Artificial graphene and artificial topological insulator.

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The aim of the project is to build an artificial topological insulator (TI) out of laterally patterned 2-dimensional electron gas in a semiconductor.

We want to use the power of modern nanotechnology to create a fully controllable and tunable TI.

- No interaction between electrons.
- I will discuss only single particle effects.
- The nontrivial physics is related to the band structure, i.e. to the interaction with external potential.
Basics

1) Band insulators,

2) Band metals,

3) Semiconductors,

4) Semimetals,

5) Gapped superconductors,

6) Gapless superconductors.
Periodic potential $\frac{p^2}{2m} \rightarrow$ band structure $\varepsilon_p$

Physical vacuum is a \textbf{band insulator} with gap $\Delta = 2mc^2 \approx 1MeV = 10^6 eV$

In a good solid state a \textbf{band insulator} $\Delta \approx 4 eV$

Semiconductor is a \textbf{band insulator} with gap $\Delta \leq 1 eV$
Metal

Has a nonzero dc conductivity \( j = \sigma E \)

Has a Fermi surface (FS).

Area of Fermi surface \( \neq 0 \).

Semimetal

Area of Fermi surface \( = 0 \).
Spectrum of a **gapped** superconductor is similar to that of an insulator.

\[
\varepsilon_p = \pm \sqrt{v^2 (p - p_F)^2 + \Delta^2}
\]

Spectrum of a **gapless** superconductor depends on direction in momentum space.

\[
\varepsilon_p = \pm \sqrt{v^2 (p - p_F)^2 + \Delta_p^2}
\]

For some direction in momentum space \( \Delta_p = 0 \)

Supercurrent flows without resistance.
Edge states and Quantum Hall effect

Mechanism of resistivity in normal metal.

Electrons scatter from impurities/defects/excitations.

Is it possible to have zero dissipation without superconducting pairing?

Quantum Hall effect (1980)

No room for electron backscattering in the edge state.
Spin orbit interaction

\[ H_{ls} = \lambda (s \cdot [p \times n]) = \lambda s_z p_x \]

Mechanism of spin orbit interaction in atomic nuclei.

In condensed matter the interaction is usually called Rashba interaction.

Is it possible to make \( H = H_{ls} \) for 1D edge state?

In this case the back scattering is forbidden if there are no magnetic impurities. This implies a **dissipationless** edge state.

blue: unit vector perpendicular to the edge  
red: momentum of the edge electron
Graphene

Graphene is an atomic-scale honeycomb lattice carbon atoms monolayer. Conduction via $\pi$-orbitals of carbon. Tight binding.
Graphene is a 2D material

Two triangular sublattices, A and B.

Brillouin zone of a triangular lattice.

**Two** Dirac cones of different parity.

\[ H = v_F \cdot p \]
\[ k = K_1 + p \]

\( \sigma \) is pseudospin \( \frac{1}{2} \) related to two sublattices A and B.

Spin orbit interaction is zero, therefore usual spin s does not appear in the Hamiltonian.
Topological insulator

*Kane & Mele, 2005*, graphene with spin orbit interaction, brute force numerical diagonalization.

\[ H = \text{graphene} + H_{ls} \]

Dirac cone of “dissipationless” edge states.

A realization of the Rashba Hamiltonian

\[ H_{ls} = \lambda (\mathbf{s} \cdot [\mathbf{p} \times \mathbf{n}]) = \lambda s_z p_x \]
Artificial Graphene

Consider a 2D electron gas in a potential \( U(r) \) with hexagonal (triangular) symmetry and spacing \( L \).

\[
H = \frac{\mathbf{p}^2}{2m} + U(r)
\]

Brillouin zone
Artificial graphene vs artificial “antigraphene”

Consider a simple periodic potential

\[
U(r) = 2W \left[ \cos(G_1 \cdot r) + \cos(G_2 \cdot r) + \cos(G_3 \cdot r) \right]
\]

\( W < 0 \) describes an array of quantum dots and this is equivalent to graphene. This regime emulates chemistry.

\( W > 0 \) describes an array of quantum antidots and this is equivalent to “antigraphene”. The “antigraphene” is more interesting and more feasible. This regime cannot be realized in a natural chemical compound.
Numerical diagonalization of the Hamiltonian is straightforward.

$$H = \frac{p^2}{2m} + U(r), \quad U(r) = 2W[\cos(G_1 \cdot r) + \cos(G_2 \cdot r) + \cos(G_3 \cdot r)]$$

Two major advantages of the artificial “antigraphene” compared to artificial graphene:

(i) Second pair of Dirac points.
(ii) Larger energy scale.
Let us switch on the spin orbit interaction $H_{so}$. Whatever is the microscopic mechanism of the interaction the matrix element between two plane waves must be of the following form

$$\langle \mathbf{p}_2 | H_{so} | \mathbf{p}_1 \rangle \propto i(\mathbf{p}_1 \times \mathbf{p}_2) \cdot \mathbf{s}$$

An additional condition follows from the **Bloch’s theorem**. Since the spin-orbit interaction has the period of the potential, the matrix element is nonzero only if

$$\mathbf{p}_2 - \mathbf{p}_1 = \pm \mathbf{G}_i$$

The interaction is small for electrons in GaAs, but for holes it is comparable with kinetic energy. Therefore we need hole doped GaAs.
Spin orbit opens a gap in the 2D bulk Dirac cone.
The 2D “sample” is restricted by electrostatic gates. The gating results in the edge states and in the edge 1D “Dirac” cone.

$S_z = +1/2$ is the right mover and $S_z = -1/2$ is the left mover.

These states are responsible for edge current without dissipation.
Points to note

1. Modern nanotechnology allows to create a **tunable topological insulator** using laterally patterned semiconductor heterostructure.

2. The ultimate **goal is a creation of dissipationless electronics** without superconductivity.

3. Artificial “antigraphene” is better than artificial graphene, 2\textsuperscript{nd} pair of Dirac points.

4. The artificial topological insulator cannot be built with electron doped semiconductors, but it can be built with hole doped.
Thank you