# VHE* Cross Sections

One of the oldest questions...

An Undertanding and a TEST

*VHE=Very High Energy

## pp cross sections:

$\downarrow LHC$ $\downarrow Auger$

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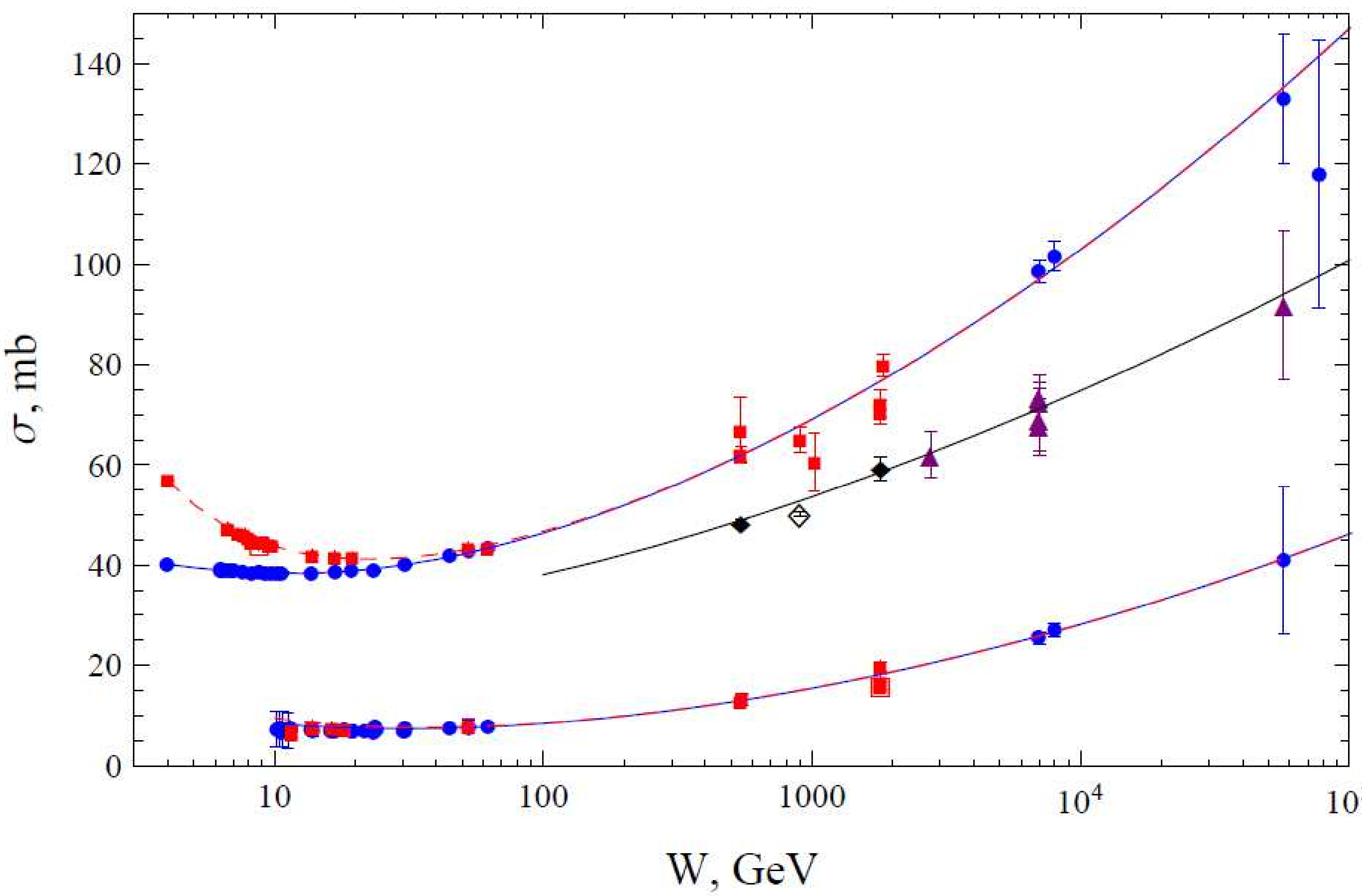



See anything simple here? (—There is! —)

Upper–total cross section

Middle –inelastic cross section

Lower –elastic cross section

Lines: Fits $c_2 ln^2 W + c_1 ln\, W + ...$

From Block, Halzen...[1]

What you're seeing is:

A 'Black Disc' $\sim ln^2 W$ (Heisenberg, 1952)[2]

with

a Smooth Edge $\sim constant$ (Us, 2015)[3]

The $\sim ln^2 W$

'Relativistic Rise' in ionization: An electron (charged particle) can interact further and further away due to relativistic boost of fields $\mathcal{E} \sim \gamma$.[4]

Similar for proton. But Coulomb $\rightarrow$ Yukawa.

$$\frac{1}{r} \rightarrow \frac{e^{-\mu r}}{r}$$

Maximum range of interaction $r_{max}$:

$$W^p \times e^{-\mu r_{max}} \geq threshold$$

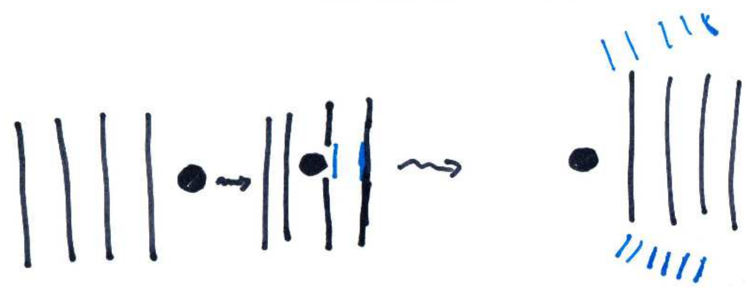
or $r_{max} = const. \times lnW$

then $\sigma \sim r_{max}^2 \sim ln^2 W + ...$

Froissart: $\sigma \sim ln^2 W$ fastest possible behavior (analyticity)

Lukaszuk-Martin: $\sigma \leq \frac{\pi}{m_\pi^2} ln^2 W$
(closest t-channel particle) [5]

The 'Black Disc'



Complete absorbtion creates 'hole' in the wavefront

Result $\sigma_{inelastic} = \sigma_{elastic}$

or

$$\sigma_{total} = 2 \times \sigma_{elastic}$$

Bloch and Halzen found in their fits [8]

$$\sigma_{total} = 1.1\, mb \times ln^2 W + ...$$

$$\sigma_{inelastic} = 0.56\, mb \times ln^2 W + ...$$

Looks like 'Black Disc' *asymptotically* !

But big non-leading terms—Asymptopia is still far far, away

# Puzzle

What is natural parameter for hadron physics?

Proton mass= 1 GeV?

$\Lambda_{QCD} = 0.2\,GeV$?

So why the devil do we need 10 000 GeV (LHC) or 100 000 GeV (Auger) to begin to see Asymptopia?

Is there a new mass scale hiding at VHE??

## The 'Edge' of the Proton

A truly hard 'disc' with sharp edge seems unphysical– expect some sort of smooth edge.

Impact parameter representation of scattering amplitude

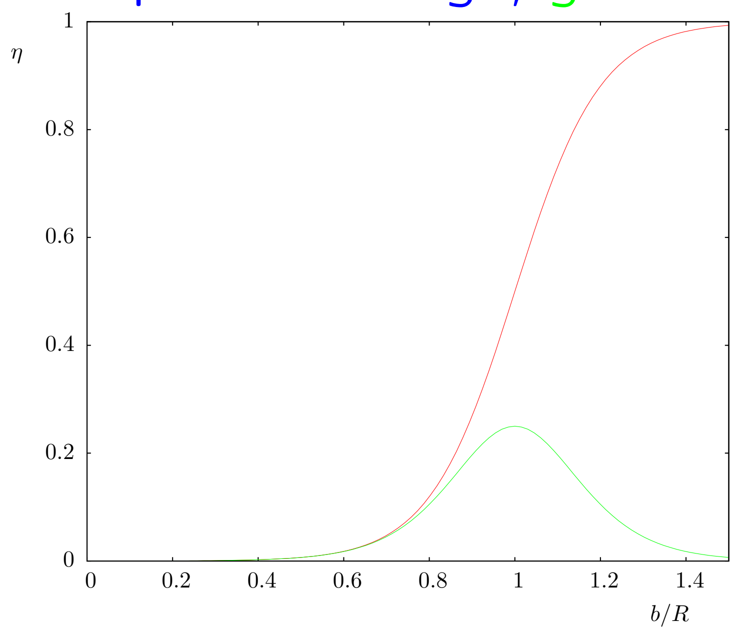
Let

$\eta(b)$ = "transparency" at impact parameter $b$.
$\sigma^{TOT}$ and $\sigma^{EL}$ are given by the same thing– $\eta$.
(Neglecting small Re part)

To isolate edge consider

$$\sigma^{TOT} - 2\sigma^{EL} = 4\pi \int_0^\infty \eta(1-\eta)\, b\, db$$

Integrand peaks at edge, green curve




Red curve=typical transparency

Normalizing to circumference

$$(\sigma^{TOT} - 2\sigma^{EL}) \frac{1}{\sqrt{(\pi/2)\sigma^{TOT}}} \approx \text{thickness}$$

## ——-Prediction——-

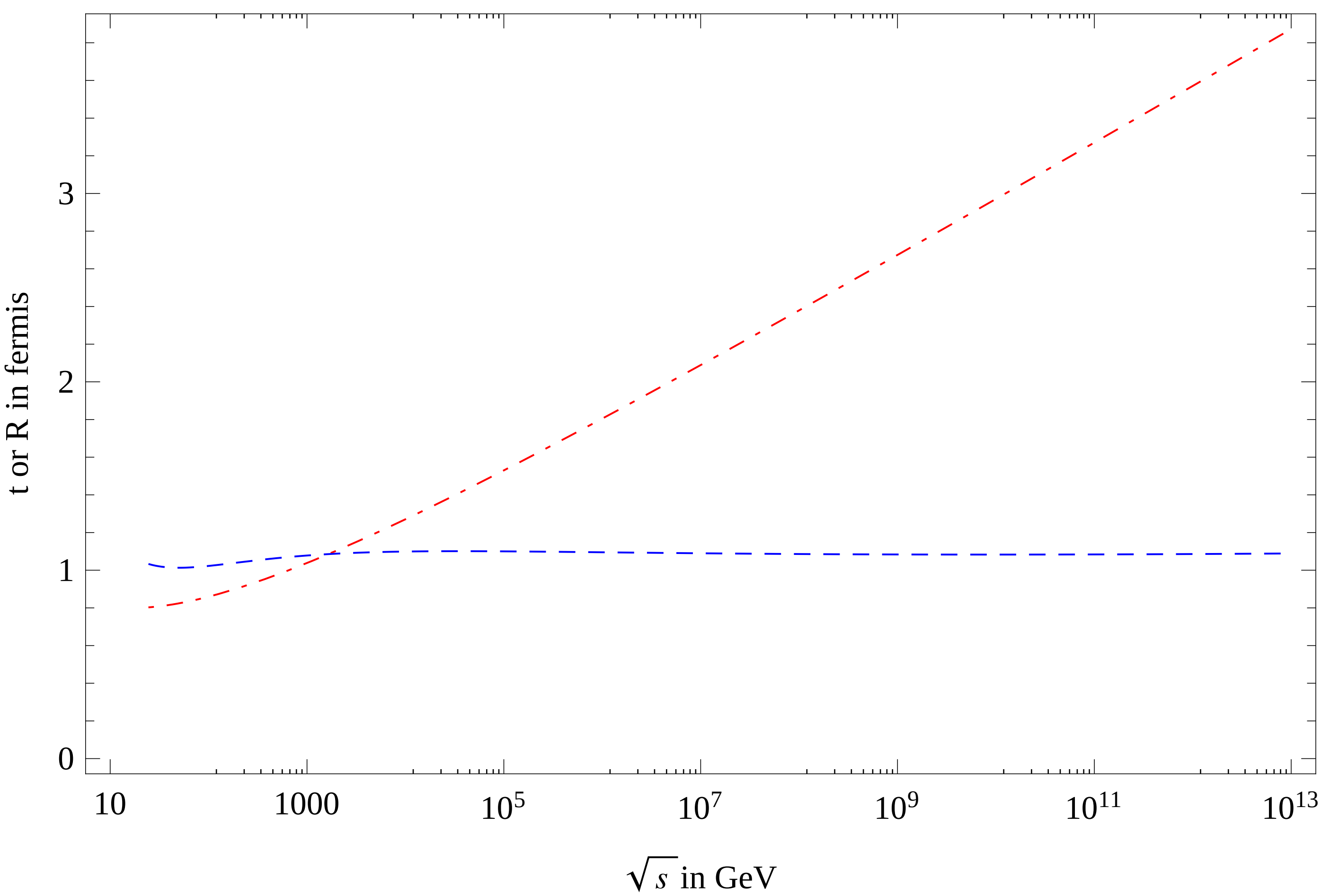
$$(\sigma^{TOT} - 2\sigma^{EL})\frac{1}{\sqrt{(\pi/2)\sigma^{TOT}}} \approx constant$$

and has ‘reasonable’ value

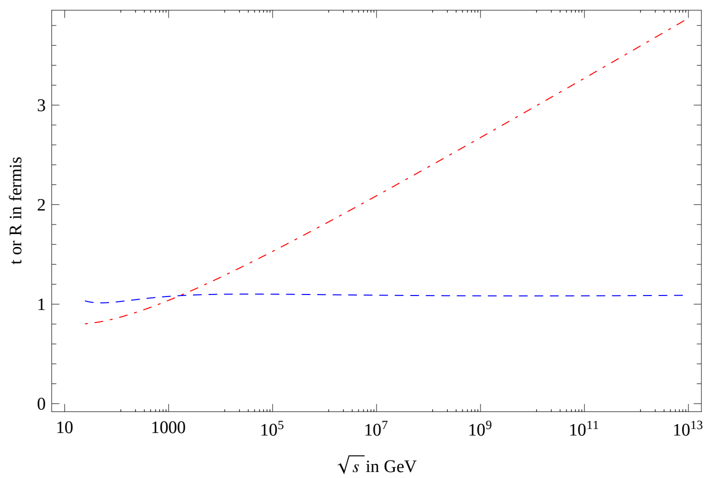
Plot this from fits [3]

Lo and behold! A constant edge with thickness 1 f!

Blue ='edge thickness',




Red= radius from $\sigma^{TOT} = 2\pi R^2$



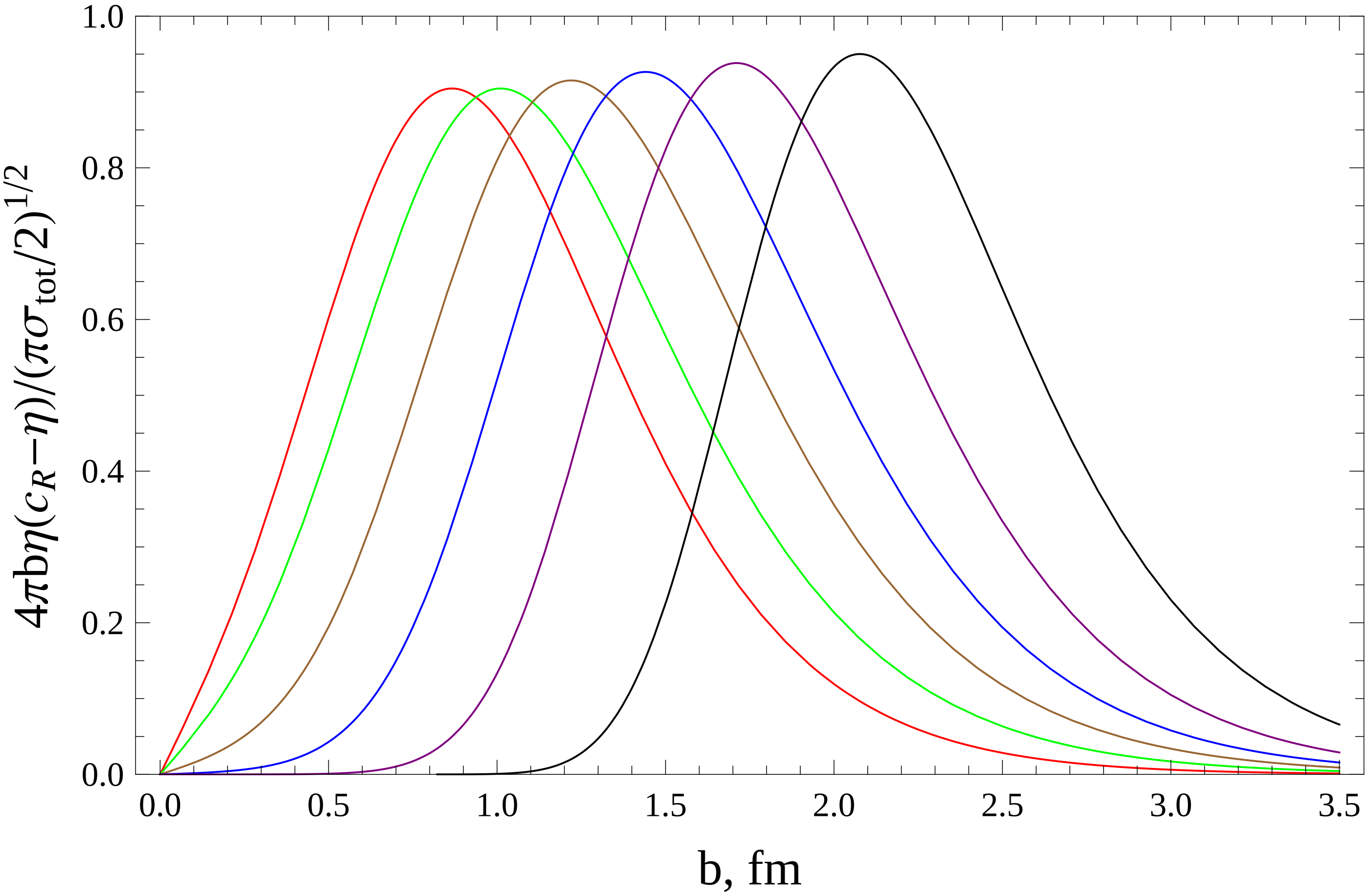

Answer to Puzzle.

'Edge' is quite big.

Asymptopia starts only above 1-10 TeV!

(Where $R_{disc} >> R_{edge} = thickness$)

Can also include information from shape of elastic diffraction peak [8]




'Edge' slowly moving out, remaining approx. constant

——————But——————

Why is edge so big/disc so small ?

Edge: $\pi(1f)^2 = 20mb$

Disc: $1.1mb \times ln^2 W$

What is 'edge' made of?

Attractive thought: pion exchange— $1/m_\pi \approx 1.4f$

Chiral symmetry says $m_\pi \approx 0$

So edge is big because pion is light!

Indeed $1.1mb \sim \pi/GeV^2 \sim \pi/m_p^2$

Edge— $\sim pion$

Disc— $\sim proton$

Suggests answer to another 'paradox'. What happens at VHE when $m_\pi \to 0$?

Lukaszuk- Martin-Froissart bound:

$$\sigma^{TOT} \leq (\pi/m_\pi^2) ln^2 W$$

Do cross sections blow up??

Our answer: As $m_\pi \to 0$ , edge grows– "Asymptopia" moves further away!!.

For final asymptoptic behavior $ln^2 W$ ok (but not coefficient)

# Challenge to Theory

The 1.1 mb is a simple, fundamental, parameter of strong interactions.

We now have $W >> any\ plausible\ scale$.

Please calculate (from Strings, Lattice QCD,...).

# Recently
# a New Thought[10]

Consider p-nucleus cross sections

Expanding proton radius should simply add to ordinary nuclear radius.

(Because nucleus is anyway 'black' to central collisions, only nucleons on outside matter)

Ordinary nucelar radius $R_A \approx 1.2 f A^{1/3}$

Add 'expanding proton'' $R_A^{vhe} \approx R_A + R_{pp}^{vhe}$

Cross section $\sigma = \pi(R_A^{vhe})^2 =$

$const. + 0.26\pi R_A f \times ln(W/m) + \sigma_{pp}^{disc} =$

$const. + 0.26\pi R_A f \times ln(W/m) + 0.55\, mb \times ln^2(W/m)$

Evaluate cross term

$= 0.26\pi\, R_A f \times ln(W/m) = 24\, mb \times ln(W/m)$

Surprisingly large!

# Tests

a) The cross sections on nuclei vary with energy approximately as $const. + ln(W/m)$

b) When comparing different nuclei the coefficient of the $ln(W/m)$ is linearly proportional to the radius of the nucleus

Cross term can double cross section on Nitrogen!

Influences interpretation of highest energy cosmic rays

True $W \gtrsim 10TeV$ –need p-nucleus collider to test.

Advantage of varying nucleus and so $R_A$

## REFERENCES


1. M. M. Block, L. Durand, P. Ha and F. Halzen, "Comprehensive fits to high energy data for $\sigma$, $\rho$, and $B$ and the asymptotic black-disk limit," Phys. Rev. D **92**, no. 11, 114021 (2015) doi:10.1103/PhysRevD.92.114021 [arXiv:1511.02406 [hep-ph]].

2. See H. G. Dosch, P. Gauron and B. Nicolescu, "Heisenberg's universal $\ln^2 s$ increase of total cross-sections", Phys. Rev. D **67**, 077501 (2003) [hep-ph/0206214].

3. M. M. Block, L. Durand, F. Halzen, L. Stodolsky and T. J. Weiler, "Evidence for an energy-invariant 'edge' in proton-proton scattering at very high energies," Phys. Rev. D **91**, no. 1, 011501 (2015) doi:10.1103/PhysRevD.91.011501 [arxiv:1409.3196 [hep-ph]].

4. See the discussion of $b_{max}$ in Chapt 2 of Fermi's *Nuclear Physics*, Univ. of Chicago Press, or in section 13 of J. D. Jackson, *Classical Electrodynamics*, John Wiley and Sons.

5. A. Martin, "The Froissart bound for inelastic cross-sections," Phys. Rev. D **80**, 065013 (2009) doi:10.1103/PhysRevD.80.065013 [arXiv:0904.3724 [hep-ph]].

6. L. Stodolsky, "Physical Basis For An Expanding High-energy Interaction Radius," SLAC-PUB-0864 (1971).

7. M. M. Block and F. Halzen, "New experimental evidence that the proton develops asymptotically into a black disk", Phys. Rev. D 86, 051504 (2012), [arXiv:1208.4086] .

8. M. M. Block, L. Durand, P. Ha and F. Halzen, "Eikonal fit to $pp$ and $\bar{p}p$ scattering and the edge in the scattering amplitude," Phys. Rev. D **92**, no. 1, 014030 (2015) doi:10.1103/PhysRevD.92.014030 [arXiv:1505.04842 [hep-ph]].

9. For a short review of this content see L. Stodolsky, “Behavior Of Very High Energy Hadronic Cross Sections,” Mod. Phys. Lett. A **32**, no. 31, 1730028 (2017) doi:10.1142/S0217732317300282 [arXiv:1703.05668 [hep-ph]].

10. For the p-nucleus idea see L. Stodolsky “Highest Energy Proton- Nucleus Cross Sections” L. Stodolsky, “Highest energy proton-nucleus cross-sections,” Mod. Phys. Lett. A **33**, no. 40, 1850242 (2019) doi:10.1142/S0217732318502425 [arXiv:1807.08548 [hep-ph]].