

Lattice study of Chiral Magnetic Effect in QCD and Dirac semimetals

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Outline

- ▶ Introduction
- ▶ Details of the calculation
- ▶ CME in Dirac semimetals
- ▶ CME in QCD
- ▶ Conclusion

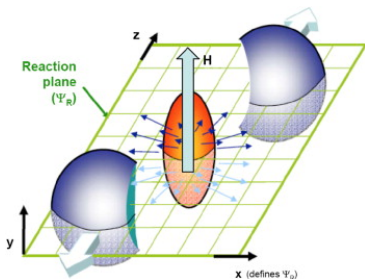
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- ▶ M. I. Katsnelson
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Chiral Magnetic Effect

"A system with a nonzero chirality responds to a magnetic field by inducing a current along the magnetic field. This is the Chiral Magnetic Effect."

[K. Fukushima, D. Kharzeev, H.J. Warringa, 2008]



$$\vec{J}_{CME} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}$$

Dynamical CME is manifested through electromagnetic conductivity

Conductivity in external magnetic field

- ▶ \vec{E}, \vec{B}
- ▶ $\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2} (\vec{E}, \vec{B}) - \frac{\rho_5}{\tau}$, τ - chirality-changing scattering time

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- ▶ $\vec{J} = \sigma \vec{E} + \frac{e^2}{2\pi^2} \vec{B} \times \mu_5$
- ▶ $\sigma_{\parallel}^{CME} \sim eB\tau$
- ▶ **Manifestation of CME: rise of the conductivity with B**
- ▶ Anomaly related quantum phenomenon (classically $\sigma_{\parallel}^{CME} = 0$)

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Q. Li et al., Nature Phys. 12 (2016) 550-554

H. Li et al., Nat. Comm. 7, 10301 (2016)

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- ▶ Observed in heavy-ion collision experiments(?)

B. I. Abelev et al. (STAR), Phys.Rev.Lett. 103, 251601 (2009)

B. Abelev et al. (ALICE), Phys.Rev.Lett. 110, 012301 (2013)

Lattice simulations of strongly correlated system

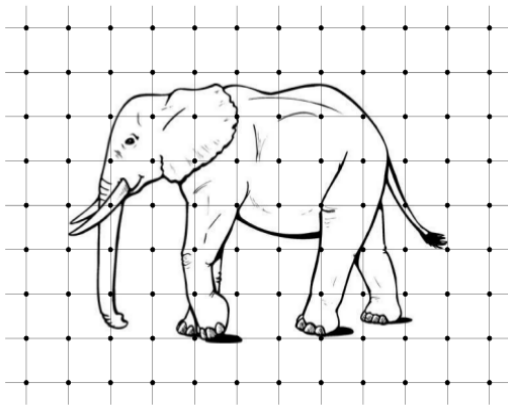
▶ Dirac semimetals (strongly correlated system)

- ▶ Two Fermi-points (Na_3Bi , Cd_3As_2)
- ▶ Dispersial relation: $E^2 = v_{\parallel}^2(k_x^2 + k_y^2) + v_{\perp}^2 k_z^2$, $v_{\parallel}, v_{\perp} \sim 10^{-3}$
- ▶ Coulomb interaction $\alpha_{\text{eff}} \sim \frac{\alpha_{\text{em}}}{v} > 1$

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 - ▶ Coulomb interaction $\alpha_{eff} \sim \frac{\alpha_{em}}{v} > 1$
- ▶ QCD (strongly correlated system)
 - ▶ Nonlinear EOM
 - ▶ $\alpha_{eff} \sim 1$

Lattice simulations of strongly correlated system



Lattice simulation

- ▶ Allows to study strongly interacting systems
- ▶ Based on the first principles of quantum field theory
- ▶ Powerful due to modern supercomputers and algorithms

Details of the simulations

- ▶ Staggered fermions (four Fermi points)

$$S_f = \bar{\Psi}_x D_{x,y} \Psi_y = \sum_x \left(m \bar{\Psi}_x \Psi_x + \frac{1}{2} \eta_4(x) [\bar{\Psi}_x \Psi_{x+\hat{4}} - \bar{\Psi}_{x+\hat{4}} \Psi_x] + \frac{1}{2} \sum_{i=1}^3 \eta_i(x) [\bar{\Psi}_x \Psi_{x+\hat{i}} - \bar{\Psi}_{x+\hat{i}} \Psi_x] \right)$$

- ▶ For Dirac Semimetals: two Fermi-points

$$Z = \int DU \exp(-S_G(U)) (\det(\hat{D}(U) + m))^{1/2}$$

- ▶ For QCD: u,d,s-quarks at physical quark masses

$$Z = \int DU \exp(-S_G(U)) (\det(\hat{D}_u(U) + m_u))^{1/4} (\det(\hat{D}_d(U) + m_d))^{1/4} (\det(\hat{D}_s(U) + m_s))^{1/4}$$

Conductivity in lattice simulations

▶ $J_i = \sigma_{ij} E_j$

- ▶ Electromagnetic conductivity

$$\sigma_{ij} = \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int_0^\infty dt \int d^3x e^{i\omega t} \langle [J_i(x), J_j(0)] \rangle$$

$$\rho_{ij} = -\frac{1}{\pi} \text{Im} G_R^{ij}(\omega, \vec{k} = 0)$$

$$\sigma_{ij} = \pi \lim_{\omega \rightarrow 0} \frac{1}{\omega} \rho_{ij}(\omega)$$

- ▶ Analytic continuation

$$G_E(\omega, \vec{p}) = -G_R(i\omega, \vec{p}), \quad \omega > 0$$

- ▶ On lattice we measure

$$C_E(\tau) = \int d^3x \langle J_i(\tau, \vec{x}) J_j(0, \vec{0}) \rangle$$

$$C_E(\tau) = \int_0^\infty d\omega \rho(\omega) \frac{\text{ch}\left(\frac{\omega}{2T} - \omega\tau\right)}{\text{sh}\left(\frac{\omega}{2T}\right)}, \quad \tau \in \left(0, \frac{1}{T}\right)$$

Conductivity with staggered fermions

- ▶ Correlation function for staggered fermions

$$C_{ij}(\tau) = \frac{1}{L_s^3} \langle J_i(\tau) J_j(0) \rangle,$$

$$J_i(\tau) = \frac{1}{4} e \sum_{\mathbf{f}} q_{\mathbf{f}} \sum_{\vec{x}} \eta_i(\mathbf{x}) (\bar{\Psi}_{\mathbf{x}}^{\mathbf{f}} U_{\mathbf{x},i} \Psi_{\mathbf{x}+\mathbf{i}}^{\mathbf{f}} + \bar{\Psi}_{\mathbf{x}+\mathbf{i}}^{\mathbf{f}} U_{\mathbf{x},i}^{\dagger} \Psi_{\mathbf{x}}^{\mathbf{f}})$$

- ▶ Two branches of staggered correlator

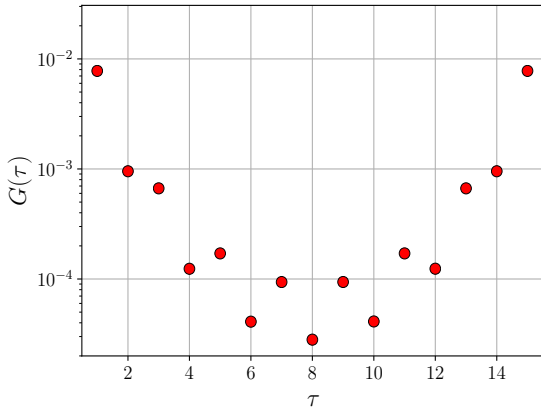
$$C_{ij}^{\mathbf{e}}(\tau = 2n \times \mathbf{a}) = \int d^3 y (\langle A_i(\tau, \vec{y}) A_j(0, \vec{0}) \rangle - \langle B_i(\tau, \vec{y}) B_j(0, \vec{0}) \rangle)$$

$$C_{ij}^{\mathbf{o}}(\tau = (2n+1) \times \mathbf{a}) = \int d^3 y (\langle A_i(\tau, \vec{y}) A_j(0, \vec{0}) \rangle + \langle B_i(\tau, \vec{y}) B_j(0, \vec{0}) \rangle)$$

$$A_i = e \sum_{\mathbf{f}} q_{\mathbf{f}} \bar{\psi}^{\mathbf{f}} \gamma_i \psi^{\mathbf{f}}, \quad B_i = e \sum_{\mathbf{f}} q_{\mathbf{f}} \bar{\psi}^{\mathbf{f}} \gamma_5 \gamma_4 \gamma_i \psi^{\mathbf{f}}$$

Conductivity with staggered fermions

- ▶ Typical plot for the staggered correlation function



Conductivity with staggered fermions

The strategy of the calculation

- ▶ Measure $C_E^{even,odd}(\tau)$ on two branches
- ▶ Reconstruct the $\rho^{even,odd}(\omega)$ (Backus-Gilbert method)

$$C_E^{even,odd}(t) = \int_0^\infty d\omega \rho^{even,odd}(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

- ▶ Calculate $\rho(\omega) = \frac{1}{2}(\rho^{even}(\omega) + \rho^{odd}(\omega))$
(what corresponds to the $\langle J_{eI}(\tau) J_{eI}(0) \rangle$)
- ▶ Calculate the conductivity $\sigma = \pi \frac{\rho(\omega)}{\omega} \Big|_{\omega \sim 0}$

Backus-Gilbert method for the spectral function

- ▶ Problem: find $\rho(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega \rho(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{\text{ch}\left(\frac{\omega}{2T} - \omega x_i\right)}{\text{sh}\left(\frac{\omega}{2T}\right)}$$

- ▶ Define an estimator $\tilde{\rho}(\bar{\omega})$ ($\delta(\bar{\omega}, \omega)$ - resolution function):

$$\tilde{\rho}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega}, \omega) \rho(\omega)$$

- ▶ Let us expand $\delta(\bar{\omega}, \omega)$ as

$$\delta(\bar{\omega}, \omega) = \sum_i b_i(\bar{\omega}) K(x_i, \omega) \quad \tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

- ▶ Goal: minimize the width of the resolution function

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$

$$W_{ij} = \int d\omega K(x_i, \omega) (\omega - \bar{\omega})^2 K(x_j, \omega), \quad R_i = \int d\omega K(x_i, \omega)$$

- ▶ Regularization by the covariance matrix S_{ij} :

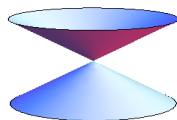
$$W_{ij} \rightarrow \lambda W_{ij} + (1 - \lambda) S_{ij}, \quad 0 < \lambda < 1$$

Chiral symmetry breaking in Dirac semimetals

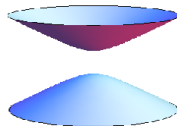
Phase transition in Dirac semimetal at $\alpha_{\text{eff}}^c \simeq 1.14$

Phys. Rev. B94, 205147 (2016), Annals Phys. 391 (2018) 278-292

- ▶ Semimetal phase $\alpha_{\text{eff}} < 1.14$: $\langle \bar{\psi}\psi \rangle = 0$, $M = 0$, $E^2 = v_F^2 \bar{p}^2$

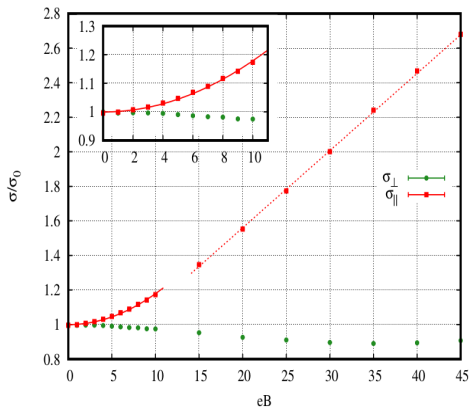


- ▶ Insulator phase $\alpha_{\text{eff}} > 1.14$: $\langle \bar{\psi}\psi \rangle \neq 0$, $M \neq 0$, $E^2 = v_F^2 \bar{p}^2 + M^2$



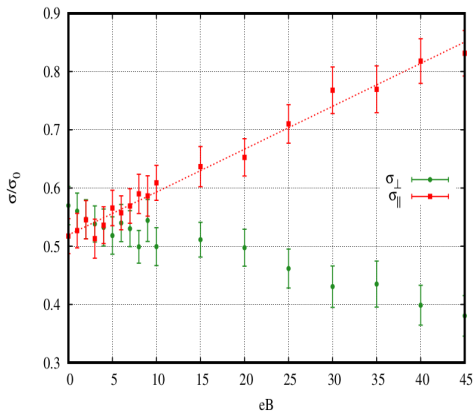
We can study CME in both phases

Conductivity in the semimetal phase



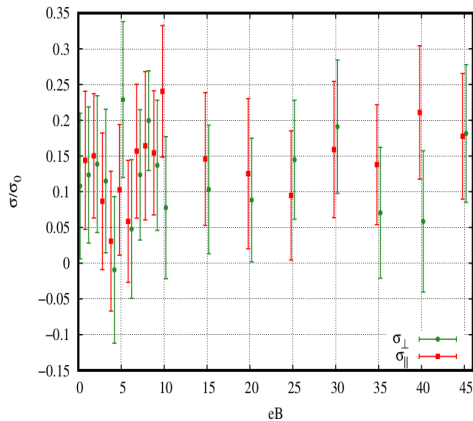
- ▶ Rise of σ_{\parallel} with $B \Rightarrow$ **we observe CME**
- ▶ Decrease of σ_{\perp} with $B \Rightarrow$ **we observe magnetoresistance**

Conductivity in the transition region



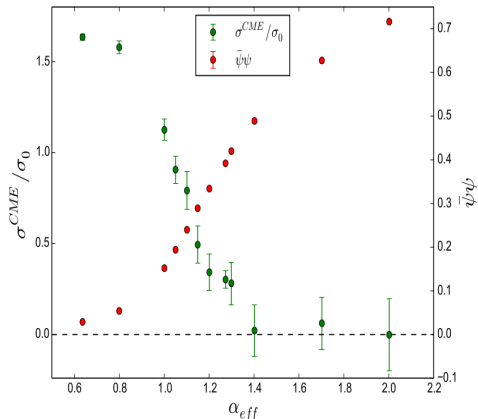
- CME in the the transition region has smaller magnitude

Conductivity in the insulator phase



- We don't observe CME in the insulator phase

CME and chiral symmetry breaking



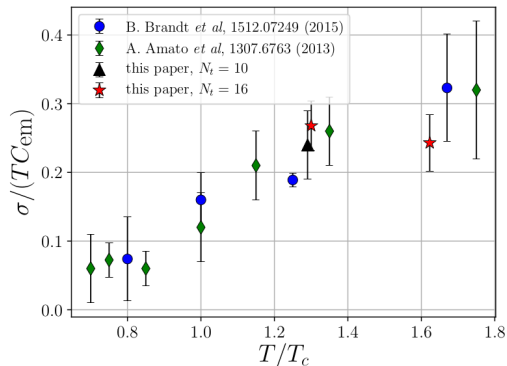
- Chiral symmetry is important for CME

Lattice simulation of QCD: details

- ▶ Stout smeared staggered 2 + 1 fermions
- ▶ Physical pion m_π and strange m_s quark masses
- ▶ $T \approx 200, 250$ MeV
- ▶ Lattice sizes and steps:

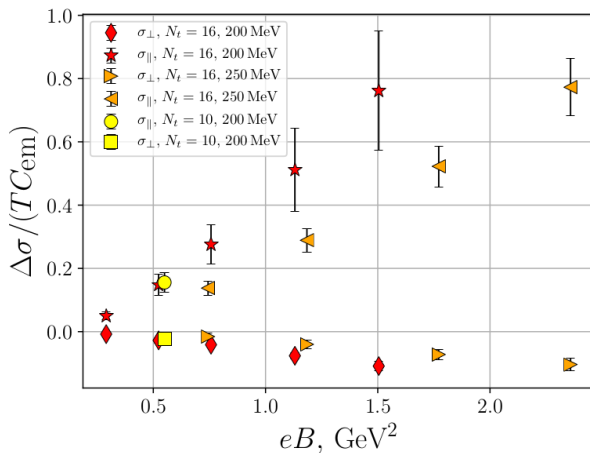
| a , fm | L_s | N_t |
|----------|-------|-------|
| 0.988 | 48 | 10 |
| 0.0618 | 64 | 16 |

Conductivity at zero magnetic field $eB = 0$



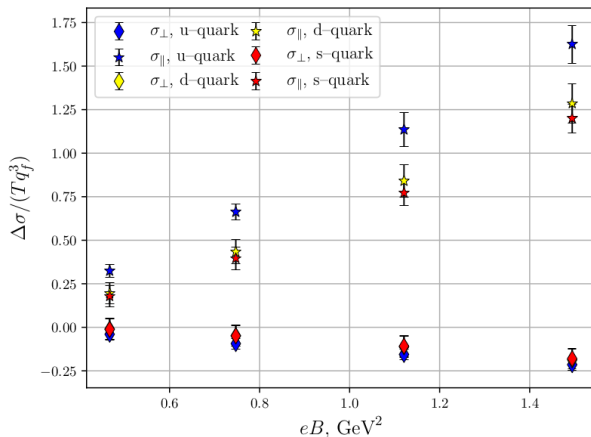
- ▶ **First calculation of the conductivity at physical pion mass**
- ▶ Agreement with previous papers

Conductivity at nonzero magnetic field $eB \neq 0$



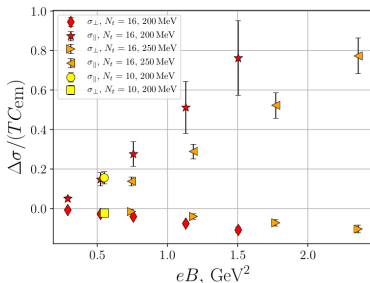
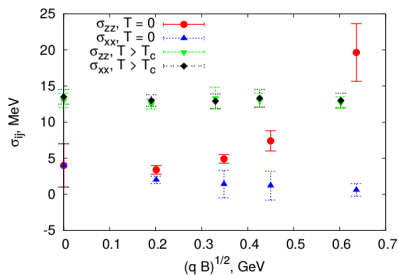
- We observe CME and magnetoresistance in QGP

The contribution of different quarks



- ▶ The conductivity scale as q_f^3
- ▶ $\sigma_d/q_d^3 \simeq \sigma_s/q_s^3$, $\sigma_u/q_u^3 > \sigma_{d,s}/q_{d,s}^3$ ($|q_u| = \frac{2}{3}$, $|q_d| = |q_s| = \frac{1}{3}$)
- ▶ Large mass of s-quark does not influence the conductivity

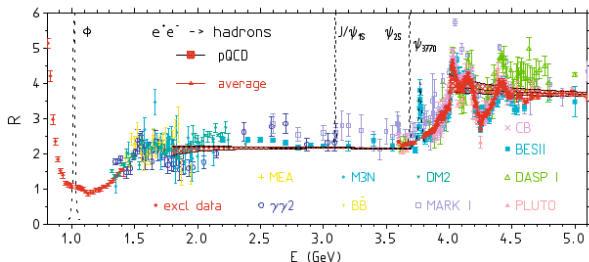
Comparison with other studies



P.V. Buividovich et al., Phys.Rev.Lett. 105 (2010) 132001

- ▶ No CME in QGP and there is CME in the confinement phase
- ▶ Disagreement with our results (small magnetic fields? $eB < 0.36$ GeV²)

CME in the confinement phase



- ▶ Complicated structure of spectral density
- ▶ In confinement phase chiral symmetry is broken
- ▶ Assumption: CME cannot be realised at sufficiently low temperature

Conclusion:

- ▶ Observe CME in Dirac semimetals (semimetal phase)
- ▶ No CME in the phase with broken chiral symmetry
- ▶ The first calculation of the conductivity in QCD at physical pion mass
- ▶ Observe CME and magnetoresistance in QGP

