Lifshitz Regime and the phase diagram of QCD

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QCD at finite density

Large $\mu$ : 1st order phase transition

$\mu = 0$ : crossover at $T = 154$ MeV

Meet at a critical end-point: 2nd order

First-principle lattice: prohibitively challenging due to sign problem
Signs of CEP?

**Lattice:** small \( \mu \), Taylor expansion & estimate radius of convergence, Cluster expansion

**No signs of CEP from lattice so far**

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Hot QCD  
Arxiv: 1701.04325

Vovchenko, Steinheimer, Philipsen, Stoecker  
Arxiv: 1701.04325
Lifshitz regime: alternative scenario?

- Quark-Gluon Plasma
- Lifshitz regime
- $T_0$
- $T$ \uparrow
- Hadronic
- $\mu$ \rightarrow
- Crossover
- 1st order line
- Chiral spirals
- Quark matter
O(N) effective model

\[ \mathcal{L} = \frac{1}{2} (\partial_0 \phi)^2 + \frac{1}{2M^2} (\partial_i \phi)^2 + \frac{Z}{2} (\partial_i \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} (\phi^2)^2 + \ldots \]

Implicitly assume finite density

\[ Z \text{ is allowed to be negative:} \]

Gap closes as \( Z \) gets sufficiently negative:

Local symmetry breaking!

\[ \phi(x) = \phi_0 (\cos(k_0 z), \sin(k_0 z)) \]

Chiral spiral

Anisotropic fluctuations

\[ \Delta^{-1} = m_{\text{eff}}^2 - 2Z(k_z - k_0)^2 + \frac{1}{M^2} (4k_0 k_z \vec{k}^2 + (\vec{k}^2)^2) \]

A’la roton condensation in superfluid?
O(N) effective model

Mean-field phase diagram

Lifshitz pt

\[ \langle \phi \rangle \neq 0 \quad \langle \phi \rangle = 0 \]

\[ \langle \phi \rangle_{CS} \neq 0 \]
Effects of quantum fluctuations: dramatic change
No Lifshitz point, but Lifshitz regime

\[ \langle \phi(x)\phi(0) \rangle \sim e^{-m_0 x} \]

\[ \langle \phi(x)\phi(0) \rangle \sim e^{-m_0 x} \cos(k_0 x) \]

\[ \Delta m^2 \sim \lambda \int d^2k_\perp dk_z \frac{1}{(k_z - k_0)^2 + m_{\text{eff}}^2 + \ldots} \sim \lambda \int_M d^2k_\perp \int_{m_{\text{eff}}}^M dk_z \frac{1}{(k_z - k_0)^2} \sim \lambda \frac{M}{m_{\text{eff}}} \]
Fate of the Chiral Spiral

*Brazovsky-type* phase transition: universal, appears due to anisotropic fluctuations

Effective dimensional reduction to 1-*dim*

\[ \phi(z) \]

*Phonon fluctuations!*

Long-range order is destroyed

*Lattice: detection of such phase is challenging*
Real-world example: inhomogeneous polymers

Mixture of polymers A & B:

- homopolymer: $A^8B^8$
- AB diblock copolymer: (co-AB)

Experiment vs. Self-consistent field theory
Relation to QCD

Fermi surface

\[ \Lambda_s \]
\[ \mu \]

\[ \alpha_s(\mu) \ll 1 \]

Quarkionyc matter: effectively 1+1-dim patches (few are shown)

Suggestive argument in 1+1 dim: \( \mu \) can be eliminated in the expense of Chiral Spiral:

\[ \bar{q}q = \cos(2\mu z)\bar{q}q + i\sin(2\mu z)\bar{q}\gamma_5 q \]

Quasi-long range order due to phonons: R. Pisarski, V. Skokov, A. Tsvelik, Arxiv:1801.08156
Conclusions

1) *Lifshitz regime instead of Lifshitz point (contrary to NJL models)*

2) *Non-perturbative generation of negative Z is essential*

3) *Relation to standard CEP is unclear: both can co-exist*

4) *Lattice SU(2) “QCD” simulation?*