Gluons in Two-Colour QCD at High Baryon Density

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- Definitions and details of simulation
- Zero-momentum gluon propagators versus μ_B and screening masses
- Comparison with perturbation theory
- Gribov-Stingl fit for the dressing function
- $D_L - D_T$ as the infrared sensitive quantity
- D_L and D_T at $p_4 \neq 0$
- Conclusions

We study QCD with $N_c = 2$, $N_f = 2$, improved gauge field action and standard staggered fermion action; $a = 0.044$ fm, $m_\pi = 740$ MeV, $N_s = 32, N_t = 32$.

Landau gauge fixing condition is

$$(\partial\mathcal{A})_x = \sum_{\mu=1}^{4} \left(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}\right) = 0 \ , \tag{1}$$

which is equivalent to finding an extremum of the gauge functional

$$F_U(g) = \ \frac{1}{4V} \sum_{x\mu} \frac{1}{2} \ \mathrm{Tr} \ U^g_{x\mu} \ , \tag{2}$$

with respect to gauge transformations g_x .

We adopt the strategy of finding gauge copies being as close as possible to the global maximum of the gauge fixing functional - so called **absolute Landau gauge**.

Features of our approach are as follows:

- efficient optimization algorithm - simulated annealing.
- many gauge copies per MC configuration with the choice of the one with maximal F_U - *best copy*.
- Gribov copy effects are small (5 and 30 copies per confuguration were used)

$$D_{\mu\nu}(p) = \frac{1}{3a^4 N_s^3 N_t} \sum_{b=1}^{3} \langle \tilde{A}_\mu^b(p) \tilde{A}_\nu^b(-p) \rangle = D_L(p) P_{\mu\nu}^L + D_T(p) P_{\mu\nu}^T + \alpha \frac{p_\mu p_\nu}{p^4}$$

$$P_{\mu\nu}^T = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \frac{p_i p_j}{|\vec{p}|^2} \end{pmatrix} \qquad P_{\mu\nu}^L = \begin{pmatrix} \frac{|\vec{p}|^2}{p^2} & -\frac{p_4 p_i}{p^2} \\ -\frac{p_4 p_j}{p^2} & -\frac{p_i p_j p_4^2}{|\vec{p}|^2 p^2} \end{pmatrix}$$

$$p_4 \neq 0, \vec{p} \neq 0 \qquad D_L \sim \frac{p^2}{|\vec{p}|^2} \langle A_4 A_4 \rangle ; D_T \sim \frac{1}{2} \left(\sum_{i=1}^{3} \langle A_i A_i \rangle - \frac{p_4^2}{|\vec{p}|^2} \langle A_4 A_4 \rangle \right)$$

$$p_4 = 0, \vec{p} \neq 0 \qquad D_L \sim \langle A_4 A_4 \rangle ; D_T \sim \frac{1}{2} \sum_{i=1}^{3} \langle A_i A_i \rangle$$

$$p_4 = 0, \vec{p} = 0 \qquad D_L \sim \langle A_4 A_4 \rangle ; D_T \sim \frac{1}{3} \sum_{i=1}^{3} \langle A_i A_i \rangle$$

$\frac{1}{3a^4 N_s^3 N_t}$ is omitted, $p^2 = p_4^2 + |\vec{p}|^2$, $\sum_{b=1}^{3} \langle \tilde{A}_\mu^b(p) \tilde{A}_\nu^b(-p) \rangle = \langle A_\mu A_\nu \rangle$

If $p_4 \neq 0, \vec{p} = 0$ then we arrive at

$$2D_T + D_L \sim \sum_{i=1}^{3} \langle A_i A_i \rangle, \qquad \text{whereas} \quad \langle A_4 A_4 \rangle = 0 \ ,$$

and general considerations are not sufficient to find D_T and D_L separately. However, in the Landau gauge

$$A_4 \ = \ - \sum_{i=1}^{3} \frac{p_i A_i}{p_4}$$

and rotation invariance implies

$$\lim_{|\vec{p}| \to 0} \left(\frac{p_m p_n}{|\vec{p}|^2} - \frac{\delta_{mn}}{3} \right) \langle A_m(p) A_n(-p) \rangle = 0$$

Thus we ontain $D_T = D_L \sim \frac{1}{3} \sum_{i=1}^{3} \langle A_i A_i \rangle$

Screening mass can be defined in either of the two ways:

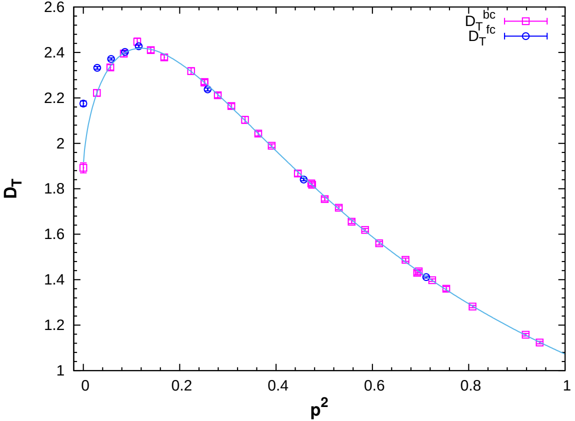
$$m^2_{Linde} \qquad m^2_{Linde} = G^{-1}(0, \vec{p} \to 0)$$

$$m^2_{FarDist} \qquad G^{-1}(0, |\vec{p}|) = \frac{1}{Z_G}(m^2_{FarDist} + |\vec{p}|^2 + \underline{O}(|\vec{p}|^4)) \qquad (3)$$

Strictly speaking, only $m_{FarDist}$ is related to the far-distant behavior

$$G(x_4, |\vec{x}|) \sim \exp(-m_{FarDist}|\vec{x}|) \quad \text{when} \quad |\vec{x}| \to \infty$$

If the Maclaurin expansion of $G^{-1}(0, |\vec{p}|)$ in $|\vec{p}|^2$ works well, then these definitions coincide up to the normalization constant $\sqrt{Z_G}$.
Otherwise it may turn out that m^2_{Linde} exists, whereas $m^2_{FarDist}$ — NOT.
An example may be provided by finite-temperature theories if ultrasoft ($0 < |\vec{p}| < 100$ MeV) gluon-field modes are not taken into account.



Pure-gauge $SU(2)$ theory: $D_T(|\vec{p}|) \sim c_0 + c_1|\vec{p}|^{2/3}$;
The magnetic screening mass, if it exists, is determined by dynamics at the scale $|\vec{p}| < 100$ MeV. In the case under study, $p_{min} = 800$ MeV

- Perturbation theory: $m_e \sim gT$, $m_M = 0$
- Linde proposal: $m_M \simeq g^2 T$
  (to provide perturbative calculability of various quantities)
- In pure glue theory at $T > 0$ and in QC_2D at $\mu \neq 0$
  - m_e can be extracted both from the propagator values at $p = 0$ and from the polynomial fit over the range $0 < p < 2$ GeV.
  - m_e can be extracted both from the propagator values at $p = 0$ using the Linde definition; however, for the analysis of far-distant behavior the propagator at ultrasoft nonzero momenta is needed.

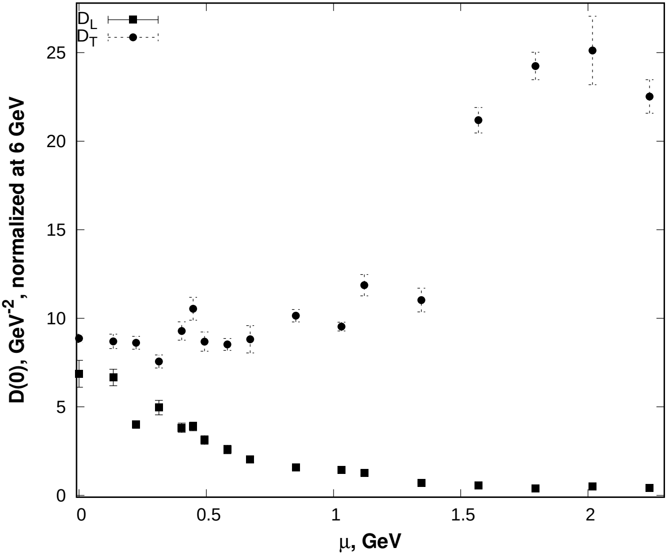
When we consider propagators only for *soft modes* $p_4 = 0$, where

$$P^T_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \dfrac{p_i p_j}{|\vec{p}|^2} \end{pmatrix} \qquad P^L_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

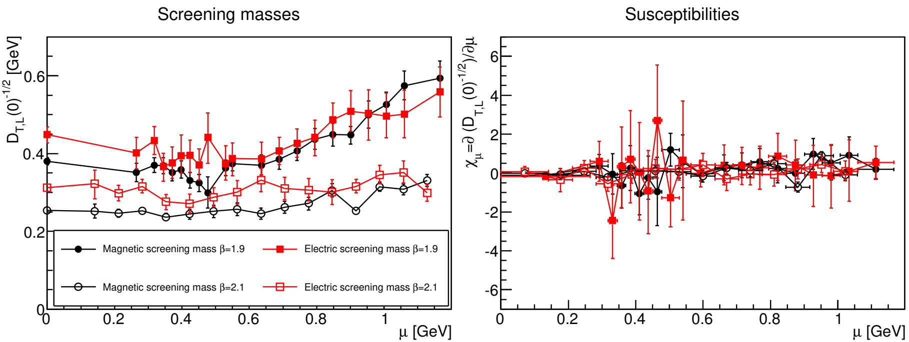
$$D_L(p) = \frac{1}{p^2 + F(p)}, \qquad D_T(p) = \frac{1}{p^2 + G(p)}$$

$D_L(0) \simeq \dfrac{1}{m_e^2} \simeq r_e^2$ — chromoelectric forces

$D_T(0) \simeq \dfrac{1}{m_m^2} \simeq r_m^2$ — chromomagnetic forces



QC_2D; 32^4

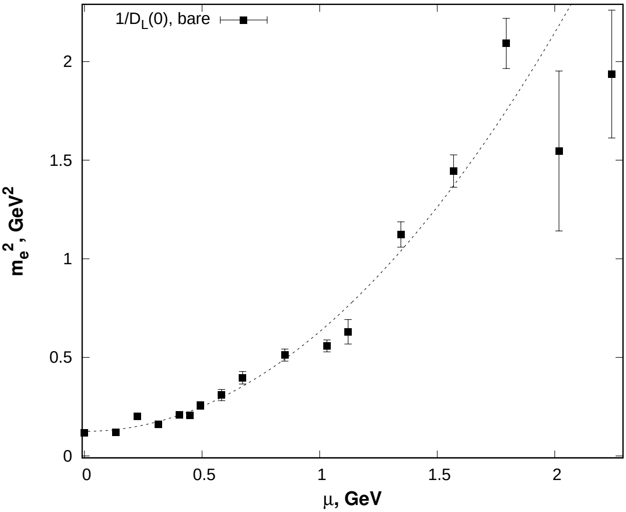


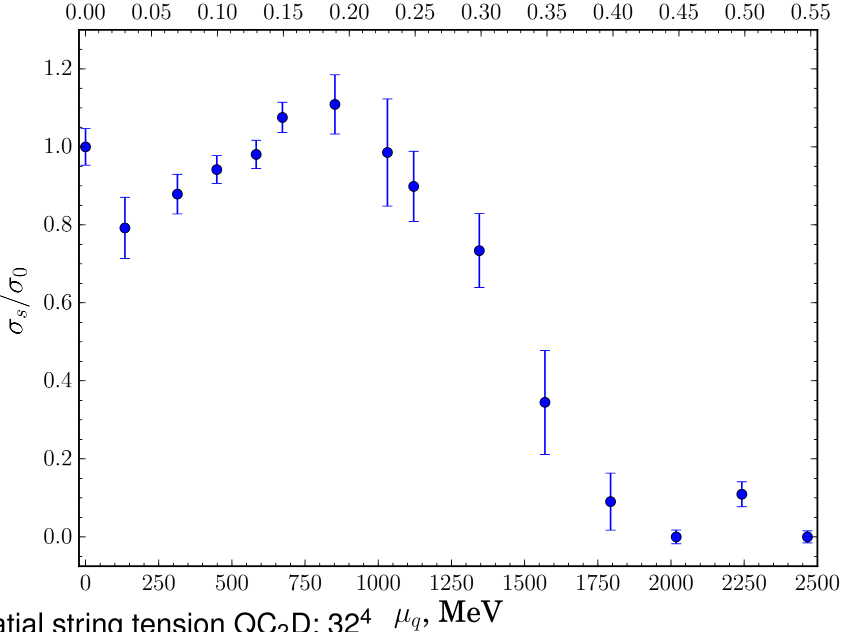
O.Hajizadeh, T.Boz, A.Maas, J.-I. Skullerud 2019

unimproved Wilson gauge action with 2 flavors of unimproved Wilson quarks $m_{pi} = 717(25)$ MeV, $a = 0.186 \div 0.138$ fm

versus our parameters:

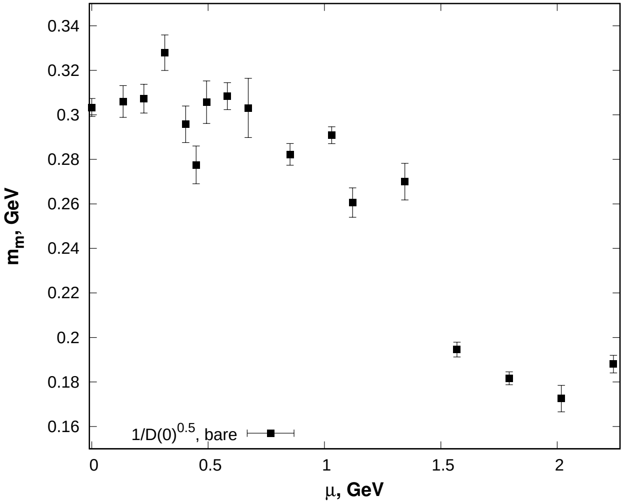
improved gauge field action and standard staggered fermion action $m_{pi} = 740$ MeV, $a = 0.044$ fm


1/D_L(0), bare
m_e^2, GeV^2
μ, GeV
0
0.5
1
1.5
2
0
0.5
1
1.5
2

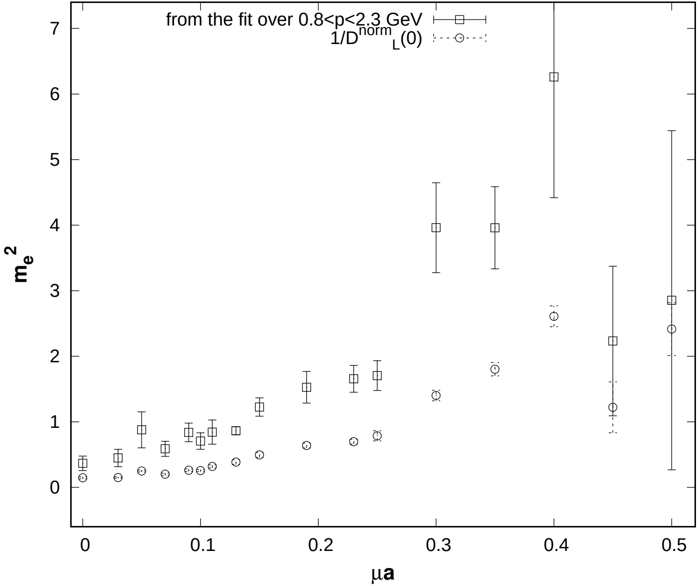


spatial string tension QC_2D; 32^4

steep decreasing at 800 MeV

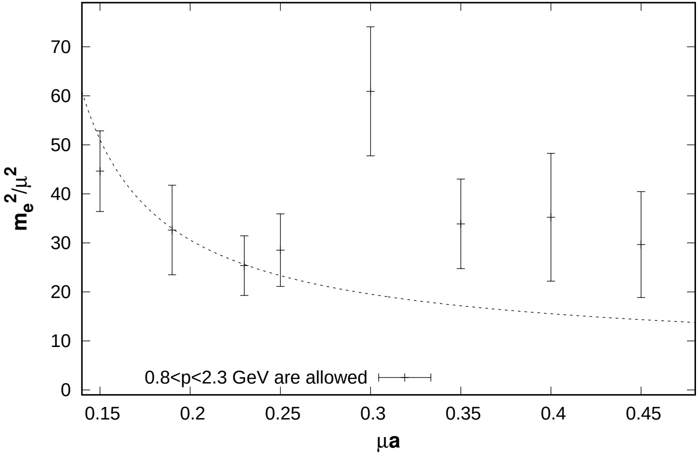


0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
m_m, GeV
0
0.5
1
1.5
2
μ, GeV
1/D(0)^0.5, bare



from the fit over 0.8<p<2.3 GeV
1/D^norm_L(0)
7
6
5
4
3
2
1
0
0
0.1
0.2
0.3
0.4
0.5
μa
m_e^2

QC_2D; 32^4



70
60
50
40
30
20
10
0
0.15
0.2
0.25
0.3
0.35
0.4
0.45
m_e^2/μ^2
μa
0.8<p<2.3 GeV are allowed

QC_2D; 32^4

The Debye screening mass in the one-loop appriximation at $T \to \infty$ and/or $\mu \to \infty$:

$$m_e^2 = \frac{g^2 N_c T^2}{3} + \sum_f \frac{g^2 \mu_f^2}{2\pi^2} . \qquad \text{where} \qquad g^2 \simeq \frac{24\pi^2}{11 \ln \frac{\mu^2}{\Lambda^2}} .$$

Our data can be fitted by such function at $p > p_{cut}$ ($T = 0$)
$\Lambda = 439(45)$ MeV
$\frac{\chi^2}{N_{d.o.f.}} = 2.66$, $p = 0.009$ $\quad p_{cut} = 630$ MeV

Perturbalively motivated fits

In the one loop approximation,
asymptotic behavior of the gluon propagator has the form

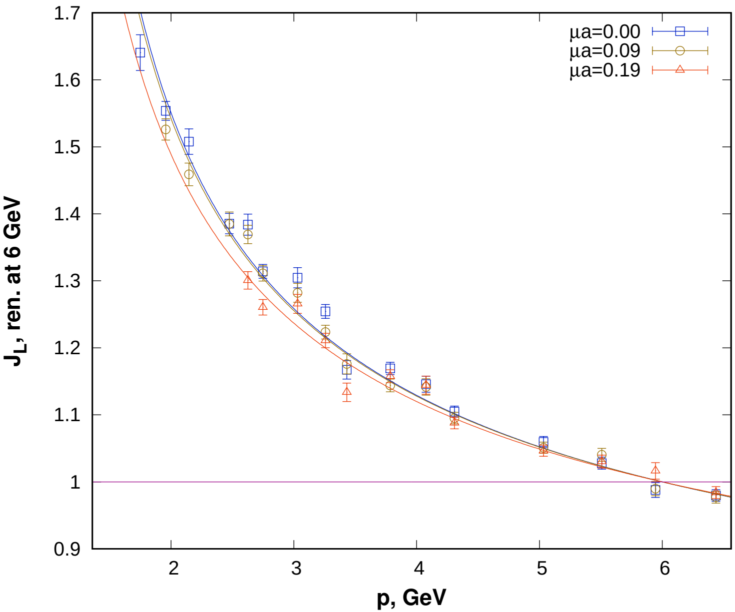
$$\lim_{p\to\infty; g=const} J(p;g) \simeq \left[\frac{\ln\left(\frac{p^2}{\Lambda^2}\right)}{\ln\left(\frac{\kappa^2}{\Lambda^2}\right)}\right]^{-\ c/(2b)} , \qquad (4)$$

c and b are the coefficients of the RG functions,

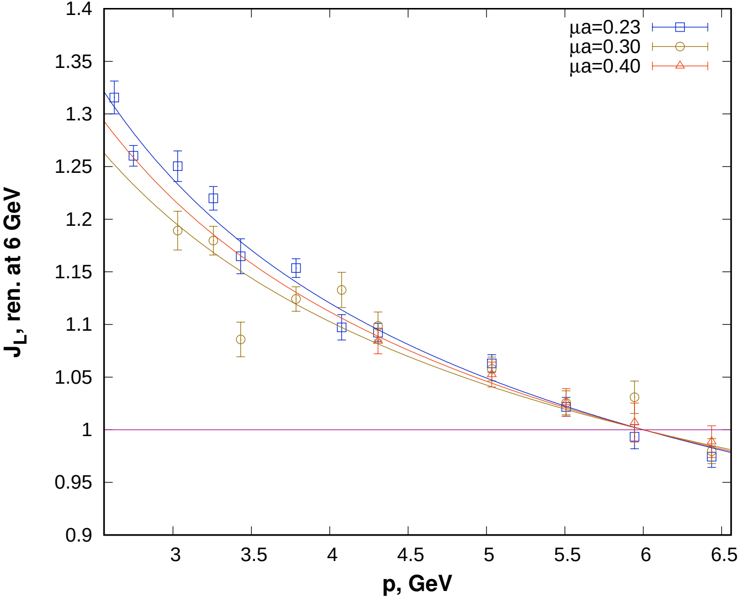
$$\beta(g) \simeq -bg^3, \qquad \gamma(g) \simeq -cg^2 .$$

In the Landau-gauge $SU(N_c)$ theories

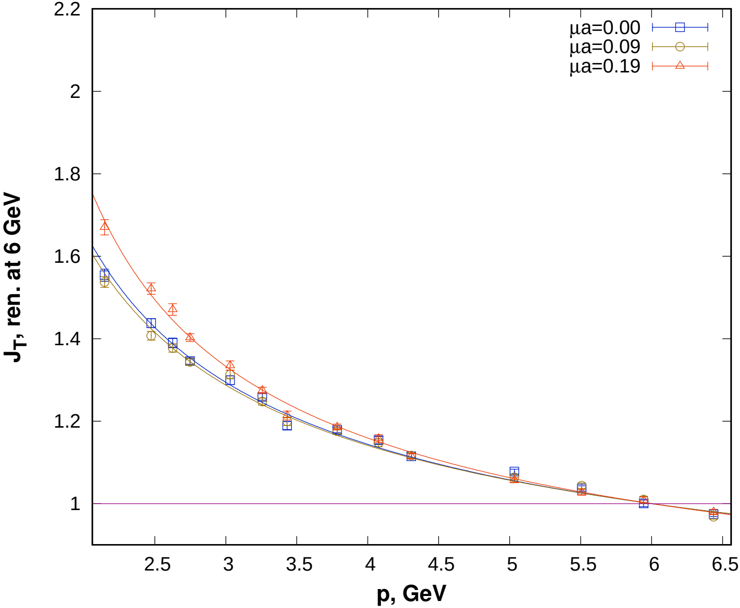
$$\frac{c}{2b} = \frac{13N_c - 4N_F}{2(11N_c - 2N_F)} = \frac{1}{2} ; \qquad (5)$$



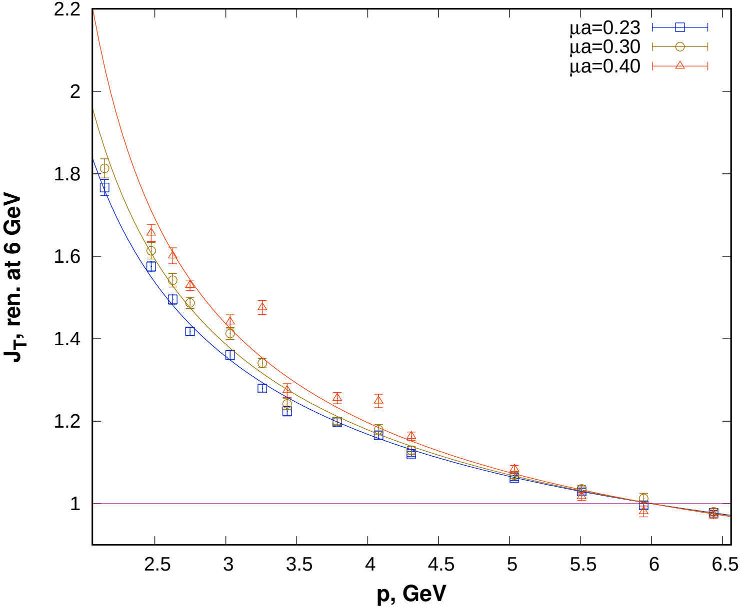
Perturbalive fit to the longitudinal dressing functions



Note the change of the perturbative domain



Perturbalive fit to the transverse dressing function



The same as in previous Fig., however, at larger values of μ

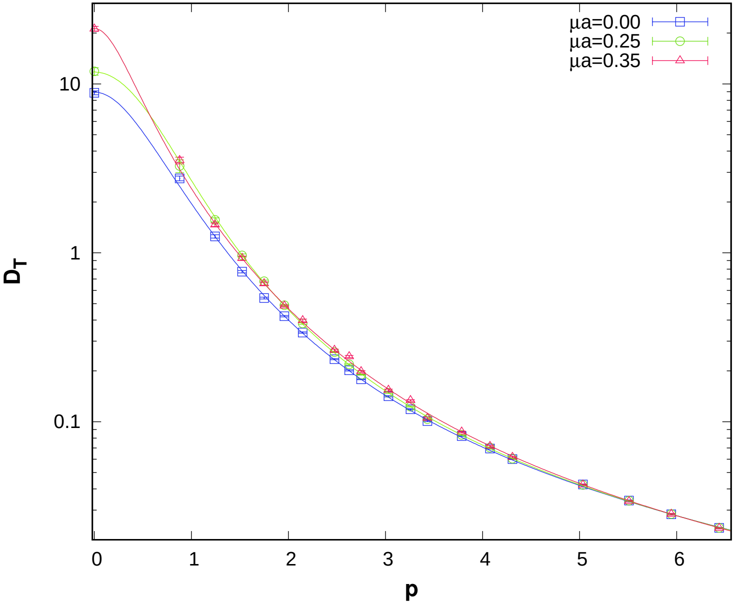
Perturbalive domain

- D_L: $p_{cut} = 1.75$ GeV $+ 0.6$ GeVμa;
- D_T: $p_{cut} = 2.7$ GeV

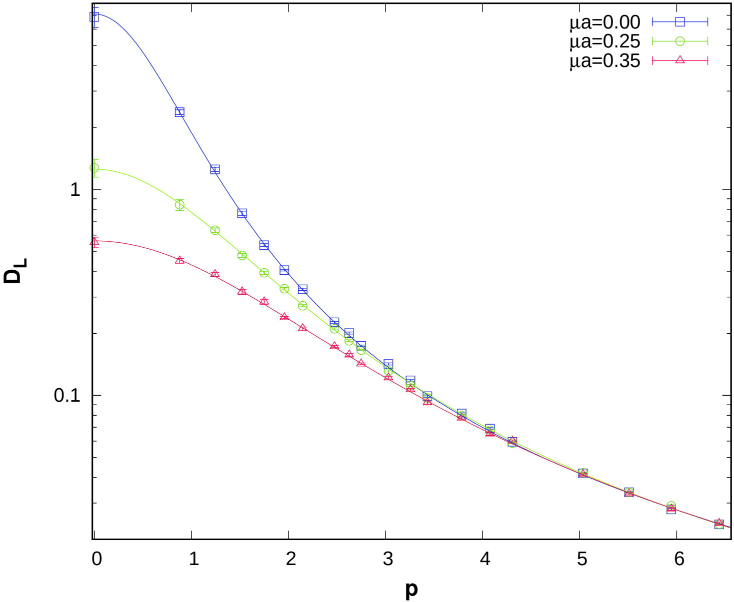
Behavior of Λ as μ increases from 0 to 2 GeV

- D_L: Λ decreases from 1.1 to 0.2 GeV
- D_T: Λ increases from 1.1 to 1.7 GeV

PRELIMINARY RESULTS



Transverse gluon propagator normalized at $p = 6$ GeV; Gribov-Stingl fit is shown.



Longitudinal gluon propagator normalized at $p = 6$ GeV; Gribov-Stingl fit is shown.

Gribov-Stingl fit was used for the renormalized propagators:

$$D(p^2) \simeq \frac{(p^2+d^2)}{(\kappa^2+d^2)} \frac{((\kappa^2+a^2)^2+b^4)}{((p^2+a^2)^2+b^4)} \tag{6}$$

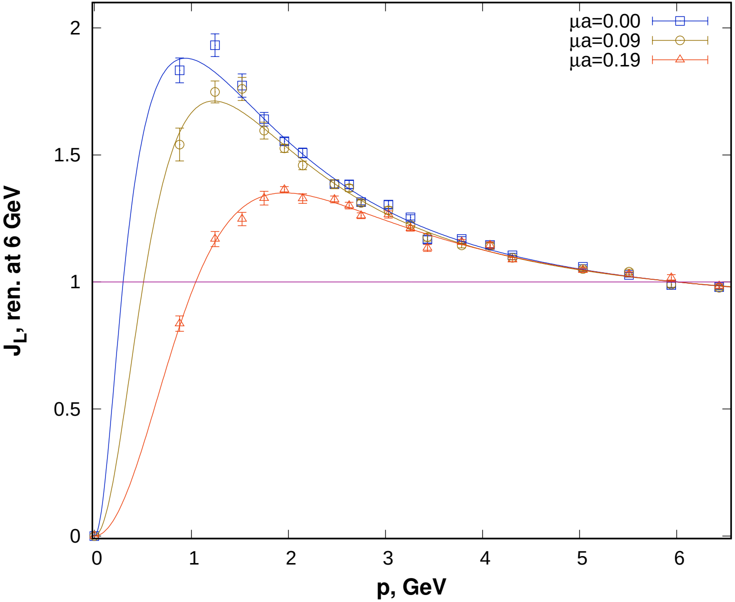
$\kappa = 6$ GeV
instead of

$$D(p^2) \simeq c \, \frac{p^2+d^2}{(p^2+a^2)^2+b^4} \tag{7}$$

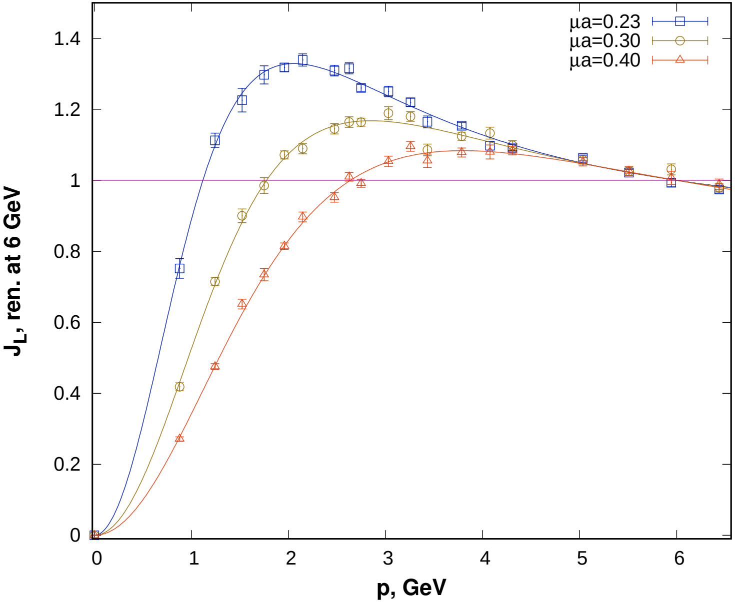
The best choice:

$$D(p^2) \simeq \frac{(\delta p^2+1)}{(\delta\kappa^2+1)} \frac{(\kappa^4+2r\kappa^2+M)}{(p^4+2rp^2+M)} \tag{8}$$

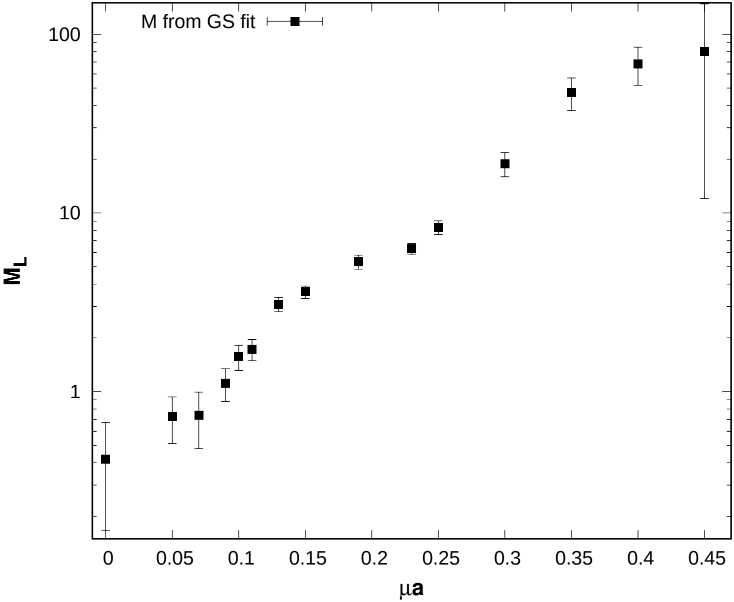
Fit parameters: M, r, δ



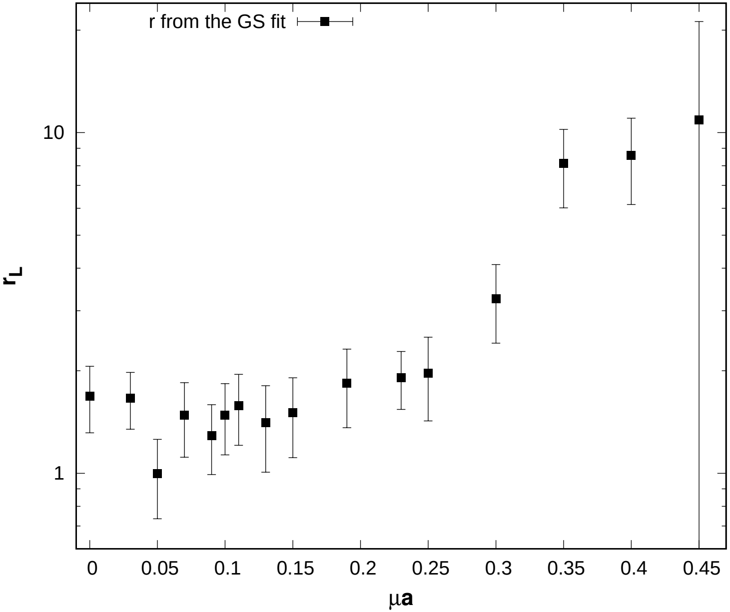
Dressing function $J_L = D_L(p_4 = 0; p^2)p^2$ at small chemical potentials



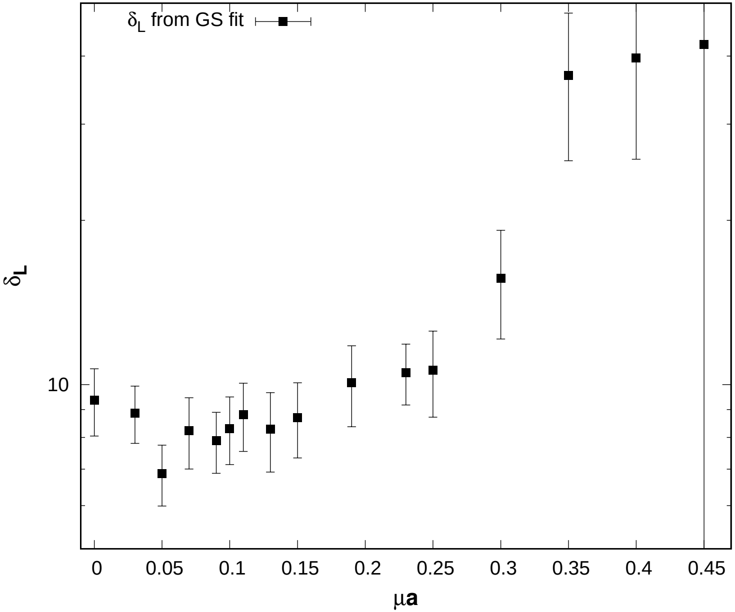
"Electric antiscreening" disappears with an increase of μ



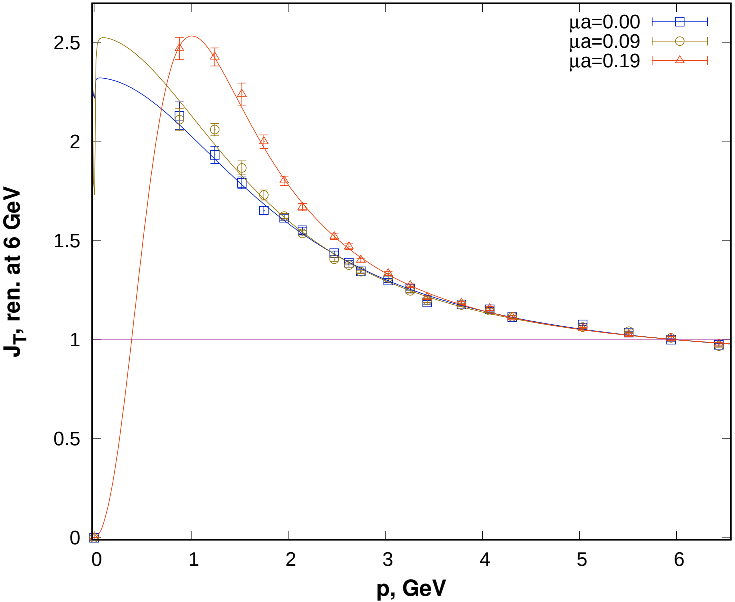
GS fit parameter $M_L = a^4 + b^4$ associated with “screening”



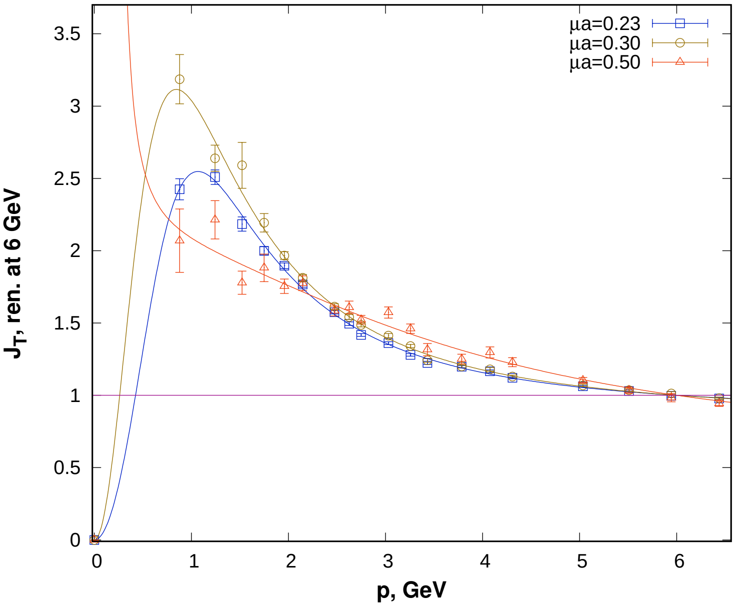
GS fit parameter $r_L = 2a^2$

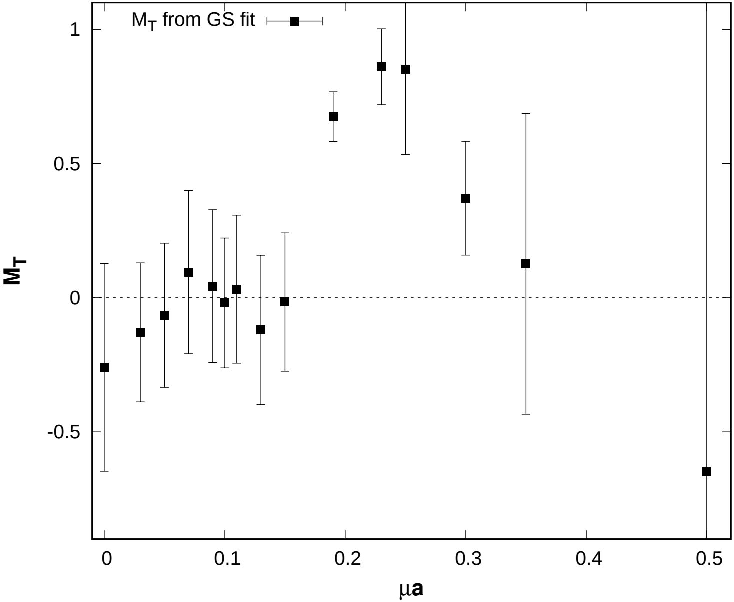


GS fit parameter $\delta_L = d^2$ shows a similar behavior to r_L

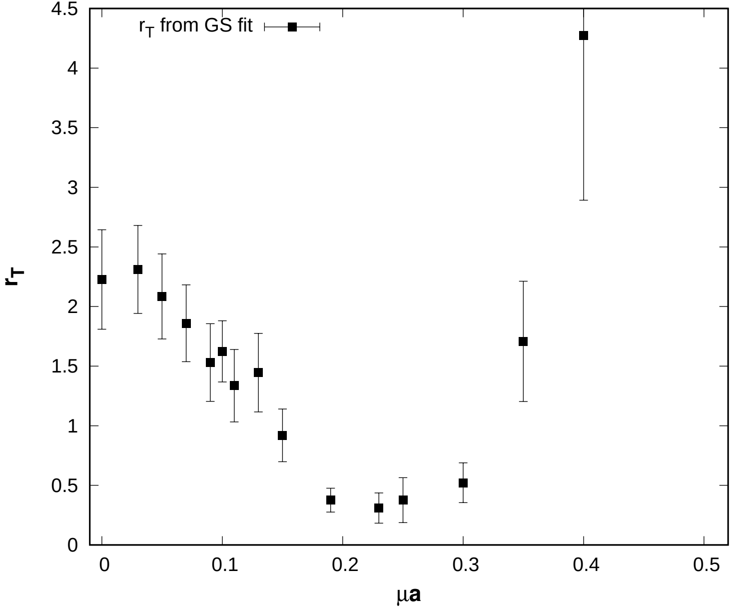


Magnetic dressing function may be consistent with the concept of massive particle only at $0.8 < \mu < 1.4$ GeV

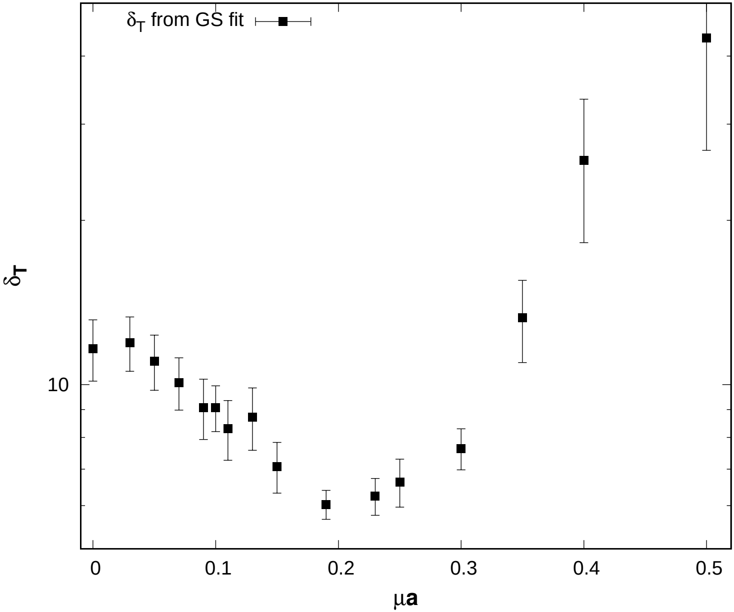

μa=0.23
μa=0.30
μa=0.50
J_T, ren. at 6 GeV
p, GeV
0
0.5
1
1.5
2
2.5
3
3.5
0
1
2
3
4
5
6



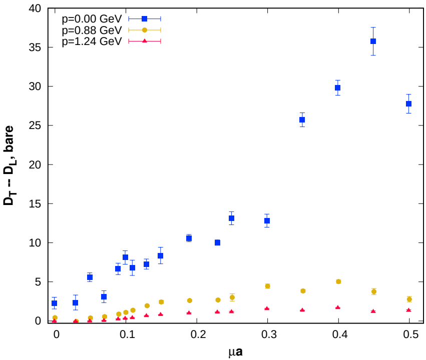
GS fit parameter $M_T = a^4 + b^4$ associated with "screening"



GS fit parameter $r_T = 2a^2$ is correlated with M_T

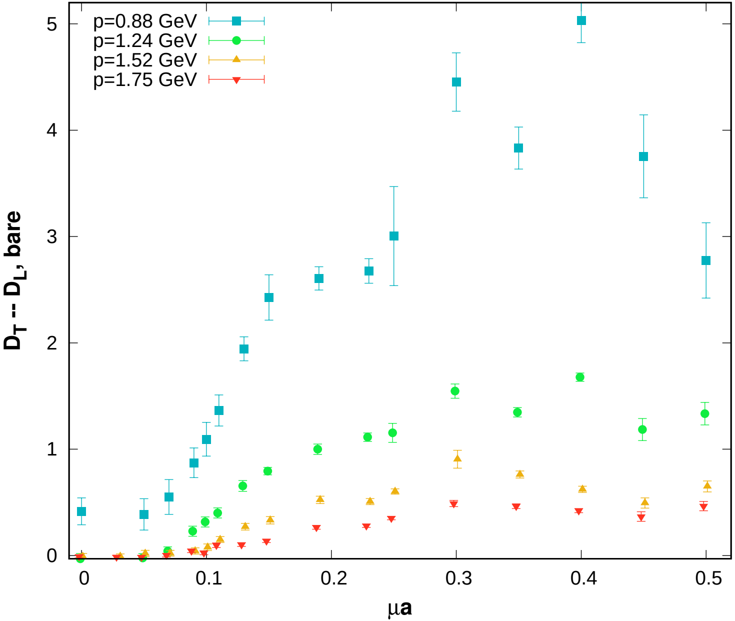


GS fit parameter $\delta_T = d^2$ shows a similar behavior to r_T

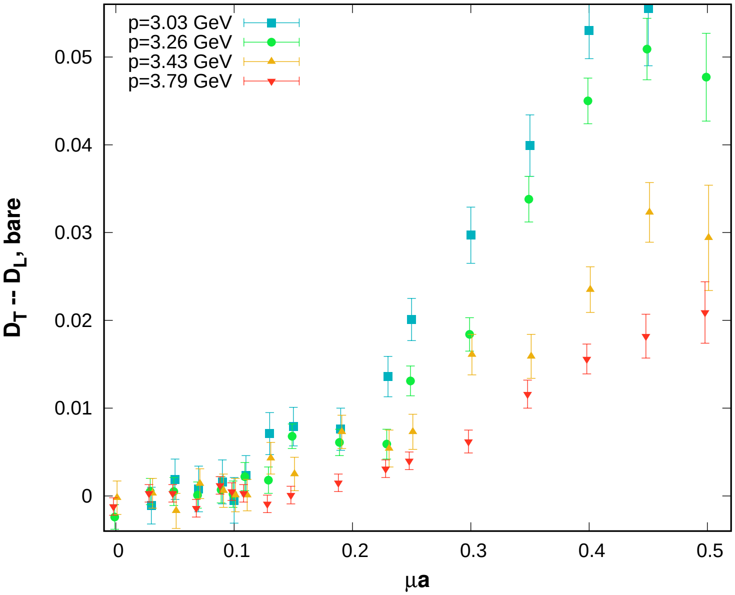


The Gribov-Stingl fit for D_T
works only partially:
$p < 10^{-2}$ at several values of μ,
parameters are
poorly determined.

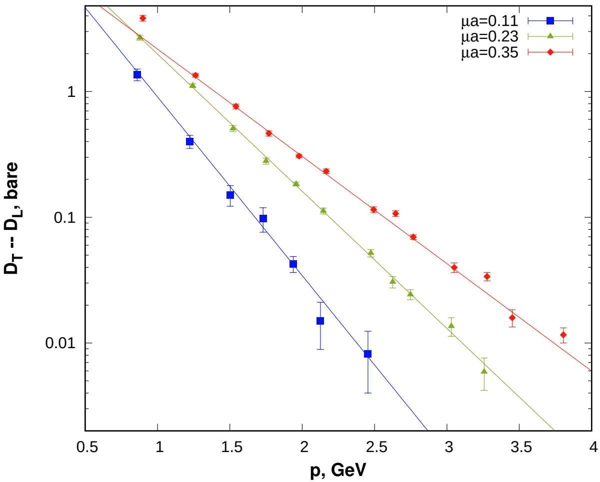
Instead of D_L and D_T,
we suggest to study D_L and $\Delta = D_T - D_L$,
because the latter quantity
shows an interesting behavior.
At nonzero momenta
it differs from zero only
at $\mu > 350 \div 400$ MeV,
and then increases
with μ



p=0.88 GeV
p=1.24 GeV
p=1.52 GeV
p=1.75 GeV
D_T -- D_L, bare
μa
0
0.1
0.2
0.3
0.4
0.5
1
2
3
4
5

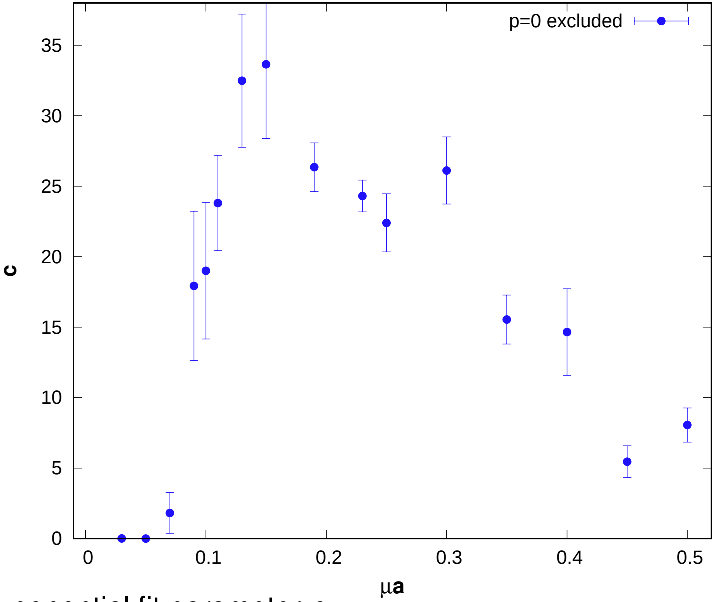


p=3.03 GeV
p=3.26 GeV
p=3.43 GeV
p=3.79 GeV
0.05
0.04
0.03
0.02
0.01
0
0
0.1
0.2
0.3
0.4
0.5
μa
D_T -- D_L, bare

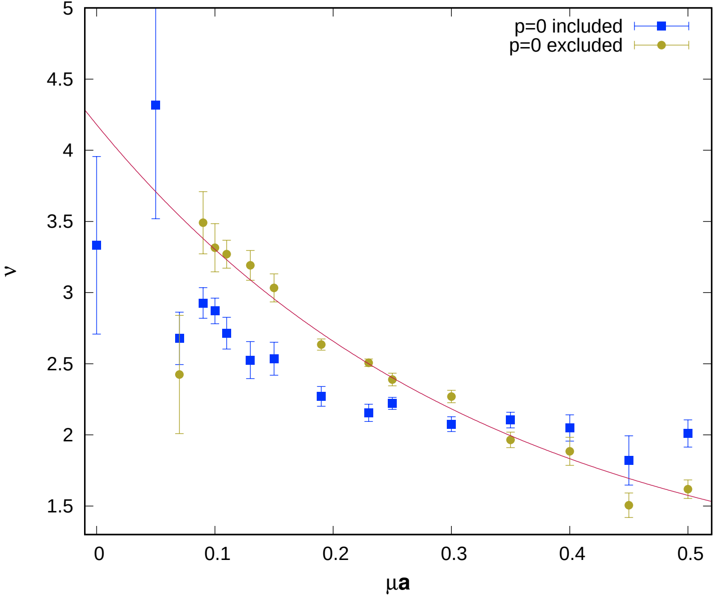


The difference between the propagators decreases exponentially:

$$(D_T(p) - D_L(p))|_{p_4=0} \simeq c\,\exp(-\nu|\vec{p}|)$$



The exponential fit parameter c



The exponential fit parameter ν

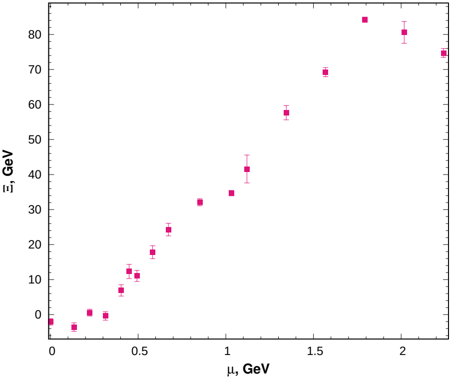
We also study the quantity

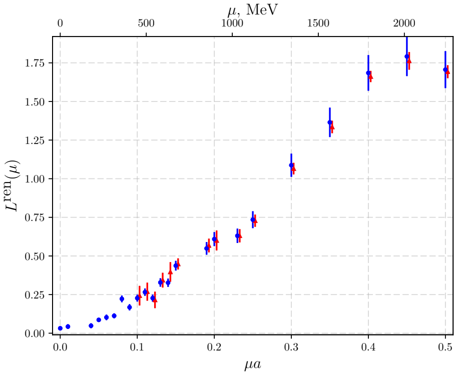
$$\Xi = \left(\frac{2\pi}{N_s a}\right)^3 \sum_{\vec{p}} (D_T(p_4 = 0, \vec{p}) - D_L(p_4 = 0, \vec{p})) \sim$$

$$\sim \sum_{\vec{x}} \sum_{x_4, y_4} \left(-\langle A_4(x_4, \vec{x}) A_4(y_4, \vec{x}) \rangle + \frac{1}{2} \sum_{i=1}^{3} \langle A_i(x_4, \vec{x}) A_i(y_4, \vec{x}) \rangle \right)$$

which is sensitive to infrared dynamics of gluon degrees of freedom.

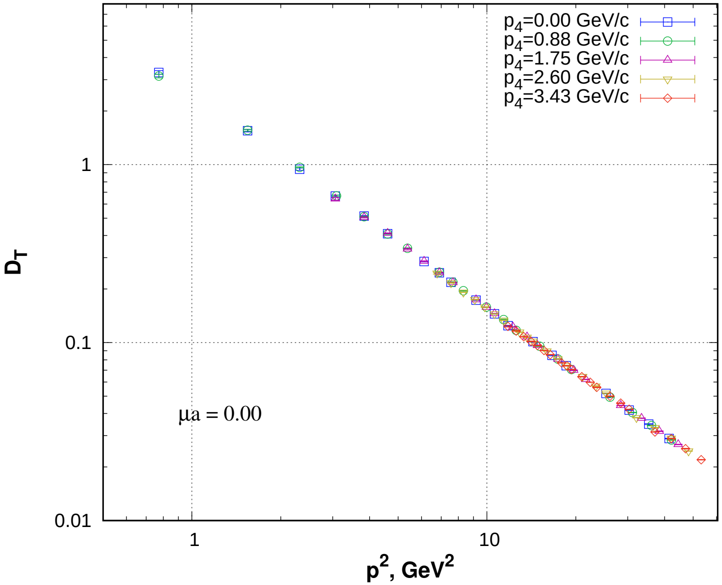
$$\Xi \simeq \int d\vec{p} (D_T(p_4 = 0, \vec{p}) - D_L(p_4 = 0, \vec{p}))$$



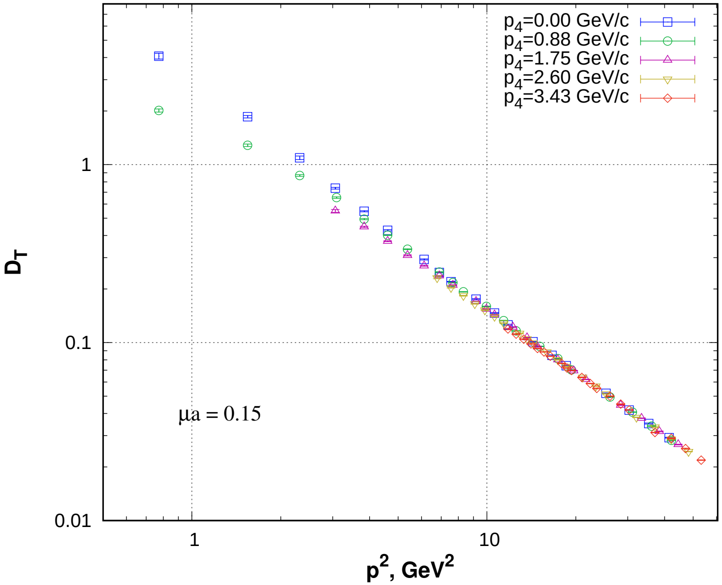


$$\Xi \simeq \int d\vec{p}(D_T(0,\vec{p}) - D_L(0,\vec{p}))$$

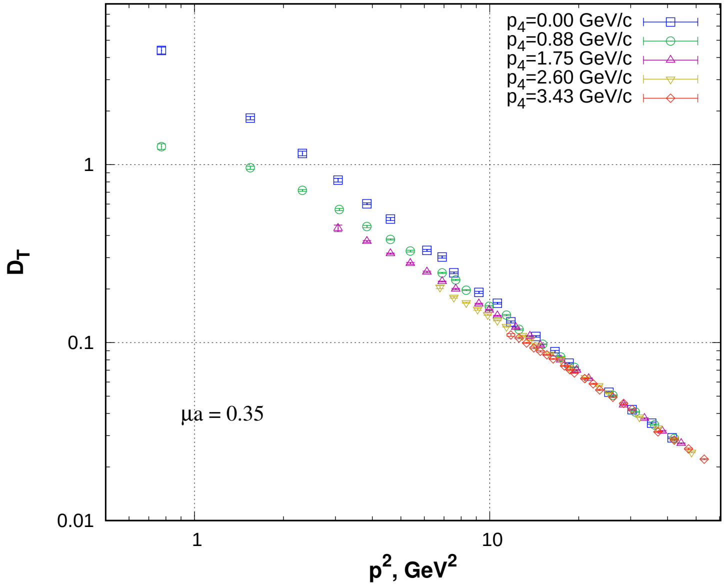
Polyakov loop



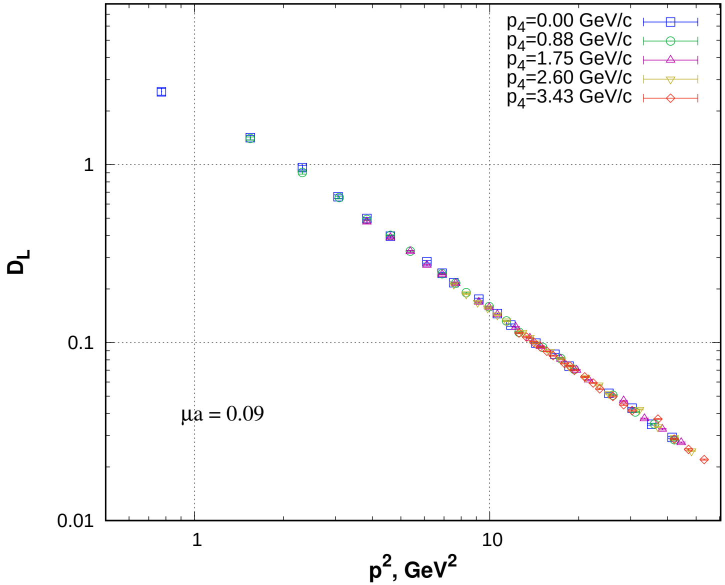
$D_T(\vec{p}^2 + p_4^2)$ at various values of p_4



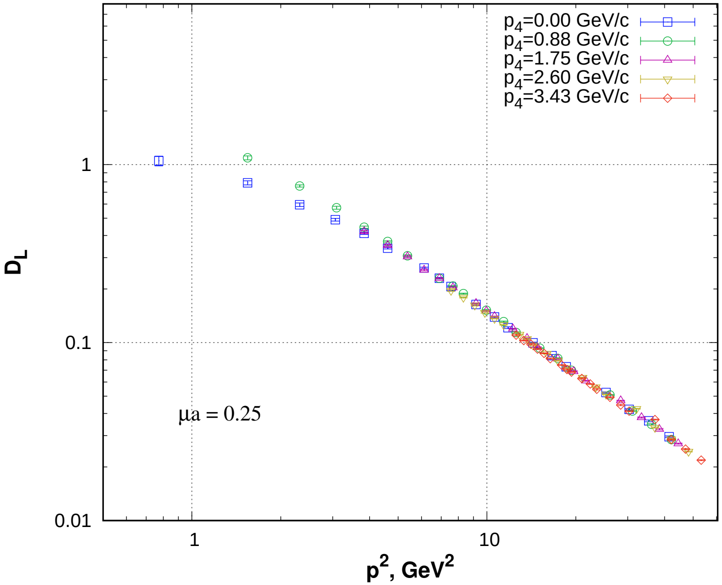
$D_T(\vec{p}^2 + p_4^2)$ at various values of p_4



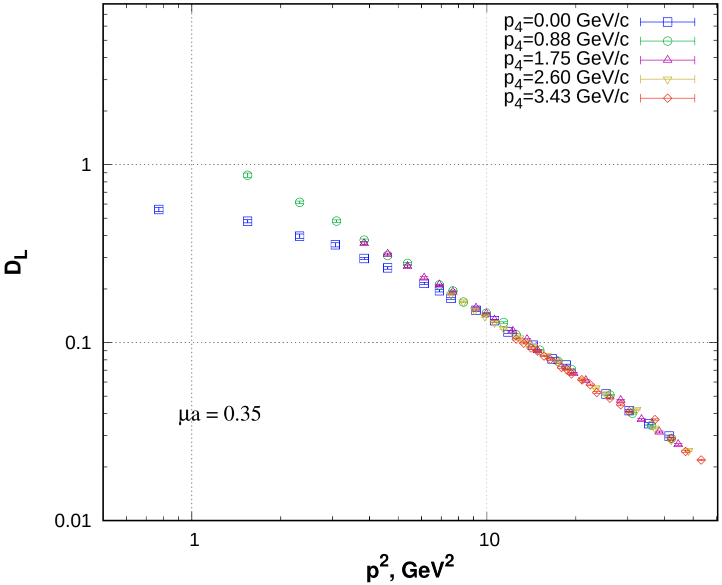
$D_T(\vec{p}^2 + p_4^2)$ at various values of p_4



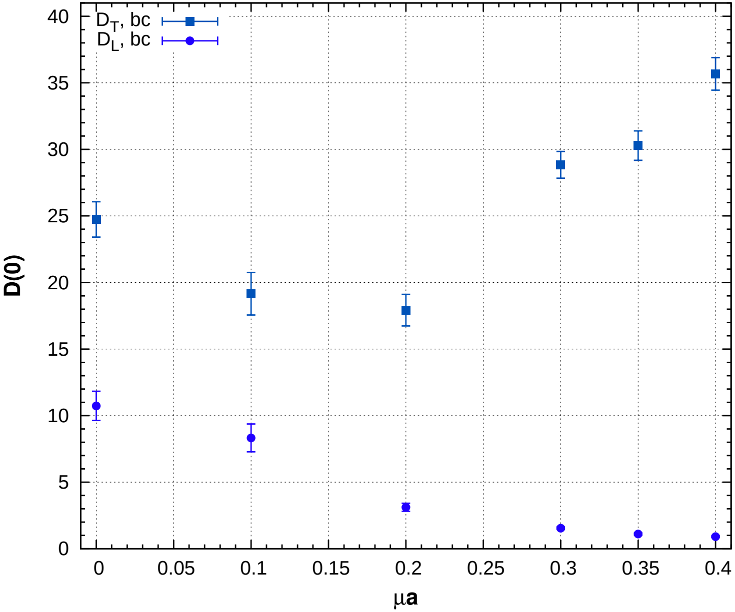
$D_L(\vec{p}^2 + p_4^2)$ at various values of p_4



$D_L(\vec{p}^2 + p_4^2)$ at various values of p_4



$D_L(\vec{p}^2 + p_4^2)$ at various values of p_4



QC_2D; $32^3 \times 24$; $T \sim 85$ MeV

Conclusions

- Debye screening mass is consistent with perturbation theory at $\mu > 650$ MeV, it increases quadratically.
- Magnetic Linde mass decreases approximately twice as μ varies over the range under study. To determine magnetic screening mass, substantially larger lattices are needed.
- The behavior of the gluon masses is in sharp disagreement with ealier findings in simulations with Wilson fermions and large a
- At large $|\vec{p}|$ gluon propagators agree well with RG-improved PT
- μ-dependence and T-dependence of D_L are similar, whereas μ-dependence and T-dependence of D_T are completely different.
- Gribov-Stingl fit parameters depend strongly on the chemical potential.
- The difference $D_T - D_L$ decreases exponentially with $|\vec{p}|$, its integral over momenta depends on μ much like the Polyakov loop.

The work is in progress