ICNFP 2019 – QCD workshop

On the generalised spectral structure of QCD propagators

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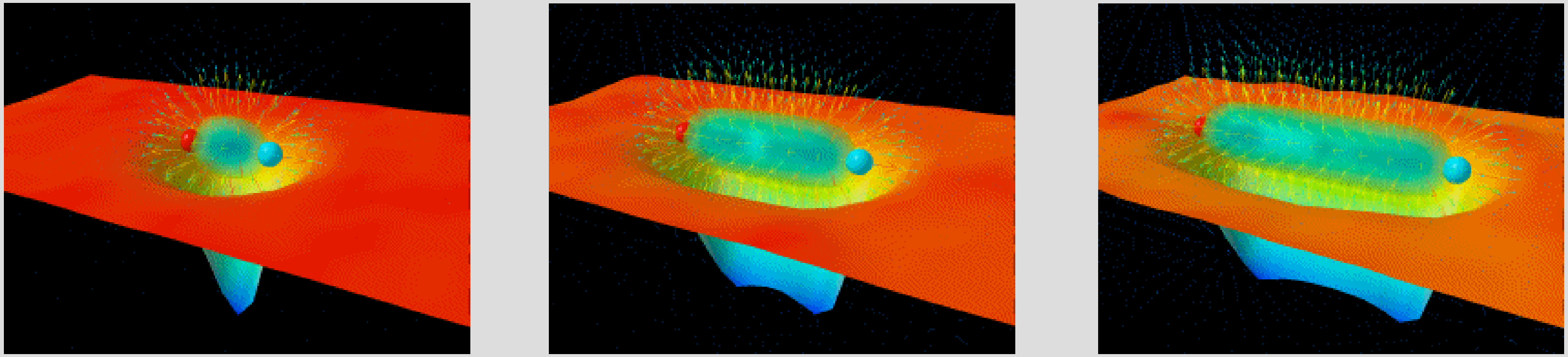
Outline

1. Correlation functions in local QFT
2. Confinement and the CDP
3. The gluon propagator
4. Summary and outlook



[The University of Adelaide (2015)]

1. Correlation functions in local QFT

- Despite their simplicity, the axioms of local QFT have many important consequences:

  → Correlation functions $\boxed{\langle 0|\varphi_{l_1}^{(\kappa_1)}(x_1)\cdots\varphi_{l_n}^{(\kappa_n)}(x_n)|0\rangle}$ are distributions

  → ***Reconstruction Theorem*** – a QFT can be reconstructed from knowledge of *all* the correlators

  → Spin-statistics theorem, *CPT* theorem, connection of Minkowski and Euclidean QFTs, existence of scattering states, ...

- Things become more complicated in gauge theories!

"Local Gauss law"

$$\boxed{\partial^{\nu}G^{a}_{\mu\nu} = J^{a}_{\mu}}$$

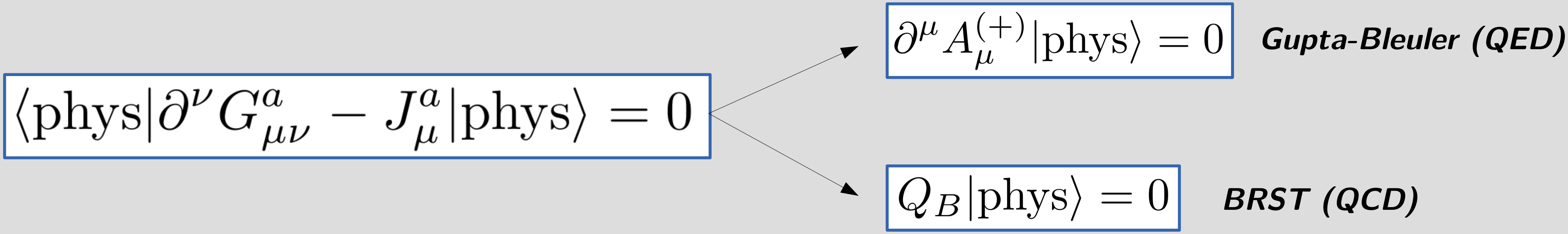
(i) *Preserve positivity, lose locality (e.g. Coulomb gauge QED)*

(ii) *Preserve locality, lose positivity (e.g. Landau gauge QCD)*

1. Correlation functions in local QFT

- For the remainder of this talk we will focus on option (ii)
- The local Gauss law implies that all charged fields are *non-local*

  $\rightarrow$ To recover locality one modifies this equation, lifting this restriction, but maintains the constraint for **physical** states



$$\boxed{\langle \mathrm{phys}|\partial^{\nu}G^{a}_{\mu\nu} - J^{a}_{\mu}|\mathrm{phys}\rangle = 0}$$

$$\boxed{\partial^{\mu}A^{(+)}_{\mu}|\mathrm{phys}\rangle = 0}$$ ***Gupta-Bleuler (QED)***

$$\boxed{Q_{B}|\mathrm{phys}\rangle = 0}$$ ***BRST (QCD)***

- This procedure necessarily introduces both zero and negative norm (ghost) states into the theory!
- Modifies QFT axioms $\rightarrow$ ***Pseudo-Wightman*** approach [Bogolubov et al.]

[N. N. Bogolubov, A. A. Logunov, A. I. Oksak and I. T. Todorov, General Principles of Quantum Field Theory, (1990).]

1. Correlation functions in local QFT

- In the Pseudo-Wightman approach many of the previous axioms are maintained, except now the full space of states is not positive-definite
- Determining the effect that this change has on the characteristics of QFTs is essential for unravelling the dynamics of gauge theories
- In particular, this can help in understanding the *non-perturbative* structure of QCD correlators

QCD correlators

- Dyson-Schwinger, Bethe-Salpeter equations, FRGEs
- Enter into the calculation of bound-state observables; meson spectra, decay constants, glueball masses,...
- QCD phase diagram: effects of finite temperature and density
- Confinement

1. Correlation functions in local QFT

→ ***What is the general structure of a Pseudo-Wightman correlation function?***

- Due to the Lorentz transformation property of the fields the Fourier transform of any field correlator can be written

$$\boxed{\widehat{T}_{(1,2)}(p) = \mathcal{F}\left[\langle 0|\phi_1(x)\phi_2(y)|0\rangle\right] = \sum_{\alpha=1}^{\mathcal{N}} Q_\alpha(p)\, \widehat{T}_{\alpha(1,2)}(p)}$$

Lorentz covariant polynomial

Lorentz invariant component

- The spectral condition implies that the Lorentz invariant components must *vanish* outside the (closed) forward light cone ($p^2 \geq 0$, $p^0 \geq 0$)

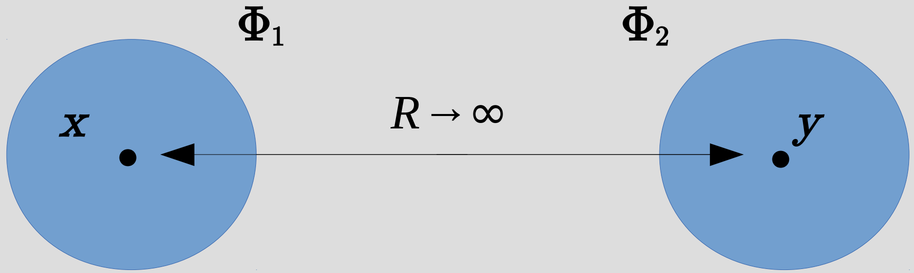
$$\boxed{\widehat{T}_{\alpha(1,2)}(p) = P(\partial^2)\delta(p) + \int_0^\infty ds\, \theta(p^0)\delta(p^2-s)\rho_\alpha(s)}$$

Purely singular component

Spectral function

2. Confinement and the CDP

- For non-gauge theories satisfying the standard local QFT axioms, the correlation strength between clusters of fields always **decreases** with space-like separation $R = -(x-y)^2$ [Araki; Araki, Hepp, Ruelle]



→ *This is called the* ***cluster decomposition property*** *(CDP)*

$$\boxed{|\langle 0|\Phi_1(x)\Phi_2(y)|0\rangle - \langle\Phi_1(x)\rangle\langle\Phi_2(y)\rangle| \xrightarrow{R\to\infty} 0}$$

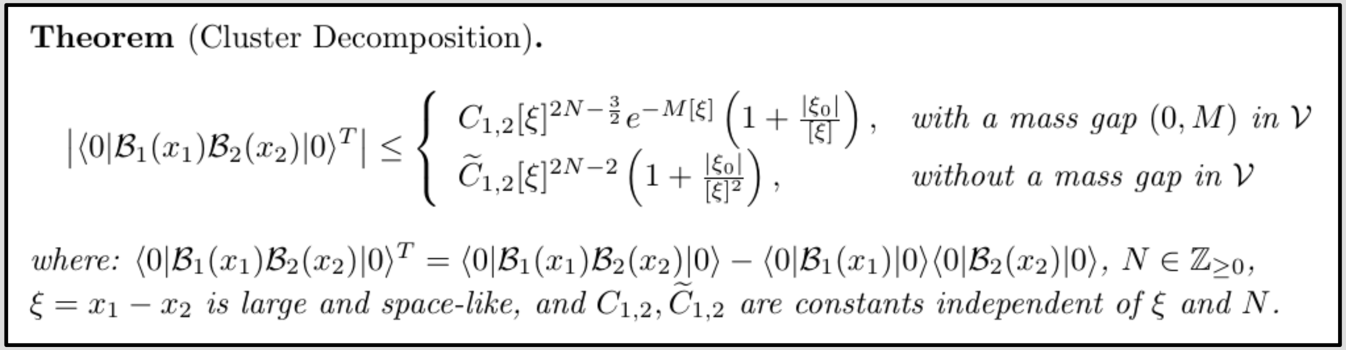
- States therefore become increasingly decorrelated the further apart they are separated!

[H. Araki, *Ann. Phys.* **11**, 260 (1960).]
[H. Araki, K. Hepp and D. Ruelle, *Helv. Phys. Acta* **35**, 164 (1962).]

2. Confinement and the CDP

- But what about QCD? If the CDP held this would allow one to pull apart coloured states!

- It turns out though that the CDP **can be violated** in QFTs satisfying the Pseudo-Wightman axioms [Strocchi 1976]:



Theorem (Cluster Decomposition).

$$\left|\langle 0|\mathcal{B}_1(x_1)\mathcal{B}_2(x_2)|0\rangle^T\right| \leq \begin{cases} C_{1,2}[\xi]^{2N-\frac{3}{2}}e^{-M[\xi]}\left(1+\frac{|\xi_0|}{[\xi]}\right), & \textit{with a mass gap } (0,M) \textit{ in } \mathcal{V} \\ \widetilde{C}_{1,2}[\xi]^{2N-2}\left(1+\frac{|\xi_0|}{[\xi]^2}\right), & \textit{without a mass gap in } \mathcal{V} \end{cases}$$

where: $\langle 0|\mathcal{B}_1(x_1)\mathcal{B}_2(x_2)|0\rangle^T = \langle 0|\mathcal{B}_1(x_1)\mathcal{B}_2(x_2)|0\rangle - \langle 0|\mathcal{B}_1(x_1)|0\rangle\langle 0|\mathcal{B}_2(x_2)|0\rangle$, $N \in \mathbb{Z}_{\geq 0}$, $\xi = x_1 - x_2$ *is large and space-like, and* $C_{1,2}, \widetilde{C}_{1,2}$ *are constants independent of* ξ *and* N.

- Depends on whether the theory has a mass gap, and the value of the positive integer parameter N

  $\rightarrow$ *for the CDP to be violated it is necessary that* $\boldsymbol{N > 0}$

- N is related to the boundedness properties of the corresponding momentum space correlator [PL, 1511.02780]

[F. Strocchi, *Phys. Lett.* B **62**, 60 (1976).]

2. Confinement and the CDP

- Any correlation function is determined by its spectral functions ρ

  → The value of N must therefore be related to the type of components appearing in ρ

- Besides ordinary massive poles $\delta(s\text{-}m^2)$ and continuum contributions, spectral functions can have more singular properties in the *Pseudo-Wightman* case

- In particular, $\rho(s)$ can possess **generalised pole** terms of the form:

$$\boxed{\delta^{(n)}(s-m_n^2) = \left(\tfrac{d}{ds}\right)^n \delta(s-m_n^2), \qquad (n \geq 1)}$$

- These components lead to a stronger IR singular behaviour

- The existence of these delta-derivative components is sufficient to prove that $N > 0$ [PL, 1511.02780]

  → **Are these components present in QCD correlators?**

3. The gluon propagator

- A QCD correlation function of particular interest is the *gluon propagator*
- Taking into account general QFT and model-dependent constraints one can write [PL, 1702.02954; 1801.09337]:

$$\boxed{\widehat{D}^{ab\,F}_{\mu\nu}(p) = i\int_0^\infty \frac{ds}{2\pi}\,\frac{\left[g_{\mu\nu}\rho_1^{ab}(s)+p_\mu p_\nu \rho_2^{ab}(s)\right]}{p^2-s+i\epsilon} + \sum_{n=0}^{N}\left[c_n^{ab}\, g_{\mu\nu}(\partial^2)^n + d_n^{ab}\partial_\mu\partial_\nu(\partial^2)^{n-1}\right]\delta(p)}$$

Taking trace, in Landau gauge

$$\boxed{D(p) = 3i\int_0^\infty \frac{ds}{2\pi}\,\frac{\rho_1(s)}{p^2-s+i\epsilon} + \sum_{n=0}^{N+1} g_n(\partial^2)^n\delta(p)}$$

- Can now use numerical data to test different propagator ansätze

Euclidean generalisation is needed to test lattice data

$$\boxed{D(p) = 3\int_0^\infty \frac{ds}{2\pi}\,\frac{\rho_1(s)}{p^2+s} + \sum_{n=0}^{N+1} g_n(-\nabla^2)^n\delta(p)}$$

3. The gluon propagator

- Using the high precision Landau gauge lattice data from [Dudal, Oliveira, Silva 1803.02281] the strategy was to perform fits of various generalised pole terms up to some IR cutoff [Li, PL, Oliveira, Silva, 1907.10073]

- Started with the simplest possible one-pole components

$$\boxed{D_0(p) = \frac{Z_0}{p^2+m_0^2}, \qquad D_1(p) = \frac{Z_1}{(p^2+m_1^2)^2}, \qquad D_2(p) = \frac{Z_2}{(p^2+m_2^2)^3}.}$$

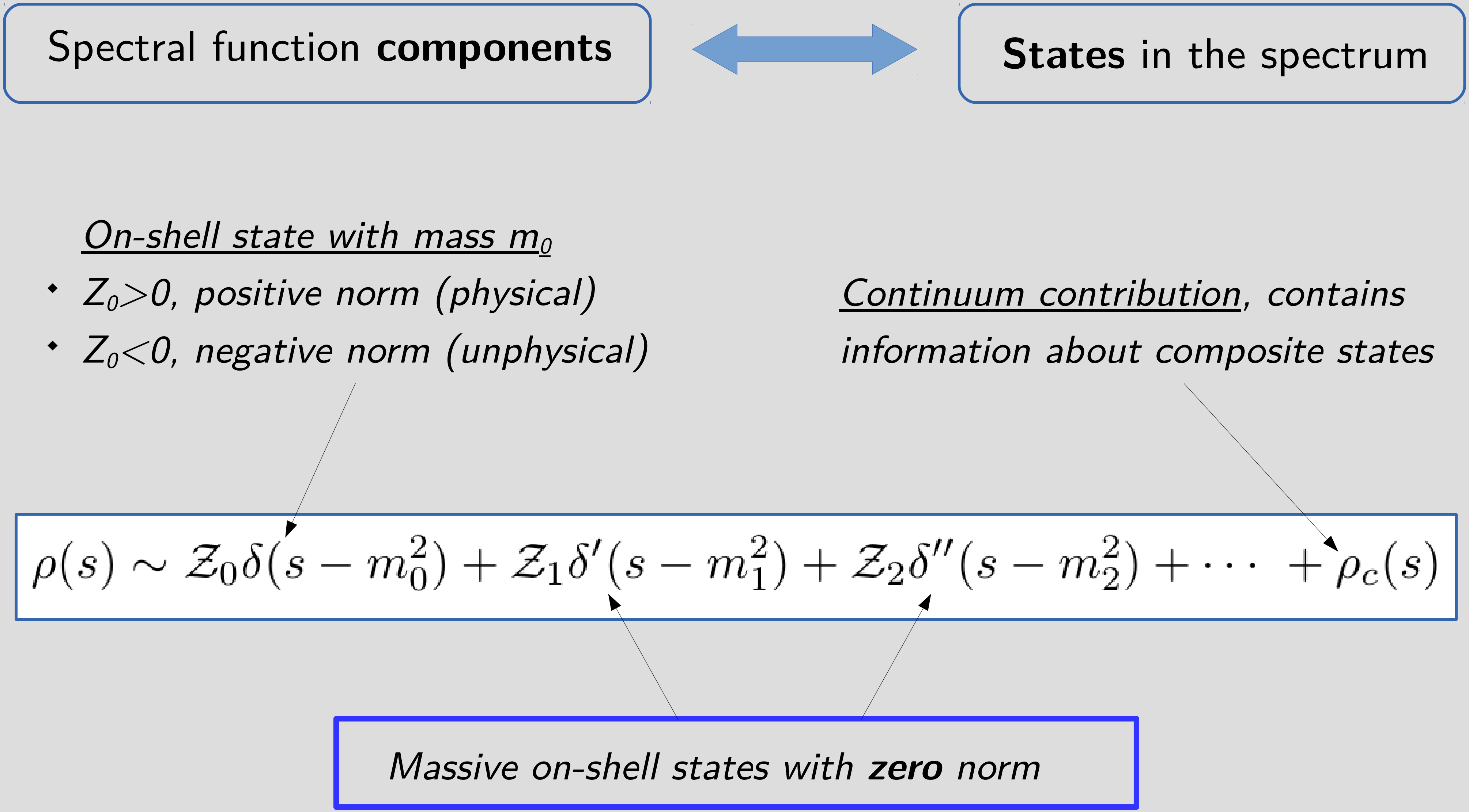
- Also tested the various two-term combinations → fits performed with different choices of (increasingly conservative) systematics

- **Results:** found that the data was consistent with the appearance of a generalised mass pole in the spectral function of the form:

$$\boxed{\mathcal{Z}_1\delta'(s-m_1^2), \qquad \left[m_1 = 0.89^{+0.09}_{-0.06} \ \text{GeV}, \quad Z_1 = 52^{+8}_{-6} \ \text{GeV}^2\right]}$$

3. The gluon propagator

→ ***How does one interpret such a spectral component?***



Spectral function **components** ⟷ **States** in the spectrum

On-shell state with mass m_0

- *$Z_0>0$, positive norm (physical)*
- *$Z_0<0$, negative norm (unphysical)*

Continuum contribution, contains information about composite states

$$\rho(s) \sim \mathcal{Z}_0\delta(s-m_0^2) + \mathcal{Z}_1\delta'(s-m_1^2) + \mathcal{Z}_2\delta''(s-m_2^2) + \cdots \ + \rho_c(s)$$

*Massive on-shell states with **zero** norm*

3. The gluon propagator

- A $\delta'(s\text{-}m_1^2)$ component in the gluon spectral function implies the existence of a **massive on-shell zero-norm state in the spectrum**

  $\rightarrow$ State dominates the *IR* behaviour of the propagator

- One has $N > 0$, but non-vanishing m_1 suggestions that that component is not singular enough at $p^2 = 0$ to violate the CDP

  $\rightarrow$ Results in an *exponential* fall-off for $R \rightarrow \infty$

- Clustered states created from *single* gluon fields do not appear to decorrelate the further apart they are separated

  $\rightarrow$ *Gluon propagator not gauge-invariant though, so doesn't contradict the expectation that asymptotic coloured states are absent from the spectrum!*

4. Summary and outlook

- Local QFT is a framework which can be used to better understand the non-perturbative characteristics of QFTs

- Due to the complications that arise in gauge theories, QCD correlators can potentially contain more singular *generalised pole* components

  → Important for understanding the asymptotic behaviour of correlators (*confinement*)

  → Constrains the spectrum of the theory (*zero-norm states*)

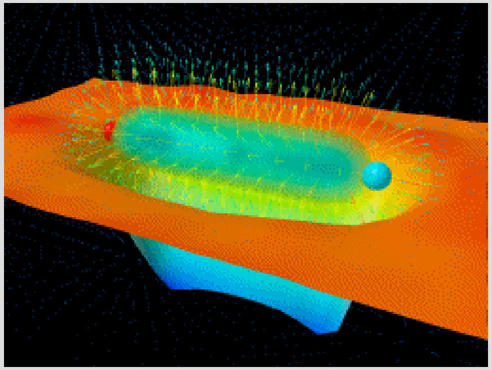
- Fits of generalised pole terms to infrared gluon propagator lattice data suggest that the data is compatible with the existence of these type of components

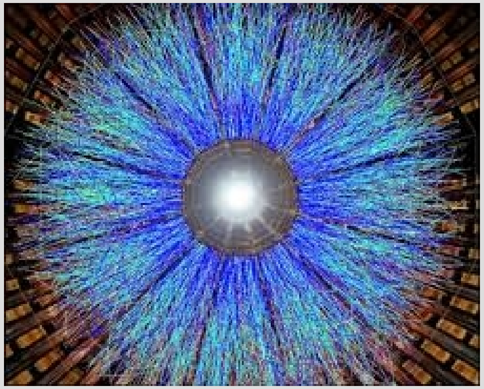
  → ***Opens a new direction for understanding QCD correlators***

4. Summary and outlook

Outlook

- *Are generalised pole terms relevant for hadron phenomenology?*
- *Can purely singular terms play a role in the CDP?*
- *What happens at finite temperature and density?*
- *What about gauge-invariant correlators?*

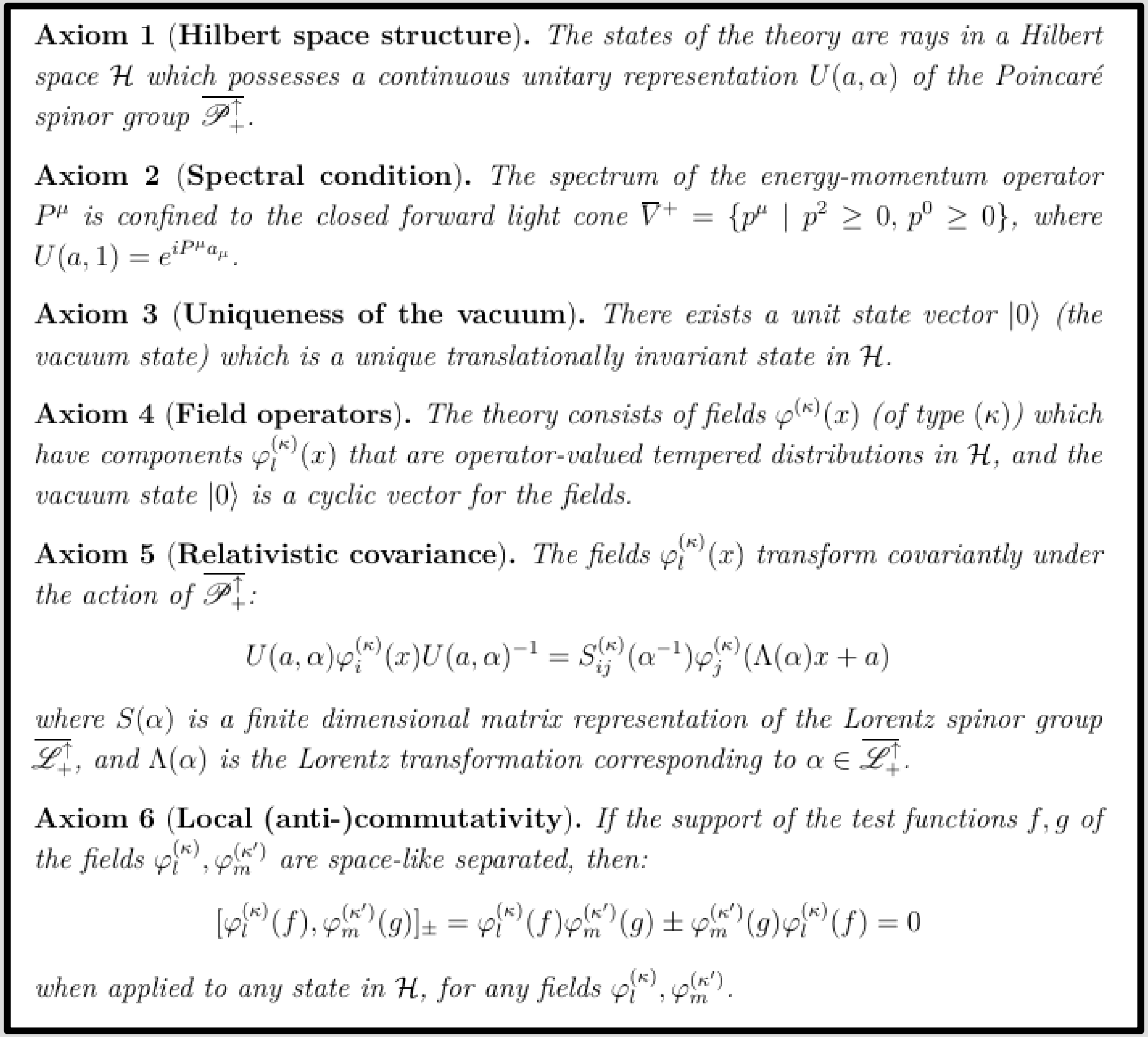




[Brookhaven National Lab]

Backup

- Local QFT is defined by a core set of axioms:



Axiom 1 (Hilbert space structure). *The states of the theory are rays in a Hilbert space $\mathcal{H}$ which possesses a continuous unitary representation $U(a,\alpha)$ of the Poincaré spinor group $\overline{\mathscr{P}}^{\uparrow}_{+}$.*

Axiom 2 (Spectral condition). *The spectrum of the energy-momentum operator P^{μ} is confined to the closed forward light cone $\overline{V}^{+} = \{p^{\mu} \mid p^{2} \geq 0,\ p^{0} \geq 0\}$, where $U(a,1) = e^{iP^{\mu}a_{\mu}}$.*

Axiom 3 (Uniqueness of the vacuum). *There exists a unit state vector $|0\rangle$ (the vacuum state) which is a unique translationally invariant state in $\mathcal{H}$.*

Axiom 4 (Field operators). *The theory consists of fields $\varphi^{(\kappa)}(x)$ (of type (κ)) which have components $\varphi^{(\kappa)}_{l}(x)$ that are operator-valued tempered distributions in $\mathcal{H}$, and the vacuum state $|0\rangle$ is a cyclic vector for the fields.*

Axiom 5 (Relativistic covariance). *The fields $\varphi^{(\kappa)}_{l}(x)$ transform covariantly under the action of $\overline{\mathscr{P}}^{\uparrow}_{+}$:*

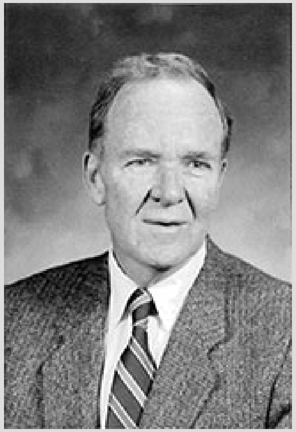
$$U(a,\alpha)\varphi^{(\kappa)}_{i}(x)U(a,\alpha)^{-1} = S^{(\kappa)}_{ij}(\alpha^{-1})\varphi^{(\kappa)}_{j}(\Lambda(\alpha)x + a)$$

where $S(\alpha)$ is a finite dimensional matrix representation of the Lorentz spinor group $\overline{\mathscr{L}}^{\uparrow}_{+}$, and $\Lambda(\alpha)$ is the Lorentz transformation corresponding to $\alpha \in \overline{\mathscr{L}}^{\uparrow}_{+}$.

Axiom 6 (Local (anti-)commutativity). *If the support of the test functions f, g of the fields $\varphi^{(\kappa)}_{l}, \varphi^{(\kappa')}_{m}$ are space-like separated, then:*

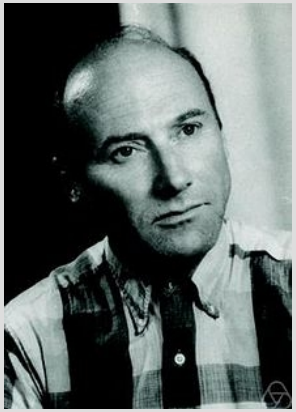
$$[\varphi^{(\kappa)}_{l}(f), \varphi^{(\kappa')}_{m}(g)]_{\pm} = \varphi^{(\kappa)}_{l}(f)\varphi^{(\kappa')}_{m}(g) \pm \varphi^{(\kappa')}_{m}(g)\varphi^{(\kappa)}_{l}(f) = 0$$

when applied to any state in $\mathcal{H}$, for any fields $\varphi^{(\kappa)}_{l}, \varphi^{(\kappa')}_{m}$.



A. Wightman

[R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and all that* (1964).]



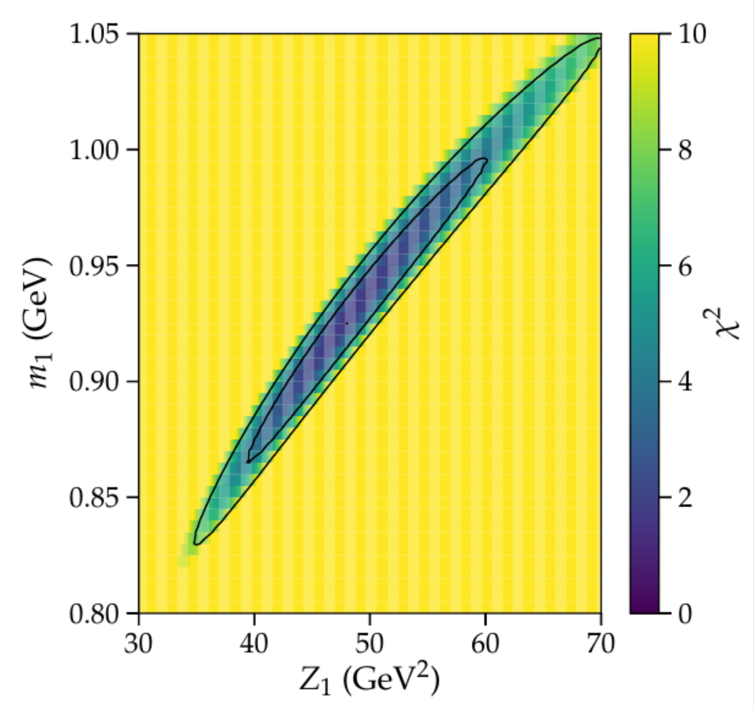
R. Haag

[R. Haag, *Local Quantum Physics*, Springer-Verlag (1996).]

Backup

Fit results for [Li, PL, Oliveira, Silva, 1907.10073]

- Performed fits using both 64^4 and 80^4 lattice data samples from [Dudal, Oliveira, Silva 1803.02281] to check for volume-dependent effects
- Found that single delta-derivative provided a good fit alone, and its inclusion in other fit ansätze resulted in an improvement of those fits



- ➔ Chi-squared map for the 80^4 $D_1(p)$ fit with statistical & polynomial shape uncertainty

Backup

Chi-squared definitions used in fits

(1) Statistical errors only:

$$\chi_1^2 = (\boldsymbol{G} - \boldsymbol{D})^{\mathrm{T}} \boldsymbol{C}^{-1} (\boldsymbol{G} - \boldsymbol{D}),$$

(2) Statistical + shape systematics:

$$f_i(p_i, a, b) = \frac{a + p_i^2}{ab + p_i^2},$$

$$\chi_2^2 = (\boldsymbol{G} \cdot \boldsymbol{f} - \boldsymbol{D})^T \boldsymbol{C}^{-1} (\boldsymbol{G} \cdot \boldsymbol{f} - \boldsymbol{D}) + \frac{(a - \alpha)^2}{\sigma_a^2} + \frac{(b - \beta)^2}{\sigma_b^2},$$

(3) Statistical + poly. shape:

$$\chi_3^2 = (\boldsymbol{G} \cdot \boldsymbol{g} - \boldsymbol{D})^T \boldsymbol{C}^{-1} (\boldsymbol{G} \cdot \boldsymbol{g} - \boldsymbol{D}) + \frac{|\boldsymbol{g} - \gamma|^2}{\sigma_g^2},$$