Transport properties of chiral fermions from real-time simulations

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Chiral plasmas

**Chiral plasma:** medium consist of chiral fermions

**Quark-gluon plasma**

**Hadronic matter**

**Leptons, neutrinos at early Universe**

**Weyl semimetals**

**Liquid He3**

**Chiral quantum anomaly:** classical action is invariant under chiral rotations, but the measure of the path integral is not:

\[
\mathcal{L} = \bar{\psi} \slashed{D} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}
\]

\[
\mathcal{Z} = \int \mathcal{D} \bar{\psi} \mathcal{D} \psi \mathcal{D} A_{\mu} e^{-i \int dx \mathcal{L} [\bar{\psi}, \psi, A_{\mu}]}
\]

\[
\rightarrow \int \mathcal{D} \bar{\psi} \mathcal{D} \psi \mathcal{D} A_{\mu} e^{-i \int dx \mathcal{L} [\bar{\psi}, \psi, A_{\mu}] - i S_0}
\]

**Non-conservation of axial current:**

\[
\partial_\mu j_\mu^A = \frac{1}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}
\]

\[
\frac{dQ_A}{dt} = \frac{e^2}{2\pi^2} \int d^3 x \vec{E} \cdot \vec{B}
\]

\[
Q_A = N_R - N_L \quad J_A = J_R - J_L
\]
Chiral magnetic effect

Chirally imbalanced medium: 
more left-handed particles than right-handed (or vice versa)

Parity is broken => new transport coefficients

\[ J = \frac{\mu_A}{2\pi^2} B \]  
Chiral Magnetic Effect (CME)

Kharzeev, Warring, Fukushima

Macroscopic effect driven by quantum axial anomaly!

Also: Chiral Separation Effect, Chiral Vortical Effect...

Axial anomaly is exact => 
CME conductivity is not renormalized by interactions?

Can be affected by non-perturbative 
or by in-medium effects
Chiral magnetic effect

**CME:** Difficult to observe directly

However, there are many manifestations:

- New massless collective excitations: **Chiral Magnetic Waves**  
  **Chiral Shock Waves**

  (Due to interplay between CME and CSE)

\[ J = \frac{\mu_A}{2\pi^2} B \]

\[ J_A = \frac{\mu}{2\pi^2} B \]

- Negative magneto-resistivity

\[ J_z = \frac{3}{8\pi^4 T^2 + \mu^2/\pi^2 \tau} E B^2 \]

- Chiral Magnetic Instability

M. Chernodub, JHEP 1601 (2016) 100

Chiral plasma instability: simplified analysis

Maxwell equations + CME current:

$$\partial_t \vec{B} = -\text{rot} \vec{E}$$
$$\partial_t \vec{E} = \text{rot} \vec{B} - \sigma \vec{E} - \frac{\mu A}{2\pi^2} \vec{B}$$

In the basis of plane waves:

$$i\omega \vec{B} = -ik \times \vec{E} \quad i\omega \vec{E} = ik \times \vec{B} - \sigma \vec{E} - \frac{\mu A}{2\pi^2} \vec{B}$$

Unstable exponentialy growing mode:

$$\omega = \frac{i\sigma}{2} \pm \sqrt{k^2 - \frac{\mu A}{2\pi^2} k - \frac{\sigma^2}{4}} \quad k < \frac{\mu A}{2\pi^2}$$

$$\frac{dQ_A}{dt} = \frac{e^2}{2\pi^2} \int d^3x \vec{E} \cdot \vec{B} \quad \vec{E} \cdot \vec{B} < 0$$

Chiral charge decays in the expense of helical long-wave electromagnetic modes?

What is the energy source for mode growing?
Chiral plasma instability

Energy source: short-wavelength EM modes?

- Source of cosmological magnetic fields
- Magnetic fields of compact stars
- THz «chiral lasers» from Dirac/Weyl semimetals?

Previous analysis was too simple: 1) CME conductivity is not a constant

2) Effects of interactions are no taken into account

More rigrous considerations:

- Chiral hydrodynamics
- Chiral kinetic theory

What is not taken into account:

- Possible inhomogeneity of axial charge
- Non-linear effects
Classical-statistical real-time simulations

\[ A + \tilde{A}/2 \]
\[ A - \tilde{A}/2 \]

\[ \langle O(t) \rangle = \text{Tr} \left[ \rho_0 U_+(0, t) O U_-(t, 0) \right] \]

\[ U_{\pm}(0, t) = \mathcal{T} \exp \left( -i \int H_{\pm}(t') dt' \right) \]

\[ H = \psi^\dagger h \psi + H_g \quad O = \psi^\dagger o \psi \]

Maxwell equations

\[ \partial_t \vec{E}(t) = -\langle j(t) \rangle - \nabla \times \vec{B}(t) \]

Fermionic current

\[ \langle j(t) \rangle = \text{tr} \left[ \rho_0 u(0, t) j u^\dagger(0, t) \right] \]

\[ \partial_t u(0, t) = -i\hbar [\tilde{A}(t)] u(0, t) \]

\[ j = \partial h/\partial A \]

Managable on lattice!

Occupation numbers of bosonic fields have to be sufficiently high

Susskind, '93
G. Aarts, '99

J. Berges, F. Hebenstreit, N. Mueller
P. Buividovich, M. Ulybyshev
Simulation setup

- **Initial state is chirally imbalanced**

- **Lattice size:**

\[ L_x \times L_y \times L_z = 20 \times 20 \times 200 \]

\[ k_{\text{lowest}} = \frac{2\pi}{L} > \sigma_{\text{CME}} \]

- **Small number of initial plane waves as a seed for instability**

\[ A_{x,i}(t = 0) = \sum_{m=1}^{n} f \frac{\cos(k_m x_3 + \phi_m)}{\sqrt{4\sin^2(k_m/2)}} \tilde{n}_m \]

1) Random polarization
2) Equal energy carried by each wave

**Previous study**

P. Buividovich, M. Ulybyshev
Observables

- Fourier components of EM fields are expressed in a helical basis

\[
E_{k,i} (t) = \frac{1}{\sqrt{L_3}} \sum_{x_3} e^{ikx_3} E_{x,i} (t)
\]

\[
B_{k,i} (t) = \frac{1}{\sqrt{L_3}} \sum_{x_3} e^{ikx_3} B_{x,i} (t)
\]

\[
B_{k,R} (t) = \frac{1}{2} (B_{k,1} (t) + B_{-k,1} (t)) + \frac{1}{2i} (B_{k,2} (t) - B_{-k,2} (t)),
\]

\[
B_{k,L} (t) = \frac{1}{2i} (B_{k,1} (t) - B_{-k,1} (t)) + \frac{1}{2} (B_{k,2} (t) + B_{-k,2} (t)).
\]

- We study energies carried by EM modes of a given momentum \( k \) and helicity

\[
I_{k,R/L}^B (t) = \left| B_{k,R/L} (t) \right|^2 / 2 + \left| B_{-k,R/L} (t) \right|^2 / 2,
\]

\[
I_{k,R/L}^E (t) = \left| E_{k,R/L} (t) \right|^2 / 2 + \left| E_{-k,R/L} (t) \right|^2 / 2.
\]
Lattice fermions and chiral symmetry

**Wilson-Dirac hamiltonian:**  
\[ h^{wd} = \gamma_0 D^{ wd} \]

\[ D^{ wd}_m = -i\gamma_i \nabla_i + m + r\Delta \]

- Fast

- Chiral symmetry is broken

**Overlap hamiltonian:**  
\[ h^{ov} = \gamma_0 D^{ ov} \]

\[ D^{ ov} = 1 + \gamma_5 \text{sign} \left[ \gamma_5 D^{ wd}_{m-1} \right] \]

- Exact lattice chiral symmetry

- Very expensive

\[ q_5 = \gamma_5 \left( 1 - \frac{D^{ ov}}{2} \right) \]

\[ [q_5, h^{ ov}] = 0 \]

\[ \{\gamma_5, D^{ ov}\} = a D^{ ov} \gamma_5 D^{ ov} \]

- We use Zolotarev rational approximation

Creutz, Neuberger hep-lat/0110009
- Instability saturates at certain point

- Only modes with lowest momenta ($k=2\pi/L$ in our case) are unstable => inverse turbulent cascade?

- Electric field is strongly suppressed
Chiral plasma instability

- Initial evolution and final state are independent of amplitude of initial perturbation

Fig. from talk of A. Dromard, Lattice 2017
Chiral plasma instability

Results of overlap and Wilson-Dirac fermions do not differ significantly.

We use expensive overlap simulations in order to control quality of simulations with Wilson-Dirac fermions.
Axial charge

- Decay of axial charge is very weak
- Axial charge is almost homogeneous, variation is less than 5%
- Axial charge decays only for strong initial EM fields
Finite volume effects

$\gamma$

$m_\mu = 0.75$

$L = 200$

$L = 400$

Continuum

Linear response growth rate of unstable mode for Wilson-Dirac fermions

$k = 2\pi / 200$

$I(t) \sim e^{\gamma t}$
Discussion and Outlook

- Currently, observed saturation of Chiral Magnetic Instability is consistent with expected finite volume artifacts

- Expand lattice so that non-linear effects start before finite volume effects become important?

- Increase coupling constant (e >> 1)?

- Simulations in expanding geometry?