Non-Abelian Proca-Dirac-Higgs theory: particle-like solutions, their energy spectrum and Mass Gap

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Outline

Main goals:

- Particle-like solutions, energy spectrum and mass gap in non-Abelian Proca-Dirac-Higgs theory.
- Understanding of the mechanism of a mass gap in PDH theory.
- Hypothesis of a similar mechanism of mass gap in QCD: nonlinear Dirac equation describes an interaction between sea quarks and gluons.
- Applications for quark - gluon plasma in QCD.
- Conclusions

Lagrangian

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\mu^2}{2}A^a_\mu A^{a\mu} +$$

$$\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda_2}{4}\left(\phi^2 - M^2\right)^2 + \frac{\lambda_1}{2}\phi^2 A^a_\mu A^{a\mu}$$

$$+i\hbar c\bar{\psi}\gamma^\mu\left(\partial_\mu - i\frac{g}{2}\sigma^a A^a_\mu\right)\psi - m_f c^2\bar{\psi}\psi + \frac{\Lambda}{2}g\hbar c\phi\left(\bar{\psi}\psi\right)^2$$

Field equations

The corresponding field equations are as follows:

$$D_\nu F^{a\mu\nu} - \left(\lambda_1 \phi^2 - \mu^2\right) A^{a\mu} = \frac{g\hbar c}{2} \bar{\psi}\gamma^\mu \sigma^a \psi,$$

$$\Box\phi - \lambda_1 A^a_\mu A^{a\mu}\phi - \lambda_2 \phi \left(M^2 - \phi^2\right) = \frac{\Lambda}{2} g\hbar \left(\bar{\psi}\psi\right)^2,$$

$$i\hbar\gamma^\mu \left(\partial_\mu - i\frac{g}{2}\sigma^a A^a_\mu\right)\psi + \Lambda g\hbar\phi\psi\left(\bar{\psi}\psi\right) - m_f c\psi = 0.$$

Ansätze

$$
\begin{aligned}
A_i^a &= -\frac{1}{g}\left[1-f(r)\right]\begin{pmatrix} 0 & \sin\varphi & \sin\theta\cos\theta\cos\varphi \\ 0 & \cos\varphi & -\sin\theta\cos\theta\sin\varphi \\ 0 & 0 & -\sin^2\theta \end{pmatrix}, \\
A_t^a &= 0, \\
\phi &= \frac{\xi(r)}{g}, \\
\psi^T &= \frac{e^{-i\frac{Et}{\hbar}}}{gr\sqrt{2}}\left\{\begin{pmatrix} 0 \\ -u \end{pmatrix}, \begin{pmatrix} u \\ 0 \end{pmatrix}, \begin{pmatrix} iv\sin\theta e^{-i\varphi} \\ -iv\cos\theta \end{pmatrix}, \begin{pmatrix} -iv\cos\theta \\ -iv\sin\theta e^{i\varphi} \end{pmatrix}\right\},
\end{aligned}
$$

Equations

Thus we obtain the following equations:

$$-f'' + \frac{f\left(f^2-1\right)}{x^2} - m^2\left(1-f\right)\tilde{\xi}^2 + \tilde{g}^2\,\frac{\tilde{u}\tilde{v}}{x} = -\tilde{\mu}^2\left(1-f\right),$$

$$\tilde{\xi}'' + \frac{2}{x}\tilde{\xi}' = \tilde{\xi}\left[\frac{\left(1-f\right)^2}{2x^2} + \tilde{\lambda}\left(\tilde{\xi}^2 - \tilde{M}^2\right)\right] - \frac{\tilde{g}^2\tilde{\Lambda}}{8}\frac{\left(\tilde{u}^2-\tilde{v}^2\right)^2}{x^4},$$

$$\tilde{v}' + \frac{f\,\tilde{v}}{x} = \tilde{u}\left(-\tilde{m}_f + \tilde{E} + m^2\tilde{\Lambda}\frac{\tilde{u}^2-\tilde{v}^2}{x^2}\tilde{\xi}\right),$$

$$\tilde{u}' - \frac{f\,\tilde{u}}{x} = \tilde{v}\left(-\tilde{m}_f - \tilde{E} + m^2\tilde{\Lambda}\frac{\tilde{u}^2-\tilde{v}^2}{x^2}\tilde{\xi}\right).$$

Energy

The total energy density of the particle-like solution:

$$\tilde{\varepsilon} = \tilde{\varepsilon}_{\mathrm{Pm}} + \tilde{\varepsilon}_s,$$

where the energy density of the spinor field:

$$\tilde{\varepsilon}_s = \tilde{E}\frac{\tilde{u}^2 + \tilde{v}^2}{x^2} + m^2\frac{\tilde{\Lambda}}{2}\tilde{\xi}\frac{\left(\tilde{u}^2 - \tilde{v}^2\right)^2}{x^4}.$$

$$\tilde{\varepsilon}_{\mathrm{Pm}} = \frac{1}{\tilde{g}^2}\left\{\left[\frac{f'^2}{x^2} + \frac{\left(f^2-1\right)^2}{2x^4} - \tilde{\mu}^2\frac{(f-1)^2}{x^2}\right] + 2m^2\left[\tilde{\xi}'^2 + \frac{(f-1)^2}{2x^2}\tilde{\xi}^2 + \frac{\tilde{\lambda}}{2}\left(\tilde{\xi}^2 - \tilde{M}^2\right)^2\right]\right\}$$

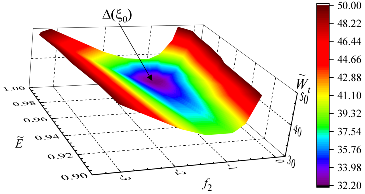
Parameters

$$
\begin{aligned}
f &= 1 + \frac{f_2}{2} x^2 + \ldots, \\
\tilde{\xi} &= \tilde{\xi}_0 + \frac{\tilde{\xi}_2}{2} x^2 + \ldots, \\
\psi^T &= \frac{e^{-i\frac{E}{\hbar}t}}{gr\sqrt{2}} \left\{ \begin{pmatrix} 0 \\ -u \end{pmatrix}, \begin{pmatrix} u \\ 0 \end{pmatrix}, \begin{pmatrix} iv \sin\theta e^{-i\varphi} \\ -iv\cos\theta \end{pmatrix}, \begin{pmatrix} -iv\cos\theta \\ -iv\sin\theta e^{i\varphi} \end{pmatrix} \right\},
\end{aligned}
$$

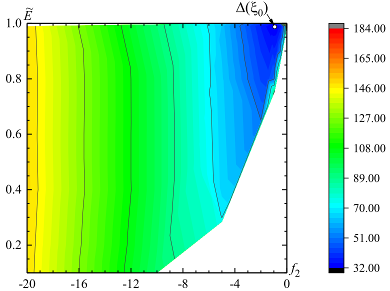
Parameters that affect the solution and energy:

$$
f_2, \quad \tilde{\xi}_0, \quad E
$$

Energy spectrum



(a) 3D plot of the energy.



(b) Contour plot of the energy.

Figure: 3D and contour plots of the energy $\tilde{W}_t$ for $\tilde{\xi}_0 = 0.5$ as a function of f_2 and $\tilde{E}$.

Energy profile has the shape of an inverted bell.

Mass gap mechanism

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\mu^2}{2}A^a_\mu A^{a\mu} +$$

$$\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda_2}{4}\left(\phi^2 - M^2\right)^2 + \frac{\lambda_1}{2}\phi^2 A^a_\mu A^{a\mu}$$

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Mass gap mechanism

The mechanism of the occurrence of a mass gap (for fixed ξ_0) in Proca-Dirac-Higgs theory: the nonlinear Dirac equation. Bearing this in mind, one can assume that a similar mechanism may also be responsible for the appearance of a mass gap in QCD. For this, however, one has to understand how the nonlinear Dirac equation may occur in QCD. This can happen as follows.

Yang - Mills and Mass Gap

Almost half a century ago, Yang and Mills introduced a remarkable new framework to describe elementary particles using structures that also occur in geometry. Quantum Yang-Mills theory is now the foundation of most of elementary particle theory, and its predictions have been tested at many experimental laboratories, but its mathematical foundation is still unclear. The successful use of Yang-Mills theory to describe the strong interactions of elementary particles depends on a subtle quantum mechanical property called the "mass gap": the quantum particles have positive masses bounded below by the mass gap value Δ. This property has been discovered by physicists from experiment and confirmed by computer simulations, but it still has not been understood from a theoretical point of view. Progress in establishing the existence of the Yang-Mills theory and a mass gap will require the introduction of fundamental new ideas both in physics and in mathematics.

Mass gap mechanism

In $SU(3)$ Lagrangian, the interaction between quarks and gluons is described by the term $\hat{\bar{\psi}}\lambda^B\hat{A}_\mu\hat{\psi}$, where $\hat{\psi} = \langle\hat{\psi}\rangle + \widehat{\delta\psi}, \hat{A}_\mu = \left\langle\hat{A}_\mu\right\rangle + \widehat{\delta A}_\mu$, $\langle\hat{\psi}\rangle, \left\langle\hat{A}_\mu\right\rangle$ are valence quarks and gluons, and $\widehat{\delta\psi}, \widehat{\delta A}_\mu$ are sea quarks and gluons. One can assume that the quantum average of the term $\left\langle\widehat{\delta\bar{\psi}}\lambda^B\widehat{\delta A}_\mu\widehat{\delta\psi}\right\rangle$ will *approximately* look like $\left\langle\widehat{\delta\bar{\psi}}\lambda^B\widehat{\delta A}_\mu\widehat{\delta\psi}\right\rangle \approx \phi\left(\bar{\xi}\xi\right)^2$, where the field ϕ *approximately* describes the sea gluons and the spinor field ξ *approximately* describes the sea quarks. Thus, in QCD, there can occur the nonlinear Dirac equation which *approximately* describes the interaction between sea quarks and gluons.

Mass gap mechanism for QCD

Probable mass gap mechanism for QCD

Summarizing, one can say that the following mechanism of the occurrence of a the nonperturbative interaction between sea quarks and gluons is approximately described by a nonlinear spinor field, the presence of which leads in turn to the occurrence of a mass gap in QCD.

Part 2: Application for quark - gluon plasma

The analogy between QCD and nonabelian Proca - Dirac - Higgs theory: application from plasma in Proca - Dirac - Higgs theory to quark - gluon plasma in QCD.
In Proca - Dirac - Higgs plasma there exist quasi-particles as above obtained particle-like solutions: Proca monopoles (hedgehogs), Proca monopoles + spinor field, spinballs (quasi-particles form non-linear spinor field).
We think that such plasma may mimic some properties of QGP in QCD.

Statistical integral

Kinetic energy of 1 non-relativistic quasi-particle is

$$W = W_q + \frac{p^2}{2m_q} = W_q + \frac{c^2 p^2}{2W_q},$$

where $m_q = W_q/c^2$.
To obtain statintegral we will use Van der Waals ideology that the particles move in the volume $V - V_q$:

$$V_q = \sum_i v_i \approx N\overline{v_q} = Vn_q\overline{v_q} = V\chi$$

where v_i is the volume of one quasi-particle.

Using some approximations we can write the statintegral as

$$Z(T) = Z_{\text{quarks + gluons}} \left[\int dV dp_x dp_y dp_z d\vec{\beta} e^{-\frac{W_q(\vec{\beta}) + \frac{c^2 p^2}{2W_q(\vec{\beta})}}{T}} \right]^N = Z_{\text{quarks + gluons}} Z_{\text{quasiparticles}}$$

After integration over coordinates we have

$$Z_{\text{quasiparticles}}(T) = Z_0 T^{3N/2} (V - V_q)^N \left[\int \tilde{W}_q^{3/2}(\vec{\beta}) e^{-\frac{\tilde{W}_q(\vec{\beta})}{\tilde{T}}} d\vec{\beta} \right]^N = Z_0 T^{3N/2} (V - V_q)^N (Z_q)^N = Z_0 T^{3N/2} V^N (1 - \chi)^N (Z_q)^N$$

where $\chi = n_q \overline{v_q}$ is the part of volume occupied by quasi-particles.

The internal energy and equation of state are

$$\frac{U_{\text{quasiparticle}}}{N} = -\frac{\partial \ln Z}{\partial \beta} = T^2 \frac{\partial \ln Z}{\partial T} = \frac{3}{2} T + T^2 \frac{\partial \ln(1-\chi)}{\partial T} + T^2 \frac{\partial \ln Z_q}{\partial T},$$

$$\frac{p_{\text{quasiparticle}}}{N} = -\frac{\partial F}{\partial V} = \frac{T}{V - V_q} = \frac{T}{V(1-\chi)} = \frac{T}{V\left(1 - n_q \overline{v_q}\right)}$$

In the first part we considered particle-like solutions. One of them is the solution with non-linear spinor field which probably describes the interaction between sea quarks and gluons. Corresponding non-linear Dirac equation is

$$i\hbar\gamma^{\mu}\partial_{\mu}\psi + \Lambda\psi(\bar{\psi}\psi) - m_f c\psi = 0.$$

The ansatz for the particle-like solution is

$$\psi^T = \frac{e^{-i\frac{\tilde{E}t}{\hbar}}}{r\sqrt{2}}\left\{\begin{pmatrix}0\\-u\end{pmatrix}, \begin{pmatrix}u\\0\end{pmatrix}, \begin{pmatrix}i\tilde{v}\sin\theta e^{-i\varphi}\\-iv\cos\theta\end{pmatrix}, \begin{pmatrix}-iv\cos\theta\\-iv\sin\theta e^{i\varphi}\end{pmatrix}\right\},$$

Corresponding equations are

$$\begin{aligned} v' + \frac{v}{x} &= u\left(-1 + E + \frac{u^2 - v^2}{x^2}\right), \\ u' - \frac{u}{x} &= v\left(-1 - E + \frac{u^2 - v^2}{x^2}\right) \end{aligned}$$

The energy density for spinball is given as

$$w_q = \frac{m_f^4 c^5}{\hbar^3 \tilde{\Lambda}^{1/2}} \left[E \frac{u^2 + v^2}{x^2} + \frac{1}{2} \left(\frac{u^2 - v^2}{x^2} \right)^2 \right] = \frac{m_f^4 c^3}{\hbar^3 \tilde{\Lambda}^{1/2}} \tilde{w}_q .$$

The energy of spinball

$$W_q(E) = 4\pi m_f c^2 \int_0^\infty x^2 \tilde{w}(E) dx = 4\pi m_f c^2 \tilde{W}_q$$

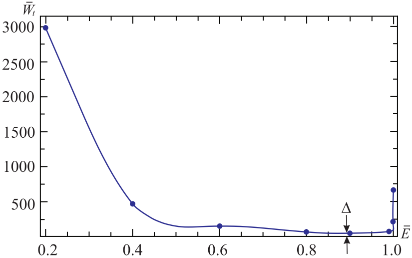

Figure: The dependence of the spinball total energy $\bar{W}_t$ on $\bar{E}$ and the location of the mass gap Δ.

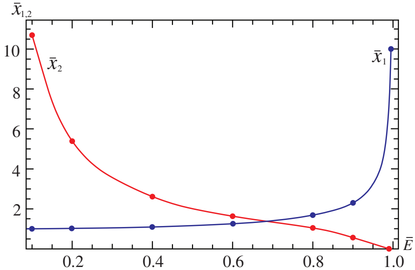

Figure: The dependence of the characteristic sizes $l_{1,2}$ of the spinball for the left- and right-hand branches of the energy spectrum shown in the left figure.

Spinball plasma

The equation of state, as compared to standard Van der Waals equation, is changed

$$p_q = kN\frac{T}{V - V_q} = kn_q\frac{T}{1 - n_q\overline{(v_q)}(T)} = kn_q f(T)$$

The internal energy that is connected with the internal structure of the quasi-particle is

$$\frac{U_q}{N} = T^2\frac{\partial \ln Z_q}{\partial T} + T^2\frac{\partial \ln(1-\chi)}{\partial T} = \frac{U_1}{N} + \frac{U_2}{N}.$$

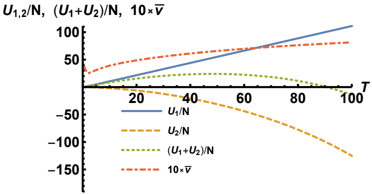

Figure: Profile of the internal energies $U_{1,2}(T)/N$, $[U_1(T)+U_2(T)]/N$ and averaged volume $\bar{v}(T)$ as the function on temperature for spinball plasma.

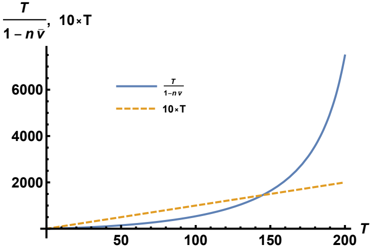

Figure: Comparison of plasma EoS for spinball plasma taking into account of an internal structure of quasi-particle and without this structure.

Conclusions

- Particle-like solutions of the type Proca-monopole-plus-spinball have been found and their energy spectra for some values of the parameter $\tilde{\xi}_0$ have been constructed.
- The main reason for a mass gap appearance in this theory is analyzed: non-linear Dirac equation.
- A mechanism for the appearance of the mass gap in QCD is proposed: The interaction between sea quarks and sea gluons can be approximately described by the non-linear Dirac equation.
- Thermodynamic calculations for quark-gluon plasma based on possible analogy with plasma in non-Abelian Proca - Dirac - Higgs theory are given.

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Thanks for your attention !