Azimuthal anisotropy Fourier harmonic correlations and initial-state fluctuations from HYDJET++ and AMPT models

arXiv:1907.02588

arXiv:1907.05450

ICNFP 2019
Crete

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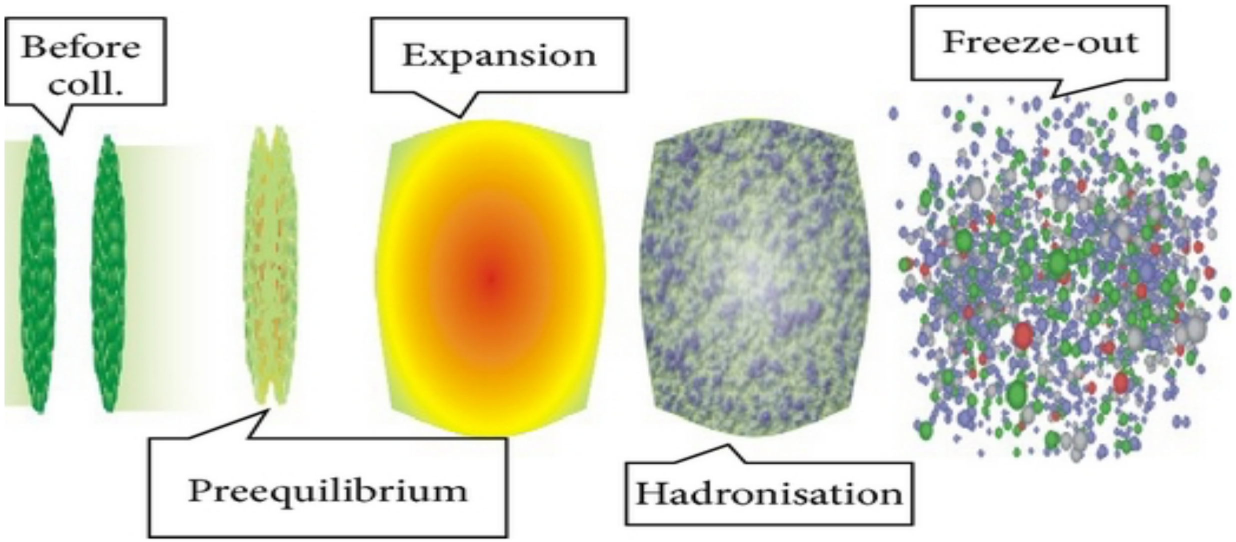
Heavy Ion Collisions



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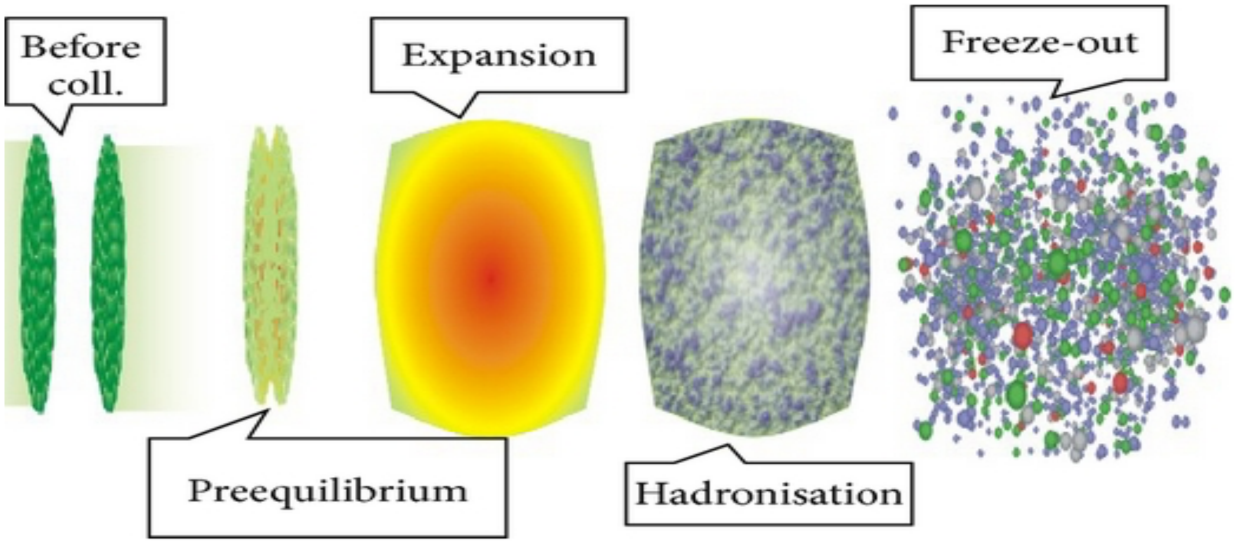
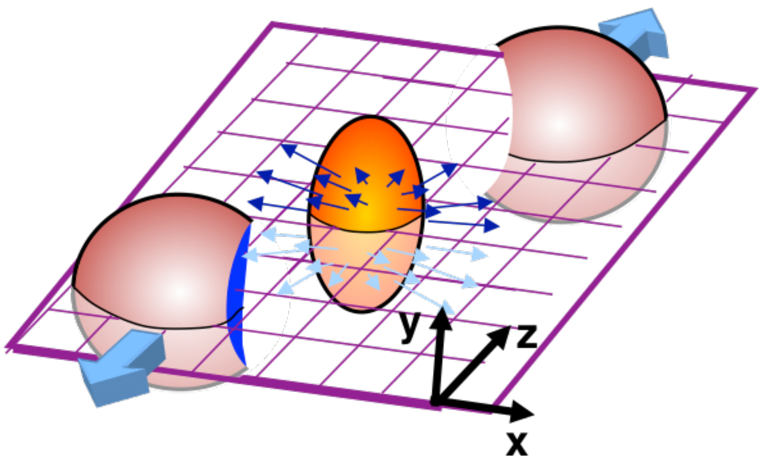
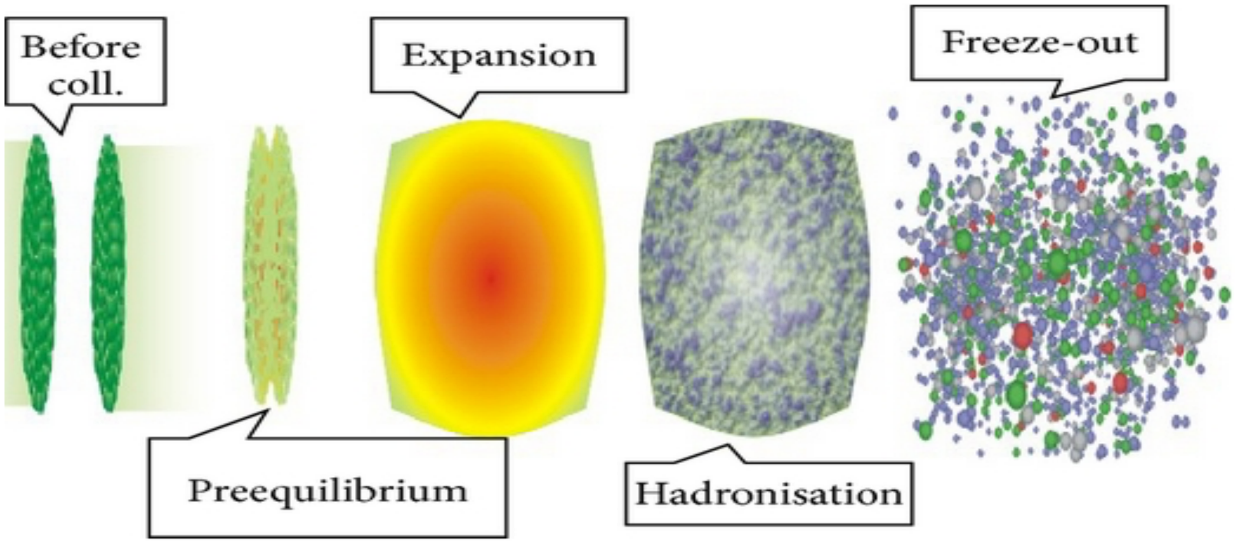
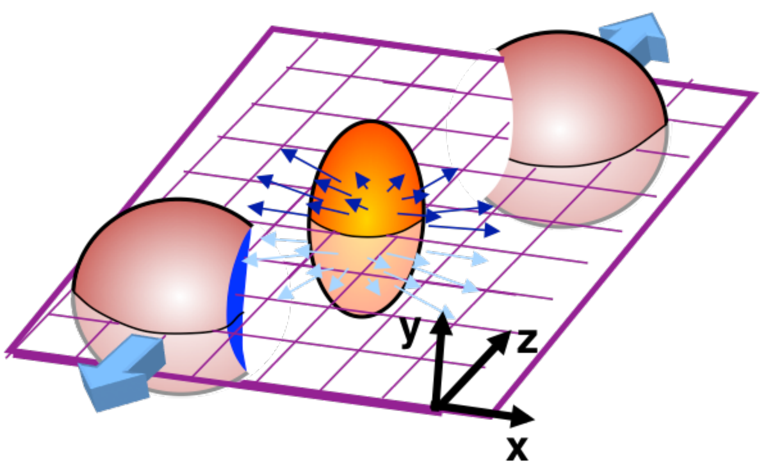


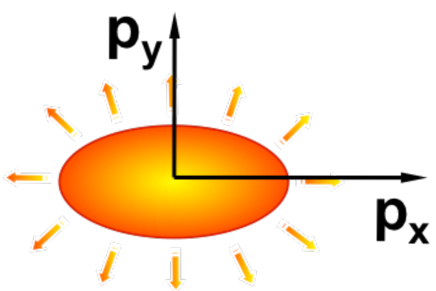


Lenticular shape → space anisotropy

Heavy Ion Collisions



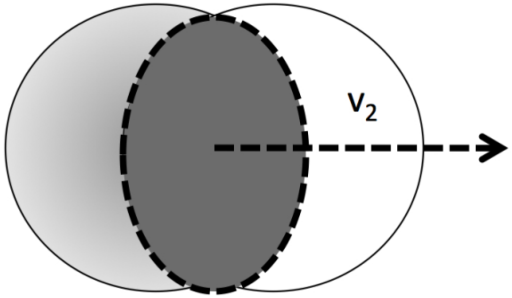




Lenticular shape → space anisotropy → momentum anisotropy

Elliptic Flow

Naïve picture:



System symmetry → Elliptic flow

$$v_2 = \langle (p_x/p_T)^2 - (p_y/p_T)^2 \rangle = \langle \cos 2(\phi - \Psi) \rangle$$

Eccentricity:

$$\varepsilon_{2,RP} = \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle}$$

Linearity:

$$v_2 = K_2 \varepsilon_2$$

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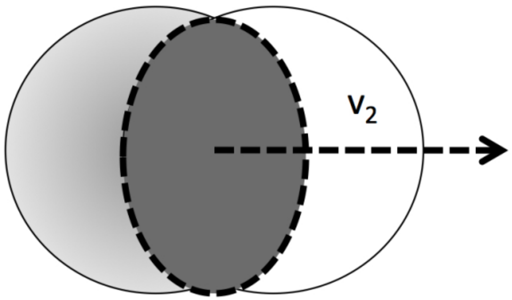

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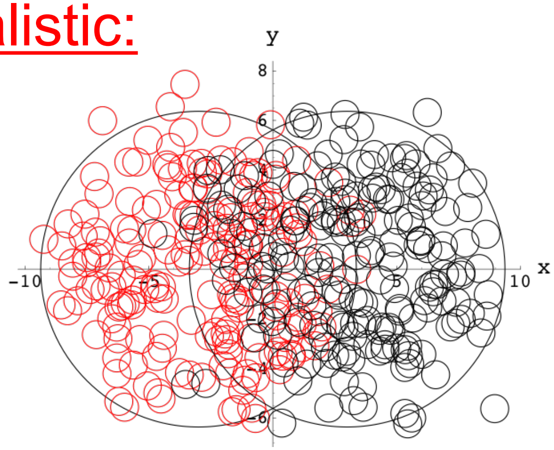
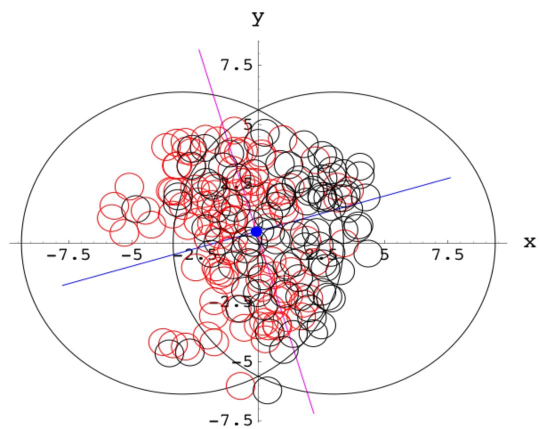
More realistic:





$$\varepsilon_2 = \frac{\sqrt{(\langle y^2 \rangle - \langle x^2 \rangle)^2 + 4\sigma_{xy}^2}}{\langle x^2 + y^2 \rangle}$$

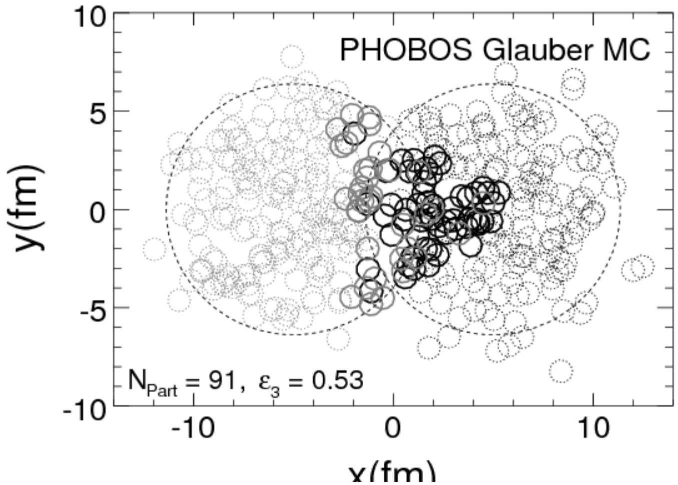
$$\sigma_{xy} = \langle xy \rangle - \langle x \rangle \langle y \rangle$$

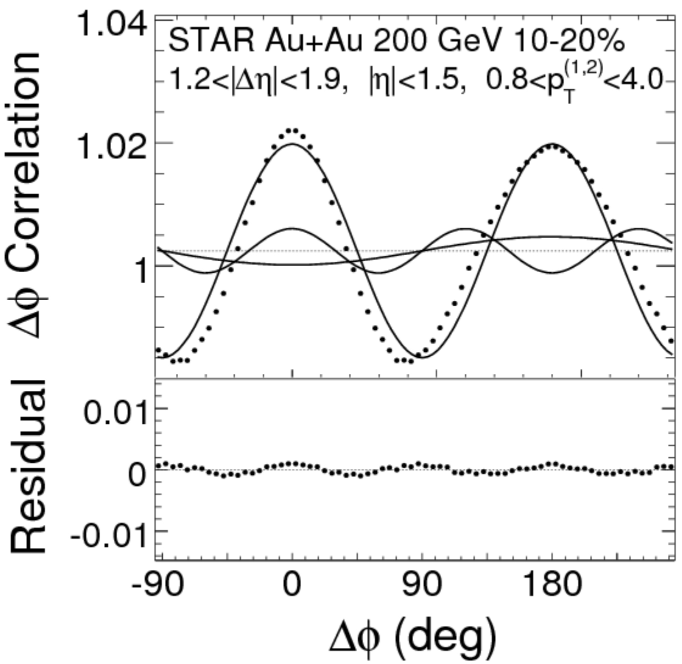
Finite # participant → Event-by-event fluctuations. → Eccentricity redefinition

$$\varepsilon_2 = \frac{\sqrt{\langle r^2 \cos(2\phi_{\text{part}}) \rangle^2 + \langle r^2 \sin(2\phi_{\text{part}}) \rangle^2}}{\langle r^2 \rangle}$$

Linearity restored: $v_2 = K_2 \varepsilon_2$

Triangular Flow





Event-by-event fluctuations → Triangular eccentricity

Phys.Rev. C78, 014901 (2008)

$$\varepsilon_3 \equiv \frac{\sqrt{\langle r^2 \cos(3\phi_{\rm part})\rangle^2 + \langle r^2 \sin(3\phi_{\rm part})\rangle^2}}{\langle r^2 \rangle}$$

$$v_3 \equiv \langle \cos(3(\phi - \psi_3)) \rangle$$

$$v_3 = K_3 \varepsilon_3$$

$$\frac{dN}{d\phi} \propto 1 + \sum_n 2 v_n \cos[n(\phi - \Psi_n)]$$

Phys. Rev. C 81, 054905 (2010); C 82 039903 (2010)

Flow Correlations & Fluctuations

$$
\begin{aligned}
V_4 &= V_{4L} + \chi_{422}(V_2)^2 \\
V_5 &= V_{5L} + \chi_{523}V_2V_3 \\
V_6 &= V_{6L} + \chi_{6222}(V_2)^3 + \chi_{633}(V_3)^2 \\
V_7 &= V_{7L} + \chi_{7223}(V_2)^2V_3,
\end{aligned}
$$

$$V_n \equiv v_n e^{in\Psi_n}$$

Phys. Lett. **B** 744 (2015) 82

Linear part (from ε_n)

Nonlinear part (from ε_2,ε_3)

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Phys. Lett. **B** 744 (2015) 82

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- v_n coefficients driven by:
  - Initial geometry;
  - Medium properties.

Flow Correlations & Fluctuations

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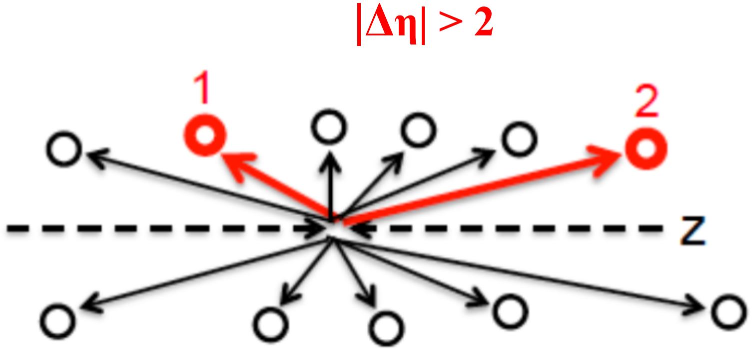
Phys. Lett. **B** 744 (2015) 82

Linear part (from ε_n) Nonlinear part (from $\varepsilon_2,\varepsilon_3$)

- v_n coefficients driven by:
  - Initial geometry;
  - Medium properties.
- Initial eccentricity fluctuations → Flow fluctuations
- Different averaging of flow over the events: way to probe initial fluctuations

Two-particle correlations method

2D correlation function: $\frac{1}{N_{trig}} \frac{d^2 N^{pair}}{d\Delta\eta d\Delta\phi}$



Fourier fit of 1D distribution:

$$\frac{1}{N_{trig}} \frac{dN^{pair}}{d\Delta\phi} = \frac{N_{assoc}}{2\pi} \left\{ 1 + \sum_n 2V_{n\Delta} \cos(n\Delta\phi) \right\}$$

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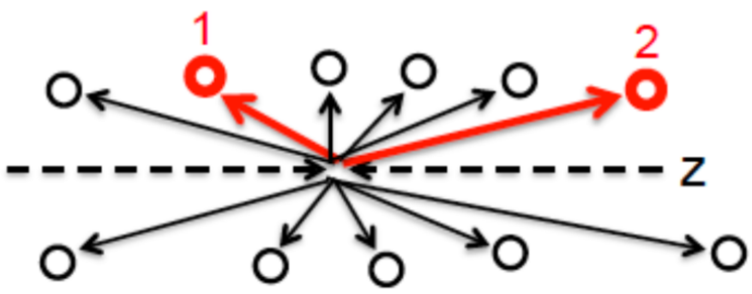

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Flow harmonics: $v_n = \sqrt{V_{n\Delta}}$

Multi-particle cumulants method

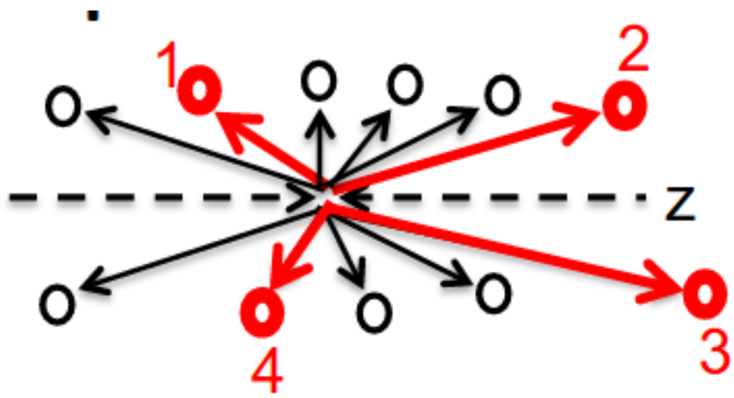
Multi-particle correlators:

$$\langle\langle 2\rangle\rangle = \langle\langle e^{in(\phi_1-\phi_2)}\rangle\rangle,$$

$$\langle\langle 4\rangle\rangle = \langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle,$$

$$\langle\langle 6\rangle\rangle = \langle\langle e^{in(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)}\rangle\rangle,$$

$$\langle\langle 8\rangle\rangle = \langle\langle e^{in(\phi_1+\phi_2+\phi_3+\phi_4-\phi_5-\phi_6-\phi_7-\phi_8)}\rangle\rangle$$



Multi-particle cumulants:

$$c_n\{4\} = \langle\langle 4\rangle\rangle - 2\langle\langle 2\rangle\rangle^2,$$

$$c_n\{6\} = \langle\langle 6\rangle\rangle - 9\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle + 12\langle\langle 2\rangle\rangle^3,$$

$$c_n\{8\} = \langle\langle 8\rangle\rangle - 16\langle\langle 2\rangle\rangle\langle\langle 6\rangle\rangle - 18\langle\langle 4\rangle\rangle^2 + 144\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle^2 - 144\langle\langle 2\rangle\rangle^4.$$

Flow harmonics:

$$v_n\{4\} = \sqrt[4]{-c_n\{4\}},$$

$$v_n\{6\} = \sqrt[6]{\frac{1}{4}c_n\{6\}},$$

$$v_n\{8\} = \sqrt[8]{-\frac{1}{33}c_n\{8\}}$$

Flow Fluctuations

$$v_2\{2\} = \sqrt{(\bar{v}_2)^2 + \sigma_x^2 + \sigma_y^2},$$

Phys. Rev. C **95** (2017) 014913

Flow Fluctuations

$$v_x \equiv \frac{1}{2\pi}\int_0^{2\pi} P(\varphi)\cos 2\varphi\, d\varphi,$$
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$$v_2\{2\} = \sqrt{(\bar{v}_2)^2 + \sigma_x^2 + \sigma_y^2},$$

$$s_1 \equiv \langle (v_x - \bar{v}_2)^3 \rangle,$$
$$s_2 \equiv \langle (v_x - \bar{v}_2) v_y^2 \rangle.$$

$$v_2\{4\} \simeq \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{s_1 + s_2}{(\bar{v}_2)^2},$$
$$v_2\{6\} \simeq \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{\frac{2}{3}s_1 + s_2}{(\bar{v}_2)^2},$$
$$v_2\{8\} \simeq \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{\frac{7}{11}s_1 + s_2}{(\bar{v}_2)^2},$$

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$$v_2\{6\} - v_2\{8\} = \frac{1}{11}(v_2\{4\} - v_2\{6\}).$$

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$$v_2\{6\} - v_2\{8\} = \frac{1}{11}(v_2\{4\} - v_2\{6\}).$$

$$\gamma_1^{\text{expt}} \equiv -6\sqrt{2}\, v_2\{4\}^2 \frac{v_2\{4\} - v_2\{6\}}{(v_2\{2\}^2 - v_2\{4\}^2)^{3/2}}$$

Phys. Rev. C **95** (2017) 014913

HYDJET++ & AMPT models

HYDJET++

- Event-by-event generator;
- Based on PYTHIA and PYQUEN initial parton-parton collisions;
- Ideal hydrodynamic evolution of the system;

Comput. Phys. Commun. 180 (2009) 779-799

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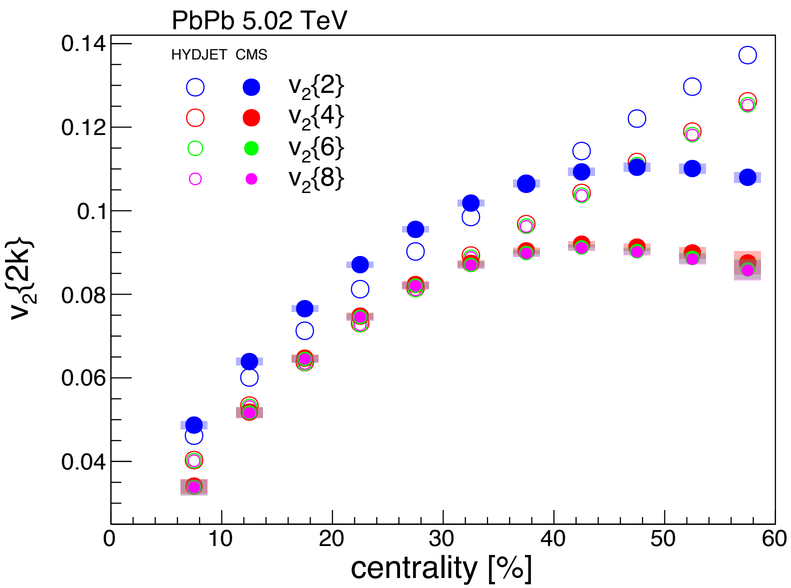
Comput. Phys. Commun. 180 (2009) 779-799

AMPT

- Event-by-event generator;
- Based on HIJING initial parton-parton collisions;
- Zhang's parton cascade;

Phys. Rev. C **72** (2005) 064901

Results – elliptic flow skewness



- $v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$

Data: Phys. Lett. **B** 789 (2019) 643

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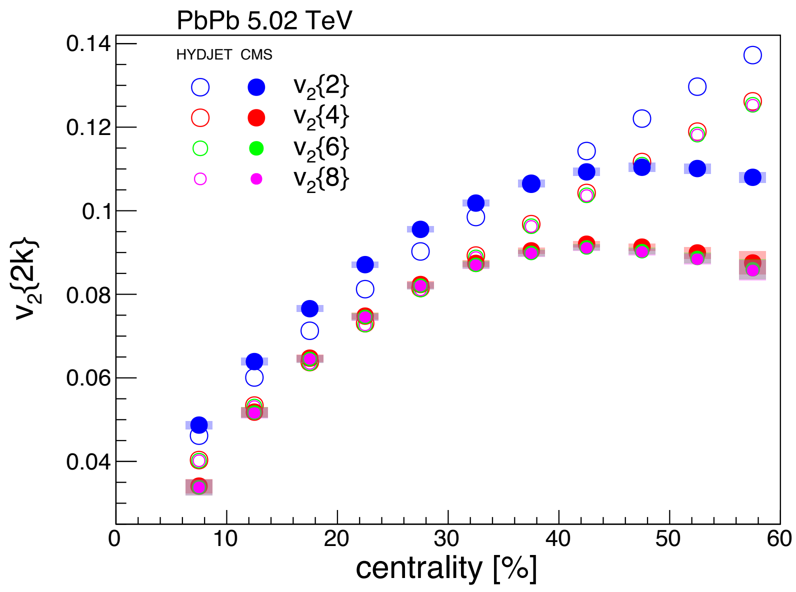
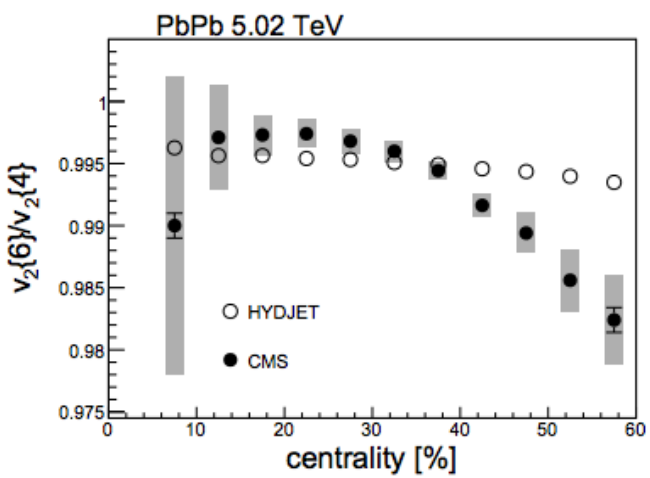
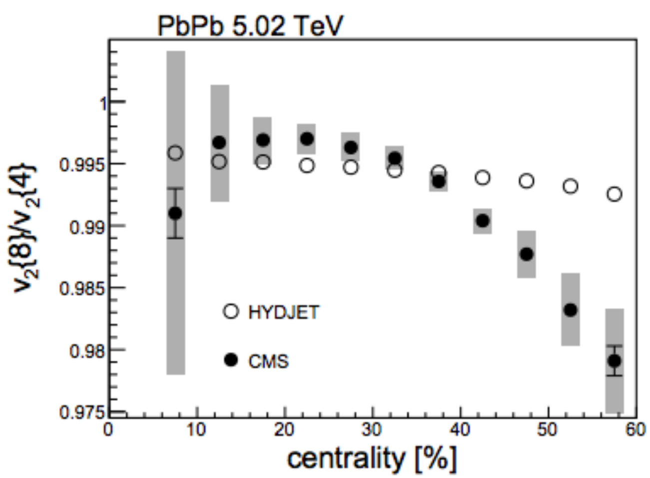
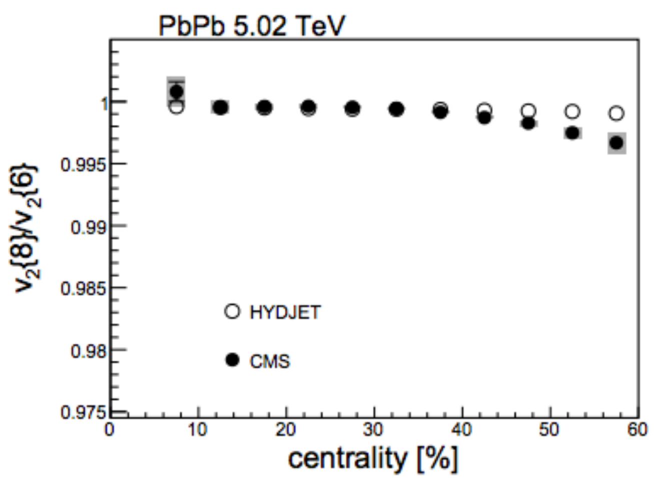


- $v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$
- Skewness observed in HYDJET
- Discrepancies with data







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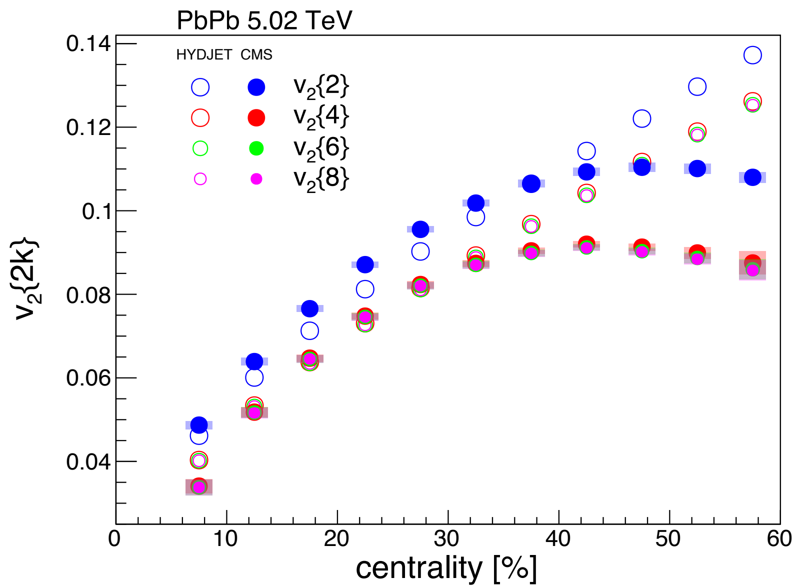
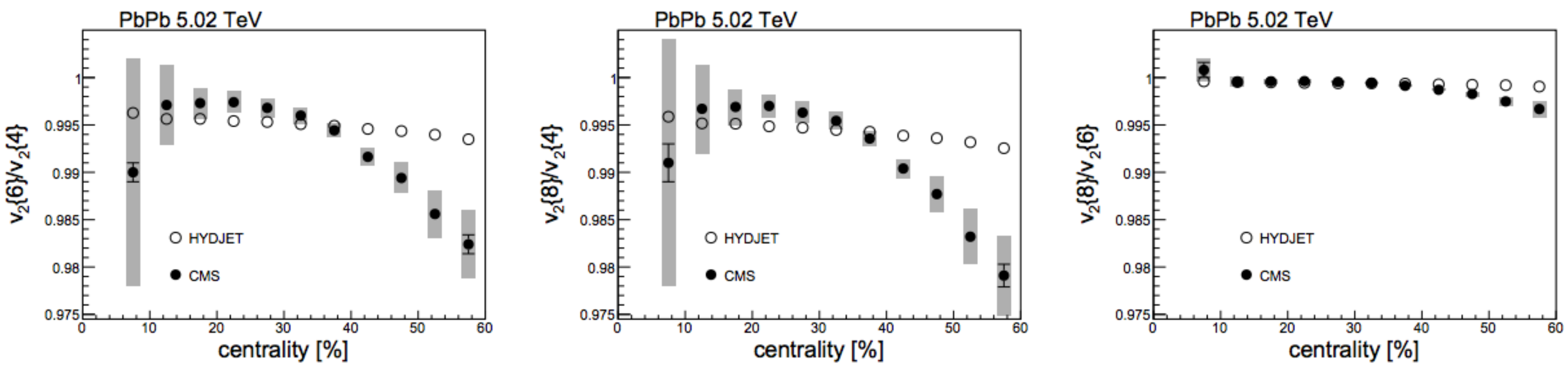


- $v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$
- Skewness observed in HYDJET
- Discrepancies with data
- Sensitivity to hydro evolution



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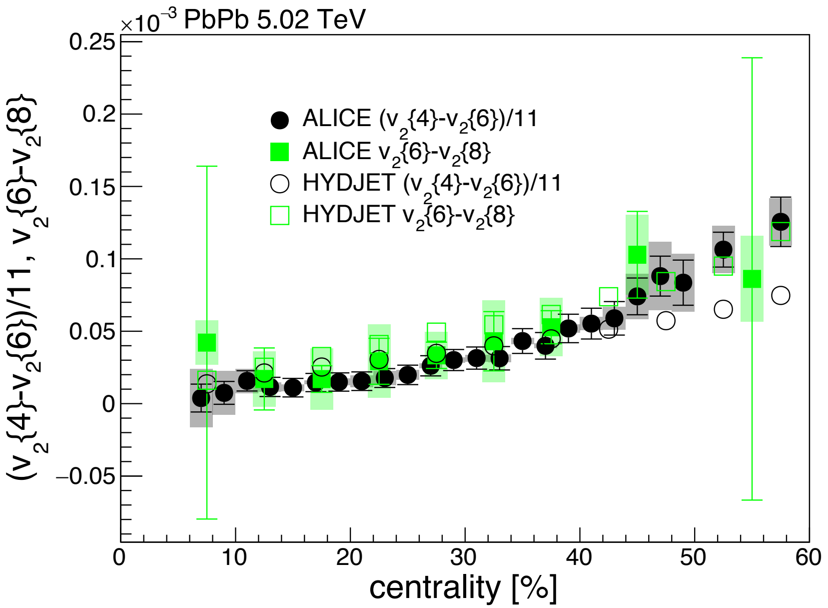
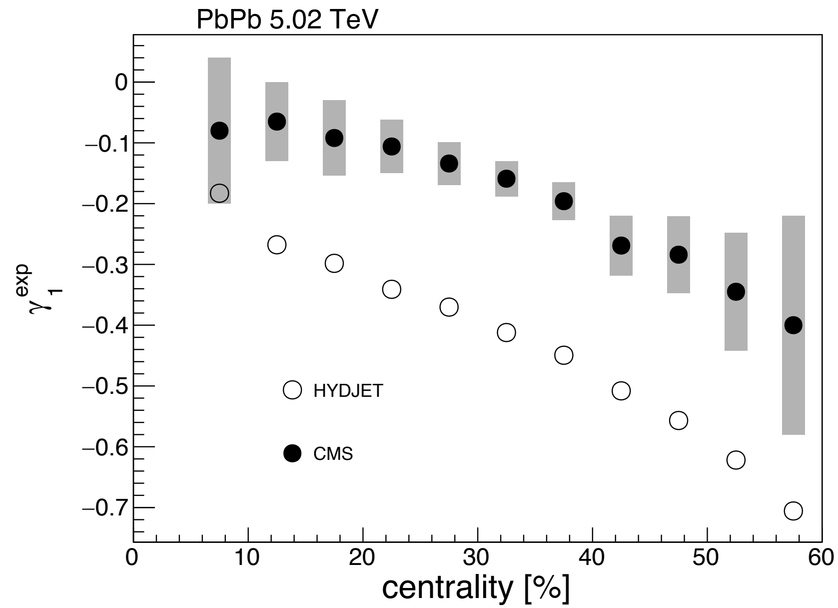




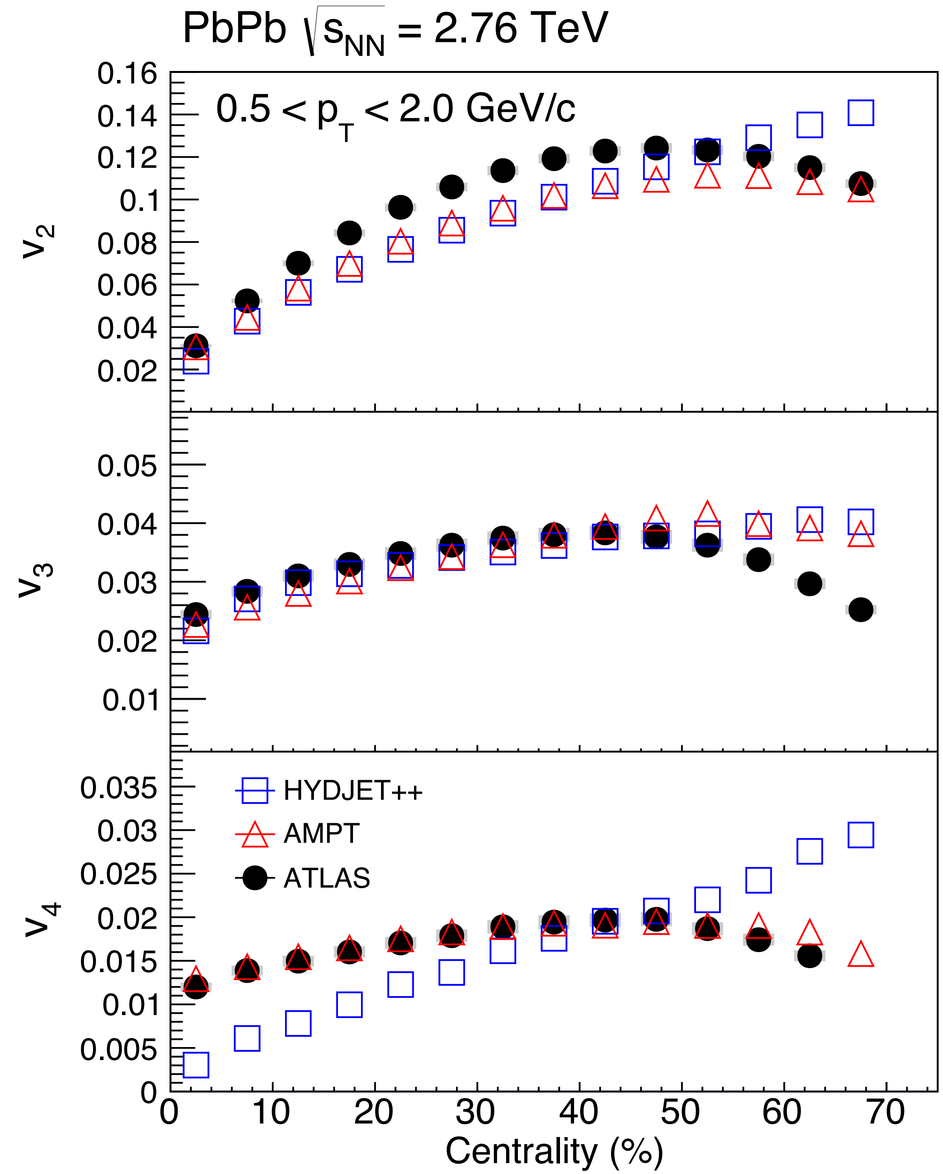
ALICE: JHEP **07** (2018) 103

CMS: Phys. Lett. B **789** (2019) 643

- Good agreement in absolute difference
- HYDJET underestimates the skewness of the data

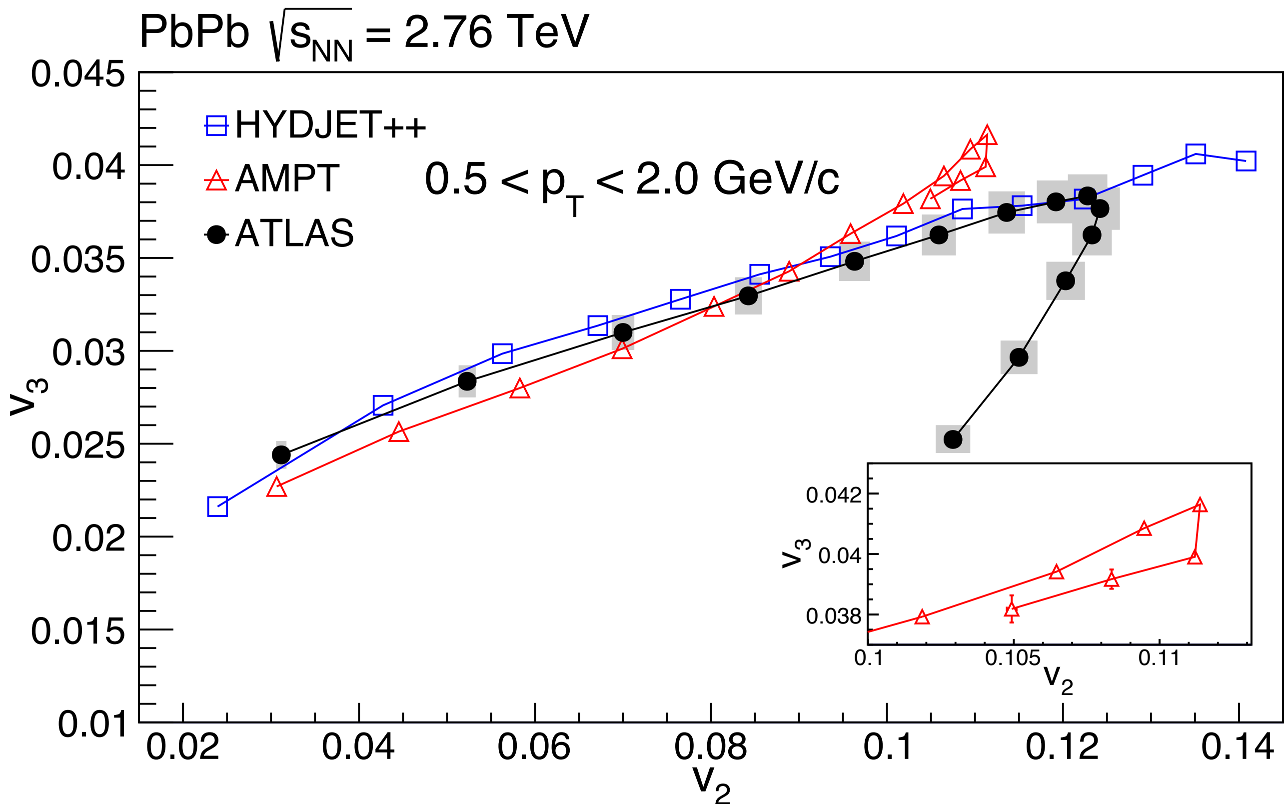
Results – flow harmonic correlations

- Both models give similar v_2 predictions in central and mid-central events;
- Similar case for v_3 with better agreement with data, except in peripheral collisions
- AMPT v_4 very close to data, unlike HYDJET



Data: Phys. Rev. C **92** (2015) 034903

Results – flow harmonic correlations



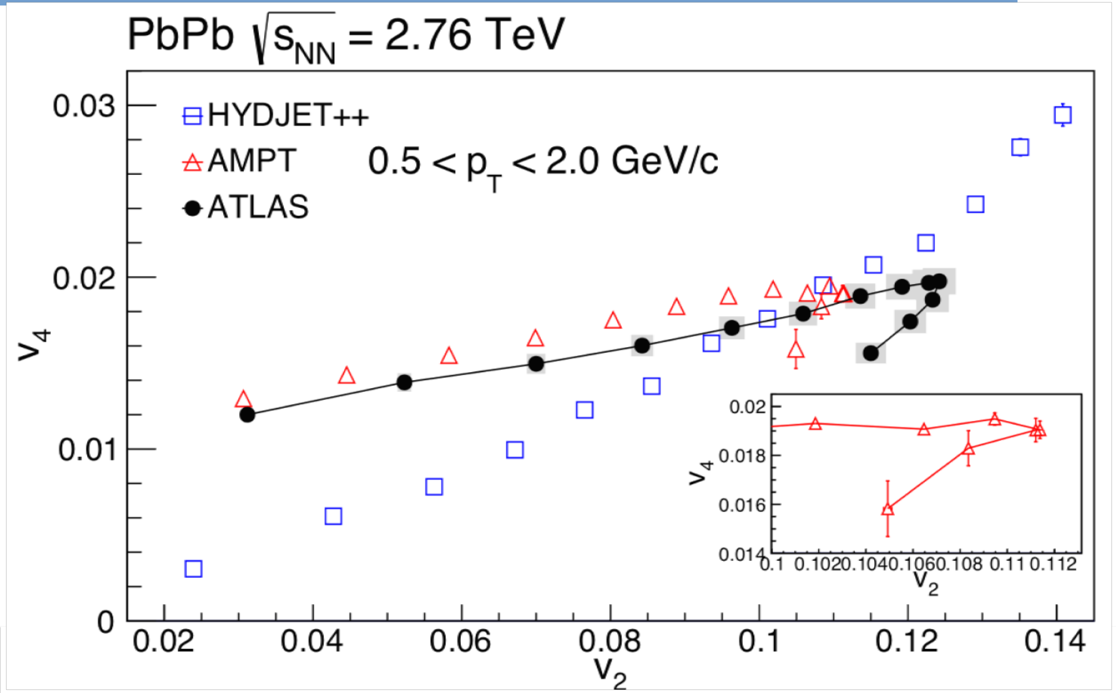
Data: Phys. Rev. C **92** (2015) 034903

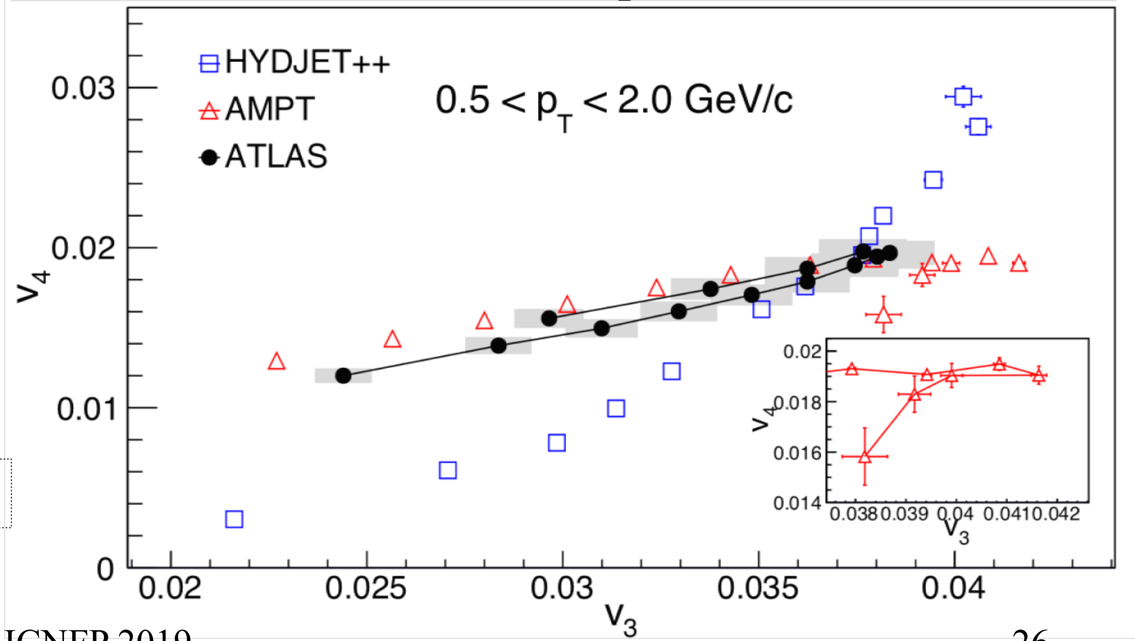
- Good agreement between data and models in v_3 vs v_3 slope in central and mid-collisions
- Each source shows different behavior in peripheral events

Results – flow harmonic correlations

- Good agreement between AMPT and data

- HYDJET slope disagrees with data





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Summary

- $v_2\{m\}/ v_2\{n\}$ ratio sensitive to the medium properties
- Skewness predictions from HYDJET++ below data results
- AMPT very good in describing higher harmonics (v_4)