Effective locality and chiral symmetry breaking in QCD

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What is effective locality (2010 +..) ?

EL : An exact non-perturbative property of *QCD*. Derived by means of functional methods of quantization in the Lagrangian Quantum Field Theory context.

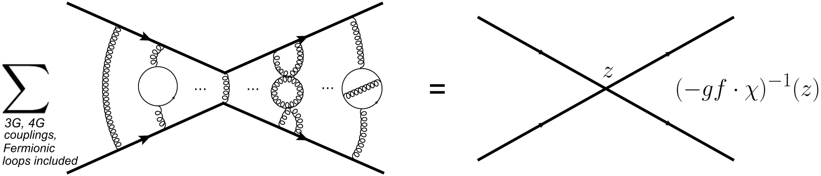
A formal statement of *EL*

- *For any fermionic* $2n$*-point Green's functions (and related amplitudes?), the full gauge-fixed sum of cubic and quartic gluonic interactions, fermionic loops included, results in a local contact-type interaction.*

- *This local interaction is mediated by a tensorial field antisymmetric both in Lorentz and color indices. The resulting sum is fully gauge-fixing independent, that is, gauge-invariant.*

- *The gauge-invariant summation of gluonic degrees at the origin of EL does not (seem to) meet the Gribov copies issue.*

Effective locality in a *symbolic* picture



$$e^{-\frac{i}{4}\int F^2} = \mathcal{N}\int \mathrm{d}[\chi]\, e^{\frac{i}{4}\int\left(\chi^a_{\mu\nu}\right)^2 + \frac{i}{2}\int \chi_a^{\mu\nu} F^a_{\mu\nu}}$$

$$[\lambda^a, \lambda^b] = f^{abc}\lambda^c$$

What is effective locality? ..

In *EL* elementary perturbative degrees, gluonic, are integrated out.

Now Is it possible to derive *QCD* g^2-perturbative expansions out of *EL* ?

In similitude to other non-perturbative attempts (*Light Front QCD*, S. Brodsky *et al.*, *Dyson-Schwinger Eqs.* , D. Kreimer *et al.*, *axiomatic analysis*, P. Lowdon ?),

answer seems to be negative

At large enough scattering sub-energies, $\hat{s}$, still, there is an *EL* form of *non-perturbative Asymptotic Freedom*.

- Not the perturbative Asymptotic Freedom of *QCD* ..

- But the expression that non-perturbative effects are washed out beyond a certain energy scale ($\sim 400 MeV$).

What effective locality is *not* ..

Taking a look at 'the coupling's scaling laws' for $2n$-pts Green's functions

- Original (perturbative) forms of couplings (to A^a_μ-fields and quark's spins, $\mathcal{O}(g^2)$)

$$\left[\mathcal{O}(g) + \mathcal{O}(g^2)\right]$$

- *EL* couplings (to $\chi^a_{\mu\nu}$, $\mathcal{O}(g) = gf^{abc} \cdot \chi^a_{\mu\nu}$, and quark's spins, $\mathcal{O}(g^2)$)

$$\left[\frac{1}{\mathcal{O}(g) + \mathcal{O}(g^2)}\right] \left\{A + B\,g + C\,g^2\right\}$$

No quark fields in pure *YM* : $B = C = \mathcal{O}(g^2) = 0$, the famous duality rule $g \to g^{-1}$ is recovered.
Now, contrarily to the pure *YM* case (H. Reinhardt *et al.*'91),

EL form is *not dual* to the original *QCD* one.

What is Effective Locality? ..

EL could be the very mode non-abelian gauge-invariance is realised in the non-perturbative regime of QCD (2017).

- *EL* gauge invariance is a direct consequence of its full gauge-fixing independance (*EL* formal statement).

- Most rewarding consequence : The never-ending *Gribov's copies issue* may be irrelevant to the non-perturbative regime of *QCD*.

- Gribov's copies are bound to perturbative *QCD* (C. Becchi !)

- Gribov's issue practical/theoretical intractability could be highly suggestive that (again!) non-perturbative *QCD* cannot be reached out of Perturbation Theory.

What is Effective Locality? ..

EL goes along with an *enigmatic* mass scale μ coming into play through an unavoidable, still non sensical, $\delta^{(2)}(b)$ where $b = |\vec{b}|$ is the (transverse) inter-quark separation in a 2-by-2 scattering quark process.

Now in a *QCD* theory where *confinement* and *chiral symmetry breaking* hold, then necessarily b must fluctuate (Casher'79, Brodsky, Shrock' 09)

$$\delta^{(2)}(\vec{b}) \longrightarrow \varphi(b) = \frac{\mu^2}{\pi} \frac{1-\xi/2}{\Gamma(\frac{1}{1-\xi/2})} e^{-(\mu b)^{2-\xi}}, \quad \xi \in \mathbb{R}^+, \ \xi \ll 1,$$

$\rightarrow$ Intriguing connections to *Levy flights*, *Lowest Landau Levels* and *non-commutative geometry* (De Moyal planes) ..

EL outputs at ‘tree level’

If an idea is good it is good *at ‘tree-level’* Wisdom says

- Almost linear confining potential for dynamical quarks

- Deuteron (Jastrow’s) potential reproduced

- QCD Green’s functions (eikonal and quenched approximations) complying with a general/formal statement (*Meijer special functions*).

- Mixture of partonic and non-perturbative dependences, as it should (also a demand of Light-Front QCD)

- Extended *AF*, as supported by a Dyson/Schwinger Eq. analysis and discontinuity *w.r.t* P.T.

- Green’s functions exhibiting the full algebraic content of $SU_c(3)$: C_2 and C_3 Casimir operators.

- Satisfying reproduction of the ISR/LHC p-p elastic differential cross sections $d\sigma/dt$ (Cf. P. Tsang talk)

Non-perturbative *QCD* as seen from *EL*

Much remains to be explored

If *EL* is relevant to non-perturbative *QCD* does it shed some light on *Chiral Symmetry Breaking* (χ*SB*)?

What could be the relation of χ*SB* to the *EL* mass scale, μ ?

What about *EL* and *Confinement* ?

Order parameter of χSB

- Contrary to first ideas bearing on the number of quark flavours, χSB, if any, is obtained out of a single quark flavor.

- Among others, the fermionic condensate $<\bar{\Psi}\Psi(x)>$ is an order parameter of chiral symmetry breaking and can be obtained out of $<\bar{\Psi}(x)\Psi(y)>$ in the limit of $x=y$

- 1^{st} step: *quenching* and *some mild eikonal* approximation will be used to deal with involved calculations. $<\bar{\Psi}(x)\Psi(y)>$ is thus,

$$\lim_{y=x} \mathcal{N} \int d[\chi]\, e^{\frac{i}{4}\int \chi^2}\ e^{\mathfrak{D}_A^{(o)}}\ e^{+\frac{i}{2}\int \chi\cdot F + \frac{i}{2}\int A\cdot \left(D_F^{(0)}\right)^{-1}\cdot A}\, G_{\mathrm{F}}(x,y|A)\Big|_{A\to 0}$$

where $\mathfrak{D}_A^{(0)}$, is the so-called *linkage operator*,
$\mathfrak{D}_A^{(0)} = -\frac{i}{2}\int \mathrm{d}^4x \int \mathrm{d}^4y \frac{\delta}{\delta A(x)} \cdot D_{\mathrm{F}}^{(0)}(x-y)\cdot \frac{\delta}{\delta A(y)}$

Basic steps of an involved calculation..

$\chi^a_{\mu\nu}$-fields : used to *linearize* the original non-abelian F^2-term.

$$\boxed{e^{-\frac{i}{4}\int F^2} = \mathcal{N}\int \mathrm{d}[\chi]\, e^{\frac{i}{4}\int \left(\chi^a_{\mu\nu}\right)^2 + \frac{i}{2}\int \chi^{\mu\nu}_a F^a_{\mu\nu}}}$$

Insertion for $G_F(x,y|A)$ of a *Schwinger-Fradkin's representation*

$$i\int_0^\infty ds\; e^{-ism^2}\, e^{-\frac{1}{2}\mathrm{Tr}\ln(2h)} \int d[u]\, e^{\frac{i}{4}\int_0^s ds'\,[u'(s')]^2}\, \delta^{(4)}(x-y+u(s))$$
$$\times\left[m + ig\gamma_\mu A^\mu_a(x)\lambda^a\right]\left(e^{-ig\int_0^s ds'\, u'_\mu(s')\, A^\mu_a(y-u(s'))\,\lambda^a}\right)_+$$

$+$ refers to s'-*Schwinger proper-time ordering*, $u_\mu(s')$, the Fradkin's fields, $\lambda^a s$ the Lie algebra generators of $SU_c(3)$ in the fundamental representation, and
$h(s_1,s_2) = s_1\Theta(s_2-s_1) + s_2\Theta(s_1-s_2)$.

Basic steps of the $<\bar{\Psi}\Psi(x)>$ calculation..

Two contributions come about whose non vanishing one as $m \to 0$, reads

$$< \bar{\Psi}\Psi(x) >= \lim_{y\to x} i\mathcal{N} \int_0^{\infty} ds\, e^{-ism^2}\, e^{-\frac{1}{2}Tr\ln(2h)}$$

$$\int d[u]\, e^{\frac{i}{4}\int_0^s ds'\,[u'(s')]^2}\, \delta^{(4)}(x-y+u(s))$$

$$\int \mathrm{d}[\alpha] \int \mathrm{d}[\Omega]\, e^{-i\int \mathrm{d}s'\, \Omega^a(s')\alpha^a(s')}\, (e^{i\int_{-\infty}^{+\infty} \mathrm{d}s\,\alpha^a(s)\lambda^a})_+$$

$$\int d[\chi]\, e^{\frac{i}{4}\int \chi^2}\;\; e^{\mathfrak{D}_A^{(0)}}\; \left[ig\gamma^\mu A_\mu^a(x)\lambda^a\right]\, e^{+\frac{i}{2}\int A_\mu^a K_{ab}^{\mu\nu} A_\nu^b}\, e^{i\int Q_\mu^a A_a^\mu}\Big|_{A\to 0}$$

with

$$K_{\mu\nu}^{ab} = gf^{abc}\chi_{\mu\nu}^c + \left(D_{\mathrm{F}}^{(0)^{-1}}\right)_{\mu\nu}^{ab}, \quad Q_\mu^a = -\partial^\nu\chi_{\mu\nu}^a + gR_\mu^a\,.$$

$$u(s) = sp\,, \quad R_\mu^a(z) = p_\mu \int \mathrm{d}s\,\Omega^a(s)\,\delta^4(z-y+sp)$$

Basic steps for $< \bar{\Psi}\Psi(x) >$ ..

At large coupling, $g >> 1$, the result of functional differentiations followed by $A \to 0$ reads

$$\mathcal{N} \int_0^\infty \mathrm{d}s\, e^{-ism^2}\, e^{-\frac{1}{2} Tr \ln(2h)} \int d[u]\, e^{\frac{i}{4}\int_0^s ds' [u'(s')]^2}\, \delta^{(4)}(x - y + u(s))$$

$$\times \int \mathrm{d}[\alpha] \int \mathrm{d}[\Omega]\, e^{-i\int \mathrm{d}s'\, \Omega^a(s')\alpha^a(s')} \left(e^{i\int_{-\infty}^{+\infty} \mathrm{d}s\, \alpha^a(s)\lambda^a}\right)_+$$

$$\times \int d[\chi]\, e^{\frac{i}{4}\int \chi^2} \frac{1}{\sqrt{\det(f\cdot\chi)}} e^{-\frac{i}{2} g \int \mathrm{d}^4 z\, R(z)\cdot(f\cdot\chi(z))^{-1}\cdot R(z)}$$

$$\times [g\gamma^\mu \lambda^a][(f\cdot\chi(x))^{-1} R(x)]^a_\mu$$

Involved enough an expression to deal with!

Basic steps for $< \bar{\Psi}\Psi(x) >$ ..

Doable, still, thanks to a standard analytic continuation of *Random Matrix* theory

Provided that the functional measure $d[\chi]$ can be taken to a measure dM on the (finite dimensional) space of real symmetric traceless $D(N_c^2-1) \times D(N_c^2-1)$ matrices : Permitted by *the measure image theorem* in *Wiener functional space*, which applies thanks to *EL* !

At real-valued *Halpern*-fields $\chi_{\mu\nu}^a$, in the adjoint representation of $SU_c(3)$ one has with $f^{abc} = i(T^a)_{bc}$, $N = D(N_c^2-1) = 32$,

$$f \cdot \chi \to \sum_{a=1}^{N_c^2-1} \chi^a \otimes i T^a = -M\,, \quad M_{ij} = M_{ji} \in \mathbb{R}\,,\ 1 \leq i, j \leq N\,,\ Tr\, M = 0.$$

$< \bar{\Psi}\Psi(x) >$ .. *RM*-measure of integration

Integration on $d[\chi]$ is now carried out with the *Random Matrix* measure

$$
\begin{aligned}
& -\mathrm{d}(\sum_{a=1}^{8} \chi^{a}{}_{\mu\nu} \otimes T^{a}) \\
= \quad & \mathrm{d}M = \mathrm{d}M_{11}\,\mathrm{d}M_{12}\cdots\mathrm{d}M_{NN} \\
= \quad & \left| \frac{\partial(M_{11},\cdots,M_{NN})}{\partial(\xi_1,\cdots,\xi_N,p_1,\cdots,p_{N(N-1)/2})} \right| \mathrm{d}\xi_1\cdots\mathrm{d}\xi_N\,\mathrm{d}p_1\cdots\mathrm{d}p_{N(N-1)/2} \\
= \quad & \prod_{i=1}^{N} \mathrm{d}\xi_i \prod_{i<j}^{N} |\xi_i-\xi_j|^{\kappa}\,\mathrm{d}p_1\,..\,\mathrm{d}p_{N(N-1)/2}\,f(p)
\end{aligned}
$$

at $\kappa = 1$ (non analytic), with a *Haar measure* of integration on the orthogonal group $O_N(\mathbb{R})$,

$$
\mathrm{d}p_1\,..\,\mathrm{d}p_{N(N-1)/2}\,f(p) \equiv \mathrm{d}\mathcal{O}(\mathbf{p}), \quad \mathcal{O}(\mathbf{p}) \in O_N(\mathbb{R})
$$

$< \bar{\Psi}\Psi(x) >$ .. *RM*-measure of integration

Integration factorises : $dM = d(Sp\,M) \times \mathrm{d}\mathcal{O}(\mathbf{p})$

While the latter, over $O_N(\mathbb{R})$, generates the full $SU_c(3)$ algebraic content of fermionic Green's functions (C_2 and C_3 *Casimir operator* dependences), the former leads to an analytic integrand ($-\infty < \xi_i < +\infty$),

$$\frac{\prod_{i<j}^{N} |\xi_i - \xi_j|^{\kappa=1}}{\sqrt{\det(M)}} = 2^{\frac{N}{2}} \left(\prod_{1\leq i<j\leq N/2}^{N/2} (\xi_i^2 - \xi_j^2)^2 \right)^{\kappa=1}$$

because *M*-matrices are *skew-symmetric*.

Now, this still entails 2^{120} *monomials* !!

$$\pm 2^{C_{q_1 \,..\, q_{N/2}}} \xi_1^{2q_1} \xi_2^{2q_2} \ldots \xi_{N/2}^{2q_{N/2}}$$

An intractable task

The contribution of a monomial ..

.. can be calculated and is *trivialised* by the *Trace*,

$$N^i = \gamma^\mu \otimes \lambda^a \,,\ Tr\; e^{\frac{i}{4}\delta s \hat{\alpha}^t \mathcal{O}(\mathbf{p})\hat{\lambda}}\, N^j(\dots) = 0 \,, \quad \forall j = 1, \dots, N$$

Not due to the eikonal neglect of $\sigma^{\mu\nu} F^a_{\mu\nu}(y - u(s'))$ in the Fradkin representation of $G_F(x, y|A)$

Trivialised a second time by the integration on $O_N(\mathbb{R})$!

$$\sum_{j=1}^{N/2} \Big(< a^2_{ij} >_{O_N} - < a^2_{N-i+1,j} >_{O_N} = \frac{1}{N} - \frac{1}{N} = 0 \Big) \times \Big(Tr\, N^j \hat{\lambda}_j = 0 \Big)$$

Each monomial contributes 0×0 !

Contribution of a monomial .. (still ?!)

As in massive $(QED)_2$, circumvented by calculating $< \bar{\Psi}\Psi(x)\bar{\Psi}\Psi(y) >$ in a definite $x - vs - y$ config.

$\longrightarrow$ a non vanishing sum of terms no longer zero identically

$$\pm \sum_{j=1}^{N/2} \sum_{k=1}^{N/2} < O^{ij} O^{ij'} O^{kl} O^{kl'} >_{O_N(\mathbb{R})} Tr \left[N^j \hat{\lambda}_{j'} \; N^l \hat{\lambda}_{l'} \right]$$

For a given monomial (the same in either cases, $2pt.$, $4pt.$, $2npt$-Green's functions), one obtains

$$< \bar{\Psi}\Psi(x) >_{partial} \times \left([\frac{1}{N} - \frac{1}{N} = 0] \times 0 \right)$$

-vs-

$$[< \bar{\Psi}\Psi(x) >_{partial}]^2 \times \left([\frac{1}{N^2}] \times C^{st} \right)$$

Now how to control the sum of so many monomials with alternate signs ?

Contribution of all of the 2^{120} monomials

Relying on *Wigner's (asymptotic, $N \to \infty$) semi-circle law*

$$(NC_{N1})^{-1} \int d\Theta_2 \ldots \int d\Theta_N \, P_{N1}(\Theta_1, \Theta_2, \ldots, \Theta_N) \equiv \sigma_N(\Theta_1)$$

including the Vandermonde determinant comprising 2^{120} monomials

$$P_{N_1}(\Theta_1, \ldots, \Theta_N) = C_{N_1} \, e^{-\Sigma_1^N \Theta_i^2} \prod_{1 \le i < j \le N} |\Theta_i - \Theta_j|^1$$

Now, as $N >> 1$,

$$\sigma_N(\Theta) \to \sqrt{2N - \Theta^2}, \, @\sqrt{2N} \ge |\Theta|; \quad \to 0, \; @ \sqrt{2N} < |\Theta|$$

and corrections to this asymptotics can be evaluated in a systematic way.

Result

$$\langle\bar{\Psi}\Psi(x)\rangle \simeq -g^2\mu^3 \cdot \frac{\mu}{\sqrt{Ep}}\sqrt{\frac{E^2-p^2}{Ep}}^{\,3} \cdot \frac{4^5(N_c-1)}{\sqrt{\pi^5 N^3}}\,\frac{I(N)}{vol(O_N(\mathbb{R}))}\,.$$

with,

$$I(N) = \int_{-\sqrt{2N}}^{+\sqrt{2N}} \frac{\mathrm{d}\Theta}{\Theta}\,\sqrt{2N-\Theta^2}\;\Phi(\Theta\sqrt{N})$$

where $\Phi(x)$ is the *probability integral*, and,

$$vol(O_N(\mathbb{R})) = \frac{2^N \pi^{\frac{N(N+1)}{4}}}{\prod_1^N \Gamma(\frac{k}{2})}$$

E, p energy and momentum of scattering quarks in their *cms*.

Conclusions

- *EL* seems to involve χSB (quenching being not a *proviso*).
- There would be a close relation of μ to $\langle \bar{\Psi}\Psi(x) \rangle \sim \mu^3$.
- A relation modulated by some *partonic damping function* accounting for the disappearance of non-perturbative effects at short distances,

$$\frac{\mu}{\sqrt{Ep}} \sqrt{\frac{E^2 - p^2}{Ep}}^{\,3} \;, \quad E, p \geq \mu \text{ scattering quarks cms } 4-\text{mom.}$$

- The Vandermonde issue circumvented by Wigner's law with available systematic corrections
- A partonic damping function requiring a 4-pt. calculation.. a possibly meaningful fact regarding the non-perturbative phase of QCD. *EL* doesn't seem to favour much the long held relation of χSB to the quark's massive pole in the dressed propagator..itself, an (axiomatic) issue !..