Artificial graphene and artificial topological insulator.

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The aim of the project is to build an artificial topological insulator (TI) out of laterally patterned 2-dimensional electron gas in a semiconductor.

We want to use the power of modern nanotechnology to create a fully controllable and tunable TI.

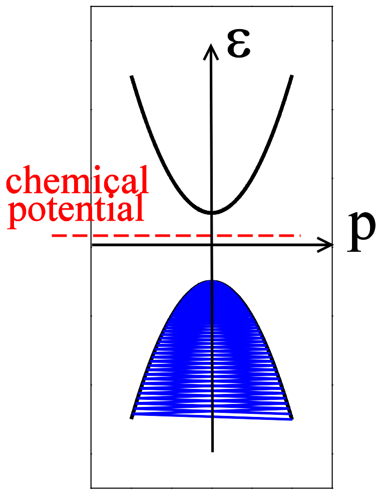
- **No interaction between electrons.**
- **I will discuss only single particle effects.**
- **The nontrivial physics is related to the band structure, i. e. to the interaction with external potential.**

Basics

1) Band insulators,
2) Band metals,
3) Semiconductors,
4) Semimetals,
5) Gapped superconductors,
6) Gapless superconductors.

U(x)

Periodic potential $\frac{p^2}{2m} \rightarrow \text{band structure } \varepsilon_{\mathrm{p}}$



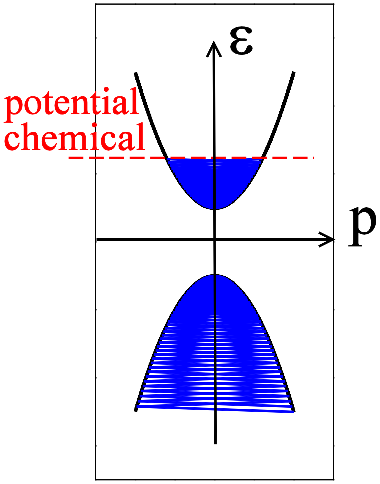
Band insulator

Physical vacuum is a ***band insulator*** with gap $\Delta = 2mc^2 \approx 1MeV = 10^6 eV$

In a good solid state a ***band insulator*** $\Delta \approx 4\,eV$

Semiconductor is a ***band insulator*** with gap $\Delta \leq 1\,eV$

Metal

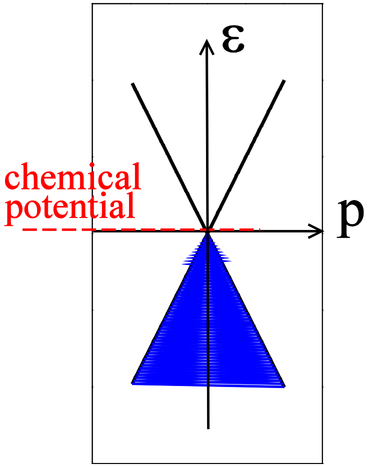


Has a nonzero dc conductivity $j = \sigma E$

Has a Fermi surface (FS).

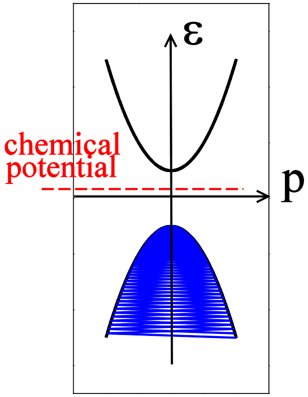
Area of Fermi surface ≠ 0.

Semimetal



Area of Fermi surface = 0.

Spectrum of a **gapped** superconductor is similar to that of an insulator.



$$\varepsilon_p = \pm\sqrt{v^2(p-p_F)^2+\Delta^2}$$

Spectrum of a **gapless** superconductor depends on direction in momentum space.

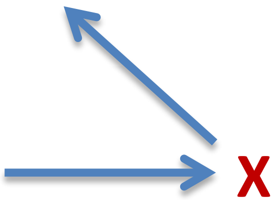
$$\varepsilon_p = \pm\sqrt{v^2(p-p_F)^2+\Delta_p^2}$$

For some direction in momentum space $\Delta_p = 0$

Supercurrent flows without resistance.

Edge states and Quantum Hall effect

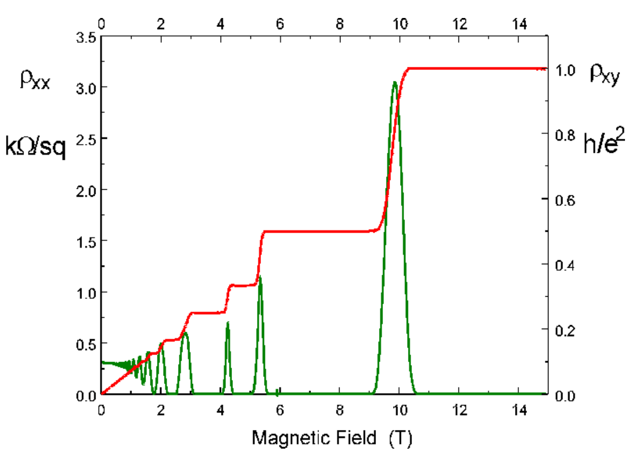
Mechanism of resistivity in normal metal.

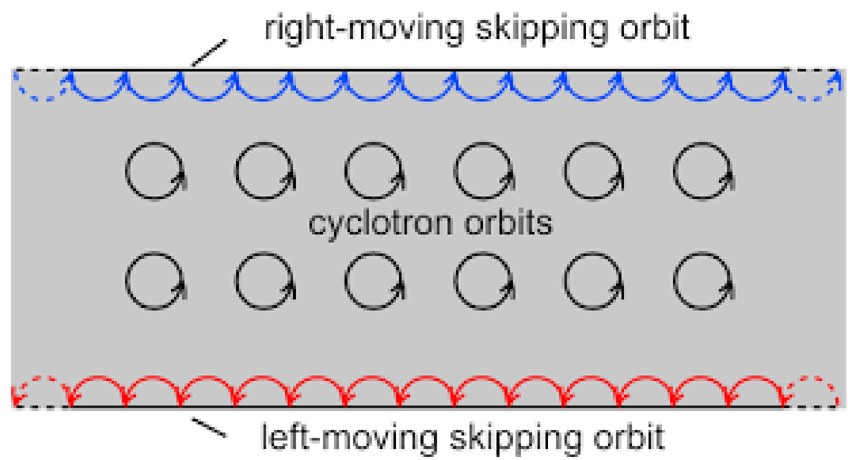


Electrons scatter from impurities/defects/excitations.

Is it possible to have zero dissipation without superconducting pairing?

Quantum Hall effect (1980)



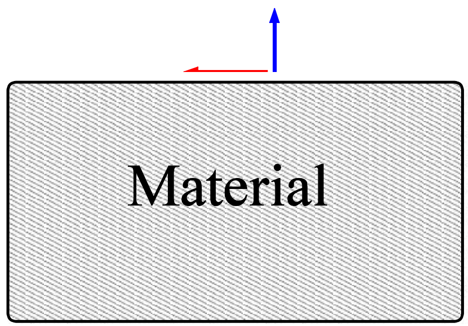


No room for electron backscattering in the edge state.

Spin orbit interaction

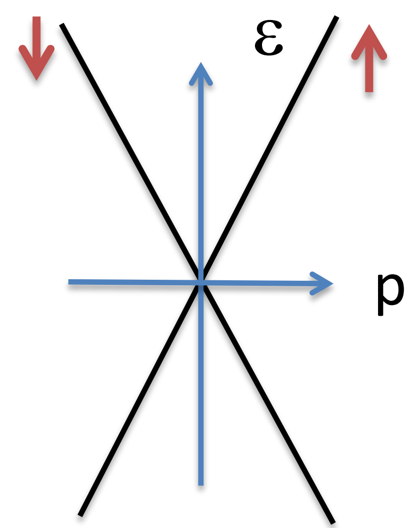
blue: unit vector perpendicular to the edge
red: momentum of the edge electron

$$H_{ls} = \lambda(\mathbf{s}\cdot[\mathbf{p}\times\mathbf{n}]) = \lambda s_z p_x$$



Mechanism of spin orbit interaction in atomic nuclei.

In condensed matter
the interaction is usually called
Rashba interaction.

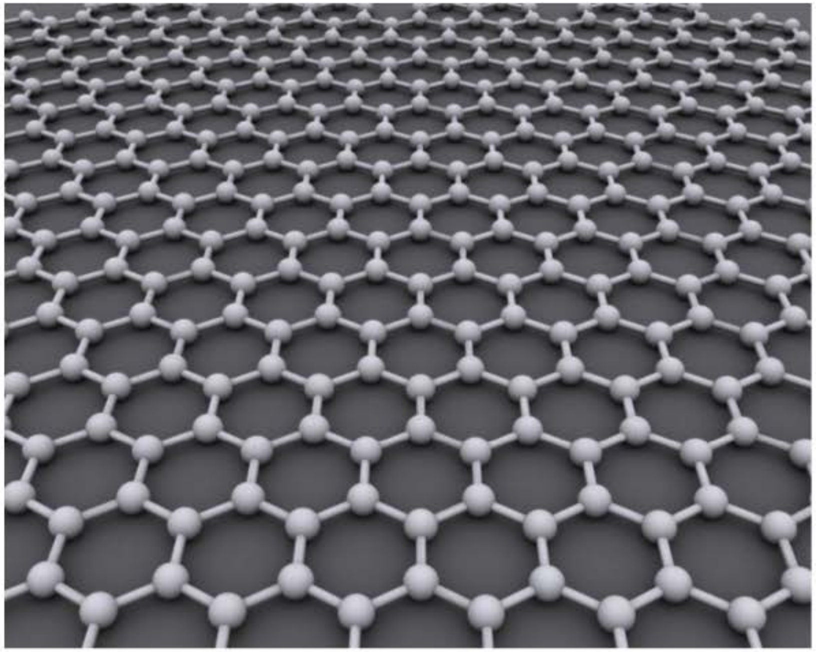


Is it possible to make $H=H_{ls}$ for 1D edge state?

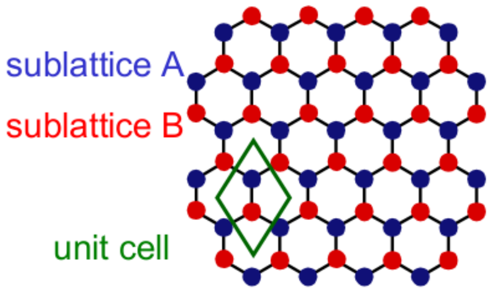
In this case the back scattering is forbidden if there are no magnetic impurities.
This implies a **dissipationless** edge state.

Graphene

Graphene is an atomic-scale honeycomb lattice carbon atoms monolayer. Conduction via π-orbitals of carbon. Tight binding.

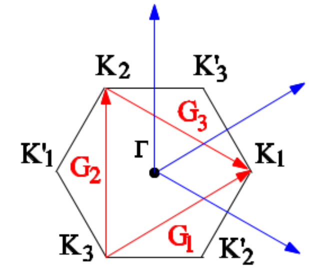


Graphene is a 2D material



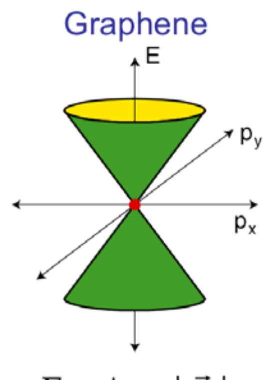
Two triangular sublattices, A and B.

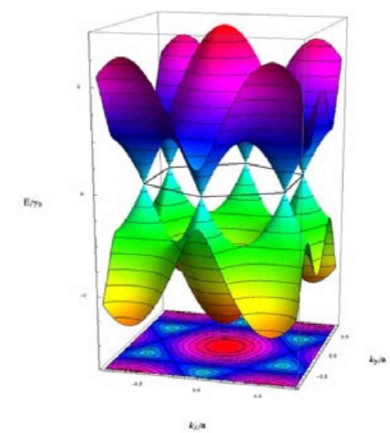
Brillouin zone of a triangular lattice.



Dispersion

Two Dirac cones of different parity.





$$H = v_F\, \boldsymbol{\sigma}\, \mathbf{p}$$

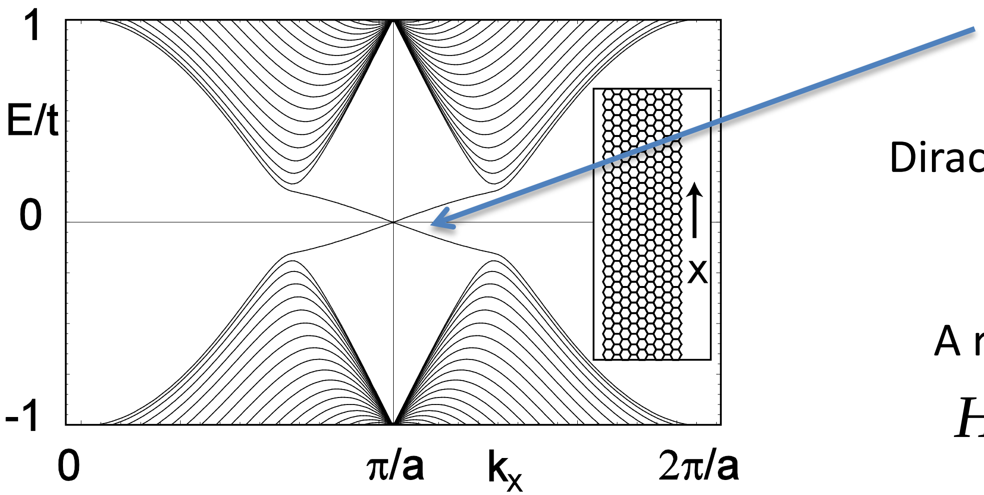
$$\mathbf{k} = \mathbf{K}_1 + \mathbf{p}$$

σ is pseudospin ½ related to two sublattices A and B.
Spin orbit interaction is zero, therefore usual spin s does not appear in the Hamiltonian.

Topological insulator

Kane & Mele, **2005**, graphene with spin orbit interaction, brute force numerical diagonalization.

$$H = graphene + H_{ls}$$



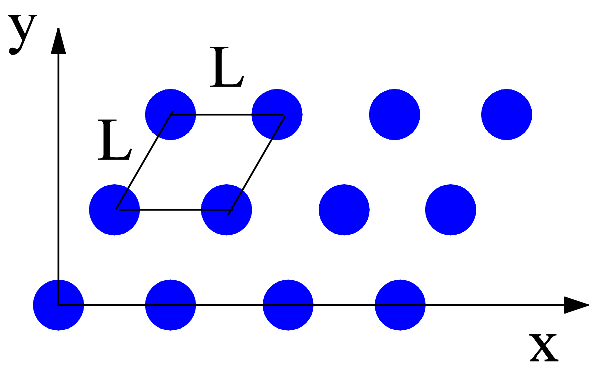
Dirac cone of “**dissipationless** ” edge states.

A realization of the Rashba Hamiltonian

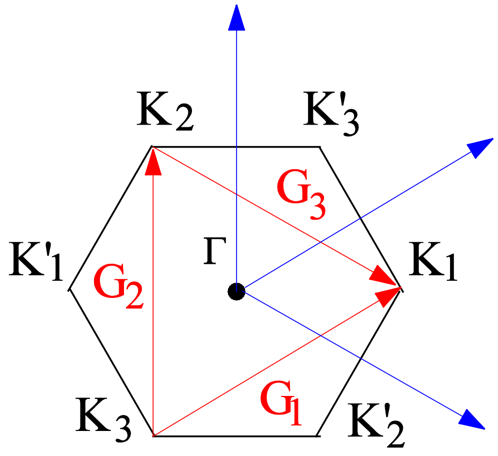
$$H_{ls} = \lambda(\mathbf{s} \cdot [\mathbf{p} \times \mathbf{n}]) = \lambda s_z p_x$$

Artificial Graphene

Consider a 2D electron gas in a potential U(r) with hexagonal (triangular) symmetry and spacing L.



$$H = \frac{p^2}{2m} + U(r)$$



Brillouin zone

Artificial graphene vs artificial "antigraphene"

Consider a simple periodic potential

$$U(r) = 2W\left[\cos\left(G_1 \cdot r\right) + \cos\left(G_2 \cdot r\right) + \cos\left(G_3 \cdot r\right)\right]$$

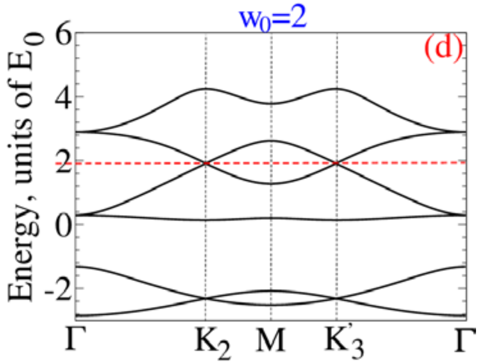
W < 0 describes an array of quantum dots and this is equivalent to **graphene**.
This regime emulates chemistry.

W > 0 describes an array of quantum antidots and this is equivalent to **"antigraphene".**
The **"antigraphene"** is more interesting and more feasible.
This regime cannot be realized in a natural chemical compound.

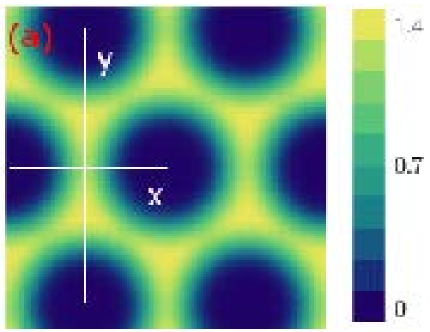
Artificial “antigraphene”

$$H = \frac{p^2}{2m} + U(r), \quad U(r) = 2W\left[\cos(G_1 \cdot r) + \cos(G_2 \cdot r) + \cos(G_3 \cdot r)\right]$$

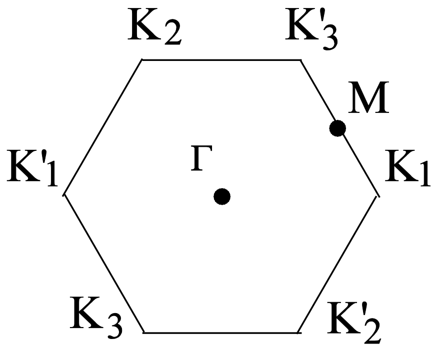
Numerical diagonalization of the Hamiltonian is straightforward.



Dispersion



Electron density map



Two major advantages of the artificial “antigraphene” compared to artificial graphene:

(i) Second pair of Dirac points.

(ii) Larger energy scale.

Topological insulator

Let us switch on the spin orbit interaction $\mathbf{H}_{so}$.
Whatever is the microscopic mechanism of the interaction the matrix element between two plane waves must be of the following form

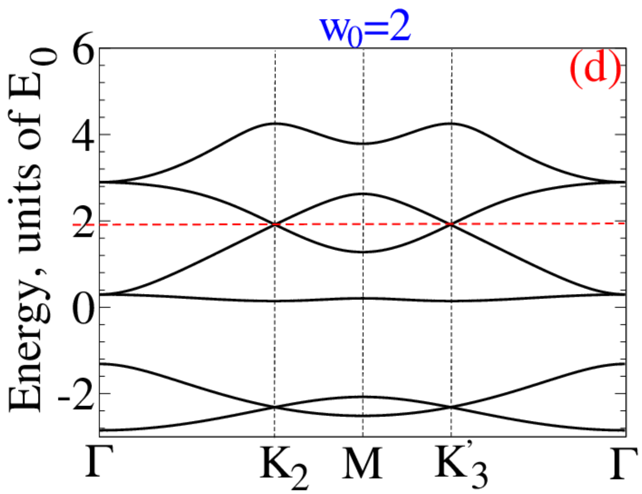
$$< \mathbf{p}_2 \,|\, H_{so} \,|\, \mathbf{p}_1 > \;\propto\; i([\mathbf{p}_1 \times \mathbf{p}_2] \cdot \mathbf{s})$$

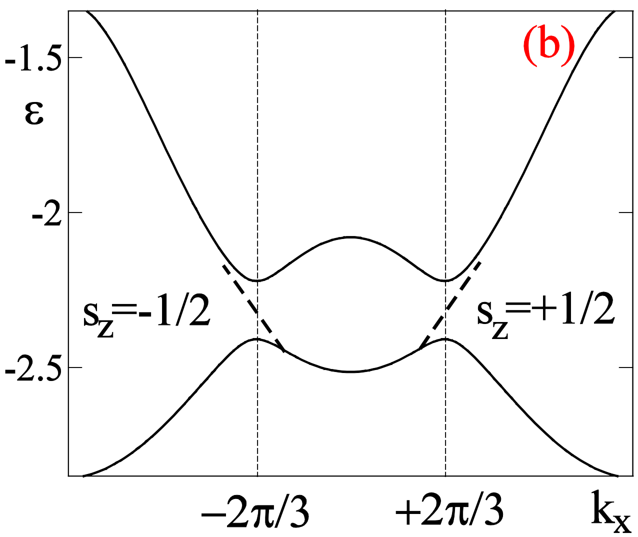
An additional condition follows from the **Bloch's theorem.**
Since the spin-orbit interaction has the period of the potential, the matrix element is nonzero only if

$$\mathbf{p}_2 - \mathbf{p}_1 = \pm \mathbf{G}_i$$

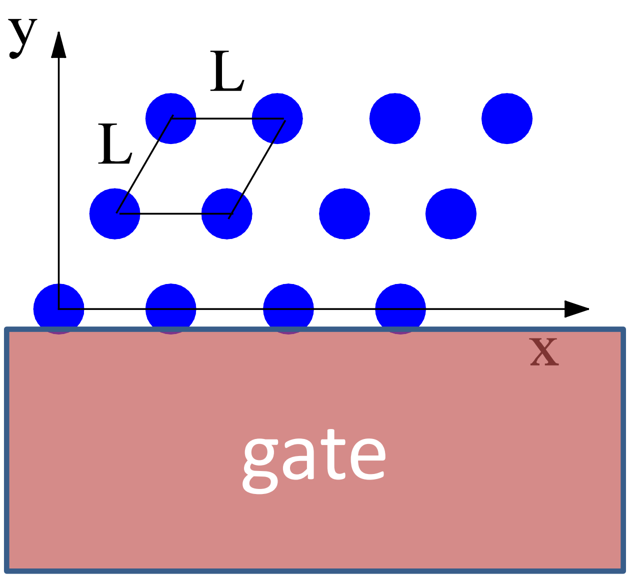
The interaction is small for electrons in GaAs, but for holes it is comparable with kinetic energy. Therefore we need hole doped GaAs.

Spin orbit opens a gap in the 2D bulk Dirac cone.

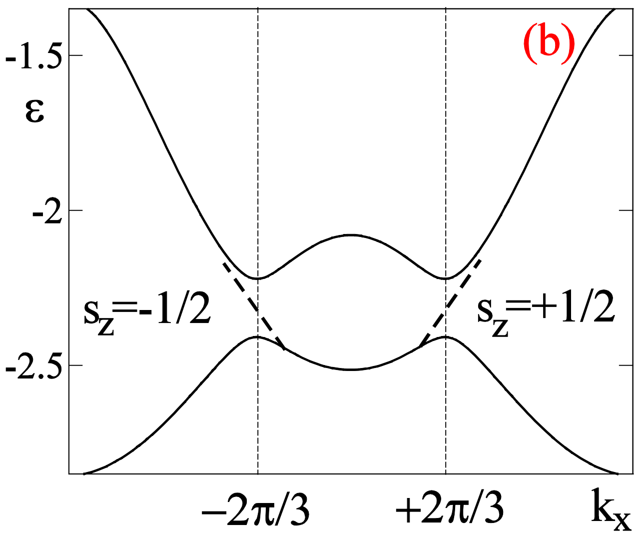




The 2D "sample" is restricted by electrostatic gates.



The gating results in the edge states and in the edge 1D "Dirac" cone.



Sz=+1/2 is the right mover and **Sz=-1/2** is the left mover.

These states are responsible for edge current without dissipation.

Points to note

1. Modern nanotechnology allows to create a **tunable topological insulator** using laterally patterned semiconductor heterostructure.

2. The ultimate **goal is a creation of dissipationless electronics** without superconductivity.

3. Artificial “**antigraphene**” is better than artificial graphene, 2^{nd} pair of Dirac points.

4. The artificial topological insulator cannot be built with electron doped semiconductors, but it can be built with hole doped.

Thank you