Schwinger based QCD formulation's derivation of elastic pp scattering. At ISR and LHC energies

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Analytic, Finite, Gauge-Invariant, Non-Perturbative QCD formulation obtained


2) A potential obtained from QCD that allows nucleons to be bounded, thus provided the first-principled model deuteron, [Fried et al. Ann. Phys. 338, 2013]


4) Obtained Chiral Symmetry Breaking for Dynamical Quarks out of Effective Locality. [Grandou, PT, arXiv:1905:05666 . to be published]

5) Extended Asymptotic Freedom as supported by other non-perturbative approaches: Dyson Schwinger Equation [Fried, Grandou, Sheu arXiv:1207.5017],

6) A qualitative description of Hadron Confinement mass scale(s), [Fried, PT 2015]

7) The full SU(3) algebraic content of non-perturbative QCD amplitudes. Both $C_2$ and $C_3$. [Granou EPL 107 (2014), 11001]


9) New physical aspects of Confinement? Work in progress….
Schwinger Generating Functional for QCD

a) Starting Point: Schwinger Generating Functional (GF) for QCD, with gluon operators in an Arbitrary (Relativistic) Gauge.

b) Re-arrange this GF in terms of a “Reciprocity Relation”, and a “Gaussian Linkage Operation”; and the GF now depends upon two functionals of A,

\[ Z_{QCD}[j, \bar{\eta}, \eta] = N e^{-\frac{1}{2} \int \frac{\delta}{\delta A} \cdot D_F^{(0)} \cdot \frac{\delta}{\delta A}} \cdot e^{-\frac{i}{4} \int F^2 + \frac{i}{2} \int A \cdot (-\partial^2) \cdot A} \cdot e^{i \int \bar{\eta} \cdot G_F[A] \cdot \eta + L[A]} \bigg|_{A = \int D_F^{(0)}} \]

\[ G_F(x, y | A) = [m + \gamma \cdot (\delta - igA\tau)]^{-1} \]

The next two steps were overlooked for decades:

1. Virtual quark loop
   \[ L[A] = \ln [1 - i\gamma A\tau] + \mathcal{O}[0] \]

2. Halpern's Representation
   \[ e^{-\frac{i}{4} \int F^2} = N \int d[\chi] e^{\frac{i}{4} \int \chi^2 + \frac{i}{2} \int F \cdot \chi} \]
   \[ \chi^a_{\mu \nu} = -\chi^a_{\nu \mu} \]
**Gauge Invariance by Gauge Independence**

Trivial rearrangement can now be made to formally insure gauge-invariance, even though the GF still apparently contains gauge-dependent gluon propagators.

Functional derivatives on Generating Functional, $Z_{QCD}$,

$$Z_{QCD}[j, \eta, \bar{\eta}] = N \int d[\chi] e^{\frac{i}{4} \int \chi^2 e^{\mathcal{D}}_A^{(0)} e^{\frac{i}{2} \int \chi \cdot F + \frac{i}{2} \int A \cdot (-\nabla^2) \cdot A} e^{i \int \bar{\eta} \cdot G_F[A] \cdot \eta + L[A]} |_{A = \int D_F^{(0)} \cdot j}$$

Calculating 2n-point Fermionic Green’s functions (e.g. n=2):

$$= N \int d[\chi] e^{\frac{i}{4} \int \chi^2 e^{\mathcal{D}}_A^{(0)} e^{\frac{i}{2} \int \chi \cdot F + \frac{i}{2} \int A \cdot (D_F^{(0)})^{-1} \cdot A} G_F(1|gA) G_F(2|gA) e^{L[A]} |_{A=0}$$

$$e^{D_A} F_1[A] = \exp \left[ \frac{i}{2} \int Q \cdot D_F^{(0)} \cdot (1 - \bar{K} \cdot D_F^{(0)})^{-1} \cdot Q - \frac{1}{2} \text{Tr} \ln (1 - D_F \cdot \bar{K}) \right]$$

$$\cdot \exp \left[ \frac{1}{2} \int A \cdot \bar{K} \cdot (1 - D_F^{(0)} \cdot \bar{K})^{-1} \cdot A + i \int Q \cdot (1 - \bar{K} \cdot D_F^{(0)})^{-1} \cdot A \right]$$

$$D_F^{(0)} \cdot (1 - \bar{K} \cdot D_F^{(0)})^{-1} = D_F^{(0)} \cdot [1 - (\bar{K} + (D_c^{(0)})^{-1}) \cdot D_F^{(0)}]^{-1}$$

$$= -(\bar{K}^{ab}_{\mu\nu} + gf^{abc} \chi_{\mu\nu}^c)^{-1} = -\bar{K}^{-1}$$

$$F_1[A] = e^{\frac{i}{2} \int A \cdot \bar{K} \cdot A + i \int Q \cdot A} \quad F_2[A] = e^{L[A]}$$

\[
\langle z|\tilde{K}_{\mu\nu}^{ab}(z') = \left[\tilde{K}_{\mu\nu}^{ab}(z) + gf^{abc} \chi_{\mu\nu}^c(z)\right] \delta^{(4)}(z - z') + \langle z| (D_F^{(0)})^{-1}]_{\mu\nu}^{ab} z'\rangle
\]

An exact property of 2n-points fermionic Green’s Fns:

\[ e^{\mathcal{O} \cdot A} F_1[A] F_2[A] = \exp\left[-\frac{i}{2} \int \bar{Q} \cdot \hat{K}^{-1} \cdot \bar{Q} + \frac{1}{2} Tr \ln \hat{K} + \frac{1}{2} Tr \ln(-D_\mathcal{F}^{(0)})\right] \]

\[ \cdot \exp\left[\frac{i}{2} \int \frac{\delta}{\delta A'} \cdot D_\mathcal{F}^{(0)} \cdot \frac{\delta}{\delta A'}\right] \]

\[ \cdot \exp\left[\frac{i}{2} \int \frac{\delta}{\delta A'} \cdot \hat{K}^{-1} \cdot \frac{\delta}{\delta A'} - \int \bar{Q} \cdot \hat{K}^{-1} \cdot \frac{\delta}{\delta A'}\right] \cdot (e^{\mathcal{O} \cdot A} F_2[A']) \]

\[ e^{\mathcal{O} \cdot A} F_1[A] F_2[A] = N \exp\left[-\frac{i}{2} \int \bar{Q} \cdot \hat{K}^{-1} \cdot \bar{Q} + \frac{1}{2} Tr \ln \hat{K}\right] \]

\[ \cdot \exp\left[\frac{i}{2} \int \frac{\delta}{\delta A} \cdot \hat{K}^{-1} \cdot \frac{\delta}{\delta A} - \int \bar{Q} \cdot \hat{K}^{-1} \cdot \frac{\delta}{\delta A}\right] \cdot \exp(L[A]) \]

\[ A \rightarrow 0 \]

No gauge dependencies left.

\[ -\hat{K}^{-1} = (f \cdot \chi)^{-1} = \]

All gluon exchanges summed!
Use Fradkin Representation for $G[A]$ and $L[A]$

$$G_F(x, y|A) = i \int_0^\infty ds \, e^{-is^2} e^{-\frac{1}{2} \ln(2\hbar)} \int d[u] \, e^{i \int_0^s ds' \left[ u'(s') \right]^2} \delta^{(4)}(x - y + u(s)) \times \left[ m - \gamma \mu \frac{\delta}{\delta u'_\mu(s)} \right] N_\Omega N_\Phi \int d[\alpha] \int d[\Xi] \int d[\Omega] \int d[\Phi] \left( e^{i \int_0^s ds' \left[ \alpha^\alpha(s') - i \sigma_{\mu \nu} \Xi^\alpha_{\mu \nu}(s') \right] \tau^\alpha} \right) +$$

$$\times e^{-i \int ds' \Omega^\alpha(s') \alpha^\alpha(s') - i \int ds' \Phi^\alpha_{\mu \nu}(s') \Xi^\alpha_{\mu \nu}(s')} \times e^{-ig \int ds' u'_\mu(s') \Omega^\alpha(s') A^\alpha_\mu(y - u(s')) + ig \int ds' \Phi^\alpha_{\mu \nu}(s') F^\alpha_{\mu \nu}(y - u(s')),$$

As advertised in Thierry Grandou’s talk in this conference, Effective Locality holds irrespective of any representation for $G[A]$ and $L[A]$

$$L[A] = -\frac{1}{2} \int_0^\infty \frac{ds}{s} \, e^{-is^2} e^{-\frac{1}{2} \ln(2\hbar)} \times N_\Omega N_\Phi \int d^4x \int d[\alpha] \int d[\Omega] \int d[\Xi] \int d[\Phi]$$

$$\times \int d[v] \delta^{(4)}(v(s)) \, e^{i \int_0^s ds' \left[ v'(s') \right]^2}$$

$$\times \left( e^{i \int_0^s ds' \left[ \alpha^\alpha(s') - i \sigma_{\mu \nu} \Xi^\alpha_{\mu \nu}(s') \right] \tau^\alpha} \right) +$$

$$\times e^{-ig \int_0^s ds' \nu'_\mu(s') \Omega^\alpha(s') A^\alpha_\mu(x - u(s')) - 2ig \int d^4z \left( \partial_\nu \Phi^\alpha_{\nu \mu}(z) \right) A^\alpha_\mu(z)$$

$$\times e^{+ig^2 \int ds' f^{abc} \Phi^a_{\mu \nu}(s') A^b_\mu(x - u(s')) A^c_\nu(x - v(s'))}.$$
All the Gaussian Linkage operations can then be carried through exactly, corresponding to the summation of all gluons exchanged between any pairs of quark (and/or antiquark) lines, including cubic and quartic gluon interactions.

Define a “Gluon Bundle” (GB) as the Sum over all gluon exchanges between any pair of quark lines,

\[
\mathcal{Q}_A^{(0)} \quad \text{is gaussian functional operation, } G[A] \text{ and } L[A] \text{ (with auxiliary variables) are gaussian in Fradkin representation}
\]

Functional derivatives operations with respect to sources \(\bar{\eta}, \eta\) can be performed exactly. This produces sum of all Feynman graphs corresponding to the exchange of infinite number of gluons between quarks.
New exact non-perturbative QCD Property: Effective Locality (E.L.)

The space-time coordinates of both ends of a GB are equal, up to fluctuations in their transverse coordinates, \( b \equiv |\vec{b}| \).

Thanks to E.L., Halpern's Functional Integrals can now be reduced to Sets of Ordinary finite dimensional integrals, Yielding vast simplification in all fermionic QCD correlation functions.

Effective Locality comes about with an mass scale $\mu$.

In addition, in QCD where confinement and chiral symmetry breaking hold, ‘b’ must fluctuate. [Casher’79, Brodsky, Shrock’09].

How do we choose a fluctuating ‘b’?

We first choose a gaussian
And/or deformed gaussians for dynamical quarks…

giving

$$\varphi(b) = \frac{\mu^2}{\pi} \frac{1 - \xi/2}{\Gamma\left(\frac{1}{1-\xi/2}\right)} e^{-\left(\mu b\right)^2 - \xi}$$

$0<\xi<< 1$
First try: A Gaussian, \( \varphi(b) \approx e^{-\left(\frac{1}{\mu}b\right)^2} \), where \( \mu^{-1} \) sets the scale of transverse fluctuations.

But this distribution gives vanishing confining quark potential: \( V(r) \).

Second try: A “deformed” Gaussian, \( \varphi(b) = \varphi(0)e^{-\left(\frac{1}{\mu}b\right)^2-\xi} \), with \( \xi \) a “deformation parameter”, real and small.

A straight-forward calculation yields, for small \( \xi \)

\[
V(r) \approx \xi \mu (\mu r)^{1-\xi}
\]
If you look up Nuclear forces on Wikipedia, you'll find the statement that there exists no derivation on the basis of QCD.

Here is an example of nucleon binding (for a Model Deuteron) proceeds from basic QCD, though neglecting electrical charge, and nucleon spins (which can always be added in); i.e. this is a Qualitative model describing, in this case, the essence of Nuclear Physics.

Question of Scale: Quark binding takes place for \( r_{ij} \sim m_\pi^{-1} \), but nucleon binding takes place at 2, or 3, or 4 times that distance. How to achieve this?

Consider:

This requires extraction and regularization of the logarithmic UV divergence of the loop, which contributes two essential features:
a) It “stretches”, so that distances larger than $m_{\pi}^{-1}$ can easily be probed (as supported by Lattice approaches).

b) It provides a crucial change of sign for the effective n-n binding potential.

This sign change can be the basis of nucleon-binding to form nuclei.

Virtual Closed-Quark Loops, their interactions with GBs.

All the basic, “radiative correction” structure of non-perturbative QCD comes from interacting closed-quark-loops with GBs. How can this be efficiently described? I will try to do this in words, describing the functional operations that need to be performed.

A single quark has Green’s Function:

\[ N \int d[\chi] e^{\frac{ix^2}{4}} (\det(gf \cdot \chi)^{-\frac{1}{2}}) e^{\hat{\mathcal{D}}_A} \cdot G_F^A[A] e^{L[A]}|_{A \to 0}, \]

While two scattering quarks are described by

\[ N \int d[\chi] e^{\frac{ix^2}{4}} (\det(gf \cdot \chi)^{-\frac{1}{2}}) e^{\hat{\mathcal{D}}_A} G_F^{(1)}[A] G_F^{(2)}[A] e^{L[A]}|_{A \to 0}, \]

\[ \hat{\mathcal{D}}_A = \frac{i}{2} \int \frac{\partial}{\partial A} (gf \cdot \chi)^{-1} \frac{\partial}{\partial A} \]

And \( \chi^a_{\mu\nu} \) is the Halpern functional tensorial field originally used to represent  

\[ e^{\frac{i}{4}} \int d^4 x F^2(x) \]
GB exchanges are accounted for by the linkage operator connecting the two $G_F[A]'s$ to each other and to themselves, and to $e^{L[A]}$.
The 'radiative corrections' of QCD enter when there is momentum transfer between one quark and another quark; and the procedure may occur when the momentum transfer passes through intermediate GBs and/or closed quark loops.

For simplicity, let us suppress possible quark binding into hadrons, and just consider two quarks exchanging momentum transfer in their CM.

A useful technique is the exact Functional Cluster Expansion,

\[ e^{\hat{\mathcal{D}}_A} \cdot e^{L[A]} = \exp \left[ \sum_{n=1}^{\infty} \frac{Q_n}{n!} \right], \quad Q_n = e^{\hat{\mathcal{D}}_A} (L[A])^n \big|_{\text{connected}} \]

With linkage operator

\[ \hat{\mathcal{D}}_A = \frac{i}{2} \int \frac{\partial}{\partial A} \hat{K} \frac{\partial}{\partial A}, \quad (\hat{K})^{ab}_{\mu\nu} = (g f_{abc} \chi^c_{\mu\nu})^{-1} \leftarrow GB \]
For example, \( Q_1 = e^{\tilde{\mathcal{D}}_A} L[A] = \bigcirc + \bigcirc \bigcirc + \bigcirc \bigcirc \bigcirc + \ldots = \bar{L}[A] \)

And \( Q_2 = \bar{L}[A](e^{\tilde{\mathcal{D}}_A} - 1) \bar{L}[A] \)

Things get complicated very quickly; e.g. \( Q_4 \) is given by

\[
Q_4 = \begin{array}{c}
\bigcirc \bigcirc \bigcirc \bigcirc + 2 \\
\bigcirc \bigcirc \bigcirc \bigcirc + 3 \\
\bigcirc \bigcirc \bigcirc \bigcirc + 4 \\
\bigcirc \bigcirc \bigcirc \bigcirc + 4 \\
\bigcirc \bigcirc \bigcirc \bigcirc + 9 \\
\bigcirc \bigcirc \bigcirc \bigcirc + 12
\end{array}
\]

(Basics of Functional Methods and Eikonal Models, H.M.Fried)
Even functionally, it is a horrid mess. But, there exists one way of reducing this to an easily-calculated set of 'chain-graph-loops',

Which form a geometric series that can be summed.

But this depends crucially on the definition of renormalization....

At this point, let's 'take stock' of where we stand. We began with a Theory of quarks and gluons; but at least at large coupling, $g \gg 1$, the gluons have “disappeared”, turned into GB sums.

What is left is GB’s and Quark loops.

What to do? Renormalize the GB's:
QCD Renormalization
Because each quark represents the “physical particle” of QCD, we'll replace the $\delta$ at each quark site by $\delta_Q$, a finite quantity. But where the $\delta$ connects to the loop, which is a virtual and not a physical particle – it remains $\delta^*$, and (very shortly) $\rightarrow 0$.

In this one-loop, 2 GB drawing, there is a net $\delta^2$ multiplying the loop.

But the loop is proportional to an expected UV log divergence, which we'll call

$\ell, \quad \ell = \ln(\Lambda/m_q)$

This loop – as well as every such loop in a chain of such GB loops – produces a factor of

$\delta^2 \cdot \ell \big|_{\ell \rightarrow \infty, \delta \rightarrow 0} = \kappa \sqrt{m^2}$

which we DEFINE to be real, finite, positive number (subsequently determined by experiment).
\[ \delta^2 \cdot \ell \big|_{\ell \to \infty, \delta \to 0} = \kappa / m^2 \]
\[ \delta^2 \cdot \ell \mid_{\ell \to \infty, \delta \to 0} = \kappa / \bar{m}^2 \]

Within this renormalization scheme only chain graphs are non-zero.
To simplify scattering problems:

1) Reduce full 6 body problem to 2 body scattering ones

2) Write $f \chi$ as $\sqrt{R}$ where $R$ is real number. (leading order Justified)
Simplifying the full 6 body problem of 2 scattering hadrons, along with using $f \cdot \chi$ as $\sqrt{R}$ where $R$ is real does not allow us to keep track of energy dependences. However, out of the 2 body problem: [Fried, Grandou, Hofmann 2016]

$$\pm \sum_{\text{monomials}} \left( \prod_{i=1}^{N} \left[ \frac{\sqrt{4iN_c} \sqrt{s(\hat{s} - 4m^2)}}{m^2} \right] [(\mathcal{OT})_i]^{-2} \frac{g\varphi(b)}{g\varphi(b)} \right) \times G_{03}^{30} \left( \left[ \frac{g\varphi(b)}{\sqrt{32iN_c} \sqrt{s(\hat{s} - 4m^2)}} \right]^2 \left[ (\mathcal{OT})_i \right] \frac{1}{2}, \frac{3 + 2q_i}{4}, 1 \right) \bigg)_{O_N(\mathbb{R})}$$

$$\simeq \pm \frac{DC_2 f}{N} I_{3 \times 3} \sum_{\text{monomials}} \left( \prod_{i=1}^{N} A_i^1 \right) \left( \sum_{i=1}^{N} A_i^3 \right) \left\{ \frac{g\varphi(b)}{\sqrt{2iN_c}} \times \left( \frac{m^2}{\hat{E}^2} \right), \quad g \frac{\mu m}{|u_0 u_3^t|} e^{-\mu b^2} \times \left( \frac{m}{\hat{E}} \right) \right\}$$

where $A_i^1 = \Gamma(\frac{1}{2}) \Gamma(\frac{2q_i + 1}{4})$, $A_i^2 = \Gamma(\frac{1 - 2q_i}{4}) \Gamma(\frac{-2q_i - 1}{4})$, $A_i^3 = \Gamma(-\frac{1}{2}) \Gamma(\frac{2q_i - 1}{4})$,

$N = D (N_c - 1) = 32$, gives $N(N-1)/2$ terms or $2^{496}$ monomials.

By symmetry, only $2^{120} !$ : untractable
Simpler problem $N = 4$ gives

$$\frac{DC_{2f}}{N} I_{3 \times 3} \left[ \frac{g \mu m}{|u'_0 \bar{u}'_3|} \left( \sum_{\text{res.mon.}} (\prod_{i=1}^{4} A_i) \left( \sum_{i=1}^{4} \frac{A_i^3}{A_i^1} \right) \right) \left( \frac{m}{E} \right) \right] \xrightarrow{\text{Left hand side fitted by}} \frac{DC_{2f}}{N} I_{3 \times 3} \left( \frac{m}{E} \right)^p,$$

Left hand side fitted by $p \sim 0.26$
Parameters to learn from experiments

- \textbf{g coupling}

- \( \kappa_i = \delta^2 \cdot \ell \) \quad \text{Loop renormalization constant}

- Gluon Bundle \( m \), Loop Chain masses \( \overline{m} \)

- \( (\lambda/m)(m/E)^p \) though can be derived in principle
Differential Cross-Section
\[ \frac{d\sigma}{dt} = \frac{m_{\text{proton}}^4}{16\pi p^2 E^2} |T|^2 \]

• Calculations

\[
\frac{d\sigma_2}{dt}(s, q^2) = K \frac{27}{4\pi} \frac{g^2}{4} \left( \frac{\lambda}{m} \right)^4 \left( \frac{6m}{\sqrt{s}} \right)^{4p} \left[ \left( \frac{1}{\sqrt{2}} e^{-q^2/4m^2} - \frac{g}{2} \frac{\lambda^2}{4\pi} \left( \frac{6m}{\sqrt{s}} \right)^{2p} e^{-q^2/8m^2} \right)^2 
+ \left( \frac{1}{\sqrt{2}} e^{-q^2/4m^2} + \frac{g}{2} \kappa \frac{q^2}{m^2} e^{-q^2/2m^2} \right)^2 \right]
\]

Each additional loop gives on more \( \kappa \), which is \( \sim 10^{-4} \)
Differential Cross-Section

\[ \frac{d\sigma}{dt} = \frac{m_{\text{proton}}^4}{16\pi p^2 E^2} |T|^2 \]

- Calculations

\[ \frac{d\sigma_2}{dt}(s, q^2) = K \frac{27}{4\pi} \frac{g^2}{4} \left( \frac{\lambda}{m} \right)^4 \left( \frac{6m}{\sqrt{s}} \right)^{4p} \left[ \left( \frac{1}{\sqrt{2}} e^{-q^2/4m^2} - \frac{g}{2} \frac{\lambda^2}{4\pi} \left( \frac{6m}{\sqrt{s}} \right)^{2p} e^{-q^2/8m^2} \right)^2 
+ \left( \frac{1}{\sqrt{2}} e^{-q^2/4m^2} + \frac{g}{2} \kappa \frac{q^2}{\bar{m}^2} e^{-q^2/2\bar{m}^2} \right)^2 \right] \]

Energy dependence of a single quark-quark subprocess yields:

\[ \lambda \left( \frac{m}{\sqrt{s}} \right)^{2p} \]

Differential Cross-Section

\[
\frac{d\sigma}{dt} = \frac{m_{\text{proton}}^4}{16 \pi p^2 E^2} |T|^2
\]

**Calculations**

1GB’s + 2GB’s + 1 loop chain:

\[
\frac{d\sigma_2}{dt}(s, q^2) = K \frac{27}{4\pi} \frac{g^2}{4} \left( \frac{\lambda}{m} \right)^4 \left( \frac{6m}{\sqrt{s}} \right)^4 p \left[ \left( \frac{1}{\sqrt{2}} e^{-q^2/4m^2} - \frac{g}{2} \frac{\lambda^2}{4\pi} \left( \frac{6m}{\sqrt{s}} \right)^{2p} e^{-q^2/8m^2} \right)^2 \\
+ \left( \frac{1}{\sqrt{2}} e^{-q^2/4m^2} + \frac{g}{2} \kappa \frac{q^2}{m^2} e^{-q^2/2m^2} \right)^2 \right]
\]
1 Gluon Bundle + 2 Gluon Bundle + 1 Loop Chain

$g=7.0$
$m \approx 1.5m_\pi$
$\bar{m} \approx 4.5m_\pi$
$
\lambda = 0.5$
$p = 0.13$
$\kappa/m^2 = -6.8 \times 10^{-4}$
1 Gluon bundle + 2 Gluon bundle + 1 Loop Chain

Note the dip moves to the left with increasing Energy. Here we input the energy dependence As lambda/m(E/m)^2p

\( \sqrt{s} = 30.7 \text{ GeV} \)
\( m_{t(\rho)} = 0.2295, \ m_{t(\rho)} = 0.64 \)
\( g = 7, \ p = 0.13 \)
\( \kappa = 0.00068 \)
\( \lambda = 0.515 \)

\( g = 7.0 \)
\( m \approx 1.5 m_\pi \)
\( \bar{m} \approx 4.5 m_\pi \)
\( \lambda = 0.5 \)
\( p = 0.13 \)
\( \kappa / m^2 = -6.8 \times 10^{-4} \)
1 Gluon bundle + 2 Gluon Bundles + 1 Loop Chain

\[ \sqrt{s} = 44.7 \text{ GeV} \]

\[ m_{\text{ext}} = 0.2295, \quad m_{\text{int}} = 0.64 \]

- \( g = 7, \quad p = 0.13 \)
- \( \kappa = 0.00068 \)
- \( \lambda = 0.515 \)

\[ g = 7.0 \]
\[ m \approx 1.5 m_{\pi}, \quad \bar{m} \approx 4.5 m_{\pi} \]
\[ \lambda = 0.5 \]
\[ p = 0.13 \]
\[ \kappa / m^2 = -6.8 \times 10^{-4} \]
$g = 7.0$
$m \approx 1.5m_{\pi}$, $\bar{m} \approx 4.5m_{\pi}$
$\lambda = 0.5$
$p = 0.13$
$\kappa/\bar{m}^2 = -6.8 \times 10^{-4}$

$\sqrt{s} = 52.8$ GeV

Equation 35 (1GB + 1CQL)
Equation 36 (1GB + 2GBs + 1CQL)
1 Gluon bundle + 2 Gluon Bundles + 1 Loop Chain

$\sqrt{s} = 62.5 \text{ GeV}$

$\sqrt{s} = 63 \text{ GeV}$

$g = 7.0$

$m \approx 1.5m_\pi$

$\bar{m} \approx 4.5m_\pi$

$\lambda = 0.5$

$p = 0.13$

$\kappa/m^2 = -6.8 \times 10^{-4}$
Extension to LHC

\[ \sigma = \frac{4\pi}{k} \Im[T(0)] \]
LHC data compared with our QCD formalism

7 TeV

g = 7.0
m ≈ 1m_π, \quad \bar{m} ≈ 3m_π
λ = 0.72
p = 0.055
\kappa / m^2 = -4.2 \times 10^{-3}
LHC data compared with our QCD formalism

Preliminary 8 TeV data and fit

\[ g = 7.0 \quad m \approx 1m_\pi \quad \bar{m} \approx 3m_\pi \]
\[ \lambda = 0.72 \]
\[ p = 0.055 \]
\[ \kappa/m^2 = -4.2 \times 10^{-3} \]

13 TeV Data and fit

\[ g = 7.0 \quad m \approx 1m_\pi \quad \bar{m} \approx 3m_\pi \]
\[ \lambda = 0.72 \]
\[ p = 0.055 \]
\[ \kappa/m^2 = -4.2 \times 10^{-3} \]
\[ \frac{\kappa}{\bar{m}^2} = \delta^2 \ell \]

\[ \frac{\kappa}{\bar{m}^2} \text{ is roughly proportional to } \sqrt{E} \]
Conclusions:

→ Out of QCD, an attempt at deriving some nuclear physics out of an effective expansion based on E.L. & Renormalization

→ After pion mass ad Deuteron binding energies,

→ quite successful again for Elastic pp scattering in the ISR → LHC energy ranges.

→ Attribute the shape of differential cross section to GB’s and, after dip, to quark loops

Prospects:

→ $\pi$-p elastic scattering

→ Total Cross-section $\sigma_{\text{total}}$

→ $\rho = \text{Re } F(z) / \text{Im } F(z)$

→ what about the Odderon?
Supplemental slides
No static approximation required! Our analysis gives \( E(b) \) explicitly, in terms of \( \varphi(b) \), so that we can calculate \( V(r) \) for any choice of \( \varphi(b) \). The minimal bound state energy representing the pion shows that most of the pion mass comes from the gluons forming the GB, and relatively little from the quark masses.
E.L & renormalization => an effective perturbation theory for Non-perturbative phase of QCD, made out of gluon bundles and quark loops.

Show 4 point greens function.

Must expand as whole bundle
It has been well-known for a half-century that, assuming a specific $V(r)$, in ordinary QM, or in Abelian QFTs, the corresponding eikonal function $E(b)$ is given by

$$E(b) = \gamma(s) \int_{-\infty}^{+\infty} dz_L V(\vec{b} + \hat{p}_L z_L),$$

where $\gamma(s)$ is a constant depending on CM energy and the type of interaction.

We can write the non-perturbative amplitude corresponding to a GB exchanged between a $\vec{q}$ and a $\vec{Q}$; and we see that the Eikonal limit of this amplitude has $E(b)$ defined in terms of $\varphi(b)$, and proportional to: $\ln\{\varphi(b)\}$. Here, $E(b) = E(b)$, and $V(\vec{r}) = V(r)$.

Our method: Calculate the 2-D Fourier transform $\tilde{E}(k_\bot^2)$ of $E(b)$. Extend $k_\bot^2 \rightarrow k_\bot^2 + k_L^2 = \vec{k}^2$, so that we now have $\tilde{E}(k^2)$; and then calculate the 3-D transform of this $\tilde{E}(k^2)$, which will yield $V(r)$.

More on Eikonal in slide 26 on Scattering Amplitude
Substituting this potential into a Schrödinger binding equation, using the “quantic” approximation, then yields $\mu \sim m_\pi, \xi \approx 0.1$. This is sensible, since the max. fluctuations should be $\leq \frac{1}{m_\pi}$. Our result encompasses two different lattice calculations, $V \sim r$ and $V \sim r \ln(r)$. But all lattice and other model calculations of binding correspond to an amplitude containing only one of the two Casimir SU(3) invariants, $C_2, C_3$; our amplitude contains both.


What method do we use to pass from $\varphi(b)$ to $V(r)$?

Imagine that a $q$ and a $\bar{q}$ are scattering at high energy. One can write an Eikonal approximation, valid in the limit of $s \gg |t|$, for the conventional scattering amplitude. (Details for QCD eikonals were worked out by HMF. YG, JA, and BMcK in two papers circa 1983.)