Gravitational waves from spin-3/2 fields
Hunting SUSY in the sky

Karim Benakli

LPTHE, Sorbonne Université - CNRS, Paris

K. B., Yifan Chen, Peng Cheng, Gaëtan Lafforgue-Marmet arXiv:1811.11774

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Outlines

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Introduction: spin 0, 1/2 and 1

- Minkowski space: rotational symmetry invariance⇒ fundamental particles are characterized by their spin
- Theoretical model developed decades ago and confirmed since.
- Fundamental quantum fields:

Spin 0: the BEH scalar fills the vacua and localizes the Compton wave lengths of particles interacting with it.

Spin 1/2 fermions are building block of matter; intrinsically quantum in nature; enforce chirality.

Spin 1 glue the fermions together and mediate distant interactions.

Introduction: spin 2

- Gravity furnishes the playground for all the fields: spacetime.
- Direct observation of GW recent (indirect signature seen before).
- Electromagnetic waves made of photons $\longrightarrow$ gravitational waves made of gravitons.
- Gravitational waves have ± 2 helicities $\longrightarrow$ gravitons have spin 2

Therefore, in our description of nature we have fundamental particles with spin: 0, 1/2, 1, ~~$\frac{3}{2}$~~, 2.

Why not a fundamental spin-3/2 particle?

The missing particle

What would be the fundamental spin-3/2 particle? Why not seen yet?

- charged massive spin 3/2 with minimal coupling in Minkowski space suffers from the "Velo-Zwanziger problem": faster-than-light propagation.
- Spin 3/2 has no e-m but gravitational interactions.
- Have only gravitational interactions? why not look at signatures of this new particle in gravitational waves ?

Spin 3/2

We construct a spin-3/2 field from the product

$$(\frac{1}{2},\frac{1}{2})\otimes(\frac{1}{2},0)=\frac{1}{2}\oplus(1\otimes\frac{1}{2})=\frac{1}{2}\oplus\frac{1}{2}\oplus\frac{3}{2}\ .$$

In general, we will therefore decompose ψ_μ into four spinors corresponding to:

a) the helicity-$\frac{3}{2}$ states $\psi^{\mu}_{\frac{1}{2}}$,

b) the helicity-$\frac{1}{2}$ states $\psi_{\frac{1}{2}}$,

c) and two remaining un-physical spinors that are projected out by two constraints:

$$\begin{aligned}\gamma^\mu\psi_\mu &= 0\\ \partial^\mu\psi_\mu &= 0\end{aligned}$$

Spin 3/2

We start with the equations of motion and constraints :

$$
\begin{aligned}
(i\partial\!\!\!/ - m_{3/2})\psi_\mu &= 0 \\
\gamma^\mu \psi_\mu &= 0 \\
\partial^\mu \psi_\mu &= 0
\end{aligned}
$$

These can be obtained from the Rarita-Schwinger Lagrangian:

$$
\mathcal{L} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_5 \gamma_\nu \partial_\rho \psi_\sigma - \frac{1}{4} m_{3/2} \bar{\psi}_\mu [\gamma^\mu, \gamma^\nu] \psi_\nu \ .
$$

the corresponding energy-momentum tensor is:

$$
\begin{aligned}
T_{\alpha\beta} &= \frac{e_{c\alpha}}{2e}\frac{\delta(e\mathcal{L})}{\delta e_c^\beta} + (\alpha \leftrightarrow \beta) \\
&= \frac{i}{4}\bar{\psi}_\mu \gamma_{(\alpha}\partial_{\beta)}\psi^\mu - \frac{i}{4}\bar{\psi}_\mu \gamma_{(\alpha}\partial^\mu \psi_{\beta)} + h.c.
\end{aligned}
$$

Spin 3/2

One can use these constraints to obtain the physical degrees of freedom $\psi^\mu_{\frac{3}{2}}$ and $\psi_{\frac{1}{2}}$. We can obtain them directly from ψ_μ by

$$\begin{aligned} \psi_{\frac{1}{2}} &= \sqrt{\frac{3}{2}}\frac{m}{k}\gamma^0\ \gamma^i\psi_i \\ \psi^\mu_{\frac{3}{2}} &= \mathcal{P}^{\mu\nu}_{\frac{3}{2}}\psi_\nu \end{aligned}$$

with k the modulus of the three-momentum and $\mathcal{P}^{\mu\nu}_{\frac{3}{2}}$ an appropriate projector. The transverse and longitudinal degrees of freedom verify a Dirac equation:

$$\begin{aligned} (i\partial\!\!\!/ - m_{3/2})\psi^\mu_{\frac{3}{2}} &= 0\ , \\ (i\partial\!\!\!/ - m_{3/2})\psi_{\frac{1}{2}} &= 0\ . \end{aligned}$$

Gravitinos

A motivated fundamental spin-3/2 particle is the "gravitino"

This is the superpartner of the graviton in supergravity.

- helicity $\pm 1/2$ originate from goldstones absorbed in a super-Higgs mechanism.
- Higgs mechanism origins insure no-pathological behavior.
- charged massive gravitino $\longrightarrow$ gauged supergravity $\longrightarrow$ curved space-time from: no Velo-Zwanziger problem.
- gravitinos can be a component of Dark Matter (for a large range of masses).

Gravitational Waves (GWs):

- Compatibility of gravity with special relativity $\Rightarrow$ changes in gravitating sources are communicated by gravitational waves
- What are gravitational waves? they are fluctuations of the metric:

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}$$

with:

a) It is "small" in a coordinate system where diagonal elements of $g^0_{\mu\nu}$ are $\mathcal{O}(1)$

$$|h_{\mu\nu}| \ll 1$$

b) $h_{\mu\nu}$ varies on time or length scales much shorter than those of $g^0_{\mu\nu}$.

Some properties of GW:

- GW are created by the acceleration of the quadrupole moments of mass distributions + weak gravity couplings $\Rightarrow$ need sources with big masses to get sensible signal.
- "GW $\leftrightarrow$ matter interactions "are much weaker than "e-m $\leftrightarrow$ matter interactions" $\Rightarrow$ difficult to detect
- "GW $\leftrightarrow$ matter interactions "are much weaker than "e-m $\leftrightarrow$ matter interactions" $\Rightarrow$ probe of parts of the Universe not possible through e-m waves.
- Wavelengths of GW of the order or bigger than of the size of sources: no image of the sources but information about their dynamics.
- We observe directly the strain: fall off as inverse distance $\Rightarrow$ slower fall off than EM

GW Sources:

Following the search techniques, GW sources are classified in different categories:

- Coalescing binaries: chirp-like signals as those seen by LIGO and Virgo.
- Unmodeled bursts: as core supernovae collapse, long transient waves Unseen.
- Continuous signals: loss of energy from rapidly rotating (not axisymmetric) neutron stars.
- Stochastic backgrounds: designating cases where h_{ij} are random variables characterized by statistical averages.

Stochastic GW Sources:

incoherent superposition of a variety of independent, unresolved, and uncorrelated sources. Different kinds:

- Astrophysical sources superposition of signals from all the binaries, neutrons stars, collapsing supernovae
- GW background from inflation: quantum primordial gravity fluctuations expanded by inflation (searches of CMB B-mode);
- Stochastic cosmological GW background after Inflation: violent processes can lead under certain circumstances to generation of GW. Examples: first-order phase transition, non-adiabatically varying fields during preheating.

GW: linear perturbations

- In the vacuum, GW have simple form in the transverse-traceless (TT) gauge:

$$h_{0\mu} = 0, \qquad h^i_i = 0, \qquad \partial^i h_{ij} = 0$$

- TT coordinates move with the wave but proper distance between two freely falling particles oscillate.
- Only two modes "transverse-transverse modes" carry energy and propagate at the speed of light: the helicities "+" and "x". These are tidal modes squeezing along one axis and stretching along another.

GW: linear perturbations

- In the presence of a source $T^{\mu\nu}$, we can not impose (TT) gauge but we use the Lorenz gauge:

$$\partial^\mu h_{\mu\nu} = 0$$

- Then, outside the source, in order to find the GW that reach the observer in the TT gauge, we act on h_{ij} with the projector:

$$\Lambda_{ij,lm}\, h_{lm} = h_{ij}^{TT}$$

- To avoid non-local operation, projection done in momentum space where the TT projection tensor satisfies

$$\Lambda_{ij,lm}(\hat{\mathrm{k}})k^l = \Lambda_{ij,lm}(\hat{\mathrm{k}})k^m = 0.$$

GW energy spectrum

- The energy carried by GW is given by:

$$\rho_{GW} = \frac{1}{32\pi G}\langle \dot{h}_{ij}(t,\mathbf{x})\dot{h}_{ij}(t,\mathbf{x})\rangle$$

- pluging the solution in the momentum space, using Green function:

$$h_{ij}\,(\mathrm{k},t) = \frac{16\pi G}{a(t)k}\int_{t_I}^{t} dt' \sin\left[k(t-t')\right] a(t')\Pi_{ij}^{TT}(\mathrm{k},t').$$

- Π_{ij}^{TT} is the anisotropic energy-momentum tensor

$$\Pi_{ij}^{TT}(\mathrm{k},t) = \Lambda_{ij,lm}(\hat{\mathrm{k}})(T^{lm}(\mathrm{k},t) - pg^{lm}), \qquad (1)$$

- Spectrum:

$$\frac{d\rho_{GW}}{d\log k} = \frac{2Gk^3}{\pi a^4(t)}\int_{t_I}^{t} dt' \int_{t_I}^{t} dt'' a(t')a(t'')\cos[k(t'-t'')]\Pi^2(k,t',t'').$$

- with the unequal-time correlator of Π_{ij}^{TT} given by:

$$\langle \Pi_{ij}^{TT}(\mathrm{k},t)\Pi^{TT\,ij}(\mathrm{k}',t')\rangle \equiv (2\pi)^3\Pi^2(k,t,t')\delta^{(3)}(\mathrm{k}-\mathrm{k}').$$

k dependence of source terms

- TT projection on the anisotropic energy-momentum tensor

Scalar : $\int d^3\mathrm{p}\ \Lambda_{ij,lm}(\hat{\mathrm{k}})[\partial^l \sigma(\mathrm{p},t)\partial^m \sigma(\mathrm{p}',t)]$,

Fermion : $\int d^3\mathrm{p}\ \Lambda_{ij,lm}(\hat{\mathrm{k}})[\bar{\psi}(\mathrm{p},t)\gamma^l \partial^m \psi(\mathrm{p}',t)]$,

Vector : $\int d^3\mathrm{p}\ \Lambda_{ij,lm}(\hat{\mathrm{k}})[\partial^l A^\mu(\mathrm{p},t)\partial^m A_\mu(\mathrm{p}',t)+\partial^\mu A^l(\mathrm{p},t)\partial_\mu A^m(\mathrm{p}',t)$
$+\ldots]$

- using $\mathrm{p}' = \mathrm{p}+\mathrm{k}$, the linear dependence on k is projected out for spin-0 and spin-1/2:

$$\begin{aligned} & \Lambda_{ij,lm}(\hat{\mathrm{k}}) && \partial^l \sigma(\mathrm{p},t)\partial^m \sigma(\mathrm{p}',t) \\ = & \Lambda_{ij,lm}(\hat{\mathrm{k}}) && p^l \sigma(\mathrm{p},t)(p^m + k^m)\sigma(\mathrm{p}',t) \\ = & \Lambda_{ij,lm}(\hat{\mathrm{k}}) && p^l \sigma(\mathrm{p},t)p^m \sigma(\mathrm{p}',t). \end{aligned}$$

- The linear dependence on k is preserved for spin-1:

$$\begin{aligned} & \Lambda_{ij,lm}(\hat{\mathrm{k}}) && \partial^\mu A^l(\mathrm{p},t)\partial_\mu A^m(\mathrm{p}',t) \\ = & \Lambda_{ij,lm}(\hat{\mathrm{k}}) && p^\mu(p_\mu + k_\mu)A^l(\mathrm{p},t)A^m(\mathrm{p}',t). \end{aligned}$$

$\Rightarrow$ different k dependance for different spins.

Spin-3/2 wave functions

Wave-functions:

$$\tilde{\psi}^{\mu}_{\mathrm{p},\lambda}(t) = \sum_{s=\pm 1, l=\pm 1,0} \langle 1, \frac{1}{2}, l, \frac{s}{2} | \frac{3}{2}, \lambda \rangle \epsilon^{\mu}_{\mathrm{p},l} \mathbf{u}^{(|\lambda|)}_{\mathrm{p},\frac{s}{2}}(t), \tag{2}$$

The linear dependence on k is preserved under projection:

$$\Lambda_{ij,lm}(\hat{\mathrm{k}}) \epsilon^{\mu}_{\mathrm{p}} (p_{\mu} + k_{\mu}) \gamma^{l} \epsilon^{m}_{\mathrm{p}'}. \tag{3}$$

Wave-function of Helicity-1/2 component:

$$\tilde{\psi}^{\mu}_{\mathrm{p},\pm\frac{1}{2}}(t) = \sqrt{\frac{2}{3}} \epsilon^{\mu}_{\mathrm{p},0} \mathbf{u}^{(\frac{1}{2})}_{\mathrm{p},\pm\frac{1}{2}}(t) + \sqrt{\frac{1}{3}} \epsilon^{\mu}_{\mathrm{p},\pm 1} \mathbf{u}^{(\frac{1}{2})}_{\mathrm{p},\mp\frac{1}{2}}(t). \tag{4}$$

The equivalence theorem: in the relativistic limit $p \gg m_{3/2}$,

$$\epsilon^{\mu}_{\mathrm{p},0} = \frac{1}{m_{3/2}} (p, \sqrt{p^2 + m^2_{3/2}} \hat{\mathrm{p}}) = \frac{p^{\mu}}{m_{3/2}} + O(\frac{m_{3/2}}{p}). \tag{5}$$

Sources: Helicity-1/2 Component

The GW spectrum is given by:

$$\frac{d\rho_{GW}}{d\log k} = \frac{2Gk^3}{\pi a^4(t)} \int_{t_I}^{t} dt' \int_{t_I}^{t} dt'' a(t')a(t'')\cos[k(t'-t'')]\Pi^2(k,t',t'').$$

- Fermionic modes fill a Fermi sphere with radius k_F. The spectrum is dominated by the most energetic, thus parameterized by k_F.
- As $\epsilon^{\mu}_{\mathrm{p},0} \sim \frac{p^{\mu}}{m_{3/2}}$, the dominant contribution to $\langle \Pi^{TT}_{ij}(\mathrm{k},t)\Pi^{TTij}(\mathrm{k}',t')\rangle$ come from terms with most $\epsilon^{\mu}_{\mathrm{p},0}$.
- Terms with $\epsilon^{\mu}_{0}\epsilon^{\mu}_{0}\epsilon^{\mu}_{0}\epsilon^{\mu}_{0}$ and $\epsilon^{\mu}_{0}\epsilon^{\mu}_{0}\epsilon^{\mu}_{0}\epsilon^{\mu}_{\pm 1}$ vanish by simply replacing ϵ^{μ}_{0} with $\frac{p^{\mu}}{m_{3/2}}$ due to the Levi-Civita symbol in $-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_5\gamma_{\nu}\partial_{\rho}\psi_{\sigma}$.
- The dominant result comes then from term with $\epsilon^{\mu}_{0}\epsilon^{\mu}_{0}\epsilon^{\mu}_{\pm 1}\epsilon^{\mu}_{\pm 1}$.

Result:

The spectrum density per logarithm of frequency $\frac{d\rho_{GW}}{d\log k}(k,t)$ is given by:

$$\frac{Gk^3}{\pi^3 a^4(t)} \times \int dp d\theta \, K(p,k,\theta,m_{3/2})\{|I_c(k,p,\theta,t)|^2 + |I_s(k,p,\theta,t)|^2\}$$

- $I_{c,s}$ cfrom wave-functions are universal for fermions with the same Fermi-radius.
- Momenta scaling factor K:

$$\begin{aligned} \text{Spin-1/2}: &\quad p^4 \sin^3\theta, \\ \text{Helicity-3/2}: &\quad p^2 k^2 f_{3/2}(\theta,\theta'), \\ \text{Helicity-1/2}: &\quad \frac{p^4 k^2}{m_{3/2}^2} f_{1/2}(\theta,\theta'). \end{aligned}$$

- For spin-1/2, spectrum scales k^3. For spin-3/2, spectrum scales k^5 near the peak and the amplitude gets enhanced by $\frac{k_F^2}{m_{3/2}^2}$.

Example: Polonyi Model

$$\mathcal{K} = |z|^2 - \frac{|z|^4}{\Lambda^2},$$
$$\mathcal{W} = \mu^2 z + \mathcal{W}_0,$$

- We use a Polony-Like model with spontaneous SUSY breaking with positive cosmological constant.
- Near the minimum, $m_{3/2} \gg \mathcal{H}$. We can use the quantization of gravitino in the flat limit.
- Equivalence theorem can be used for $m_{3/2} \ll p \ll \sqrt{F_z}$.
- Effective Yukawa coupling between scalar and helicity-1/2 gravitino:

$$\tilde{g} = \frac{m_z^2}{2\sqrt{3} m_{3/2} M_{Pl}} = \frac{m_z^2}{2F} \ll 1. \tag{6}$$

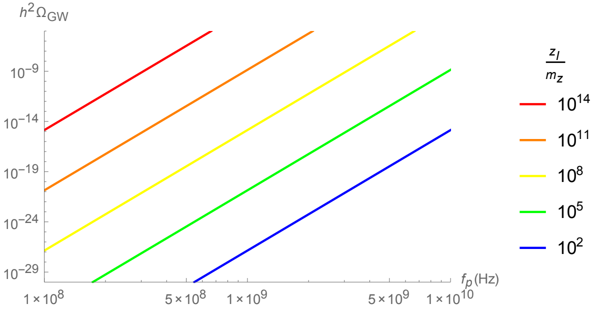
- Peak frequency now:

$$f_p \simeq 6 \cdot 10^{10} \tilde{g}^{\frac{1}{2}} \text{Hz}. \tag{7}$$

Corresponding GW spectrum

- Current spectrum:

$$h^2\Omega_{GW}(f_p) \simeq 3 \cdot 10^{-10} (\frac{f_p}{6 \cdot 10^{10}\text{Hz}})^{12} (\frac{z_I}{m_z})^2. \quad (8)$$



- Typically at ultra-high frequency, needs new design of GW detectors in the future.

Summary

- Is there a fundamental particle with spin-3/2 in Nature? May be difficult to detect because gravitationally coupled? Indirect detection signals: decays? signature in GW?
- Candidate for fundamental with spin-3/2 particle: the gravitino.
- The non-adiabatic processes during preheating could be sources of stochastic GW
- Gases made of particles with different spins lead to different dependences in momenta of the gravitational waves.
- The helicity-1/2 component dominates the GW production. Near the peak, the scaling goes k^5 instead of k^3 like spin-1/2 fermions.