Topological Objects in Holographic QCD

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Abstract: We study topological objects in holographic QCD in two-flavor case. The holographic QCD is constructed with D4 and D8branes in the superstring theory, and is equivalent to 1+3 dim infrared QCD. The holographic QCD is described as 1+4 dim non-abelian gauge theory in flavor space with a gravitational background, and its instanton solution corresponds to baryons.

First, using Witten Ansatz, we present a formalism to describe instantons in the curved space in holographic QCD. Second, we investigate merons, and propose a new-type baryon excitation of two-merons oscillation in holographic QCD.

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Introduction ~ Holographic QCD



Emergence of Gauge Theory on D-brane,

Superstring theory is formulated in 10 dimensional space-time, and has D_p -brane as a (p+1)dimensional soliton-like object of strings.

On N_c D-brane, U(N_c) Gauge Theory is constructed, where open string behaves as U(N_c) gauge field.

Around N_c D-brane, Supergravity field is formed, because D-brane is massive and is the source of gravity field.





Construction of Non-SUSY SU(*N*) gauge theory

Similar to Thermal SUSY breaking, Supersymmetry can be removed by S¹-compactification with normal boundary condition (periodic for bosons, anti-periodic for fermions). [E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998).]



Holographic QCD for 1+3 dim QCD

T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005).

Using D4/D8/D8-branes, massless 1+3 dim QCD can be constructed. Here, N_c D4-brane gives Color and Gluons and D8-brane gives Flavor. Left (Right) Quarks appear at the cross point between D4 and D8 (D8bar).



In Large N_c limit, N_c D4-brane is extremely massive and is replaced by Gravitational background, via gauge/gravity correspondence.

> D4-brane is replaced by Gravitational Background



Low-energy effective theory of the D8 brane becomes Dirac-Born-Infeld (DBI) action

 $+H(u)^{1/2}(du^{2} / f(u) + u^{2}d\Omega_{4}^{2})$ $f(u) = 1 - (u_{0} / u)^{3}$



1+4 dim SU(N_f) Gauge theory in Flavor space T. Sakai, S. Sugimoto, PTP 113, 843 (2005).

$$S = \kappa \int d^4 x \, \underline{dw} \, \mathrm{Tr} \left(-h(w) \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + k(w) F_{\mu\omega} F^{\mu\nu} \right)$$

Gravitational background $h(w) \equiv (1 + w^2)^{-1/3}$ $k(w) \equiv (1 + w^2)$

W: extra spatial dimension

Holographic QCD for 1+3 dim QCD

Thus, Infrared effective theory of 1+3 dim QCD is *derived* as 1+4 dim classical Gauge theory in Flavor space with Gravitational background (Holographic QCD).

T. Sakai and S. Sugimoto, PTP 113, 843 (2005).

Final Form in leading order of $1/N_c$ and $1/\lambda$: 1+4 dim SU(N_f) Gauge Theory in Flavor Space

$$S = \kappa \int d^4 x \, dw \, \mathrm{Tr} \left(-h(w) \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + k(w) F_{\mu\omega} F^{\mu\nu} \right)$$

W: extra spatial dimension

Gravitational background $h(w) \equiv (1 + w^2)^{-1/3}$ $k(w) \equiv (1 + w^2)$

 $M_{\rm KK}=1$ unit

K : overall constantwith mass dimension

This Holographic QCD describes hadron properties, within 30% error, and is *successful* to explain many phenomenological laws in hadron physics: Vector-Meson Dominance (VMD), Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation, Gell-Mann-Sharp-Wagner (GSW) relation, Hidden Local-Symmetry (HLS) picture, Skyrme Chiral Soliton picture, etc.

Baryons in Holographic QCD



Baryon in Holographic QCD

Holographic QCD (in leading order of $1/N_c$ and $1/\lambda$)

 \rightarrow 1+4 dim SU(N_f) YM theory in Flavor Space with Gravitational Background

$$S = -\kappa \int d^4 x \, dw \operatorname{Tr}\left(h(w) \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + k(w) F_{\mu\nu} F^{\mu\nu}\right)$$

w : extra spatial dimension

 $h(w) \equiv (1 + w^2)^{-1/3}$ $k(w) \equiv (1 + w^2)$

 $M_{\rm KK}=1$ unit

 κ : overall constant with mass dimension



Note that Holographic QCD already includes 4 dim spatial coordinates including *extra dimension w*. In fact, *instanton* can be naturally introduced in Holographic QCD without necessity of Euclidean process or Wick rotation.

Baryon from Instanton in Flavor space

Holographic QCD:

1+4 dim SU(N_f) YM-like theory in Flavor Space with Gravitational Background

$$S = -\kappa \int d^4 x \, dw \operatorname{Tr}\left(h(w) \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + k(w) F_{\mu\nu} F^{\mu\nu}\right)$$

$$h(w) \equiv (1 + w^2)^{-1/3}$$

 $k(w) \equiv (1 + w^2)$

W: extra spatial dimension

Holographic QCD has Instanton as Topological Soliton in spatial coordinates (x, y, z, w) $\Pi_3(SU(N_f))=Z$

The Instanton gives Baryon as Topological Chiral Soliton in 1+3 dim τ_2

 $\Pi_3(\mathrm{SU}(N_{\rm f})_{\rm L} \times \mathrm{SU}(N_{\rm f})_{\rm R} / \mathrm{SU}(N_{\rm f})_{\rm V}) = Z$

H. Hata, T. Sakai, S. Sugimoto, and S. Yamato, PTP117,1157(2007).

$$B_{\mu} = \frac{1}{24\pi^2 i} \varepsilon_{\mu\alpha\beta\gamma} \mathrm{Tr}(L^{\alpha}L^{\beta}L^{\gamma})$$

Baryon as Topological Chiral Soliton

K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D75, 086003 (2007).

Holographic QCD eventually becomes 1+3 dim Skyrme model including (axial) vector mesons, and Baryon eventually becomes Topological Hedgehog Soliton. In previous work, we performed first holographic study of Baryon for N_f =2.

- Hedgehog soliton with
$$B = 1$$

T.H.R. Skyrme, Proc. R. Soc. A260, 127 (1961).
 $U(\mathbf{x},t) = e^{i\tau^a \pi^a(\mathbf{x},t)}$ $U^{HH}(\vec{x}) = e^{i\tau^a \hat{x}^a F(r)}$ $\Pi_3 (SU(N_f)_A) = Z$
 $\rho_\mu(\mathbf{x},t)$ $\rho_0^{HH}(\vec{x}) = 0$, $\rho_i^{HH}(\vec{x}) = \rho_{ia}^{HH}(\vec{x})\tau_a = \left\{ \varepsilon_{iab} \hat{x}_b \tilde{G}(r) \right\} \tau_a$

For baryon, pion profile function F(r) has topological boundary condition:

Boundary condition
$$\begin{cases} F(0) = \pi \\ F(\infty) = 0 \end{cases} \qquad B = \frac{F(0) - F(\infty)}{\pi} = 1$$

Baryon number current (Goldstone-Wilczek current)

$$B_{\mu} = \frac{1}{24\pi^2 i} \varepsilon_{\mu\alpha\beta\gamma} \mathrm{Tr}(L^{\alpha}L^{\beta}L^{\gamma})$$



Baryon as Chiral Soliton

K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D75, 086003 (2007).

In the first Holographic study of Baryon, we investigated baryon mass, radius, structure and so on.



In our previous work, we have included Gravitational Background precisely, but considered Only Light-mesons (pion and ρ -meson) contribution.

Baryon from Instanton in Holographic QCD

H. Hata, T. Sakai, S. Sugimoto, and S. Yamato, PTP117,1157(2007).

Hata et al. described Baryon from Instanton in Holographic QCD for $N_f=2$, but they neglected Gravitational Background h(w) and k(w), because of the difficulty to deal with *Instanton in the Curved Space*. Π_3 (SU(N_f))=Z

$$S = -\kappa \int d^4 x \, dw \operatorname{Tr}\left(h(w) \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + k(w) F_{\mu\nu} F^{\mu\nu}\right)$$

W : extra spatial dimension

$$h(w) \equiv (1 + w^2)^{-1/3}$$

 $k(w) \equiv (1 + w^2)$

 $M_{KK} = 1$ unit K: overall constant with mass dimension

We investigate *Baryon from Instanton* in Holographic QCD including *Gravitational Background* h(w) and k(w) for N_f =2. To this end, we present *Formalism* to deal with *Instanton in the Curved space* by way of *Witten Ansatz.*



$$S_{HQCD} = -\kappa \int d^4 x \, dw \operatorname{Tr}\left(h(w) \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + k(w) F_{\mu\nu} F^{\mu\nu}\right)$$

W: extra spatial dimension

$$h(w) \equiv (1 + w^2)^{-1/3}$$

 $k(w) \equiv (1 + w^2)$

$$M_{KK} = 1$$
 unit

K : overall constantwith mass dimension

Our strategy is to use *Witten Ansatz for Instanton* in the framework of 1+4 dim Holographic QCD with

Gravitational Background h(w) and k(w) for $N_f=2$.

Here, *Witten Ansatz* respects *SO(3)* spatial symmetry, and it is useful to describe spatial Hedgehog structure of Soliton.



Witten Ansatz for Instanton in 4 dim SU(2) YM theory

Ordinary Witten Ansatz for Instanton in 4 dim SU(2) YM theory:

$$\begin{bmatrix} A_0^a(x^{\mu}) = a_0(t,r)\hat{x}_a \\ A_i^a(x^{\mu}) = \frac{\phi_2(t,r) + 1}{r} \varepsilon_{iak} \hat{x}_k + \frac{\phi_1(t,r)}{r} \hat{\delta}_{ia} + a_1(t,r)\hat{x}_i \hat{x}_a \\ r \equiv (x_i x_i)^{1/2} \qquad \hat{x}_i \equiv \frac{x_i}{r} \qquad \hat{\delta}_{ij} \equiv \delta_{ij} - \hat{x}_i \hat{x}_j \end{bmatrix}$$
E. Witten, PRL38, 121 (1977).

With Witten Ansatz, 4 dim SU(2) YM theory is reduced into 2 dim Abelian Higgs theory on a curved space with $g^{\mu\nu} = r^2 \delta^{\mu\nu}$

$$S_{YM} = \int d^4 x \frac{1}{2} \operatorname{Tr} F_{\mu\nu} F_{\mu\nu} = 4\pi \int_{-\infty}^{\infty} dt \int_{0}^{\infty} dr \left[\left| D_0 \phi \right|^2 + \left| D_1 \phi \right|^2 + \frac{1}{2r^2} (1 - \left| \phi \right|^2)^2 + \frac{r^2}{2} f_{01}^2 \right]$$

$$\begin{split} \phi(t,r) &\equiv \phi_1 + i\phi_2 \in \mathbf{C} \qquad a_{\mu}(t,r) \in \mathbf{R} \\ D_{\mu} &\equiv \partial_{\mu} - ia_{\mu} \qquad f_{\mu\nu} \equiv \partial_{\mu}a_{\nu} - \partial_{\nu}a_{\mu} \qquad \mu, \nu \in \{0,1\} = \{t,r\} \end{split}$$

$$S_{HQCD} = -\kappa \int d^4 x \, dw \, \mathrm{Tr} \left(h(w) \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + k(w) F_{\mu\nu} F^{\mu\nu} \right)$$

W: extra spatial dimension

$$h(w) \equiv (1 + w^2)^{-1/3}$$

 $k(w) \equiv (1 + w^2)$

 $M_{\rm KK}=1$ unit

K : overall constantwith mass dimension

For 1+4 dim $SU(2)_f$ Holographic QCD, we generalize Witten Ansatz :

$$\begin{cases}
A_{0}^{a}(x^{\mu}) = a_{0}(t, r, w)\hat{x}_{a} \\
A_{i}^{a}(x^{\mu}) = \frac{\phi_{2}(t, r, w) + 1}{r} \varepsilon_{iak}\hat{x}_{k} + \frac{\phi_{1}(t, r, w)}{r} \hat{\delta}_{ia} + a_{1}(t, r, w)\hat{x}_{i}\hat{x}_{a} \\
A_{w}^{a}(x^{\mu}) = a_{w}(t, r, w)\hat{x}_{a} \\
r \equiv (x_{i}x_{i})^{1/2} \quad \hat{x}_{i} \equiv \frac{x_{i}}{r} \quad \hat{\delta}_{ij} \equiv \delta_{ij} - \hat{x}_{i}\hat{x}_{j} \\
\phi(t, r, w) \equiv \phi_{1} + i\phi_{2} \in \mathbf{C} \\
a_{\mu}(t, r, w) \in \mathbf{R}$$

$$\mu, v \in \{0, 1, 2\} = \{t, r, w\}$$

By way of the generalized Witten Ansatz, 1+4 dim SU(2)_f Holographic QCD is reduced into 1+2 dim Abelian Higgs theory on a curved space.

Gravitational Background

$$h(w) \equiv (1 + w^2)^{-1/3}$$

 $k(w) \equiv (1 + w^2)$

W: extra spatial dimension

$$S_{HQCD} = -\kappa \int d^4 x \, dw \, \mathrm{Tr} \left(h(w) \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + k(w) F_{\mu\nu} F^{\mu\nu} \right)$$

= $4\pi \kappa \int_{-\infty}^{\infty} dt \int_{0}^{\infty} dr \int_{-\infty}^{\infty} dw \left\{ h(w) \left(\left| D_0 \phi \right|^2 - \left| D_1 \phi \right|^2 \right) - k(w) \left| D_2 \phi \right|^2 - \frac{h(w)}{2r^2} (1 - \left| \phi \right|^2)^2 + \frac{r^2}{2} \left[h(w) f_{01}^2 + k(w) f_{02}^2 - k(w) f_{12}^2 \right] \right\}$

$$\begin{split} \phi(t,r,w) &\equiv \phi_1 + i\phi_2 \in \mathbf{C} \qquad a_{\mu}(t,r,w) \in \mathbf{R} \\ D_{\mu} &\equiv \partial_{\mu} - ia_{\mu} \qquad f_{\mu\nu} \equiv \partial_{\mu}a_{\nu} - \partial_{\nu}a_{\mu} \end{split} \qquad \mu,\nu \in \{0,1,2\} = \{t,r,w\} \end{split}$$

Thus, using the generalized Witten Ansatz, Instanton in 1+4 dim SU(2)_f Holographic QCD is described as Vortex Soliton in Gravitational Background

$$h(w) \equiv (1 + w^2)^{-1/3}$$

 $k(w) \equiv (1 + w^2)$

1+2 dim Abelian Higgs theory on a curved space. W: extra spatial dimension

The Static Soliton Energy in temporal gauge $a_0(t, r, w) \equiv 0$ is derived as that of Two-dimensional Soliton (Abrikosov Vortex):

$$E = 4\pi\kappa \int_0^\infty dr \int_{-\infty}^\infty dw \left\{ h(w) \left| D_1 \phi \right|^2 + k(w) \left| D_2 \phi \right|^2 + \frac{h(w)}{2r^2} (1 - \left| \phi \right|^2)^2 + \frac{r^2}{2} k(w) f_{12}^2 \right\}$$

Static variables:

$$\begin{split} \phi(r,w) &\equiv \phi_1 + i\phi_2 \in \mathbf{C} \qquad a_{\mu}(r,w) \in \mathbf{R} \\ D_{\mu} &\equiv \partial_{\mu} - ia_{\mu} \qquad f_{\mu\nu} \equiv \partial_{\mu}a_{\nu} - \partial_{\nu}a_{\mu} \qquad \mu, \nu \in \{1,2\} = \{r,w\} \end{split}$$

Various Topological Description of Baryons in QCD

To summarize, there are

various topological descriptions for baryons in QCD:



Meron and Meron-pair in Holographic QCD



Meron is a curious solution of YM theory, because it is zero-size "half-instanton" with Q=1/2 and has an infinite action.

C.G. Callan, R. Dashen, D.J. Gross, PLB66 (1977) 375, PRD19 (1979) 1826.

However, a recent study shows that meron has finite action in some gravitational background. F. Canfora et al., arXiv:1812.11231

Then, we investigate Meron in Holographic QCD, where gravitational background exists.

Meron Solution in YM theory



Meron Solution





'tHooft symbol

$$\eta^a_{\mu\nu} \equiv \varepsilon_{a\mu\nu4} + \delta_{a\mu}\delta_{\nu4} - \delta_{a\nu}\delta_{\mu4}$$

$$Q = \frac{1}{16\pi^2} \int d^4 x \, \text{Tr}(G_{\mu\nu} \tilde{G}_{\mu\nu}) = \frac{1}{2}$$

$$S_L \equiv \frac{1}{2g^2} \int^L d^4 x \operatorname{Tr}(G_{\mu\nu}G_{\mu\nu}) \propto \ln L$$

Meron action is divergent, but its divergence is logarithmic and weak. Then, its divergence or finiteness is nontrivial in the presence of gravitational background. Meron Configuration in Holographic QCD

We investigate a smeared-type Meron configuration in spatial coordinates (x,y,z,w) in Holographic QCD.

Smeared-type Meron configuration



spatial coordinates

cf instanton solution $A^{\mu}_{a} = \eta^{a}_{\mu\nu} \frac{x^{\nu}}{x^{2} + a^{2}}$

As the result,

the "Magnetic energy" of Meron is Finite, owing to the reduction gravitational factor h(w), but the "Electric energy" of Meron is badly divergent, due to the increasing gravitational factor k(w).

In fact, *No half-integer baryon* apprears in Holographic QCD.

Too natural conclusion

Meron-pair Configuration in Holographic QCD

Finally, we investigate a Two-Meron configuration in spatial coordinates (x,y,z,w) in Holographic QCD.

Smeared-type Two-Meron configuration

$$A_{a}^{\mu} = \frac{1}{2} \eta_{\mu\nu}^{a} \left(\frac{(x - x_{1})^{\nu}}{(x - x_{1})^{2} + a^{2}} + \frac{(x - x_{2})^{\nu}}{(x - x_{2})^{2} + a^{2}} \right)$$

5th coordinate *w* plays a role of Euclidean time

$$S_{HQCD} = -\kappa \int d^4 x \, dw \, \mathrm{Tr}\left(\underline{h(w)} \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \underline{k(w)} F_{\mu\nu} F^{\mu\nu}\right)$$

"magnetic" part "electric" part

As the result,

The energy of this meron-pair is Finite.

Gravitational Background

х, у, Z

$$h(w) \equiv (1 + w^2)^{-1/3}$$

 $k(w) \equiv (1 + w^2)$

 $M_{\rm KK} = 1$ unit

W

K : overall constantwith mass dimension

Meron-pair Configuration in Holographic QCD

Finally, we investigate a Two-Meron configuration in spatial coordinates (x,y,z,w) in Holographic QCD.

Next, we consider time-dependent Meron-pair oscillating in 5th direction w

$$A_{a}^{\mu} = \frac{1}{2} \eta_{\mu\nu}^{a} \left(\frac{(x - l(t)\hat{w})^{\nu}}{(x - l(t)\hat{w})^{2} + a^{2}} + \frac{(x + l(t)\hat{w})^{\nu}}{(x + l(t)\hat{w})^{2} + a^{2}} \right)$$

The energy of this oscillating meron-pair is Finite. Note that this oscillation is in the 5th direction, which is independent of ordinary coordinates.

Therefore, as an interesting possibility, this type of oscillating excitation is expected to appear for any baryon universally.

A rough estimate of the this meron-pair energy is about 500MeV for a = 0.5fm. This may correspond to the Roper resonance N*(1440).

spatial coordinates

$$\mu = x, y, z, w$$

$$\psi$$

$$+l(t)$$

$$x, y, z$$

$$\Delta E \sim \left(\frac{16\pi^2 \kappa}{a^2}\right)^{1/3} \sim 500 \,\mathrm{MeV}$$

Summary and Conclusion

We have studied topological objects in holographic QCD in two-flavor.

First, using Witten Ansatz, we have present a formalism to describe instantons in the curved space in holographic QCD.

Second, we have investigated merons, and propose a new-type universal baryon excitation of two-merons oscillation in holographic QCD.



Thank you !

