Chiral effects in rotating and accelerated medium

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General remarks

Original statements are based on common work with George Prokhorov and Oleg Teryaev (JINR, Dubna)

"On axial current in rotating and accelerated medium", arXiv:1805.12029 [hep-th]

"Effects of rotation and acceleration in the axial current: density operator vs Wigner function" arXiv:1807.03584
" Unruh effect for fermions from the Zubarev density operator" arXiv:1903.09697 [hep-th]
"Instability at Unruh temperature as manifestation of singularity in complex momentum plane", arXiv:1906.03529
"From the chiral vortical effect to polarization of baryons: A model" arXiv:1801.08183 [hep-th]

Motivation

We will consider thermodynamics with $\vec{\Omega}, \vec{a} \neq 0$
Motivated by interpretation of data on heavy-ion collisions
But we take it as a theoretical issue, interesting per se
Quantum Field Theory vs Quantum Statistical Physics

As for (future) applications we have in mind:

- heavy-particle polarization in chiral theory (massless quarks)
- thermolization and Unruh effect
  The idea put forward by P. Castorina, D. Kharzeev, H. Satz, "Thermal Hadronization and Hawking-Unruh Radiation in QCD" arXiv:0704.1426

Chiral effects

Pioneering paper: D.T. Son, P. Surowka "Hydrodynamics with Triangle Anomalies" arXiv:0906.5044 [hep-th]
Hydrodynamics: expansion in derivatives plus conservation laws:

$$\partial_\mu s^\mu = 0, \partial_\mu J_V^\mu \; = \; 0, \; \partial_\mu J_A^\mu \; = \; e^2 C_{anom}(\vec{E} \cdot \vec{H}), \partial_\mu \Theta^{\mu\nu} = \ldots$$

There are extra pieces in currents uniquely fixed by C_{anom}

$$(J_A^\mu)_{hydro} \; = \; n_A \cdot u^\mu + \mu^2 C_{anom} \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

where μ is the chemical potential.
Quantum anomaly unifies micro- and macro-scopic helical motions, an amusing effect

Trading equilibrium for new interaction

In thermodynamics,

$$\hat{H} \;\rightarrow\; \hat{H} - \mu\hat{Q}$$

Instead, could think in terms of new 4d interaction:

$$\mu\hat{Q} \;\rightarrow\; \mu u_\alpha \hat{J}^\alpha, \quad or \quad eA_\alpha \;\rightarrow\; eA_\alpha + \mu u_\alpha$$

A.V. Sadofyev, V.I. Shevchenko, V.I.Z., arXiv:1012.1958

Starting from the triangle anomaly one immediately reproduces chiral effects of Son&Surowka
(+"new" conservation laws for ideal fluid)

Some details

Since a paper by Gell-Mann we know that in U(1) case chiral anomaly can be reformulated as

$$\partial_\alpha \left(J_5^\alpha - e^2 C_{anom} \epsilon^{\alpha\beta\gamma\delta} A_\beta \partial_\gamma A_\delta \right) = 0$$

Making the substitution $eA_\alpha \to eA_\alpha + \mu u_\alpha$ reproduces all the chiral effects in the current

However, we are not allowed to change the anomaly since the "effective interaction" does not change short-distance physics. This imposes extra conservation laws.

Quantum-statistical approach

The field is started by A. Vilenkin (1980), for rotation:

$$< J^\mu(\vec{x}) > = Tr\Big(\hat{\rho} J^\mu(\vec{x}, t)\Big)$$

where $J^\mu = \frac{1}{2}[\bar{\Psi}, \gamma^\mu \Psi]$ is the current density operator and

$$\hat{\rho} = C \exp\Big(-\beta(\hat{H} - \hat{\vec{M}} \cdot \vec{\Omega} - \Sigma_i \mu_i \hat{\vec{N}}_i)\Big)$$

$\beta = T^{-1}$, $\hat{\rho}$ is the statistical operator (see Landau&Lifshitz)
$\hat{\vec{M}}$ is the angular momentum, $\vec{\Omega}$ is the angular velocity
μ_i is chemical potential, $\hat{N}_i$ is number of charged particles
$\hat{\rho}$ is built on conserved operators

Sommerfeld Integrals

Vilenkin succeeded to reduce the matrix element of the axial current for massless fermions to:

$$< J^5 > = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \epsilon^2 d\epsilon \cdot \left(\frac{1}{1 + e^{\beta(\epsilon - (\mu + \Omega/2))}} - \frac{1}{1 + e^{\beta(\epsilon - (\mu - \Omega/2)}} \right)$$

The integral is one of so called Sommerfeld integrals, and

$$< J_N > = \left(\frac{\mu^2 \Omega}{4\pi^2} + \frac{\Omega^3}{48\pi^2} + \frac{T^2 \Omega}{12} \right)$$

(for a single Weyl fermion of unit charge):

Zubarev density operator

In the spirit of the discussion above one introduces the Lorentz-invariant density operator

$$\hat{\rho} \; = \; \frac{1}{Z} \exp\Big(-\beta_\alpha \hat{P}^\alpha + \frac{1}{2} \bar{\omega}_{\alpha\beta} \hat{J}^{\alpha\beta} + \mu \beta_\alpha J^\alpha \Big)$$

where $\beta_\alpha = u_\alpha / T$, $\bar{\omega}_{\alpha\beta} = -1/2 \big(\partial_\alpha \beta_\beta - \partial_\beta \beta_\alpha \big)$
D. N. Zubarev, TMF (1979)....reviewed F.Becattini et al.
e-Print: arXiv:1704.02808
Perturbatively one gets the same result as above :

$$< J^5_\alpha > = \; \Big(\frac{\mu^2}{2\pi^2} + \frac{T^2}{6} \Big) \omega_\alpha$$

where $\omega_\alpha = (1/2) \epsilon_{\alpha\beta\gamma\delta} u^\beta \partial^\gamma u^\delta$

Sommerfeld integrals vs QFT anomalies

Both Sommerfeld integrals and QFT anomalies produce one-loop exact results for the currents. Dependence on external parameters is exhausted by polynomials, with no obvious reason for that. We considered an example when both derivations are complete (chiral anomaly for axial current and a known Sommerfeld integral).

Review of further discussions see, in particular, M.Stone, arXiv:1804.08668.

We proceed to very recent examples of finite perturbative series, related to acceleration. And find further examples of one-loop exact results.

Acceleration, $a = const$

Consider first no rotation, no charge case.

$$\hat{\rho} \;=\; \frac{1}{Z}\exp\Big(-\beta_\mu \hat{P}^\mu - \alpha_z \hat{K}^z\Big)$$

where $\alpha_\mu \;=\; u^\alpha \partial_\alpha u_\mu / T$, $\hat{K}^z$ is the boost operator
The density operator looks rather paradoxical:

- constant a implies horizon, but we work in Minkowski
- first-order interaction is exponentiated, but can be put to zero by choice of coordinates

Perturbative technique is developed by F. Becattini et al
see, F. Becattini, e-Print: arXiv:1712.08031 + references
It is desirable to have independent checks.

Massless fermions. One-loop energy density

We have calculated one-loop energy density for massless spin-1/2, spin-0. As well as first correction due to a non-vanishing mass

$$\rho_{spin\ 1/2} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} + O(a^6)$$

First term is the Stefan-Boltzmann, the a^4 term is pure QFT (no temperature), second term was calculated earlier

$$\rho_{spin\ 1/2} = \frac{1}{240}\left(T^2 - \left(\frac{a}{2\pi}\right)^2\right)(17a^2 + 28\pi^2 T^2) + O(a^6)$$

Energy density vanishes at the Unruh temperature in fact, in all four cases considered

Comments on the energy density

- Temperature $T = T_{Unruh}$ implies Minkowski vacuum, and if a tensor of GR vanishes in one frame, it vanishes in any other frame as well
- No direct proof of absence of $O(a^6)$ term since no associated anomaly known. However vanishing of $O(a^6)$ was expected.
- In fact, the result could have been compared with J.Dowker hep-th/940159 who evaluated energy density in non-trivial metric. Results fully agree.

Perturbative calculation in Minkowski space reproduces exactly one-loop Casimir effect due to the horizon

Acceleration as an imaginary chemical potential

Wigner-function formalism allows to generalize Fermi-distribution to the cases of $\vec{\Omega}, \vec{a} \neq 0$.
For simplicity consider $\vec{\Omega}||\vec{a}$

Rotation (as mentioned also by some other authors):

$$\mu \rightarrow \mu \pm \frac{\Omega}{2}$$

Acceleration:

$$\mu \rightarrow \mu \pm \frac{\Omega}{2} \pm i\frac{|a|}{2}$$

Imaginary acceleration looks unexpected but in fact fits FT

Examples

To be more precise let us give examples:

$$< J^5 > = \int \frac{d^3p}{(2\pi)^3}\Big(n_F(E_p - \mu - \Omega/2) - n_F\big(E_p - \mu + \Omega/2\big) + n_F\big(E_p + \mu - \Omega/2\big) - n_F\big(E_p + \mu + \Omega/2\big)\Big)$$

Valid also for massive particles. Another example $(\mu, \Omega = 0)$

$$\rho_{Wigner} = \int \frac{d^3p}{(2\pi)^3}\Big(n_f(E_p + ia/2) + n_F(E_p - ia/2)\Big)$$

Expansion in a does not have imaginary part, it is canceled. However analytical properties are sensitive to ia

Subtraction term

Pert. th. gives calculable corrections to the Wigner-function approach. Final answer:

$$\rho = \frac{a^4}{120\pi^2} + + \frac{T^4}{\pi^2}\left(\int_{-\infty}^{+\infty} \frac{x^3 dx}{e^{x+iy}+1} + (y \rightarrow -y) \right) \quad (1)$$

$$+2iy\frac{T^4}{\pi^2}\left(\int_{-\infty}^{\infty} \frac{x^2 dx}{e^{x+iy}+1} - (y \rightarrow -y) \right) \quad (2)$$

Subtraction term +"dispersion terms" which are in fact novel Sommerfeld-type integrals

No free parameters, coincides with direct Rindler-space calculation

Closer to phenomenology: models

In case of rotation, superfluidity seems right toy model (because of low viscosity of plasma)
In the model of Son+Stephanov everything is explicit (O. Teryaev+VZ (2017)+references therein):

- Rotation is transferred to vortices. Momentum carried by the cores is in exact correspondence with (anomalous) Chiral Vortical Effect (CVE)
- CVE is assoiated with heavy particles, which are originally not included into the model
- Chiral Magnetic Effect is dissipation-free but is a displacement current (no “real” motion)

Unruh Instability ?

Probably, if one starts from say accelerated medium and no thermal particles, this state decays into the Minkowski vacuum (acceleration +Unruh temperature)

Conclusions

- Generalization of the standard density operator to include acceleration was checked and found to work numerically
- The approach can be converted into a field theory with finite perturbative expansions. With no obvious connection to anomalies. But the work is at the beginning