c}$, quartic int. $\sim 1/N_c$
- meson masses $\sim \Lambda_{QCD}$
- baryons: N_c quarks, baryon masses $\sim N_c \Lambda_{QCD}$
- baryon interactions: $\sim N_c$

Witten 1979

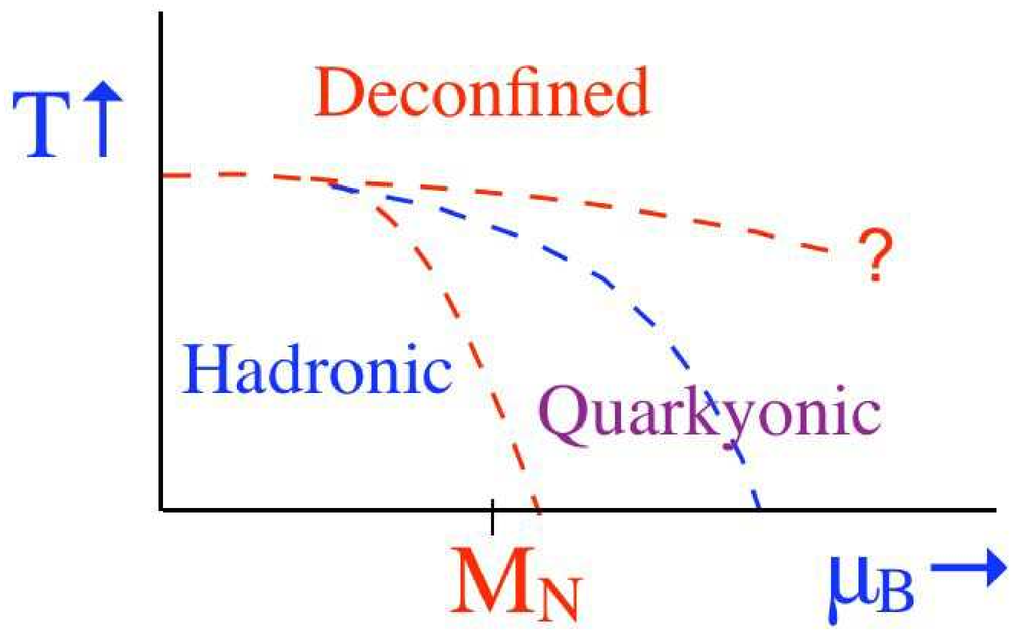
Implications on the phase diagram

McLerran, Pisarski 07:

large Nc

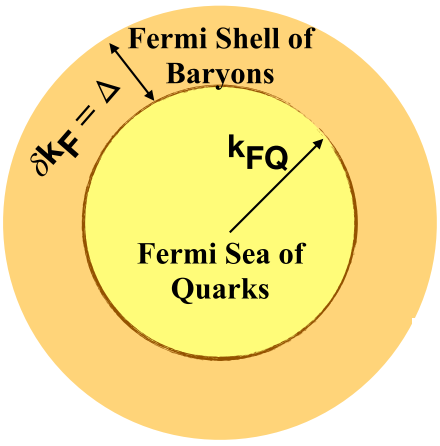


QCD, conjectured



Quarkyonic matter:

can smoothly vary from baryons to quark matter



The effective theory for large N_c

Recalculate for general N_c, start with strong coupling limit, need new SU(N) integrals!

Static determinant:

$$\int_{SU(N)} \mathrm{d}U \det(1+h_1 U)^{2N_f} = \sum_{p=0}^{N_f} \left(\prod_{i=1}^{p} \frac{(i-1+2N_f-p+N)^{\underline{2N_f-p}}}{(i-1+2N_f-p)^{\underline{2N_f-p}}} \right) \left(h_1^{pN} + h_1^{(2N_f-p)N} \right) \left(1 - \frac{\delta_{p,N_f}}{2} \right)$$

And corrections: $$\int_{SU(N)} \mathrm{d}U \det(1+h_1 U)^{2N_f} \operatorname{tr}\left(\frac{(h_1 U)^n}{(1+h_1 U)^m} \right)$$

$$= h_1^{N(2N_f+1)} \sum_{r=\max(0,N-m)}^{2N_f+N-m} (-1)^{r+N+1} \binom{N+r-1}{r} (r+m-1)^{\underline{N-1}} \frac{(2N_f)^{\underline{2N_f+1-r-m}}}{(N+2N_f-r-m)}$$

$$+ \sum_{p=0}^{2N_f} h_1^{Np} \det_{1\leq i,j\leq N} \left[\binom{2N_f}{i-j+p} \right] \sum_{\mu=1}^{N} \sum_{r=\max(0,\mu-m)}^{\mu+p-m} (-1)^r \binom{r+n-1}{r}$$

$$\times \frac{(-1)^{\mu+1}}{r+m} \frac{(r+m+N-\mu)^{\underline{r+m}}}{(r+m-\mu)!(\mu-1)!} \frac{(\mu+p-1)^{\underline{r+m}}}{(N+2N_f-p+r+m-\mu)^{\underline{r+m}}}.$$

Results for $N_f = 2$:

Static determinant:

$$z_0 = 1 + \frac{1}{6}(h_1^N + h_1^{3N})(N+3)(N+2)(N+1) + \frac{1}{12}h_1^{2N}(N+3)(N+2)^2(N+1) + h_1^{4N}$$

Curious: spin degeneracy of a baryon determined by N!

Correction:

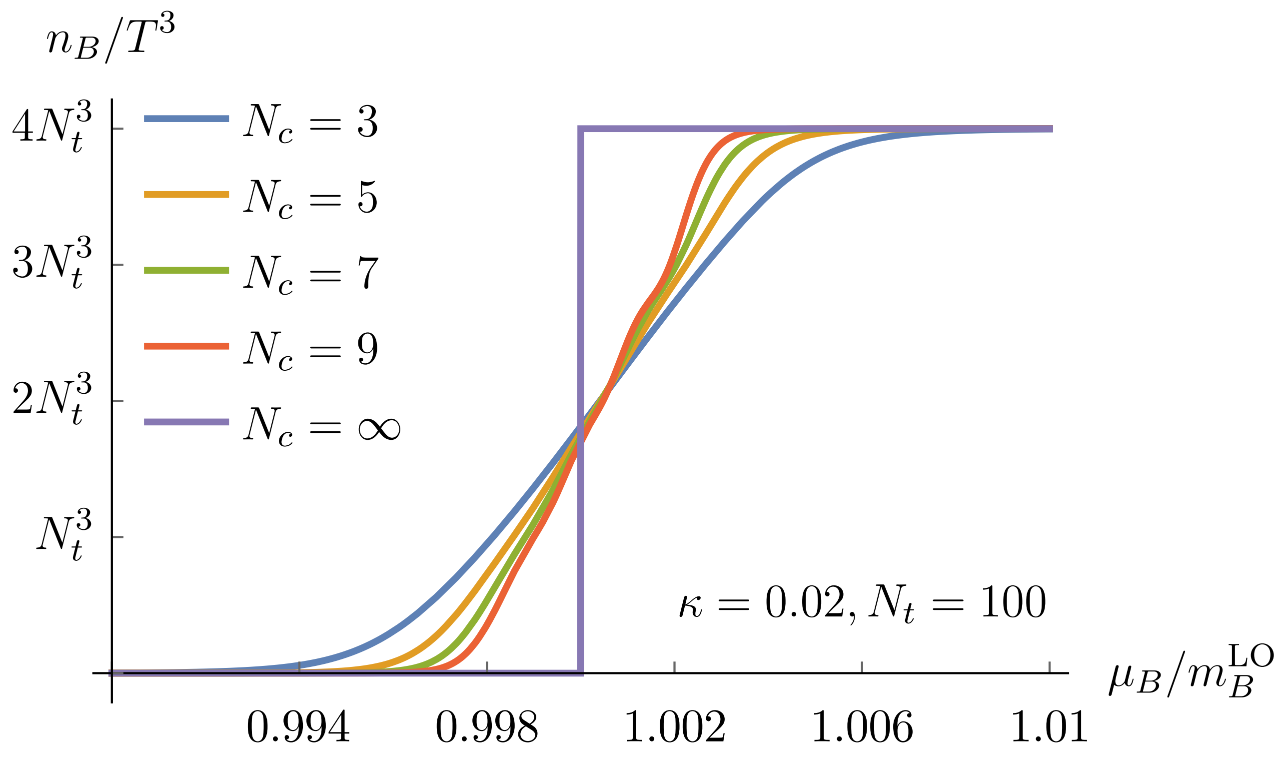
$$z_{11} = \frac{1}{24}h_1^N(N+3)(N+2)(N+1)N + \frac{1}{24}h_1^{2N}(N+3)(N+2)^2(N+1)N$$
$$+ \frac{1}{8}h_1^{3N}(N+3)(N+2)(N+1)N + h_1^{4N}N$$

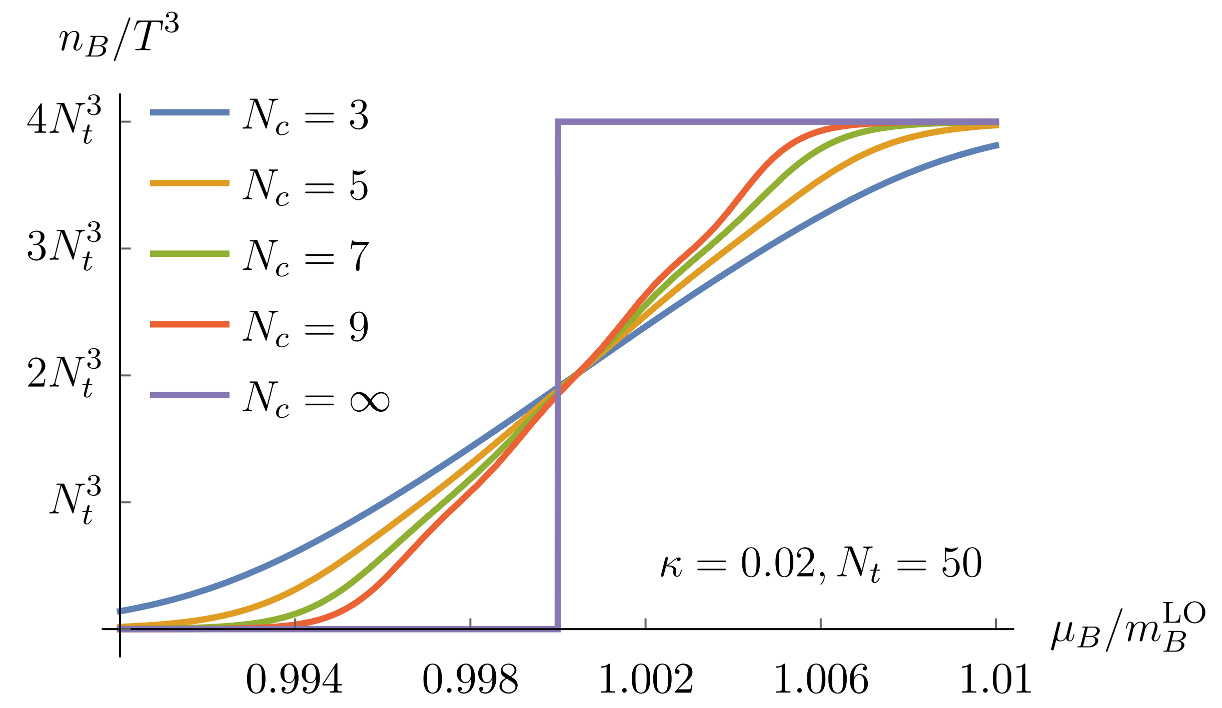
Thermodynamic functions for large N_c

Order hopping expansion		κ^0	κ^2	κ^4
$h_1 < 1$	$a^4 p$	$\sim \frac{1}{6N_\tau} N_c^3 h_1^{N_c}$	$\sim -\frac{1}{48} N_c^7 h_1^{2N_c}$	$\sim \frac{3N_\tau \kappa^4}{800} N_c^8 h_1^{2N_c}$
	$a^3 n_B$	$\sim \frac{1}{6} N_c^3 h_1^{N_c}$	$\sim -\frac{N_\tau}{24} N_c^7 h_1^{2N_c}$	$\sim \frac{(9N_\tau+1)N_\tau}{1200} N_c^8 h_1^{2N_c}$
	$a^4 e$	$\sim -\frac{\ln(2\kappa)}{6} N_c^4 h_1^{N_c}$	$\sim \frac{N_\tau \ln(2\kappa)}{48} N_c^8 h_1^{2N_c}$	
	ϵ	0	$\sim -\frac{1}{4} N_c^3 h_1^{N_c}$	
$h_1 > 1$	$a^4 p$	$\sim \frac{4\ln(h_1)}{N_\tau} N_c$	$\sim -12 N_c$	$\sim 198 N_c$
	$a^3 n_B$	~ 4	$\sim -N_\tau \frac{N_c^4}{h_1^{N_c}}$	$\sim -\frac{(59N_\tau-19)N_\tau}{20} \frac{N_c^5}{h_1^{N_c}}$
	$a^4 e$	$\sim -4\ln(2\kappa) N_c$	$\sim 24\ln(2\kappa) N_c$	
	ϵ	0	~ -6	

Beyond the onset transition: $p \sim N_c$ definition of quarkyonic matter!

The baryon onset transition for growing N_c





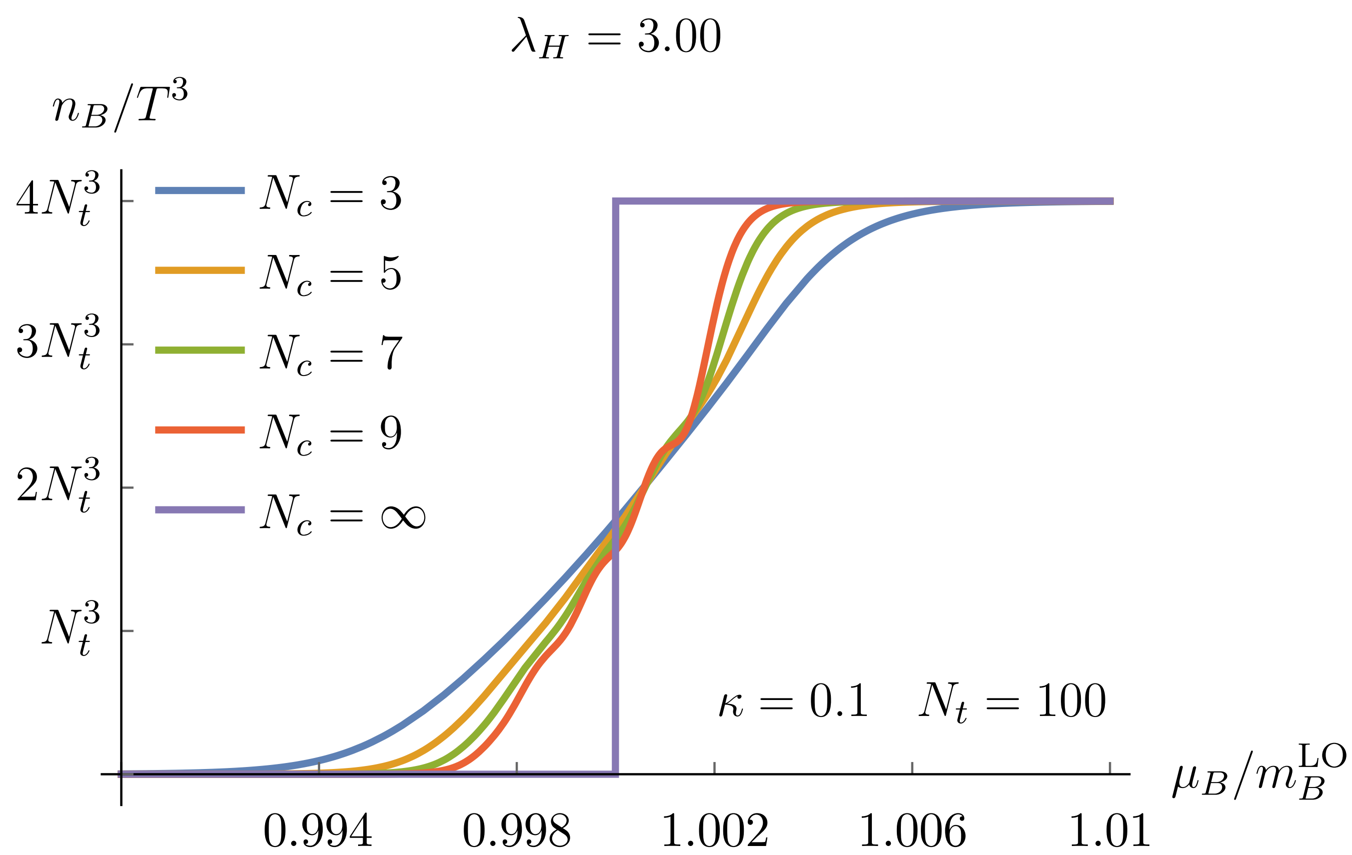
Transition becomes more strongly first-order!

Gauge corrections

So far strong coupling limit, not consistent with 't Hooft scaling

$$u(\beta) = \frac{1}{\lambda_H} = \frac{1}{g^2 N_c} < 1$$

Gross, Witten 80



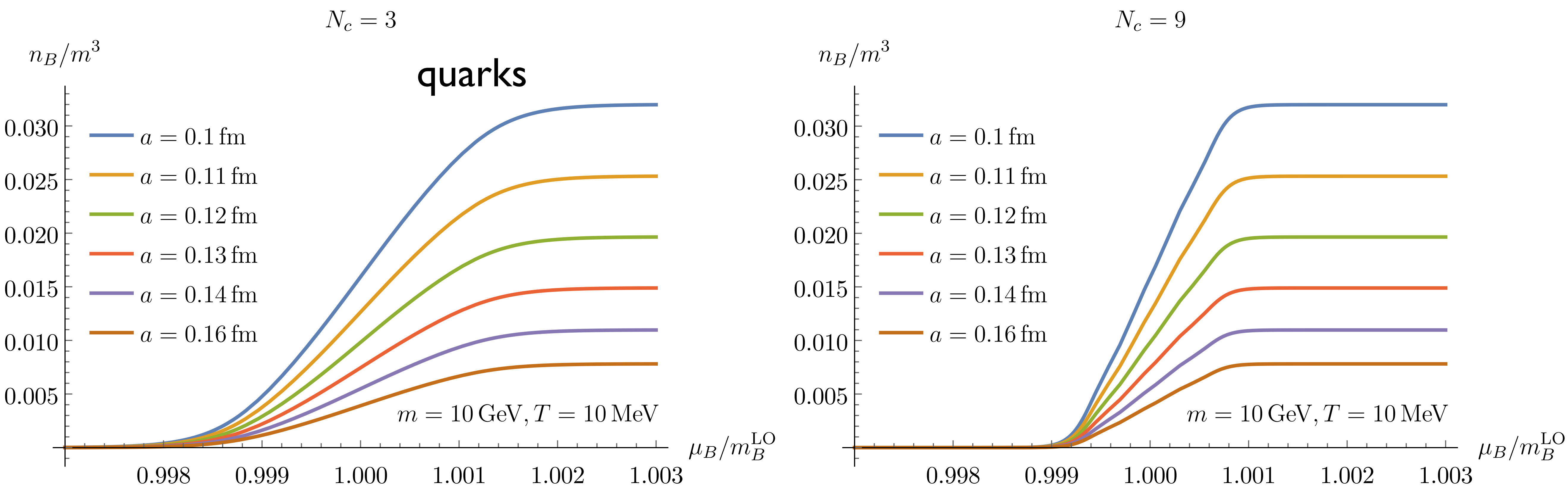
Transition still steepens, Nc-scaling in condensed phase unaffected

Continuum approach

Gross, Witten 80: interchange of strong coupling and large Nc-limit “highly suspicious” in 1+1d

Same here: system immediately jumps to lattice saturation, unphysical

take continuum limit first!

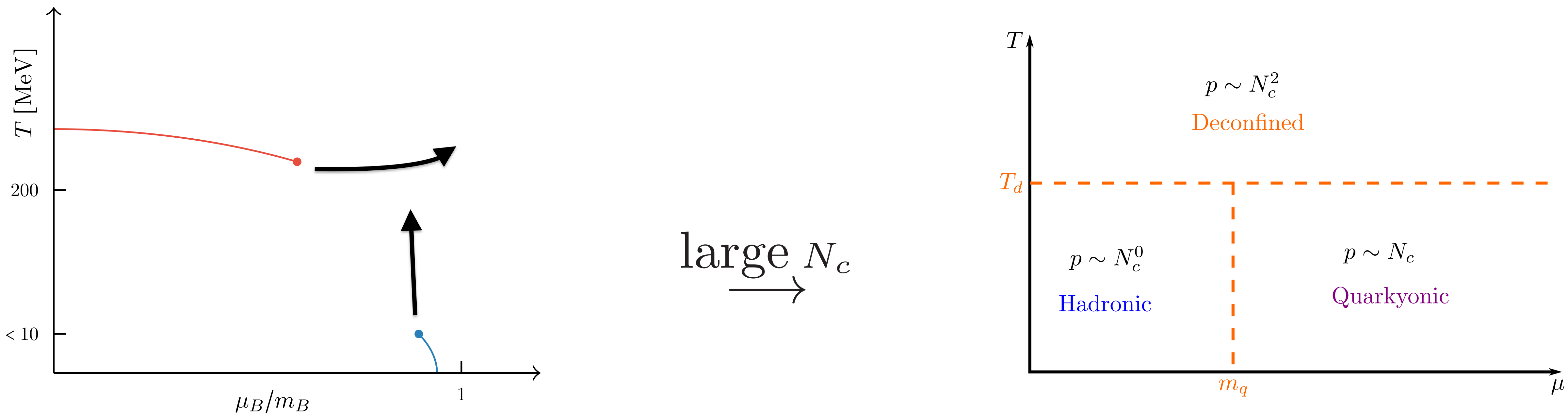


baryons

Not enough orders to take limits, but steepening of transition clearly observed!

Quarkyonic matter on the lattice?

Altogether:



Smooth transition of phase diagram to conjectured limit, scaling beyond baryon onset!

Large N_c limit independent of current quark masses, the same starting from physical QCD

Conclusions

- Sign problem beaten by effective lattice theory for heavy quarks
- Nuclear liquid gas transition and equation of state calculable in the heavy mass region
- Varying Nc: dense QCD is consistent with quarkyonic matter

Backup slides

Subleading couplings

Subleading contributions for next-to-nearest neighbours:

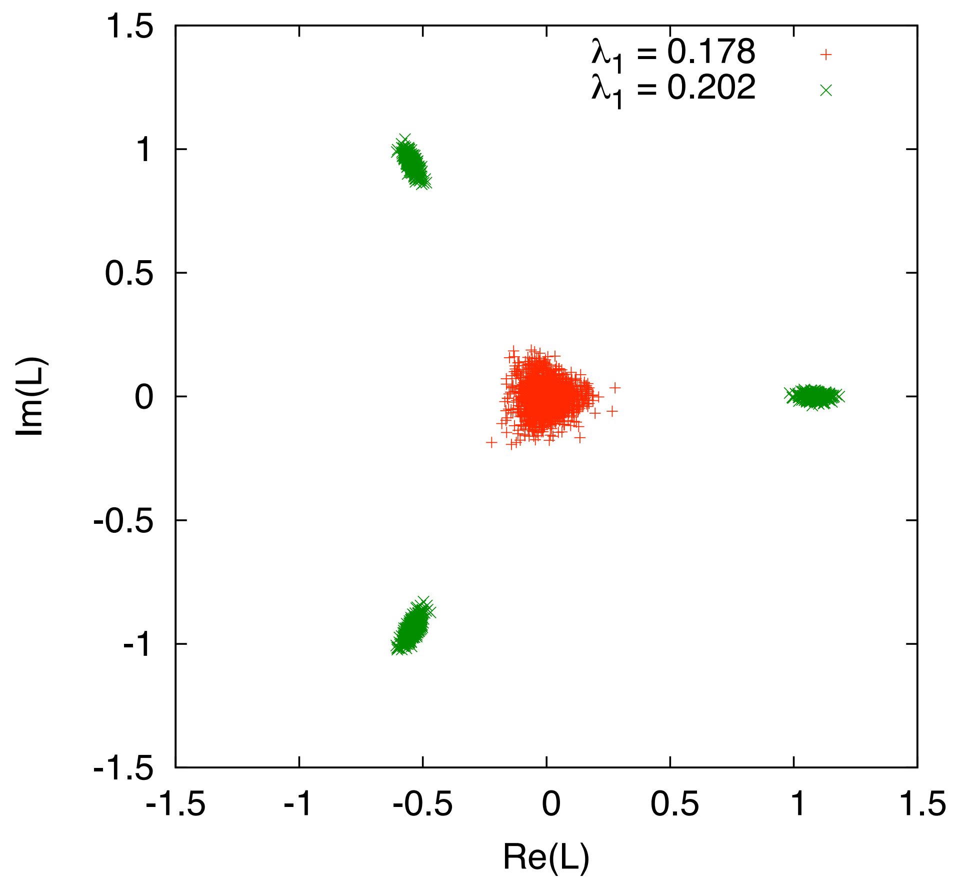
$$\lambda_2 S_2 \quad \propto \quad u^{2N_\tau+2} \sum_{[kl]}' 2\mathrm{Re}(L_k L_l^*) \quad \text{distance} \; = \sqrt{2}$$

$$\lambda_3 S_3 \quad \propto \quad u^{2N_\tau+6} \sum_{\{mn\}}'' 2\mathrm{Re}(L_m L_n^*) \quad \text{distance} \; = 2$$

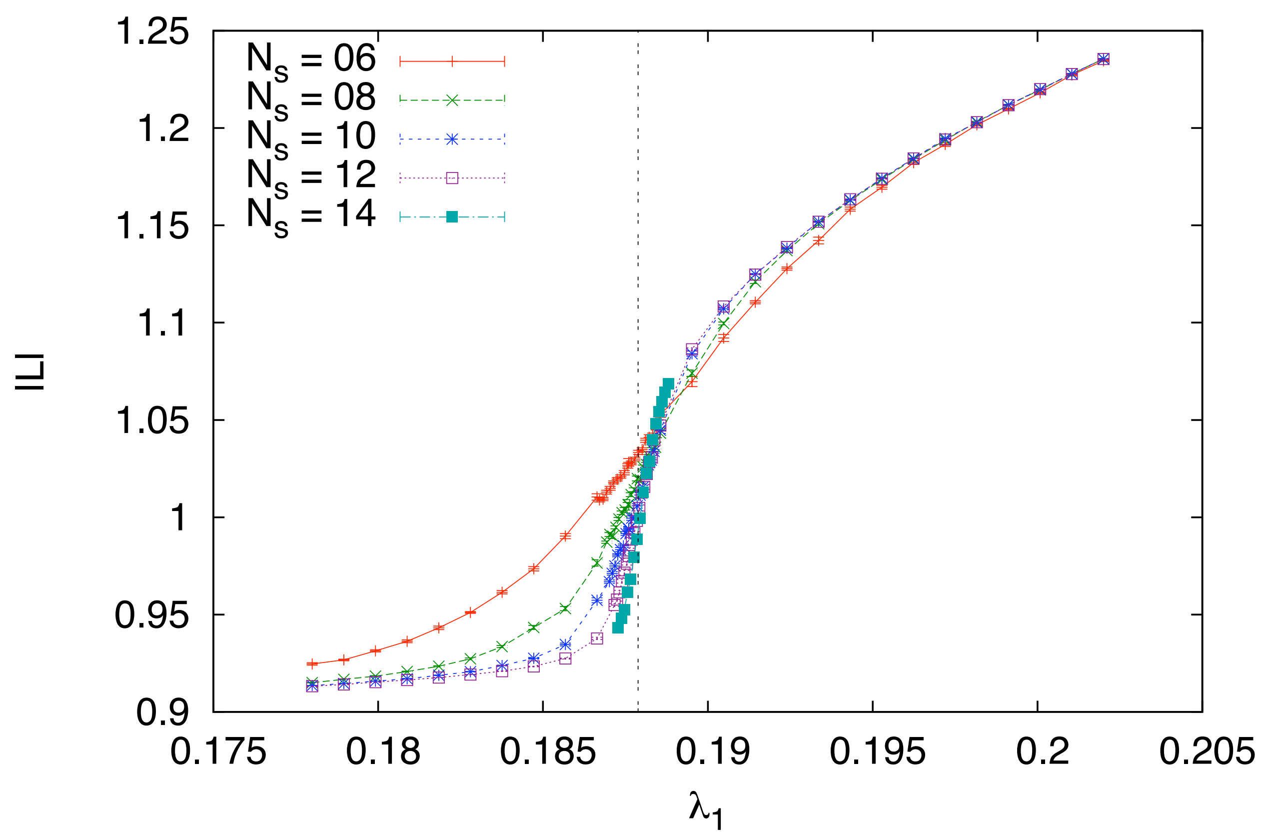
as well as terms from loops in the *adjoint* representation:

$$\lambda_a S_a \propto u^{2N_\tau} \sum_{<ij>} \mathrm{Tr}^{(a)} W_i \mathrm{Tr}^{(a)} W_j \quad ; \quad \mathrm{Tr}^{(a)} W = |L|^2 - 1$$

Numerical results for SU(3), one coupling

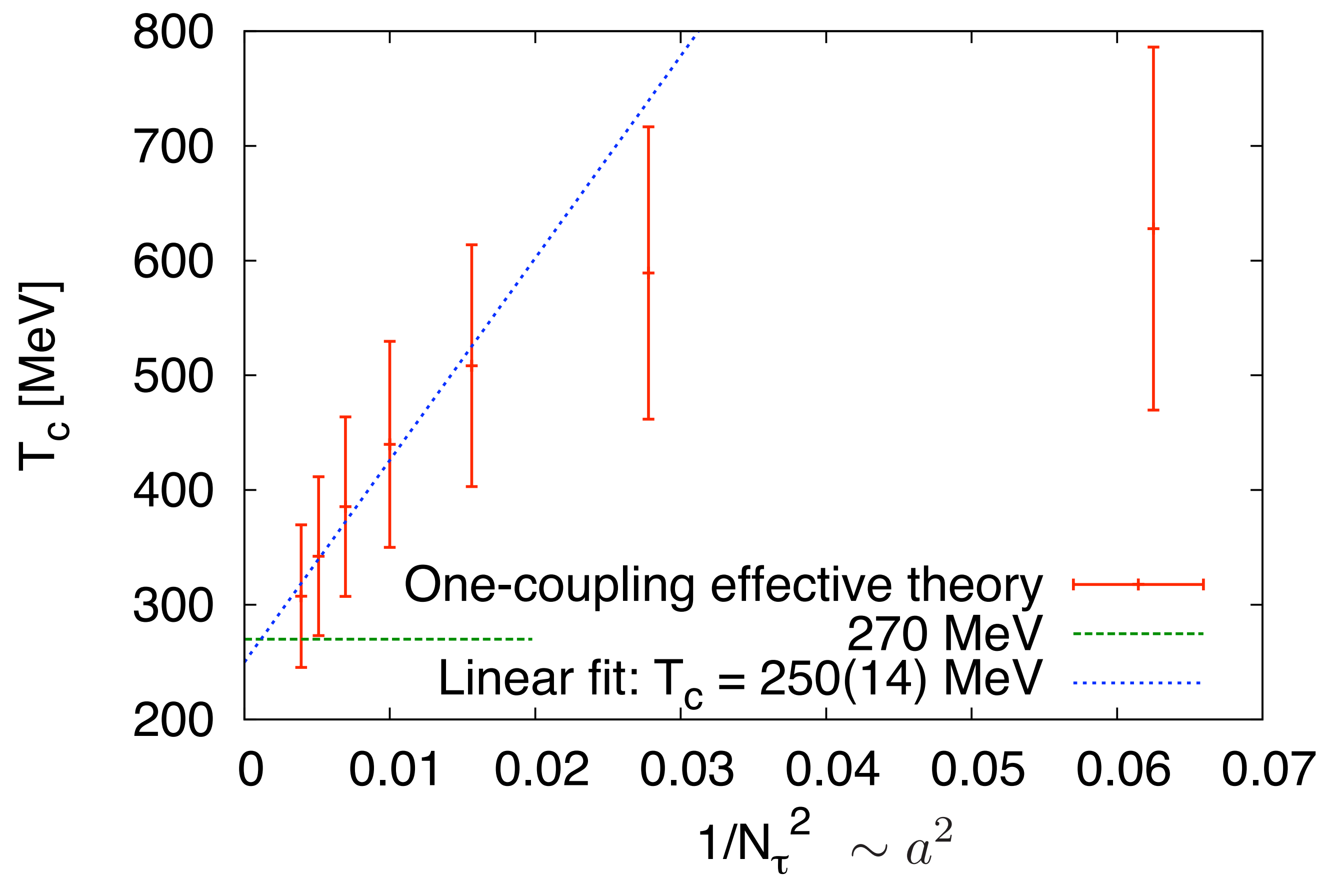


Order-disorder transition
=Z(3) breaking



Mapping back to 4d Yang-Mills

$$\lambda_1(N_\tau, \beta) \to \beta_c(\lambda_{1,c}, N_\tau)$$



-error bars: difference between last two orders in strong coupling exp.

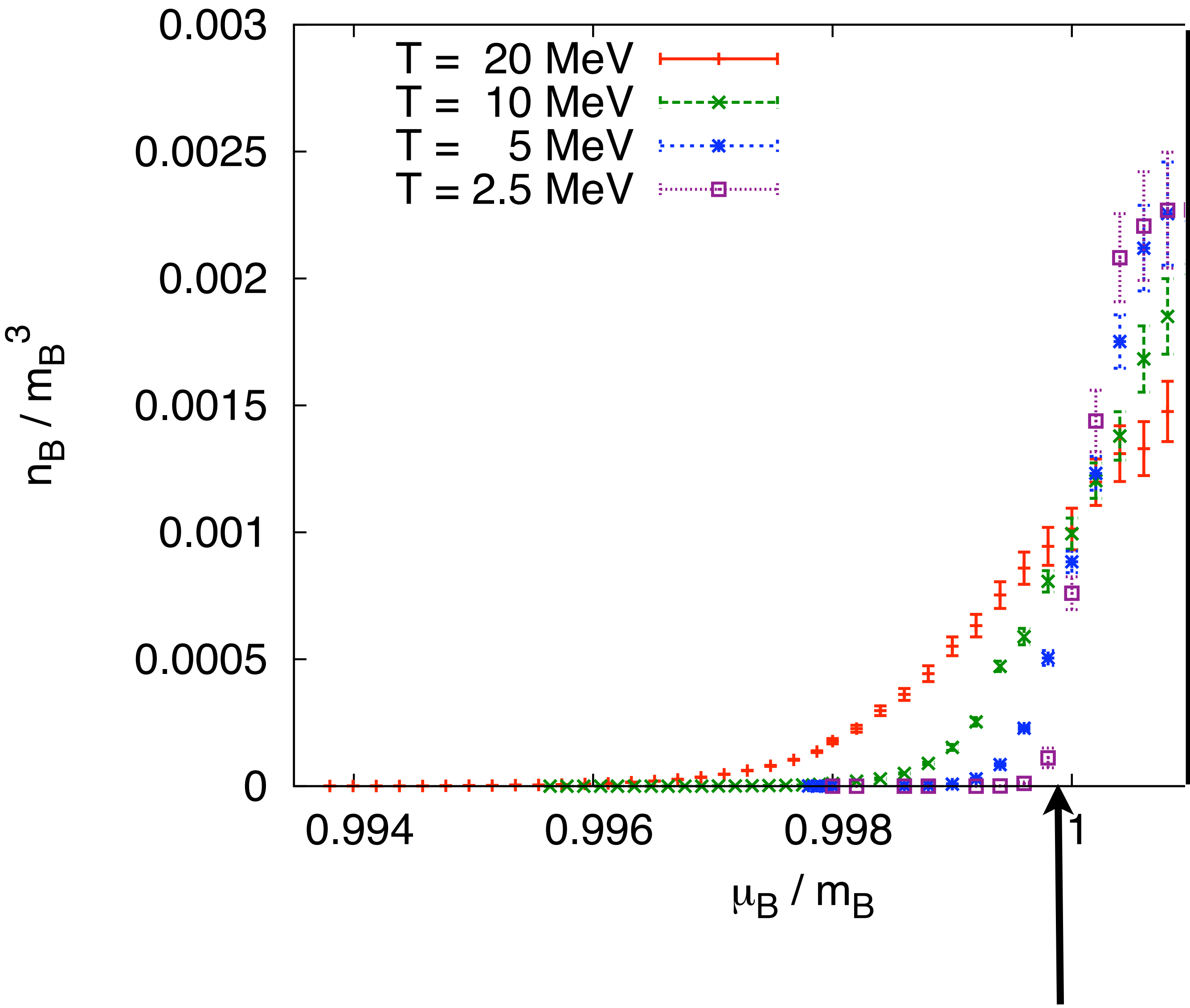
-using non-perturbative beta-function (4d T=0 lattice)

-all data points from one single 3d MC simulation!

Cold and dense, interacting: onset to nuclear matter

continuum extrapolated

$$m_\pi = 20\ \mathrm{GeV}$$



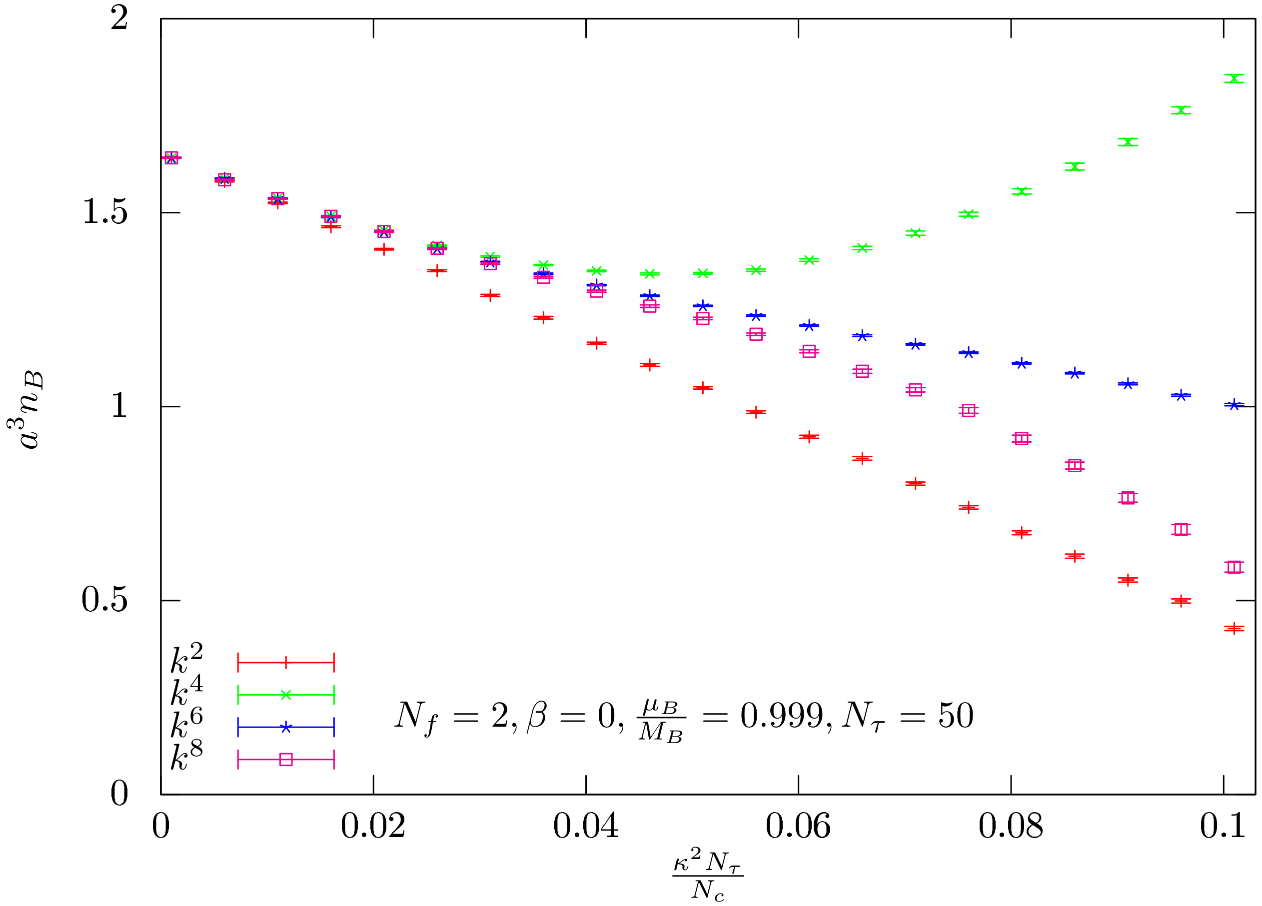
$$\frac{\mu}{T} \sim 4000$$

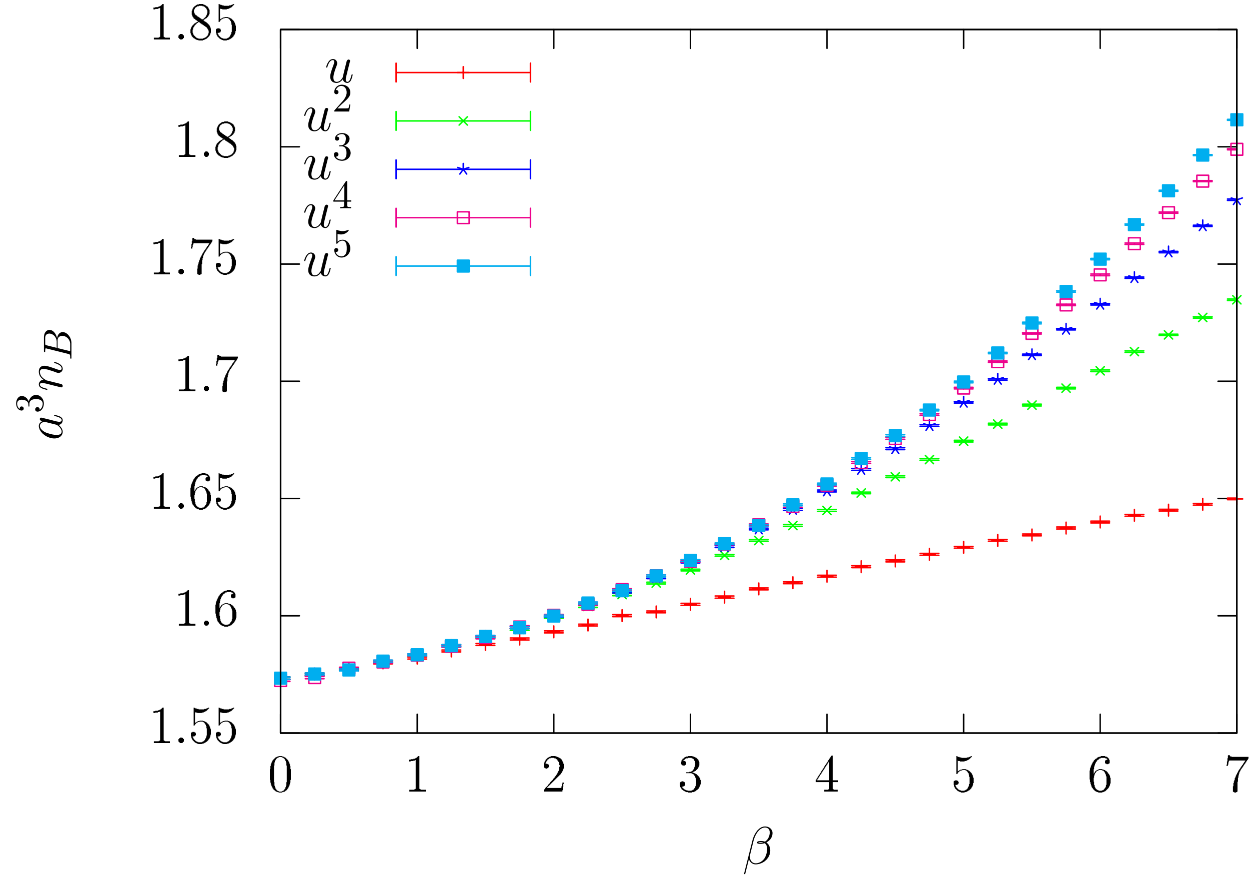
Effect of binding between baryons: $\mu_c < m_B$

Binding energy per nucleon: $\epsilon = \frac{\mu_c - m_B}{m_B} \sim 10^{-3}$

Transition is smooth crossover: $T > T_c \sim \epsilon\, m_B$

Convergence of the effective theory





hopping expansion in strong coupling limit

strong coupling expansion at κ^8

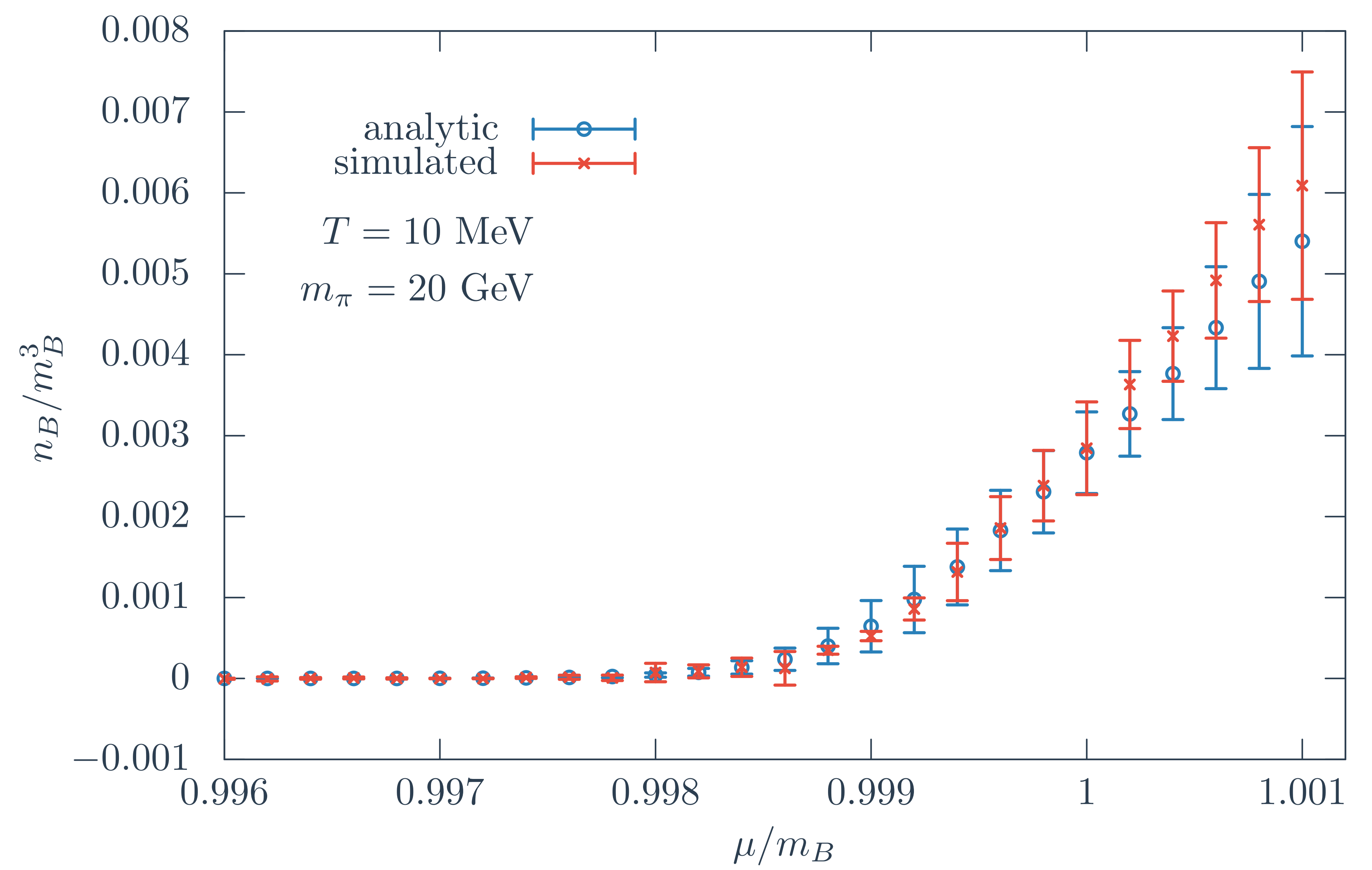
Linked cluster expansion of effective theory

$$\mathcal{Z} = \int \mathcal{D}\phi \, e^{-S_0[\phi] + \frac{1}{2}\sum v_{ij}(x,y)\phi_i(x)\phi_j(y) + \frac{1}{3!}\sum u_{ijk}(x,y,z)\phi_i(x)\phi_j(y)\phi_k(z) + \ldots}$$

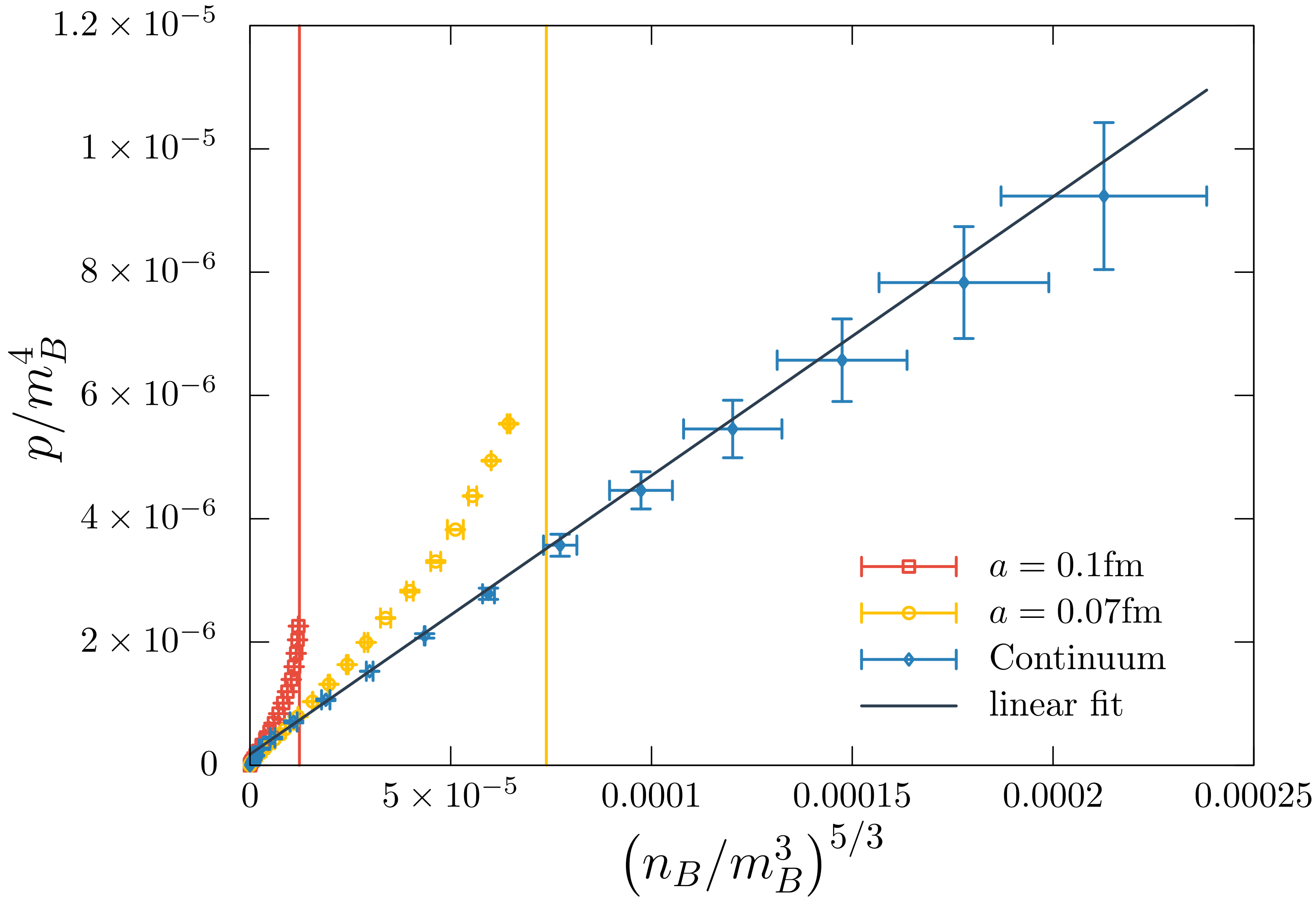
"perturbation theory" in effective couplings

Glesaaen, Neuman, O.P. 15

through $u^5\kappa^8$



Equation of state of heavy nuclear matter, continuum



- EoS fitted by polytrope, non-relativistic fermions!
- Can we understand the pre-factor? Interactions, mass-dependence…