Transport properties of chiral fermions from real-time simuations

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Chiral plasmas

Chiral plasma: medium consist of chiral fermions

Quark-gluon plasma

Hadronic matter

Leptons, neutrinos at early Universe

Weyl semimetals

Liquid He3

Chiral quantum anomaly: classical action is invariant under chiral rotations, but the measure of the path integral is not:

$$\mathcal{L} = \bar{\psi}\not{D}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

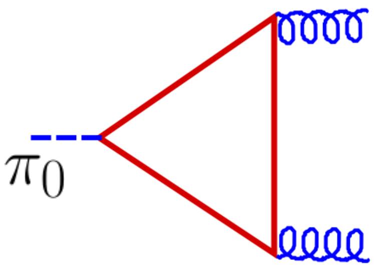
$$\mathcal{Z} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-i\int dx_\mu \mathcal{L}[\bar{\psi},\psi,A_\mu]}$$

$$\xrightarrow[e^{i\theta\gamma_5}]{} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-i\int dx_\mu \mathcal{L}[\bar{\psi},\psi,A_\mu]-iS_\theta}$$

Non-conservation of axial current:

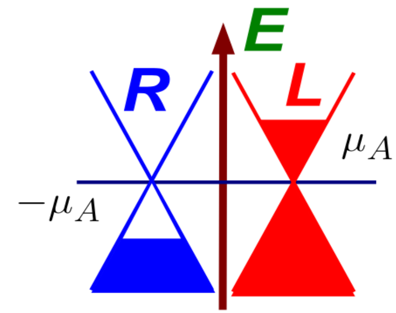
$$\partial_\mu j_A^\mu = \frac{1}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$$

$$\frac{dQ_A}{dt} = \frac{e^2}{2\pi^2}\int d^3x \vec{E}\cdot\vec{B}$$



$$Q_A = N_R - N_L \qquad J_A = J_R - J_L$$

Chiral magnetic effect



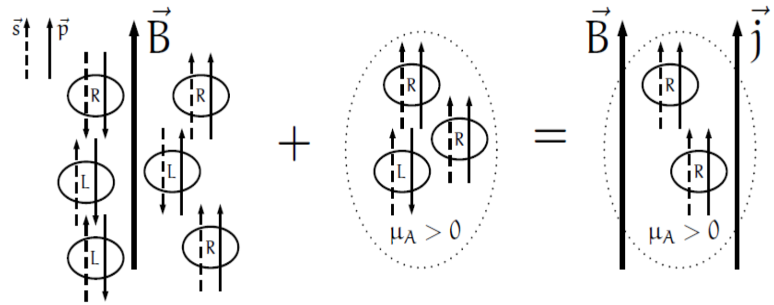
Chirally imbalanced medium:
more left-handed particles than right-handed (or vice versa)

Parity is broken => new transport coefficients

$$J = \frac{\mu_A}{2\pi^2} B$$

Chiral Magnetic Effect (CME)

Kharzeev, Warring, Fukushima

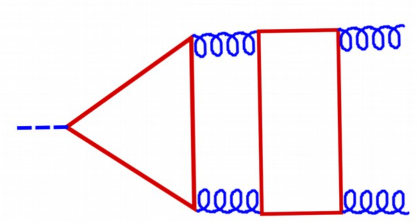


Macroscopic effect driven by quantum axial anomaly!

Also: Chiral Separation Effect, Chiral Vortical Effect...

Axial anomaly is exact =>
CME conductivity is not renormalizaed by interactions?

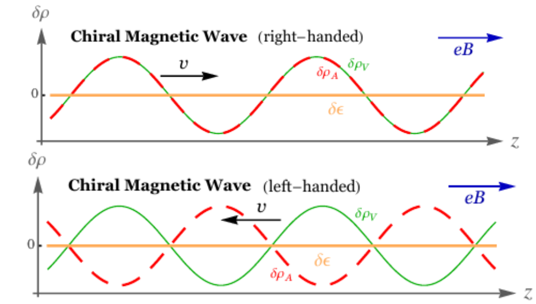
Can be affected by non-perturbative
or by in-medium effects



Chiral magnetic effect

CME: ***Difficult to observe directly*** ***However, there are many manifestations:***

- New massless collective excitations: Chiral Magnetic Waves
Chiral Shock Waves



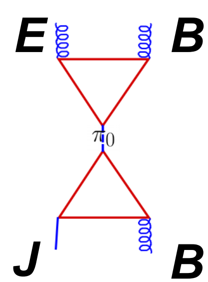
M. Chernodub, JHEP 1601 (2016) 100

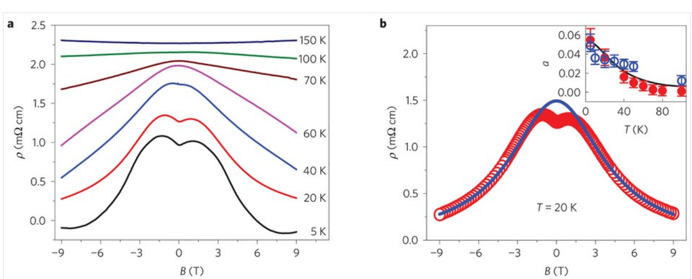
(Due to interplay between CME and CSE)

$$J = \frac{\mu_A}{2\pi^2} B \qquad J_A = \frac{\mu}{2\pi^2} B$$

- Negative magneto-resistivity

$$J_z = \frac{3}{8\pi^4} \frac{E B^2}{T^2 + \mu^2/\pi^2} \tau$$





Qiang Li et al,Nature Physics 12, 550–554 (2016)

- Chiral Magnetic Instability

Chiral plasma instability: simplified analysis

Maxwell equations + CME current:

$$\partial_t \vec{B} = -\mathrm{rot}\,\vec{E}$$

$$\partial_t \vec{E} = \mathrm{rot}\,\vec{B} - \sigma \vec{E} - \frac{\mu_A}{2\pi^2}\vec{B}$$

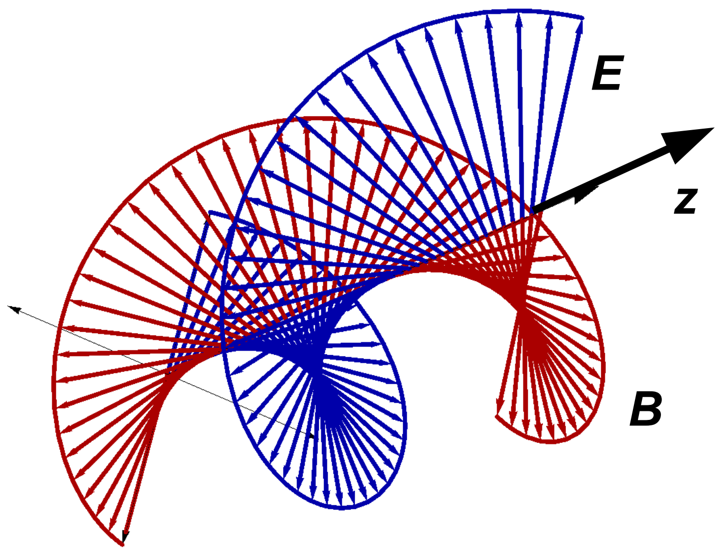
In the basis of plane waves:

$$i\omega \vec{B} = -i\vec{k} \times \vec{E} \qquad i\omega \vec{E} = i\vec{k} \times \vec{B} - \sigma \vec{E} - \frac{\mu_A}{2\pi^2}\vec{B}$$

Unstable exponentialy growing mode:

$$\omega = \frac{i\sigma}{2} \pm \sqrt{k^2 - \frac{\mu_A}{2\pi^2}k - \frac{\sigma^2}{4}} \qquad k < \frac{\mu_A}{2\pi^2}$$

$$\frac{dQ_A}{dt} = \frac{e^2}{2\pi^2}\int \underbrace{d^3x \vec{E}\cdot\vec{B}}_{\text{Helicity}} \qquad \vec{E}\cdot\vec{B} < 0$$



Chiral charge decays in the expense of helical long-wave elctromagnetic modes?

What is the energy source for mode growing?

Chiral plasma instability

Energy source: short-wavelength EM modes?

Inverse turbulent cascade?

- Source of cosmological magnetic fields

- Magnetic fields of compact stars

- THz «chiral lasers» from Dirac/Weyl semimetals?

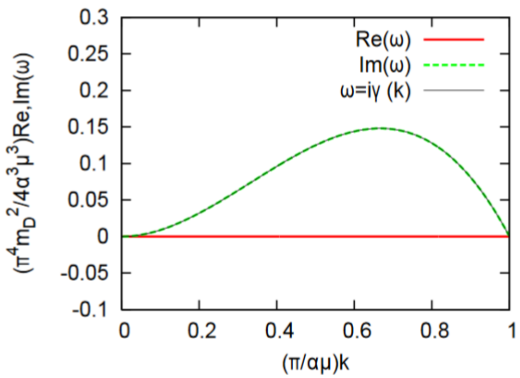
Previous analysis was too simple: 1) CME conductivity is not a constant

2) Effects of interactions are no taken into account

More rigrous considerations:

- Chiral hydrodynamics

- Chiral kinetic theory



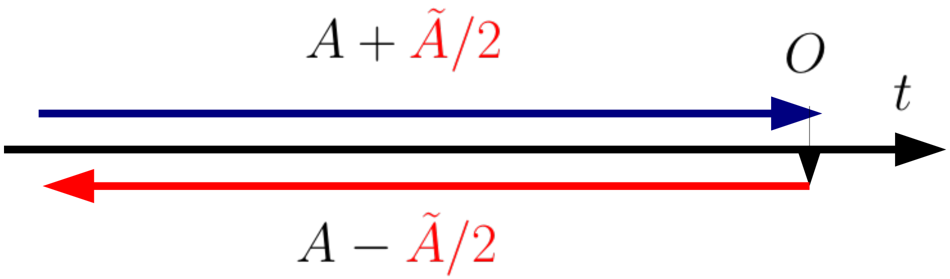
Phys.Rev.Lett.111:052002,2013

What is not taken into account:

- Possible inhomogeniuty of axial charge

- Non-linear effects

Classical-statistical real-time simulations



$$\langle O(t) \rangle = \mathrm{Tr}\left[\rho_0 U_+(0,t) O U_-(t,0)\right]$$

$$U_\pm(0,t) = \mathcal{T} \exp\left(-i \int H_\pm(t')dt'\right)$$

$$H = \psi^\dagger h \psi + H_g \qquad O = \psi^\dagger o \psi$$

$$\xrightarrow[\tilde{A} \ll A]{}$$

Maxwell equations

$$\partial_t \vec{E}(t) = -\langle j(t) \rangle - \nabla \times \vec{B}(t)$$

Fermionic current

$$\langle j(t) \rangle = \mathrm{tr}\left[\rho_0 u(0,t) j u^\dagger(0,t)\right]$$

$$\partial_t u(0,t) = -ih[\vec{A}(t)]u(0,t)$$

$$j = \partial h / \partial A$$

Managable on lattice!

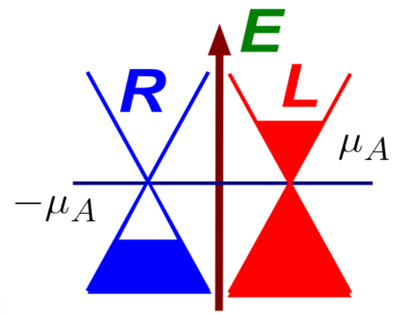
Occupation numbers of bosonic fields have to be sufficiently high

Susskind, '93
G. Aarts, '99

J. Berges, F. Hebenstreit, N. Mueller
P. Buividovich, M. Ulybyshev

Simulation setup

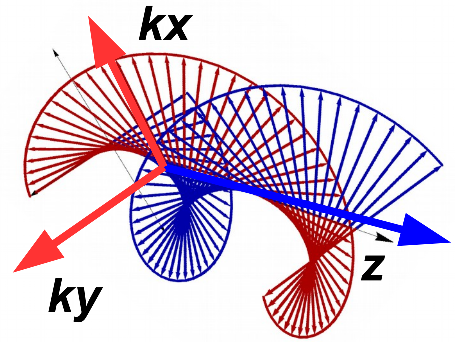
- Initial state is chirally imbalanced



- Lattice size:

$$L_x \times L_y \times L_z = 20 \times 20 \times 200$$

$$\boxed{k_{\text{lowest}} = 2\pi / L > \sigma_{\text{CME}}}$$



- Small number of initial plane waves as a seed for instability

$$A_{x,i}(t=0) = \sum_{m=1}^{n} f \frac{\cos\left(k_m x_3 + \phi_m\right)}{\sqrt{4\sin^2\left(k_m/2\right)}} \vec{n}_m$$

1) Random polarization
2) Equal energy carried by each wave

Previous study

Phys. Rev. D 94, 025009 (2016)
P. Buividovich, M. Ulybyshev

Observables

- Fourier components of EM fields are expressed in a helical basis

$$E_{k,i}(t) = \frac{1}{\sqrt{L_3}} \sum_{x_3} e^{ikx_3} E_{x,i}(t)$$

$$B_{k,i}(t) = \frac{1}{\sqrt{L_3}} \sum_{x_3} e^{ikx_3} B_{x,i}(t)$$

$$\begin{aligned} B_{k,R}(t) &= \frac{1}{2}\left(B_{k,1}(t) + B_{-k,1}(t)\right) + \\ &+ \frac{1}{2i}\left(B_{k,2}(t) - B_{-k,2}(t)\right), \\ B_{k,L}(t) &= \frac{1}{2i}\left(B_{k,1}(t) - B_{-k,1}(t)\right) + \\ &+ \frac{1}{2}\left(B_{k,2}(t) + B_{-k,2}(t)\right). \end{aligned}$$

- We study energies carried by EM modes of a given momentum k and helicity

$$I^B_{k,R/L}(t) = \left|B_{k,R/L}(t)\right|^2/2 + \left|B_{-k,R/L}(t)\right|^2/2,$$

$$I^E_{k,R/L}(t) = \left|E_{k,R/L}(t)\right|^2/2 + \left|E_{-k,R/L}(t)\right|^2/2$$

Lattice fermions and chiral symmetry

Wilson-Dirac hamiltonian: $h^{wd} = \gamma_0 D^{wd}_m$

$$D^{wd}_m = -i\gamma_i \nabla_i + m + r\Delta$$

+ Fast

- Chiral symmetry is broken

$$\nabla_{i,xx'} = \frac{1}{2}\left(\delta_{x+e_i,x'} e^{iA_{x,i}} - \delta_{x-e_i,x'} e^{-iA_{x-e_i,i}}\right)$$

$$\Delta_{xx'} = \delta_{x,x'} - \frac{1}{2}\sum_i \left(\delta_{x+e_i,x'} e^{iA_{x,i}} + \delta_{x-e_i,x'} e^{-iA_{x-e_i,i}}\right)$$

Overlap hamiltonian: $h^{ov} = \gamma_0 D^{ov}$

Creutz, Neuberger hep-lat/0110009

$$D^{ov} = 1 + \gamma_5 \mathrm{sign}\left[\gamma_5 D^{wd}_{m-1}\right]$$

+ Exact lattice chiral symmetry

- Very expensive

$$q_5 = \gamma_5 \left(1 - \frac{D_{ov}}{2}\right) \qquad [q_5, h^{ov}] = 0 \qquad \{\gamma_5, D^{ov}\} = a D^{ov} \gamma_5 D^{ov}$$

- We use Zolotarev rational approximation

Chiral plasma instability

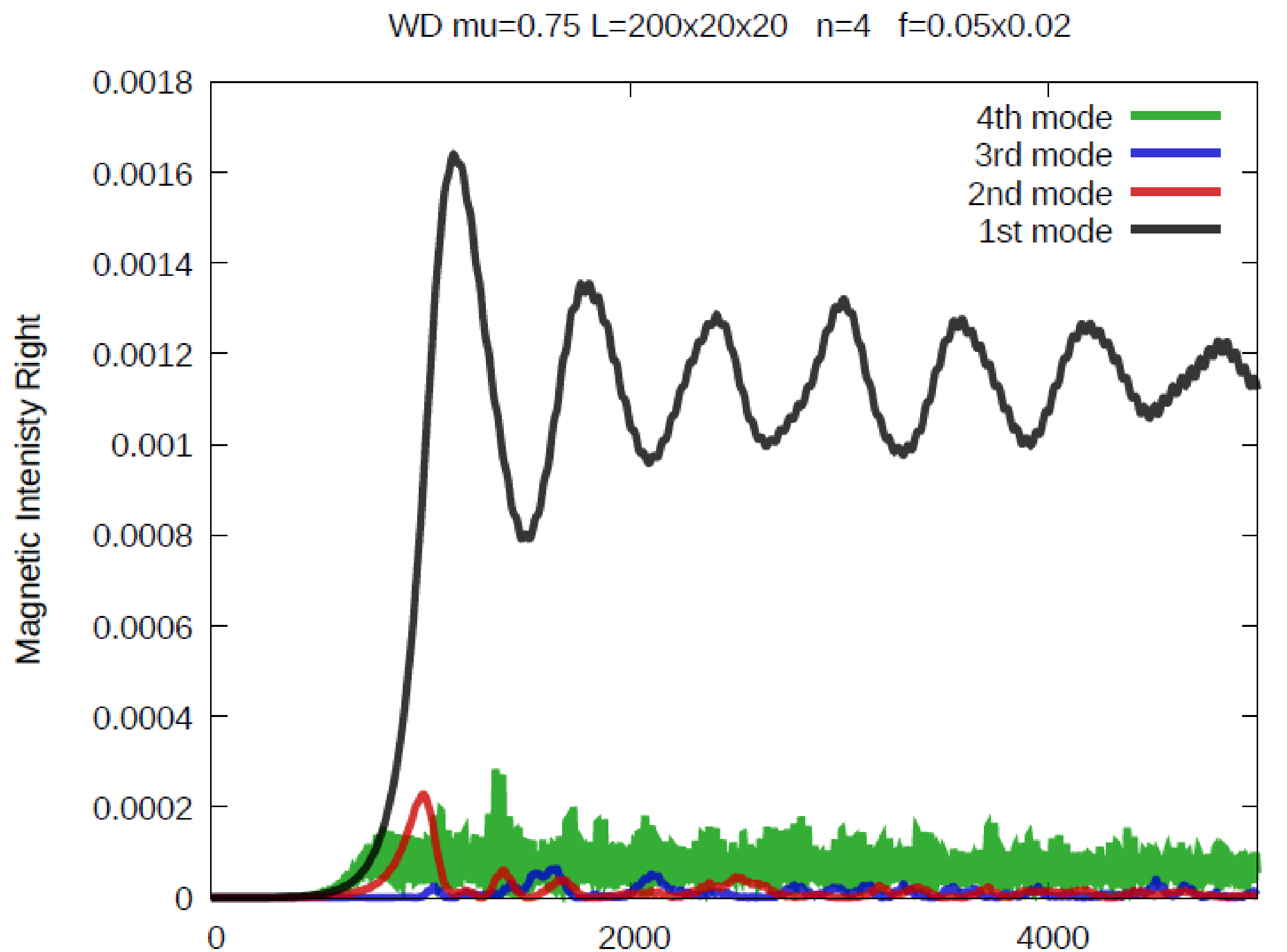


Fig. from talk of A.Dromard, Lattice 2017

- Instability saturates at certain point

- Only modes with lowest momenta (k=2pi/L in our case) are unstable => inverse turbulent cascade?

- Electric field is strongly suppressed

Chiral plasma instability

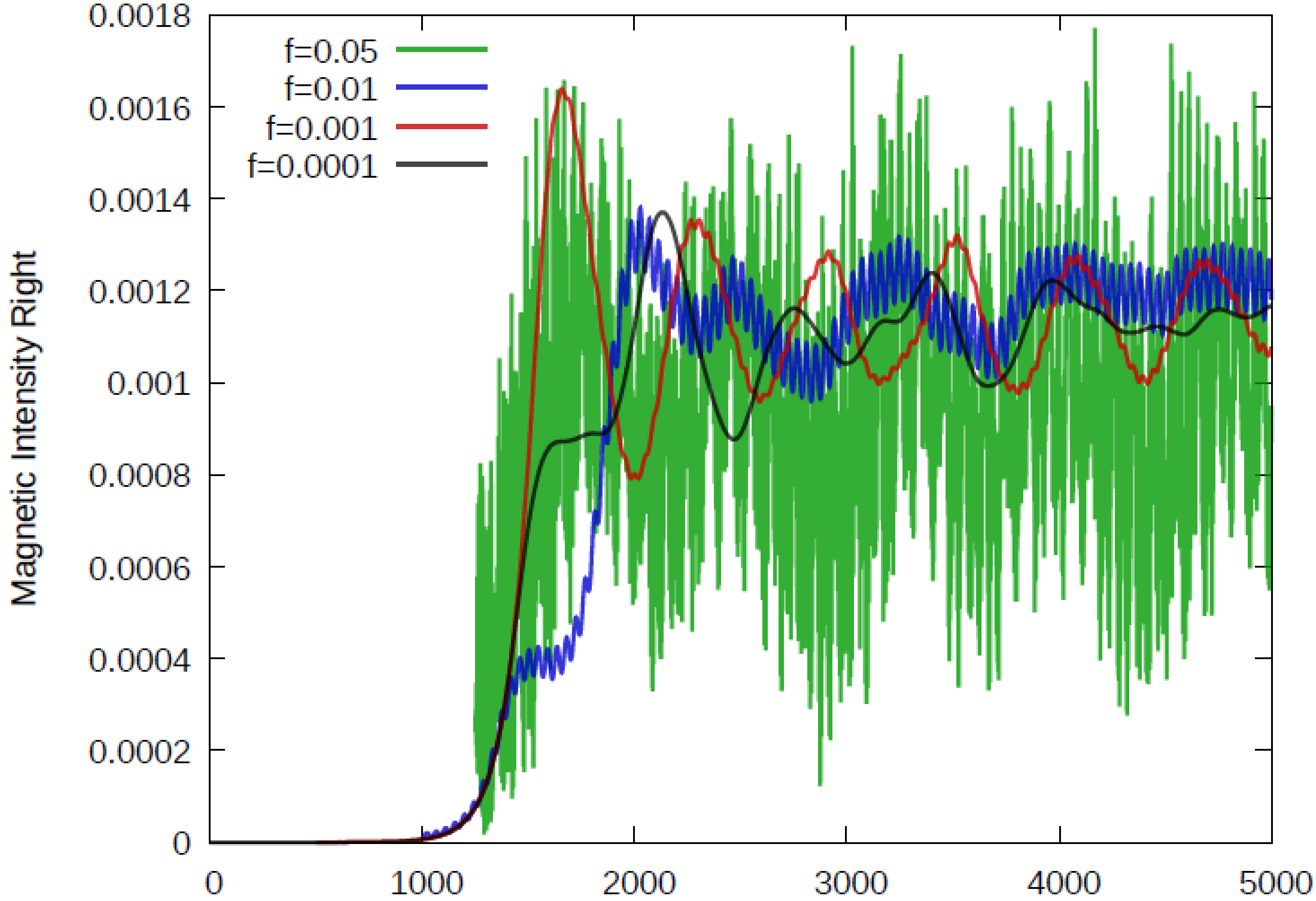


Fig. from talk of A.Dromard, Lattice 2017

- Initial evolution and final state are independet of amplitude of initial perturbation

Chiral plasma instability

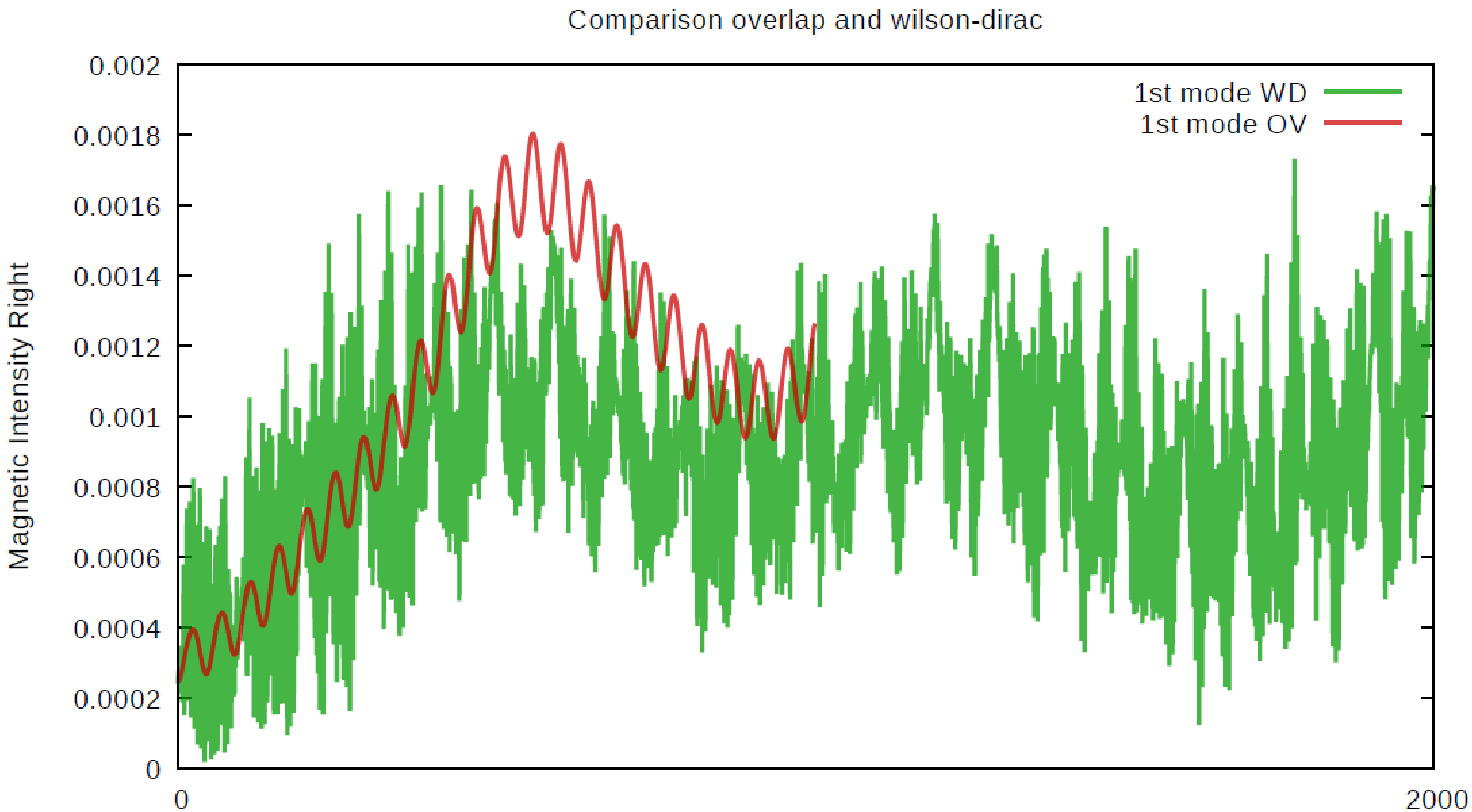


Fig. from talk of A.Dromard, Lattice 2017

- Results of overlap and Wilson-Dirac fermions do not differ significantly

- We use expensive overlap simulations in order to control quality of simulations with Wilson-Dirac fermions

Axial charge

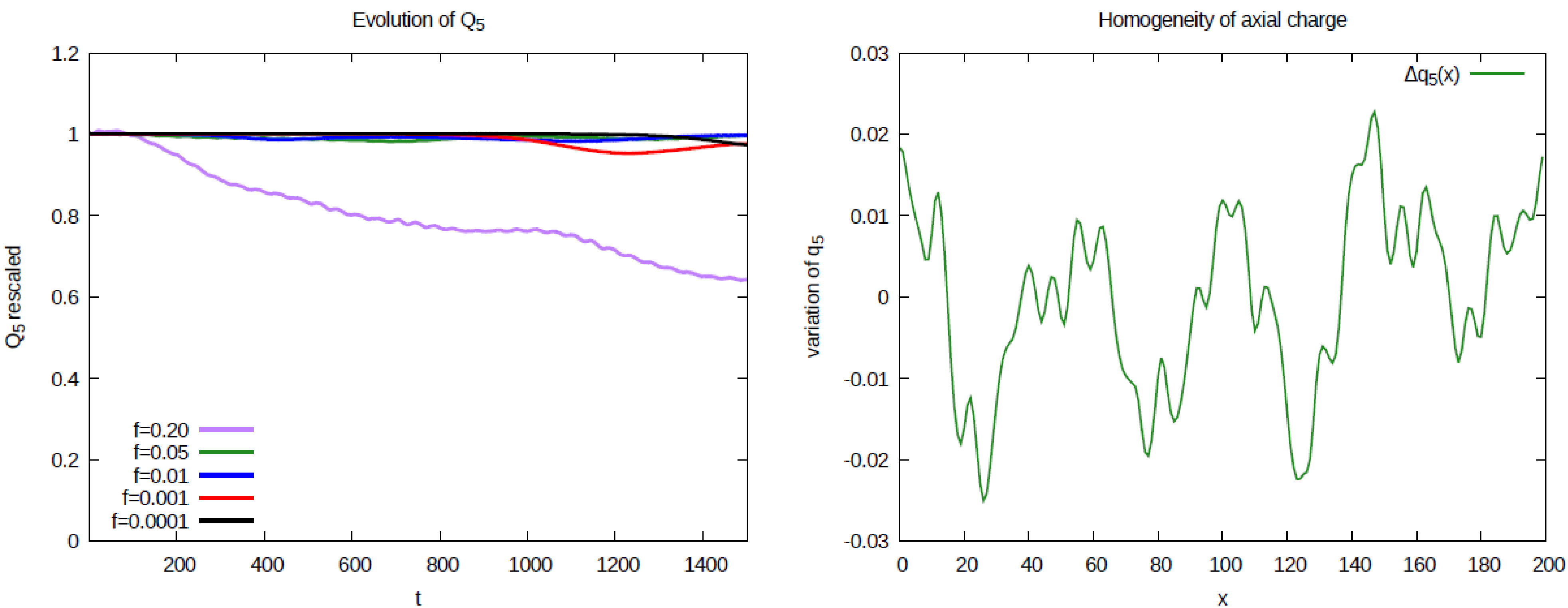


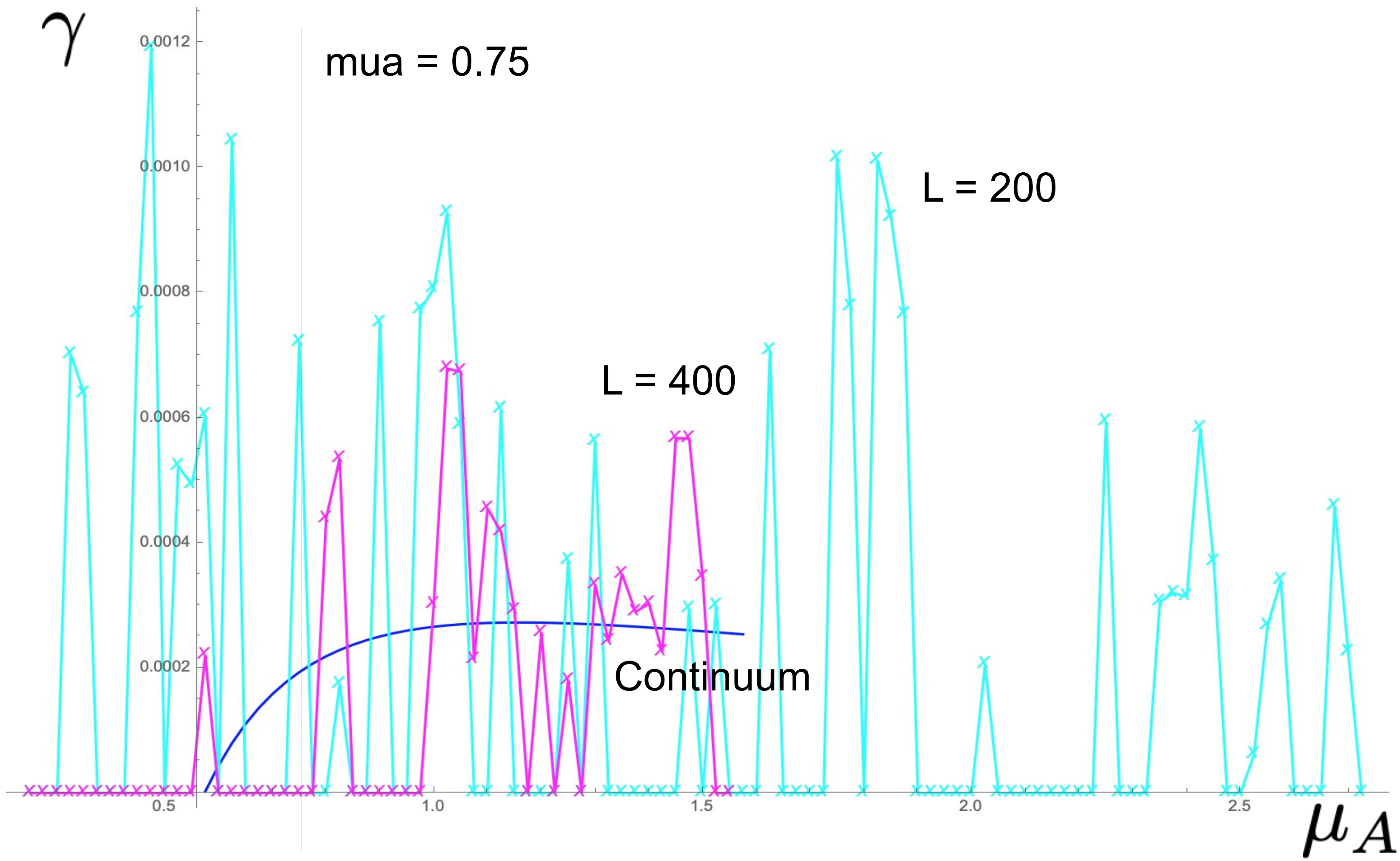
Fig. from talk of A.Dromard, Lattice 2017

- Decay of axial charge is very weak

- Axial charge is almost homogeneous, variation is less than 5%

- Axial charge decays only for strong initial EM fields

Finite volume effects



Linear response growth rate of unstable mode for Wilson-Dirac fermions

k = 2 pi / 200

$$I(t) \sim e^{\gamma t}$$

Discussion and Outlook

- Currently, observed saturation of Chiral Magnetic Instability is consistent with expected finite volume artifacts

- Expand lattice so that non-linear effects start before finite volume effects become important?

- Increase coupling constant (e >> 1)?

- Simulations in expanding geometry?