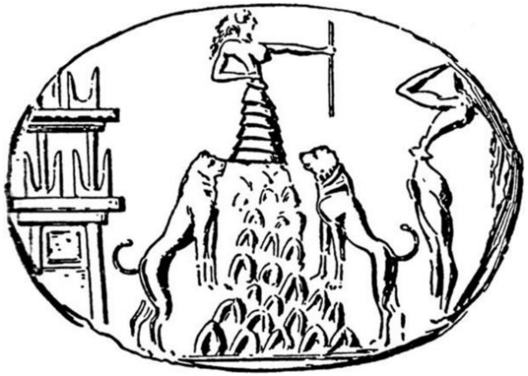
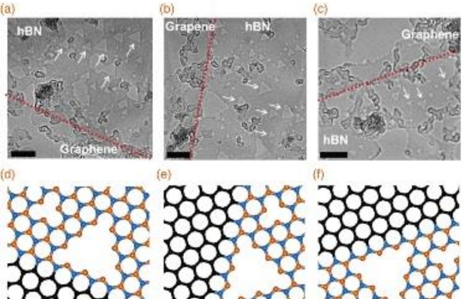
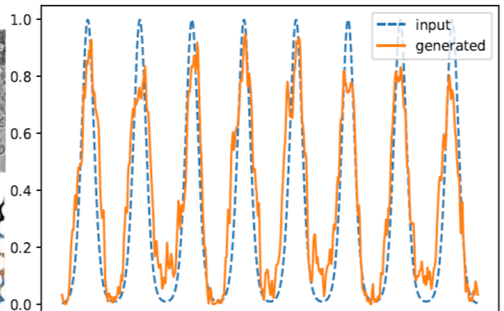
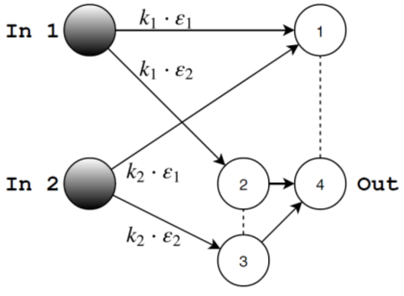


# Monte-Carlo simulation of the artificial quantum neural network

**Oleg Pavlovsky**
**ITEP & MSU, Moscow, Russia**



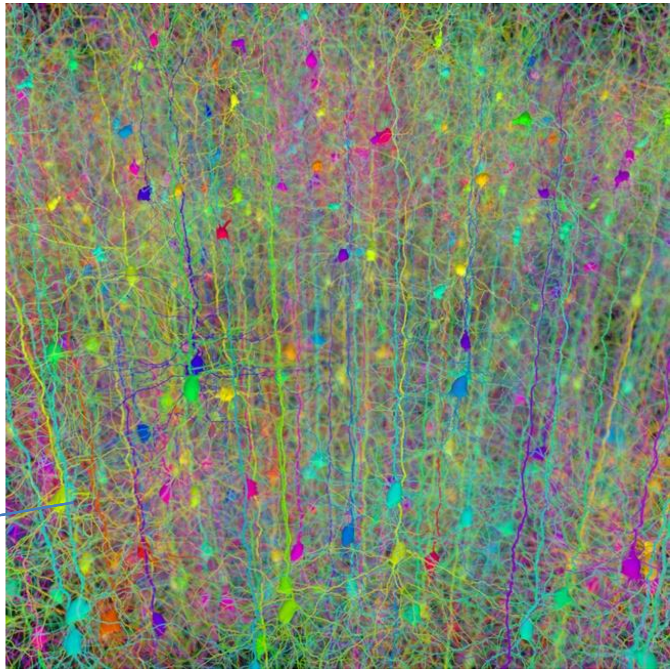
# Programmers / Biologists

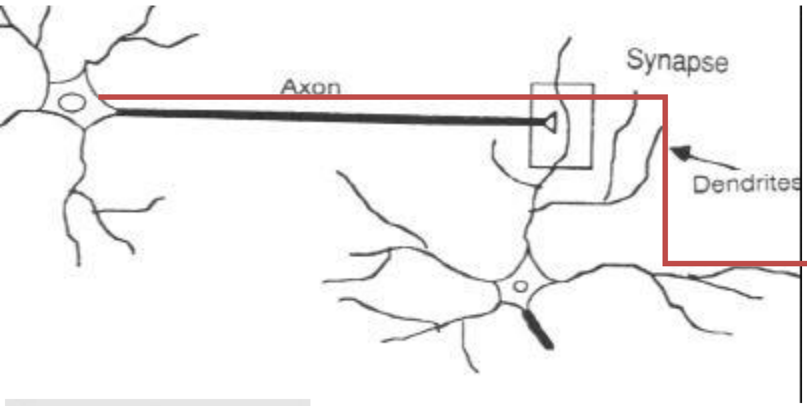
**Two point of view on neural networks**

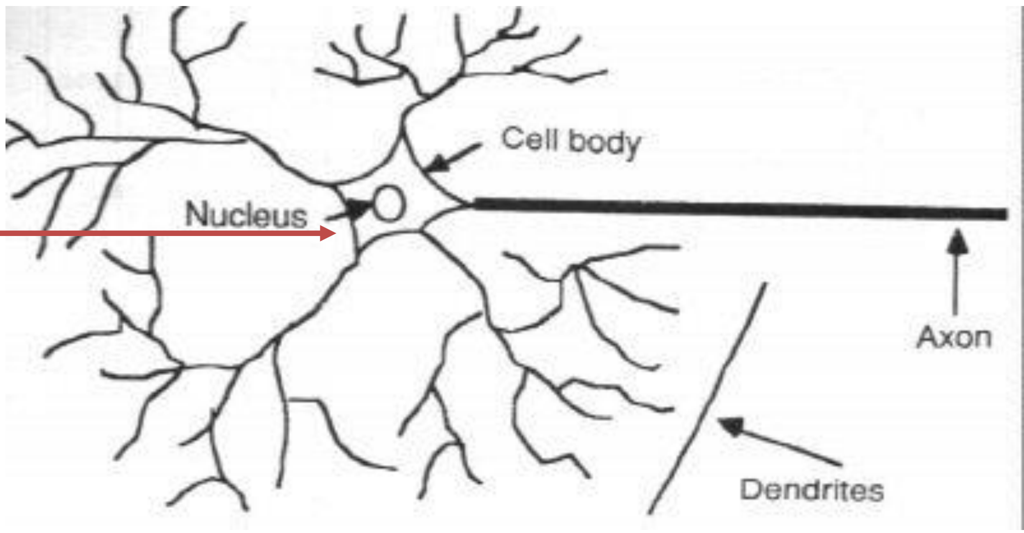
**Programmers**

**Biologists**

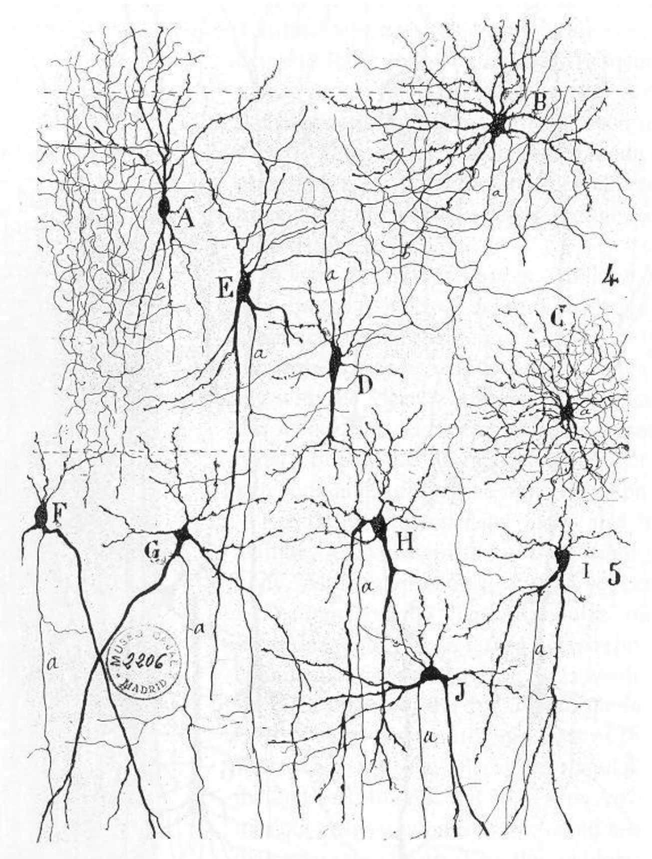
# Neural network: Biology

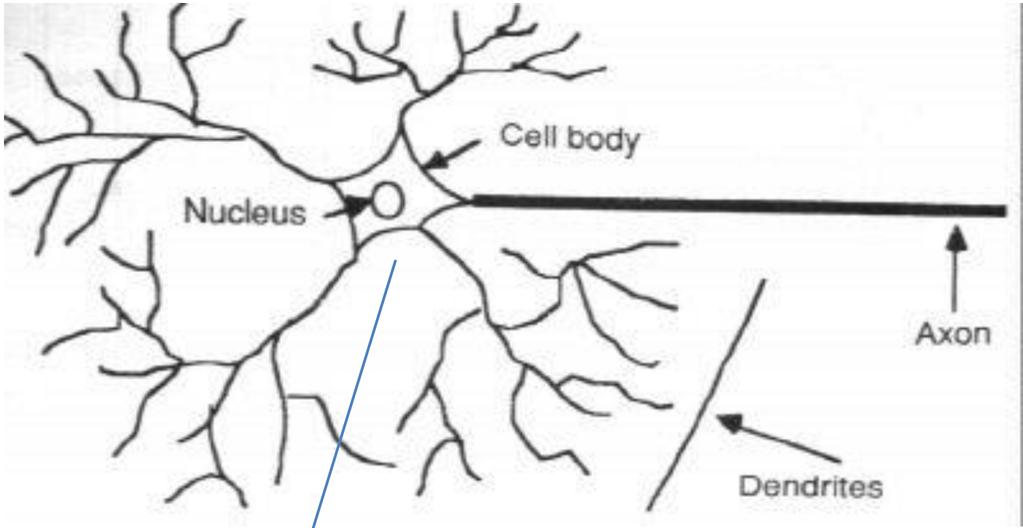


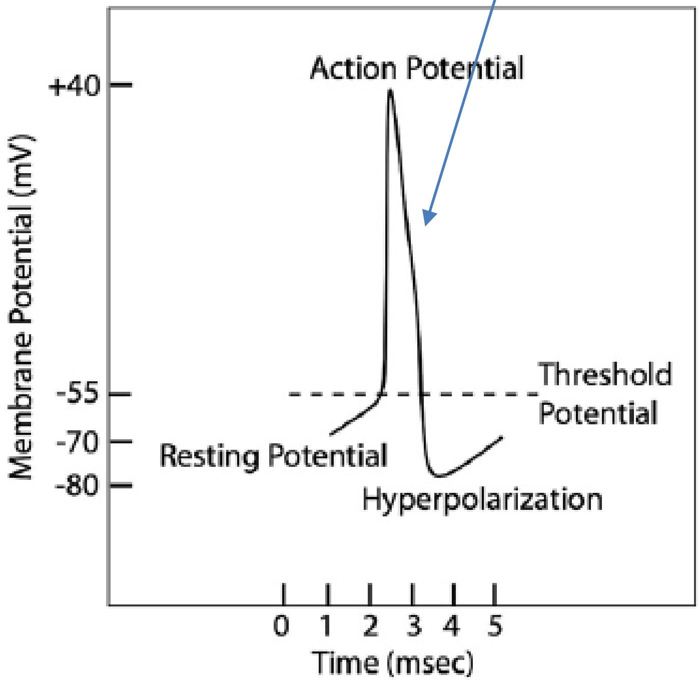


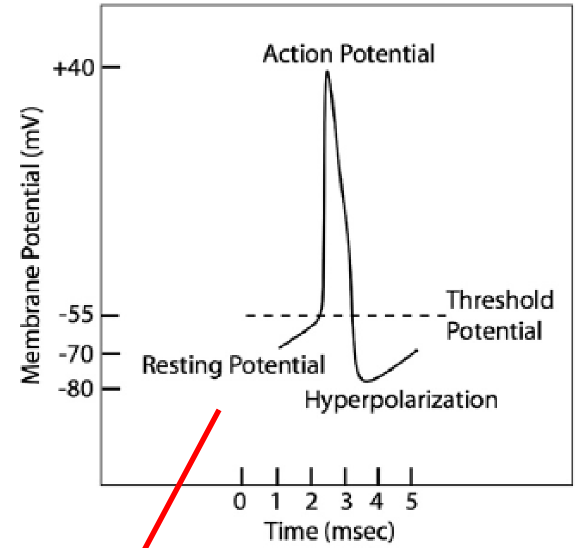

# How Neuron Works: Biology






# The models of Biological Neuron


$$I(t) = C_{\mathrm{m}} \frac{dV_{\mathrm{m}}}{dt}$$

**Different for Different Models**

# Artificial Neural Network: main idea

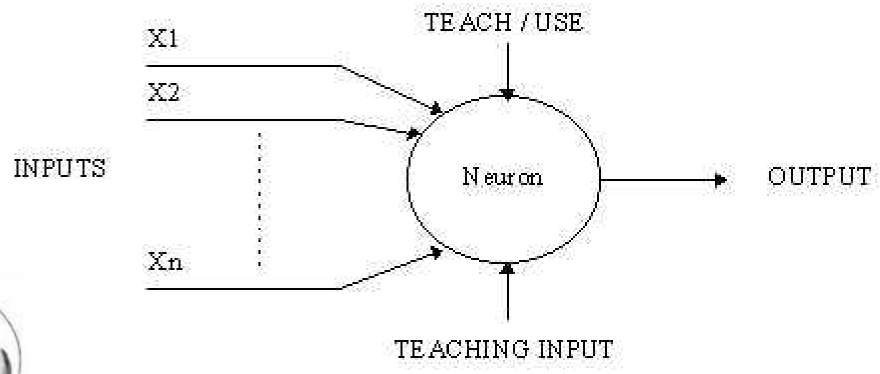
**Main idea of Artificial Neural Network:**

**Artificial Neural Network (ANN) is an information system that is inspired by the biological nervous systems, such as the brain.**

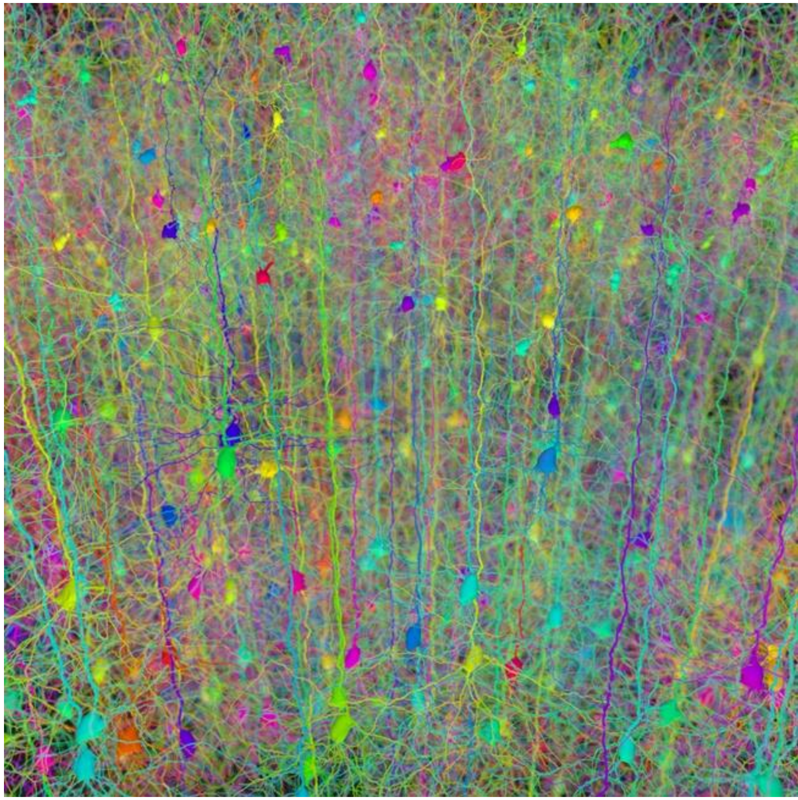
**The key element of ANN is a large number of highly interconnected processing elements (neurones) working in unison to solve specific problems.**

**ANN, like brain, learn by examples or patterns.**

**Typical ANN problems: pattern recognition, data classification and so on.**




# New era in biological neural networks



**Biological (real) neural networks are VERY complicated systems.**

**Models of neuron – stochastic diff. equations.**

**So…**

**Only qualitative analysis is possible now.**

**Statistical mechanics approach can be used**

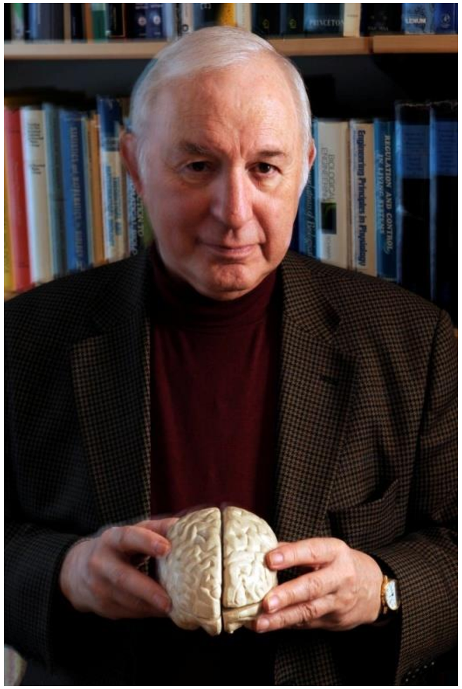
**Idea of the universality classes:  biological details may be not so essential in contrast with Symmetries and Topology of the Network.**

# Conformity in neural networks

**Main question is:**

**What is the mechanism of the big correlation on the neural network?**

**Statistical mechanics gives the possible answer: Conformity near the phase transition. (Michael A. Buice and Jack D. Cowan, 2008-2009)**



# Conformity in neural networks

**Jack D. Cowan: "Strange and interesting things happen in the neighborhood of a phase transition"**

**Statistical mechanics gives the possible answer: Conformity near the phase transition. (Michael A. Buice and Jack D. Cowan, 2008-2009)**

Progress in Biophysics and Molecular Biology 99 (2009) 53–86

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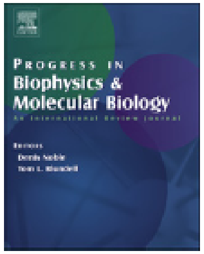
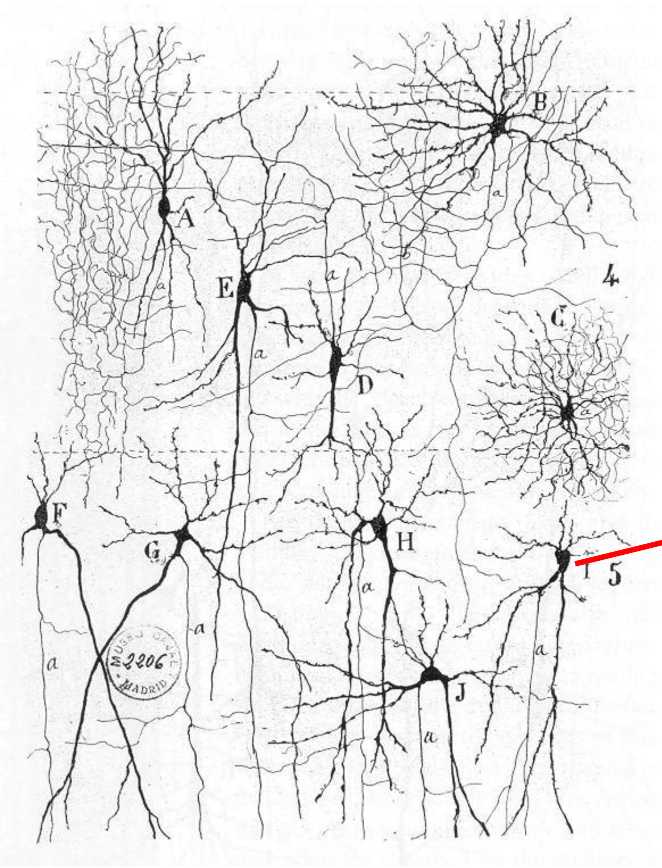


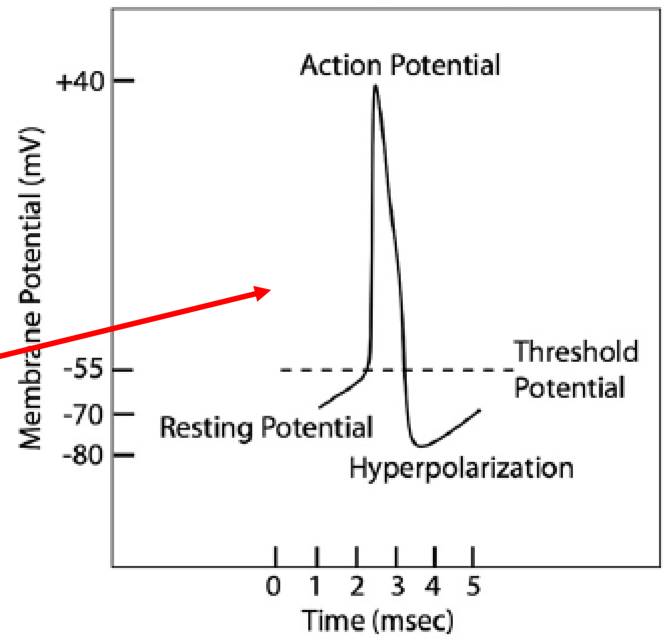

Review

Statistical mechanics of the neocortex

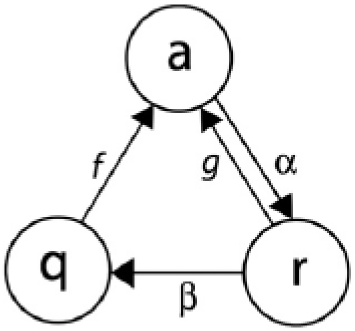
Michael A. Buice [a], Jack D. Cowan [b,*]

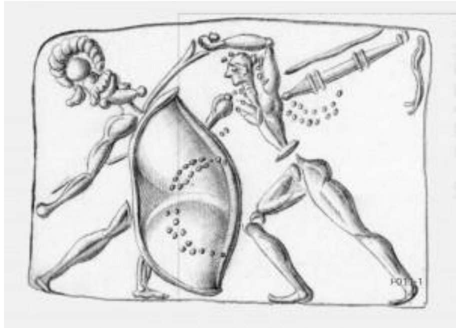
# Neural network as statistical model






**3–state model of neuron**



# Main idea

**If the brain is statistical machine,**
**Why ANN is classical one?**

# Main idea

**If the brain is statistical machine,**
**Why ANN is classical one?**

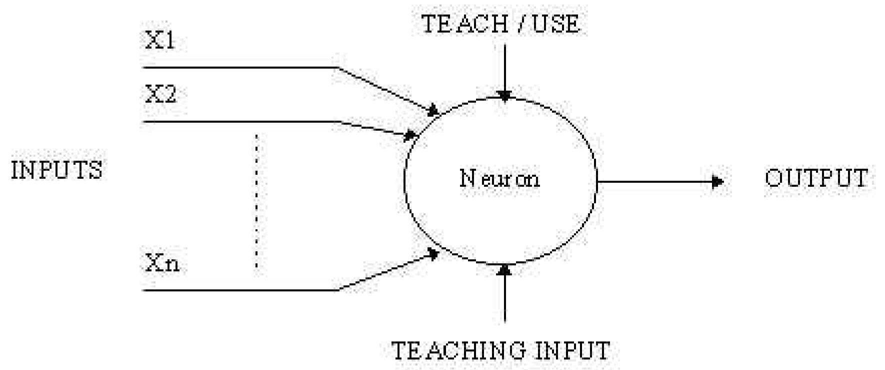
**Artificial Neuron must be**
**stochastic ….**

# Main idea

**If the brain is statistical machine,
Why ANN is classical one?**

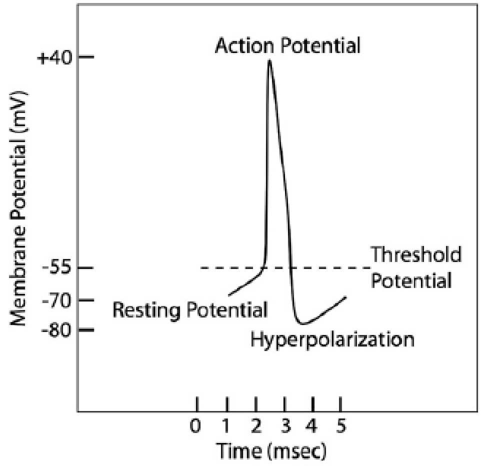
**Artificial Neuron must be
stochastic or Quantum.**

# Neuron: classical VS stochastic



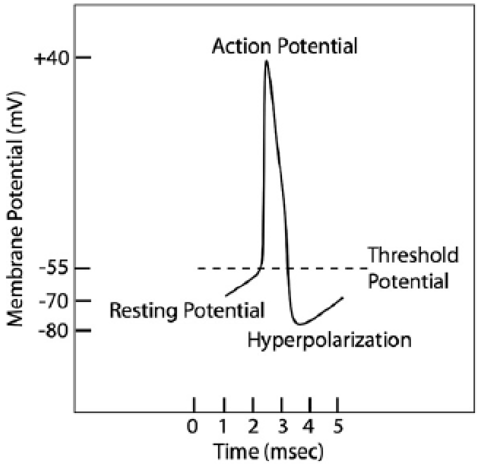

If Σ ( W_1 X_1 + W_2 X_2 + … + W_n X_n ) > I_treshold

then neuron generates spike

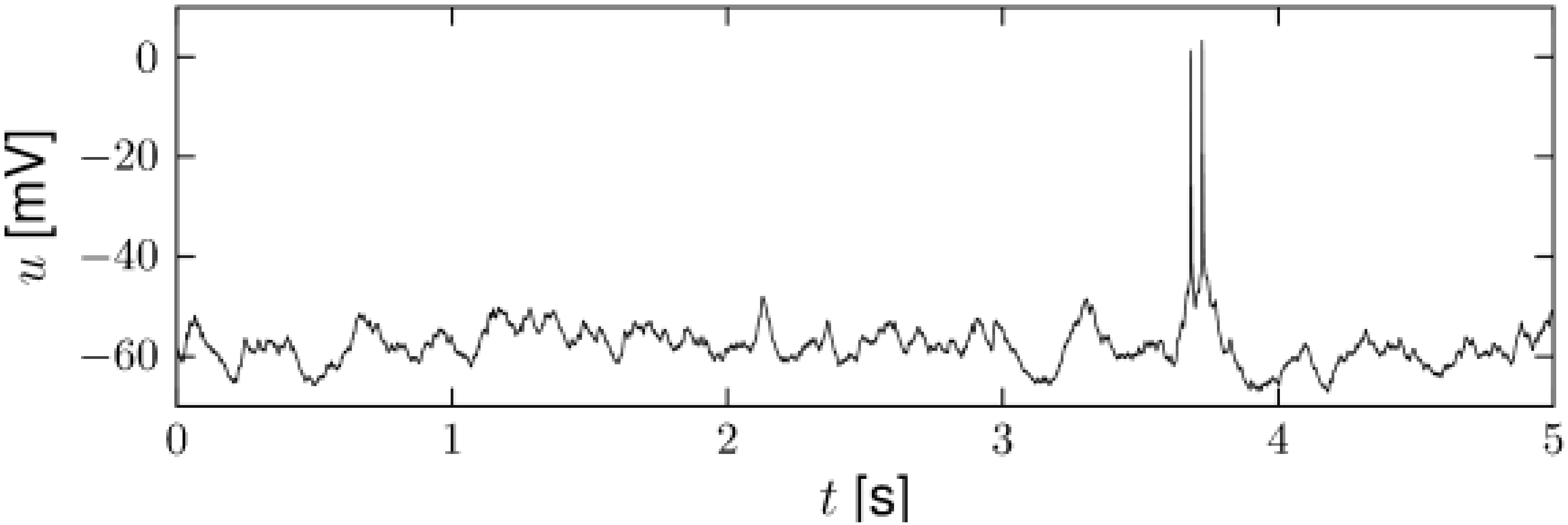


# Neuron: classical VS stochastic

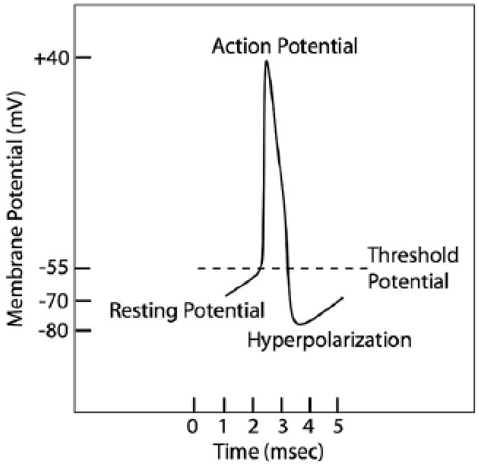
Simplification




Real neuron potential

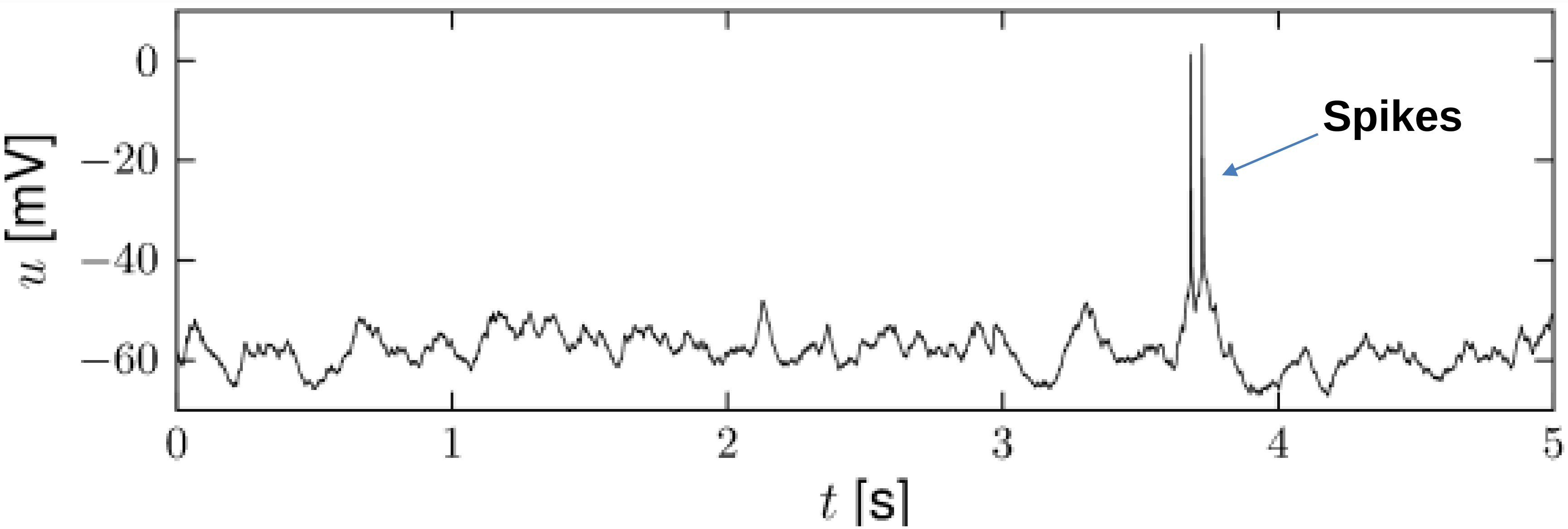


# Neuron: classical VS stochastic



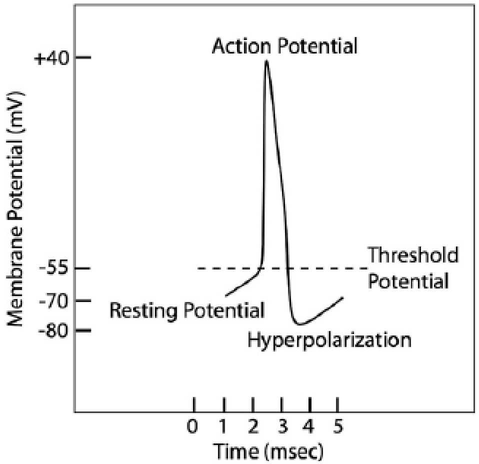

**Simplification**

**Real neuron potential**



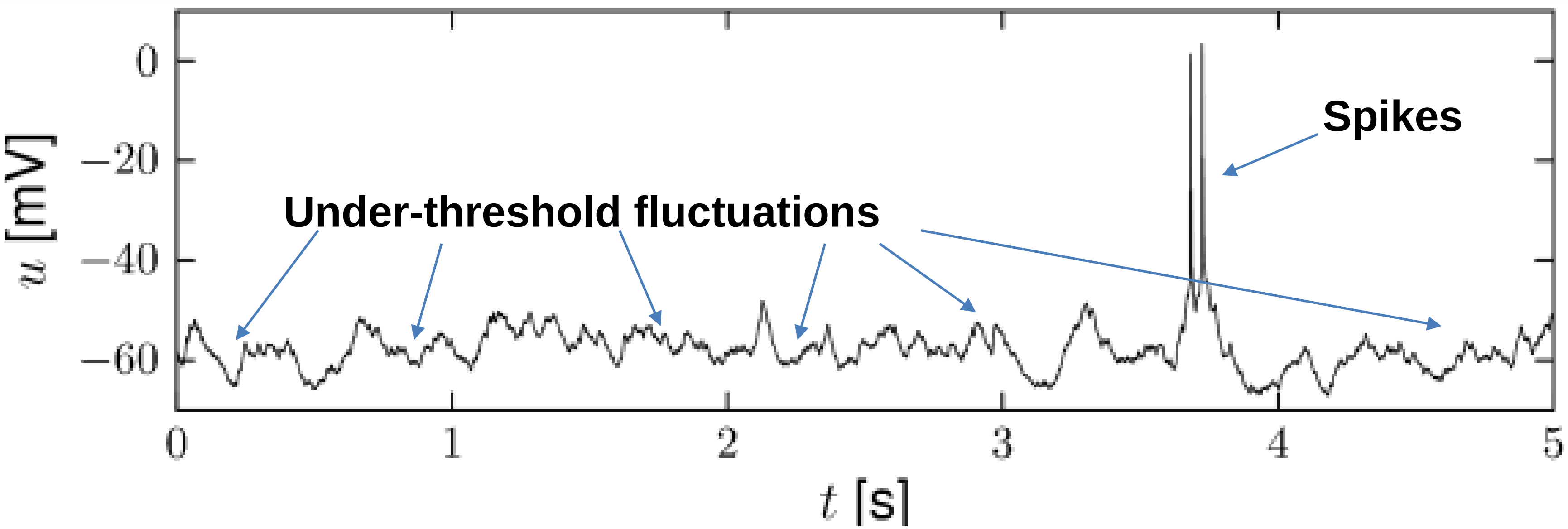

**Spikes**

# Neuron: classical VS stochastic

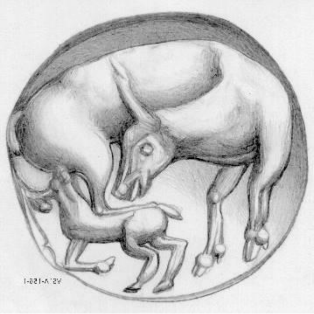


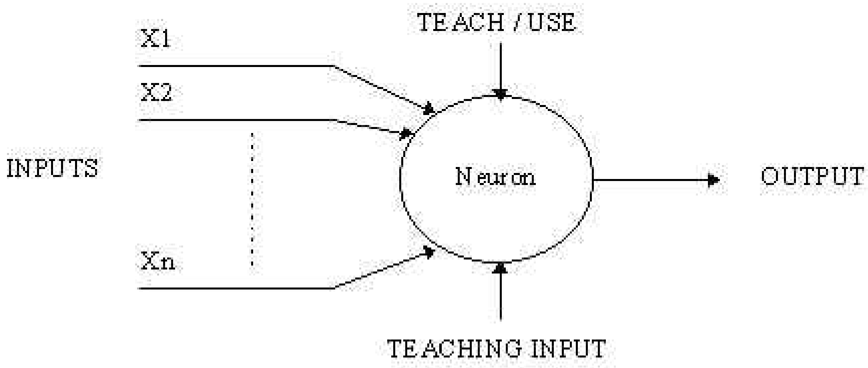

**Simplification**

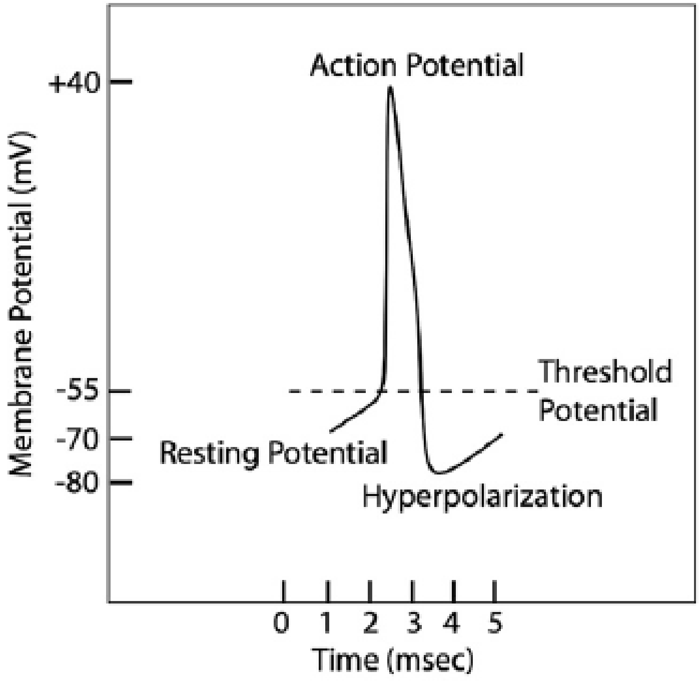
**Real neuron potential**

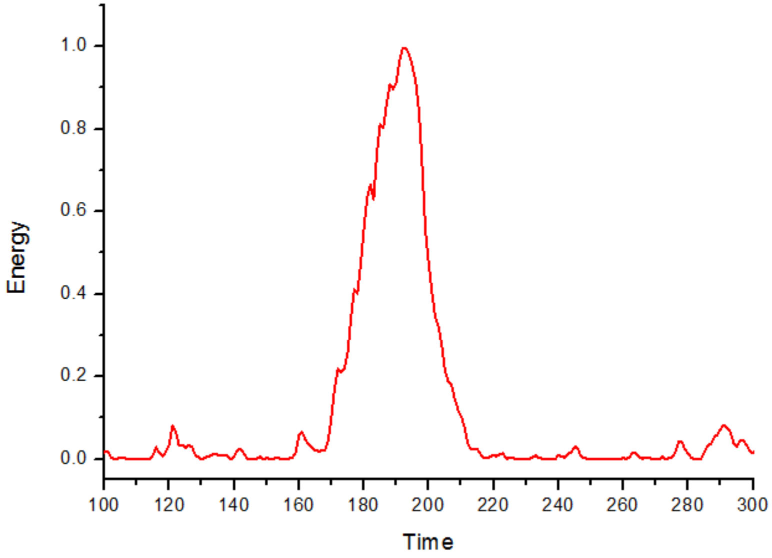


# Quantum neuron = Q-neuron

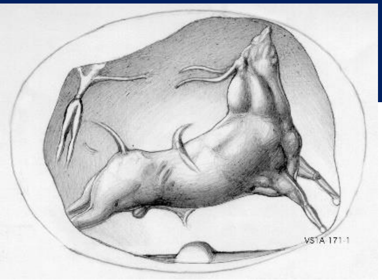


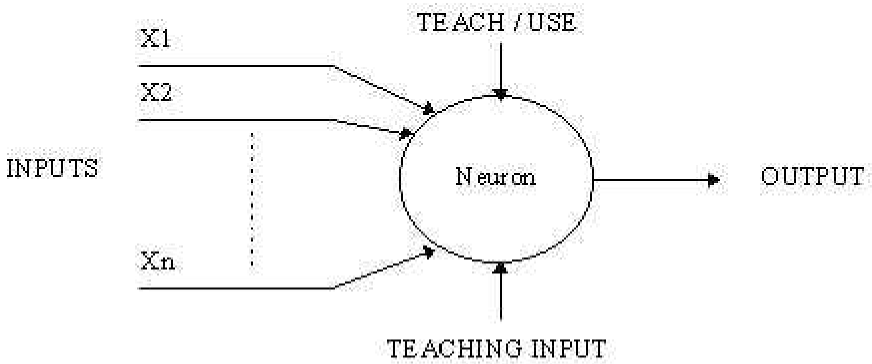


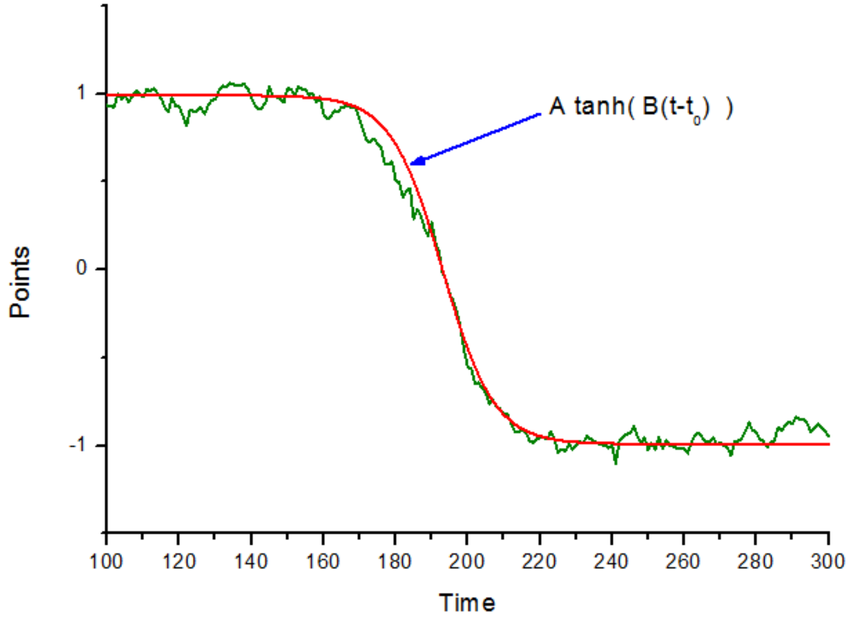



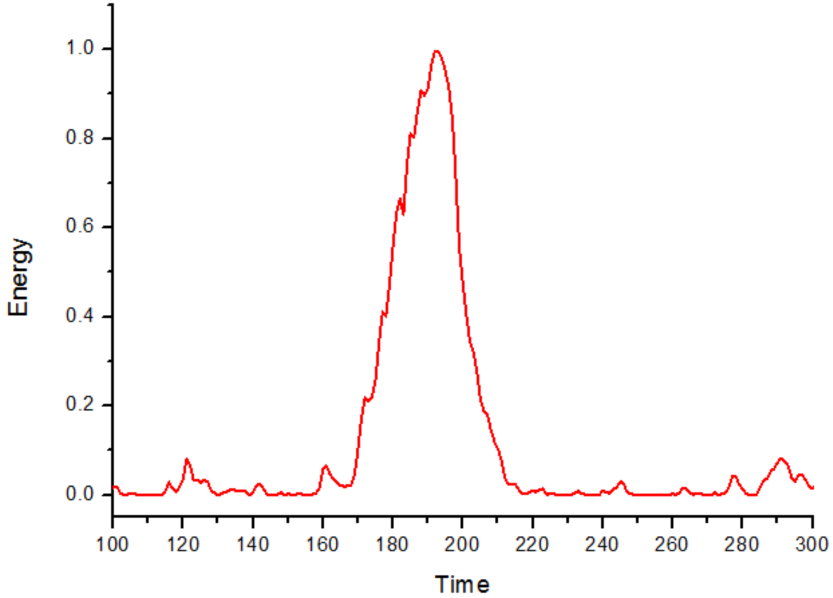



# Quantum neuron = Q-neuron





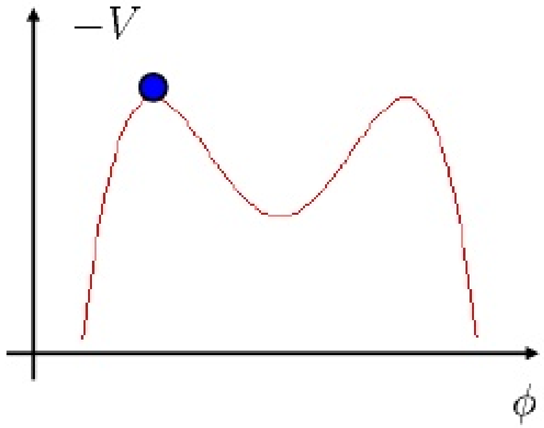



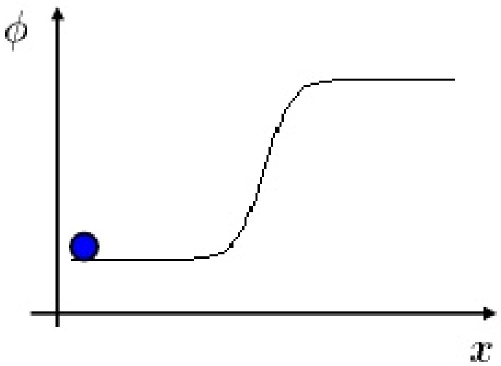



# Quantum neuron

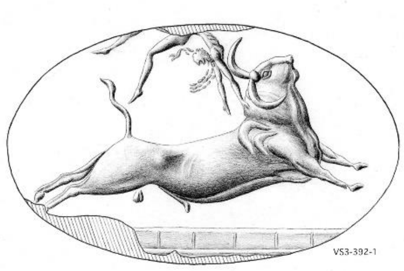
$$\hat{H}_i = \frac{1}{2}\hat{p}_i^2 + V_0\left(\hat{\varphi}_i\right).$$

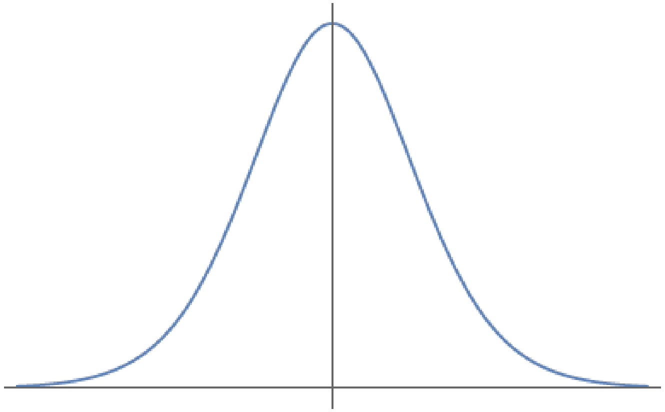
$$V_0(\varphi_i) = \frac{\Lambda}{4}\left(\varphi^2 - \frac{\mu^2}{\Lambda}\right)^2.$$



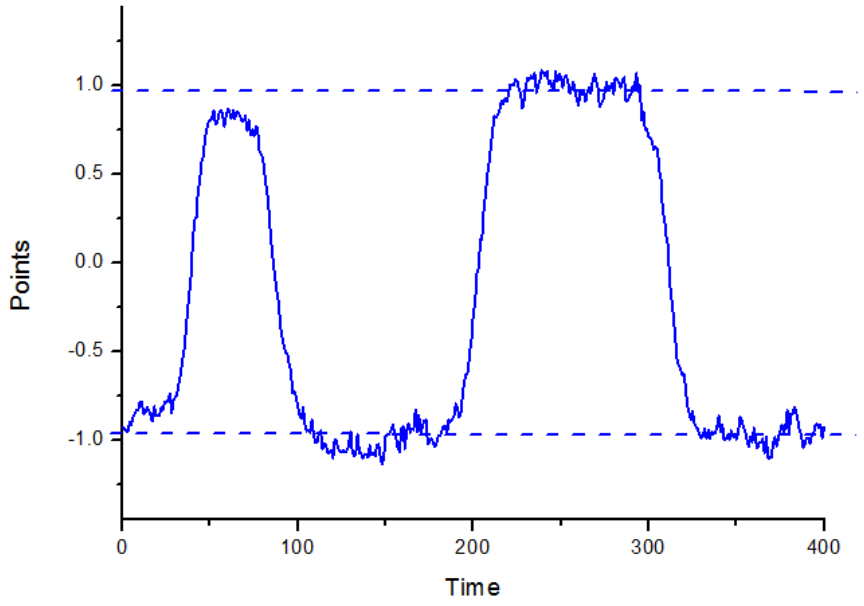



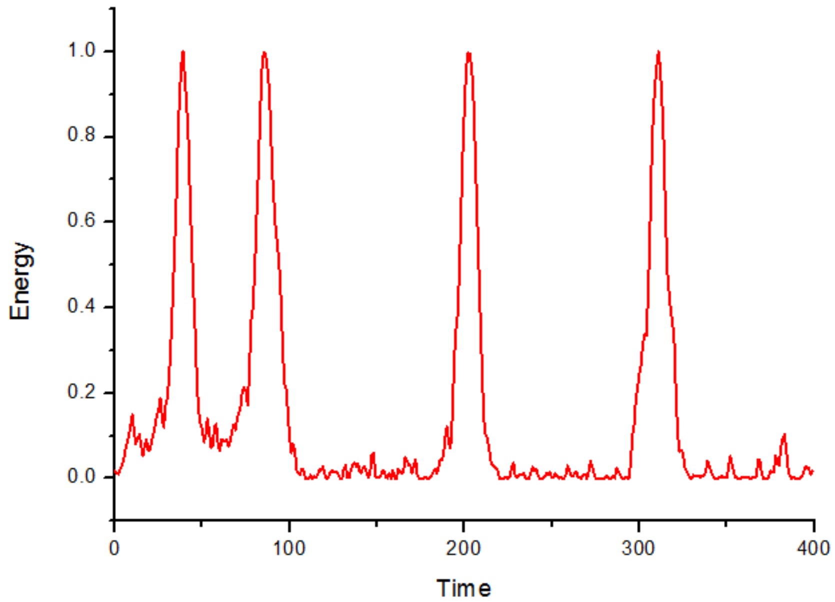

$$\phi(x,t) = \frac{m}{\sqrt{\lambda}}\tanh\left(\frac{m}{\sqrt{2}}(x - x_0)\right)$$



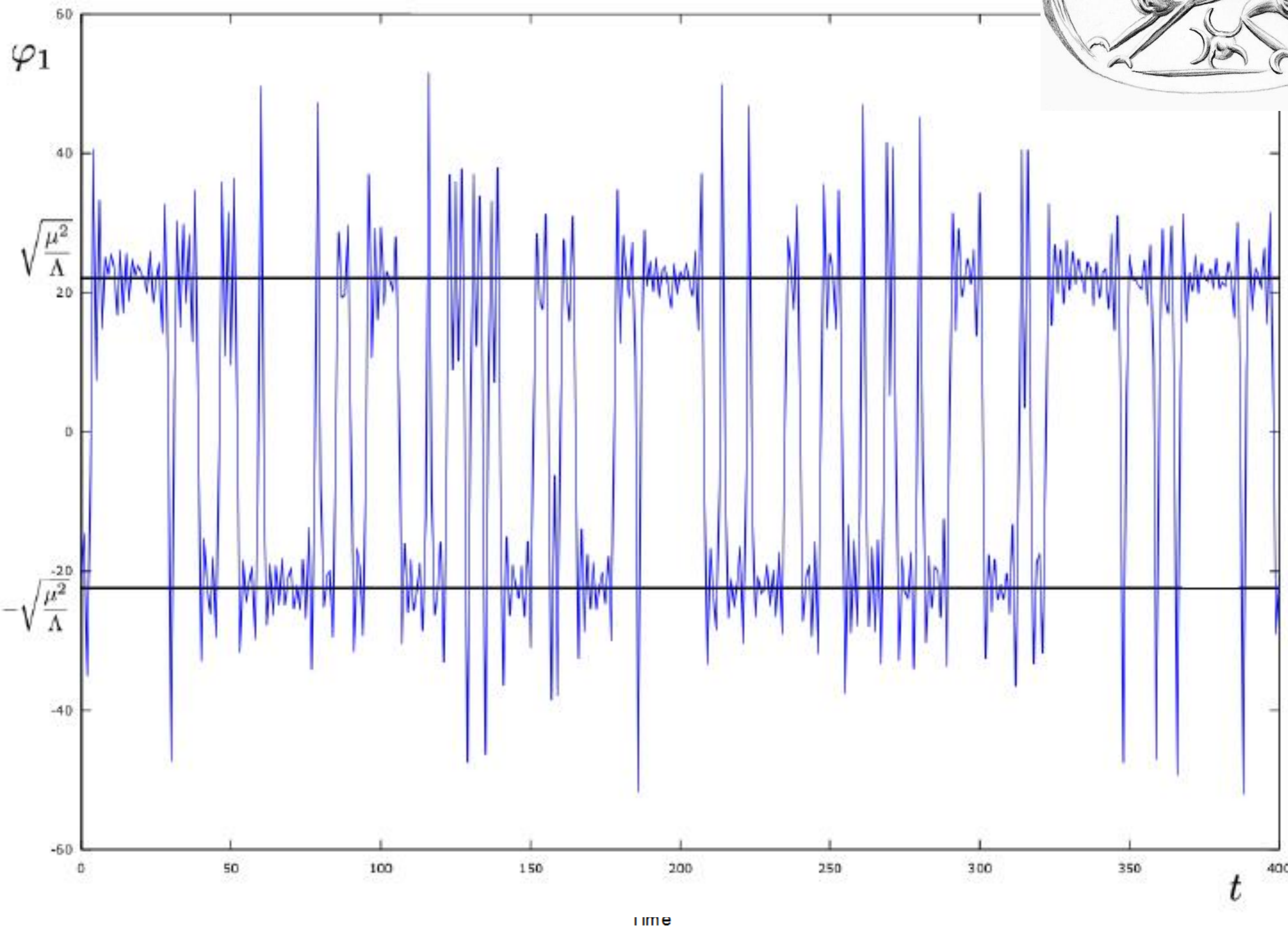



# Quantum neuron = Q-neuron

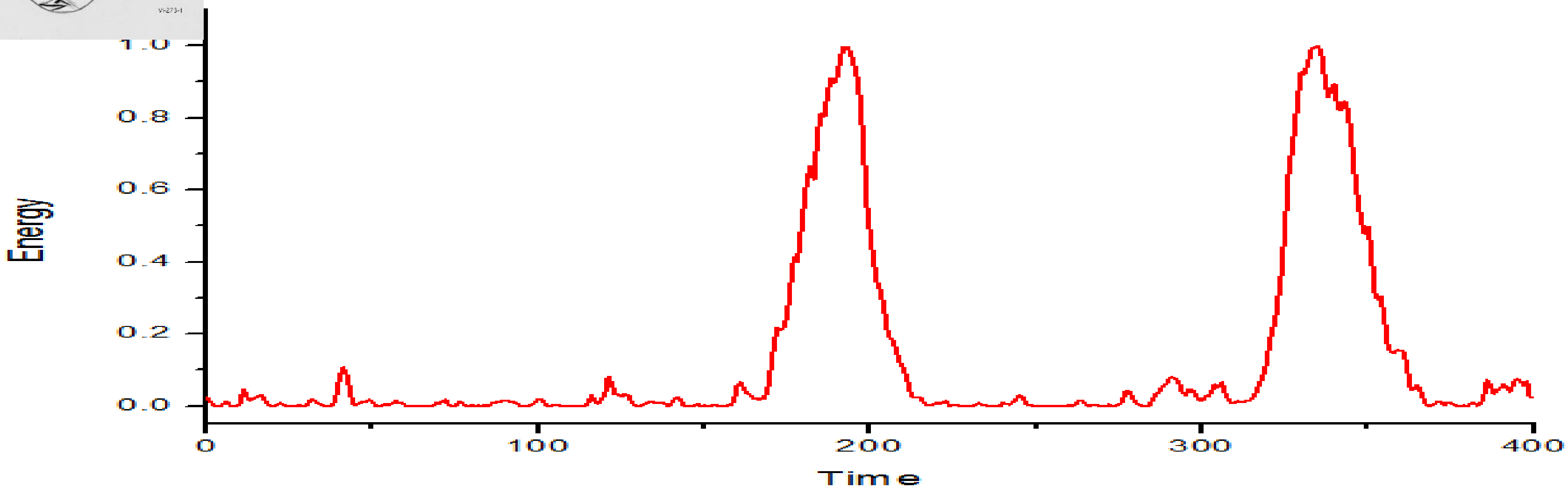


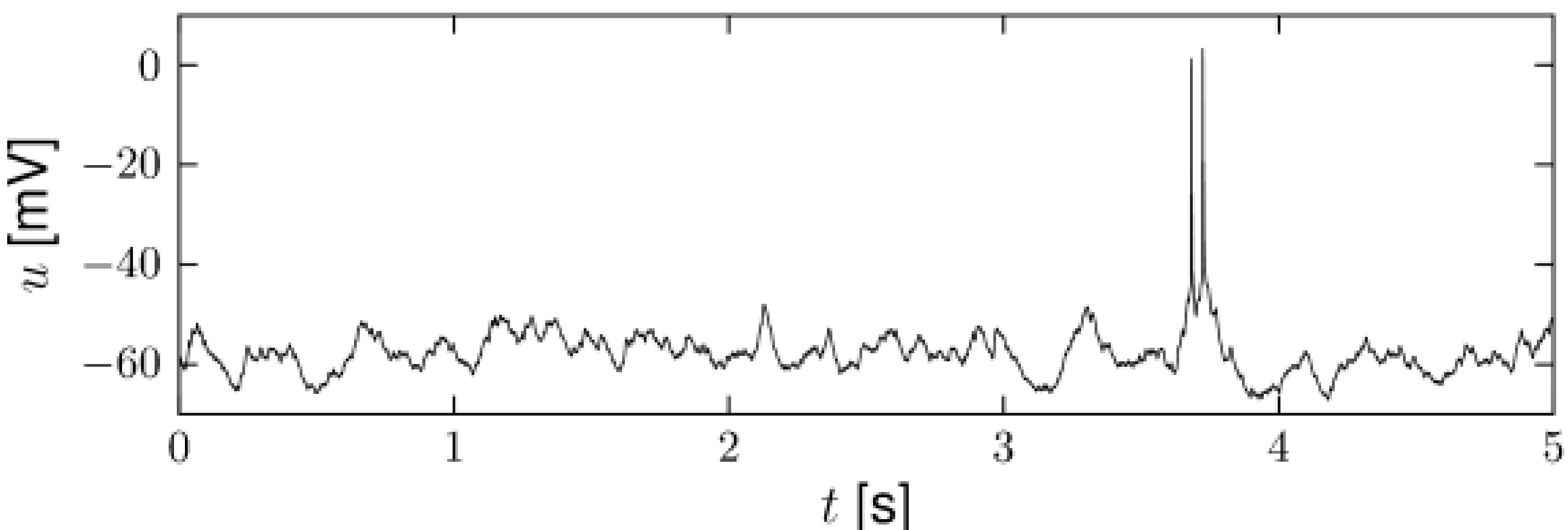



# Quantum neuron = Q-neuron




# Quantum neuron = Q-neuron

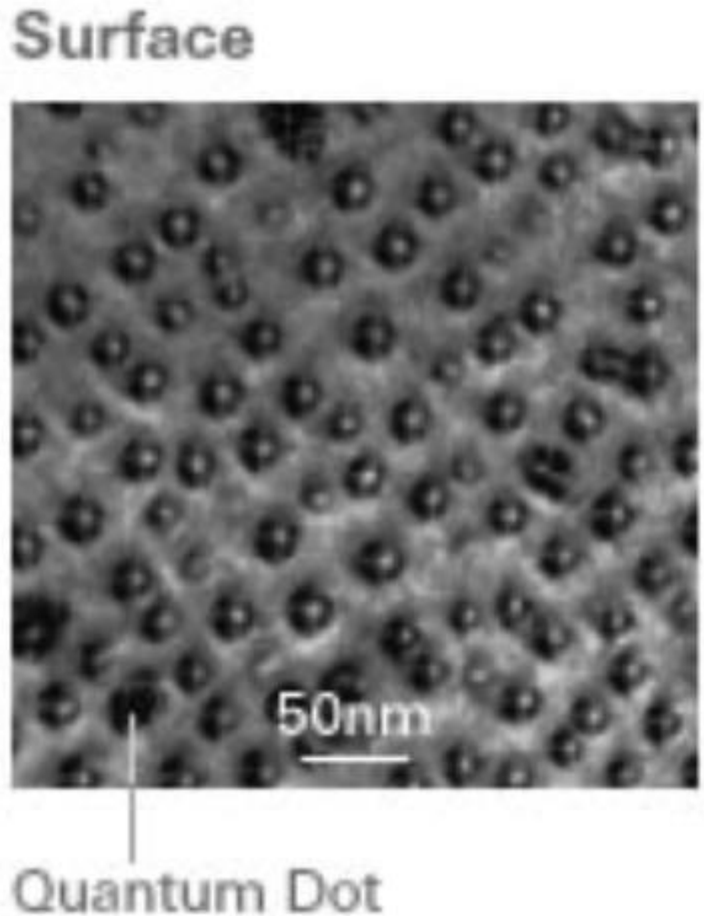


# Nano-technological realizations

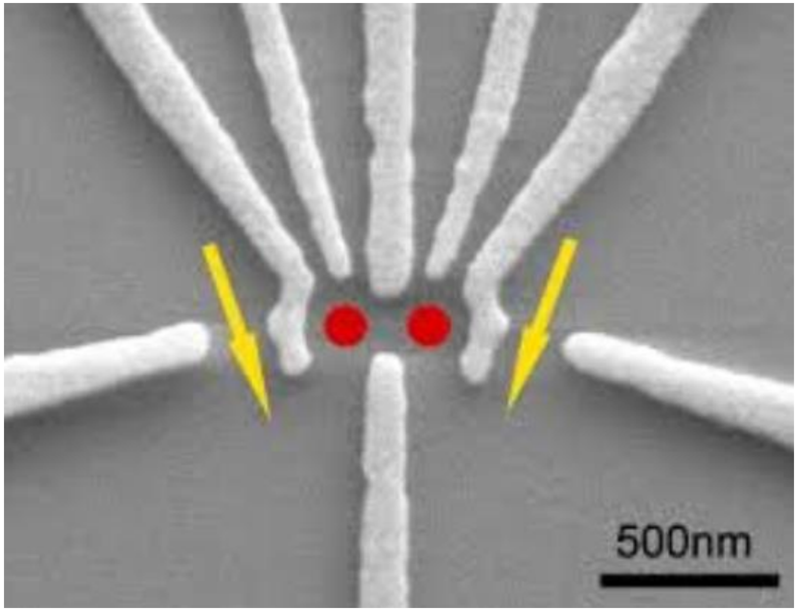
We need in Nano-technological platform for realization of QNN.

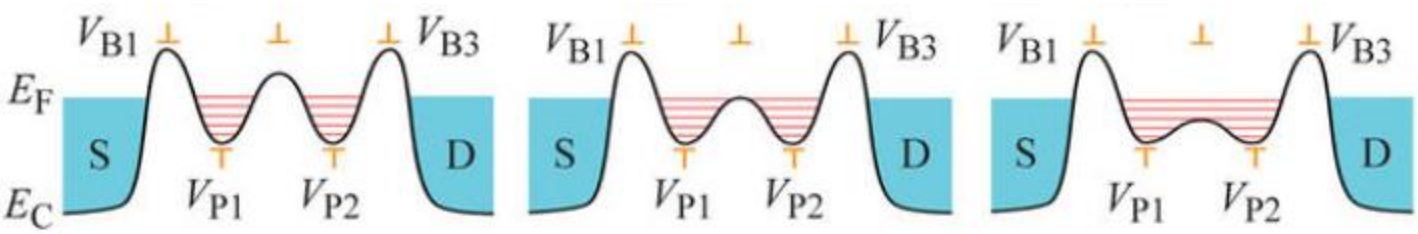
One possible way:  **quantum double dots**.



# Nano-technological realizations

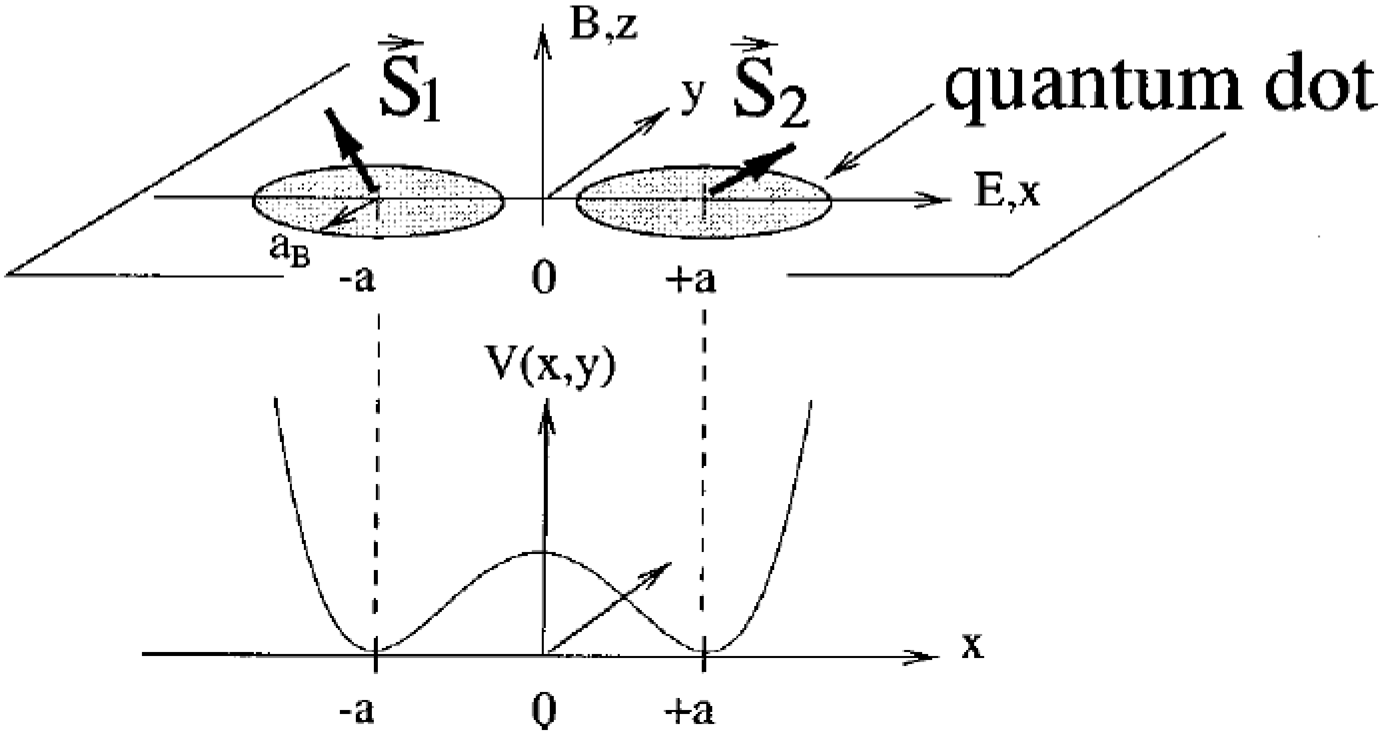
**Quantum double dots**.




*Scientific Reports* **1**, Article number: 110 (2011)

## Quantum double dots.



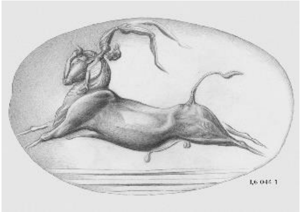

PHYSICAL REVIEW B VOLUME 59, NUMBER 3 15 JANUARY 1999-I

**Coupled quantum dots as quantum gates**

Guido Burkard* and Daniel Loss†
*Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland*

David P. DiVincenzo‡
*IBM Research Division, Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598*

(Received 3 August 1998)

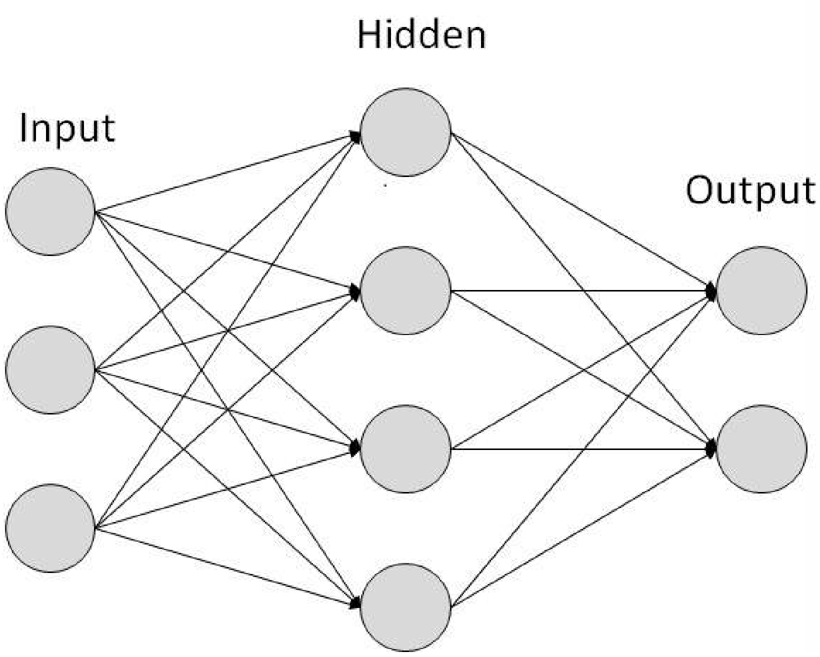


# Quantum neural network as quantum many body system

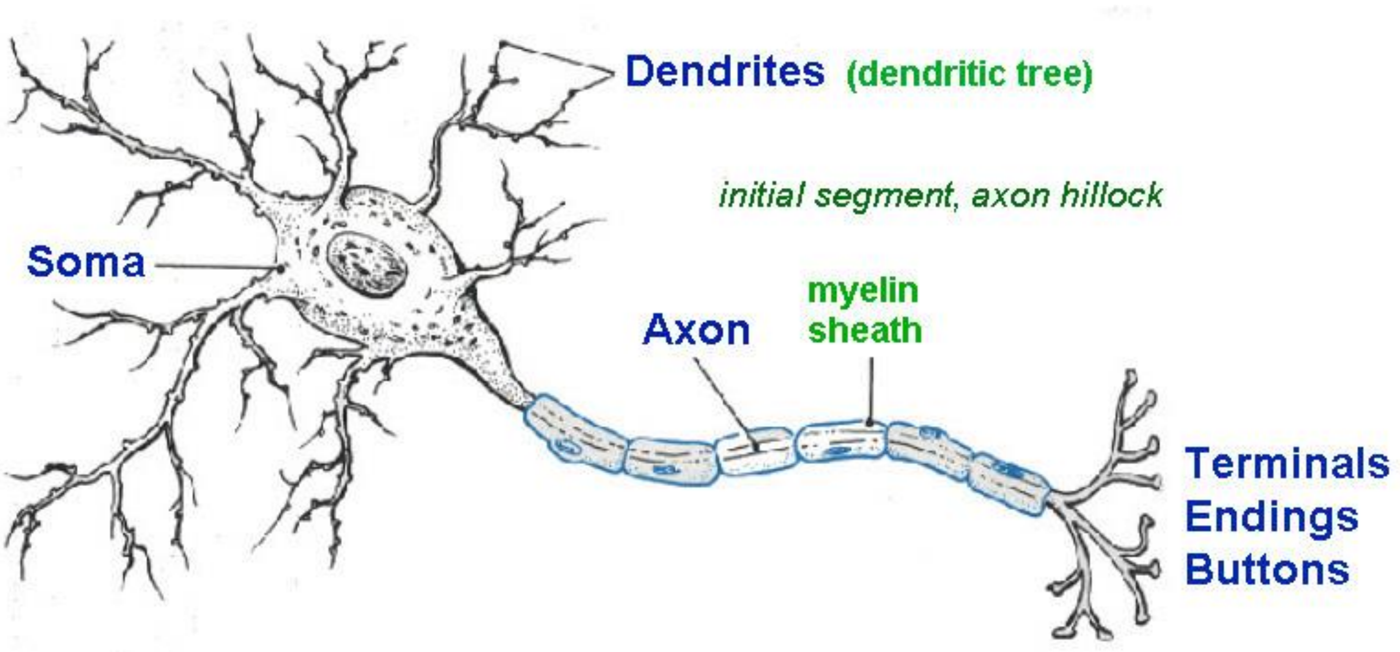
$$Z = \int \prod_i \mathcal{D}\varphi_i\,(\tau)\exp(-S(\varphi_i(\tau))),\, \varphi_i(0) = \varphi_i(T)$$

$$S = \int_0^T d\tau \left[\sum_i \left(\frac{1}{2}\dot{\varphi}_i^2 + V_0(\varphi_i)\right) + \sum_{i>j} V_{int}(\varphi_i, \varphi_j)\right]$$

$$\langle \mathcal{O}(\varphi_1, ..., \varphi_i)\rangle = \frac{1}{Z}\int \prod_i \mathcal{D}\varphi_i(\tau)\mathcal{O}(\varphi_1, ..., \varphi_i)\exp(-S(\varphi_i))$$



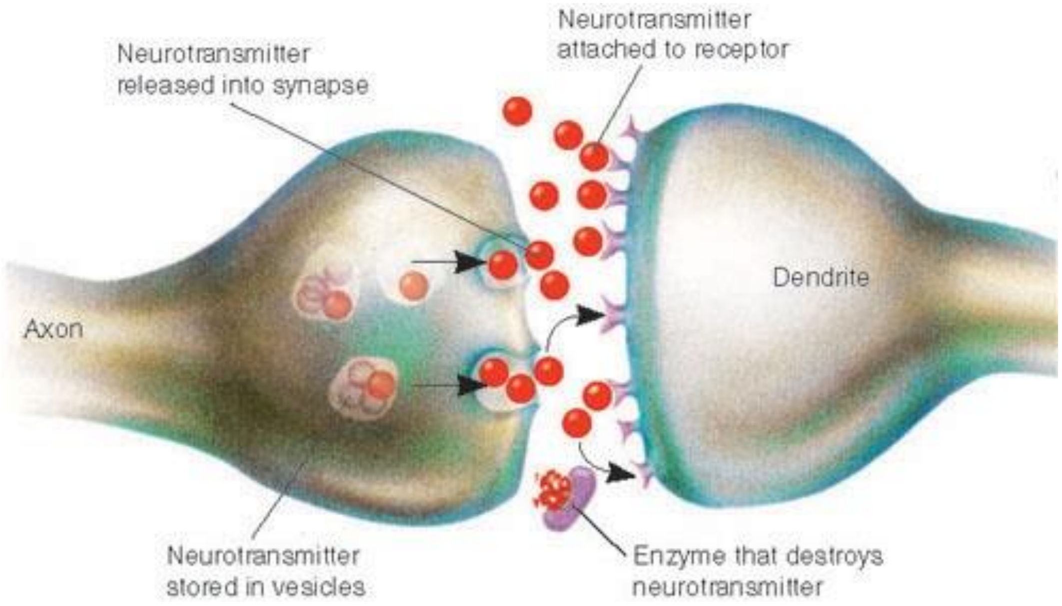
# Axons in neural net

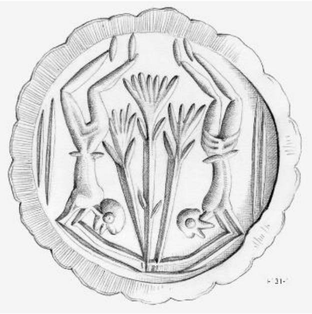



**"Axon" is output information line from neuron.**
**So neural net is very non-local system.**

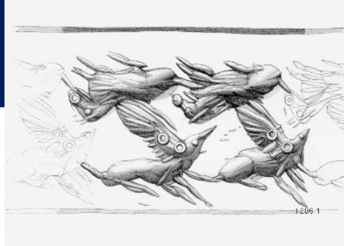
# Role of Synepse

**Role of Synepse is the contact coefficient, the measure of neuron connection.**

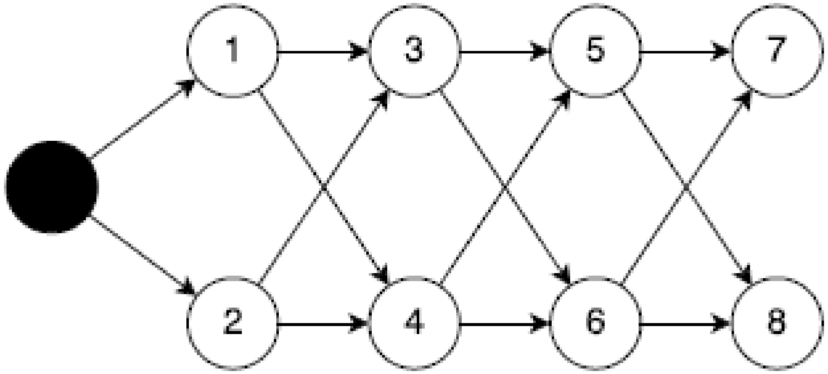


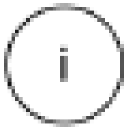



# Excitation connection

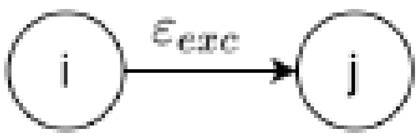


$$S = \int_0^T d\tau \left[ \sum_i \left( \frac{1}{2}\dot{\varphi}_i^2 + V_0(\varphi_i) \right) + \sum_{i>j} V_{int}(\varphi_i, \varphi_j) \right]$$



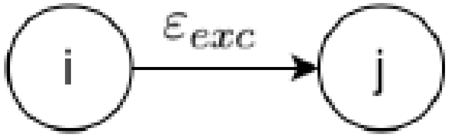

1. 


$$-\ \mathcal{L}_0 = \frac{1}{2}\dot{\varphi}_i^2 + \frac{\Lambda}{4}\left(\varphi_i^2 - \frac{\mu^2}{\Lambda}\right)^2$$

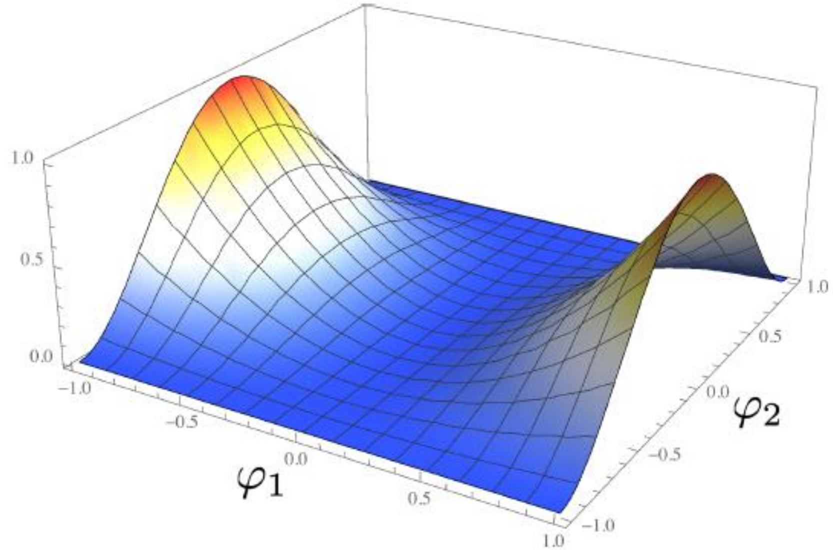
2. 


$$-\ \mathcal{L}_{int} = \varepsilon_{exc}\varphi_j^2 \left(\varphi_i^2 - \frac{\mu^2}{\Lambda}\right)^2$$

# Excitation connection

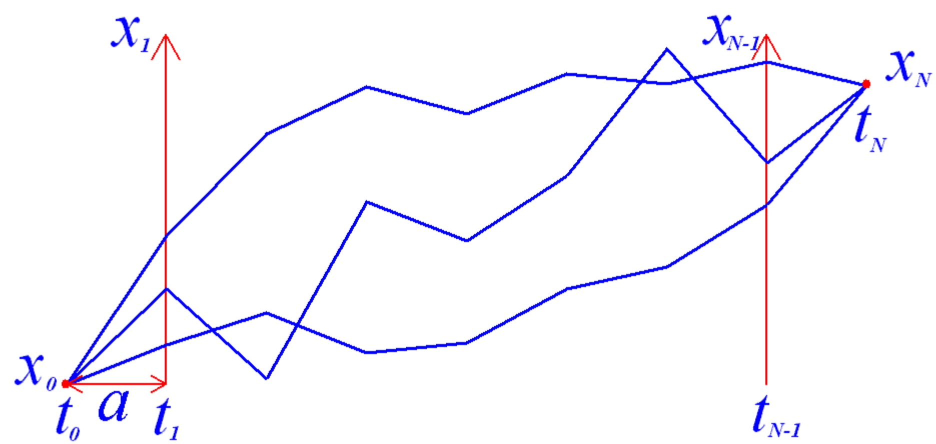



$$-\mathcal{L}_{int} = \varepsilon_{exc}\varphi_j^2\left(\varphi_i^2 - \frac{\mu^2}{\Lambda}\right)^2$$

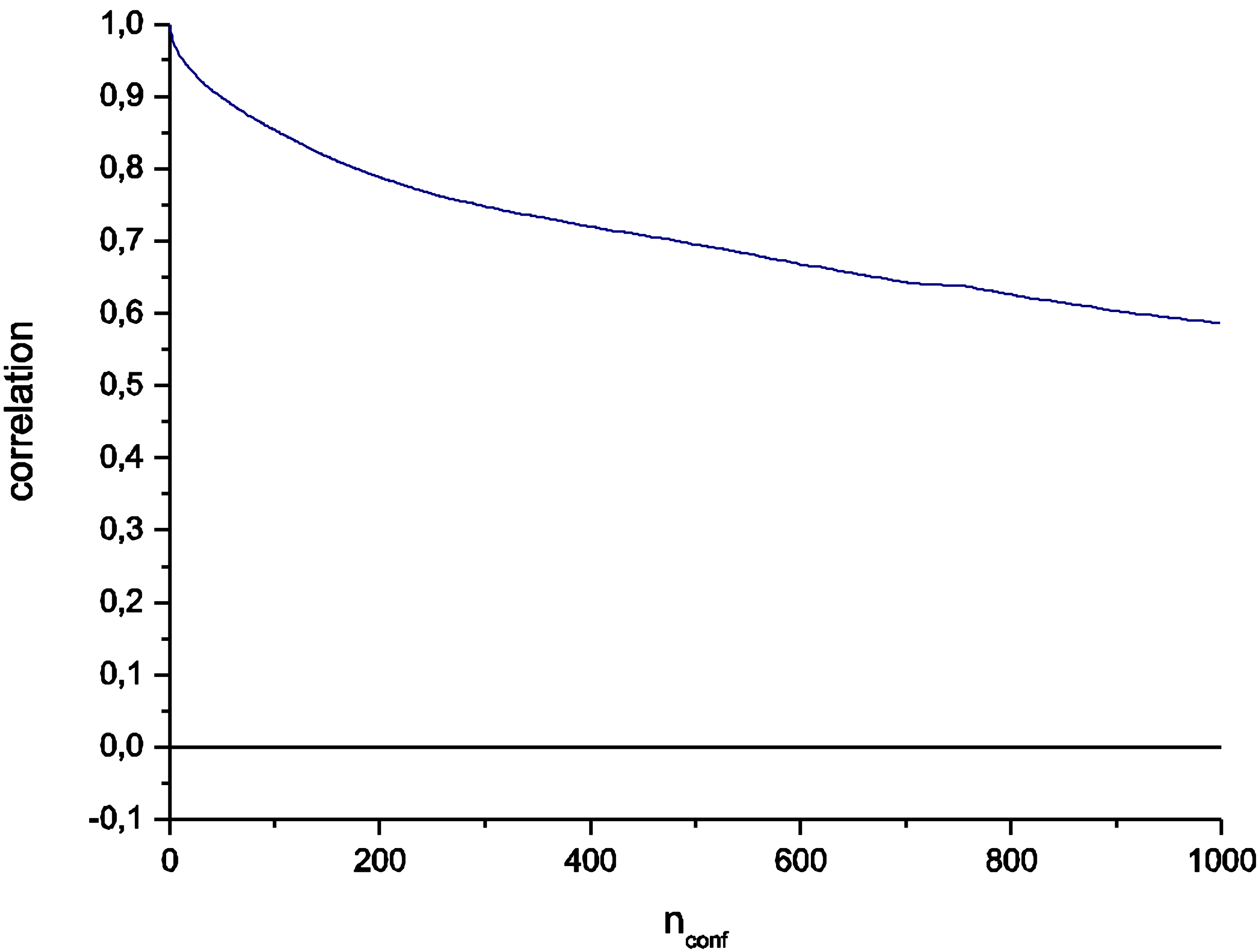



$$\langle A \rangle = \frac{\int \mathcal{D}x A(x) e^{iS_m(x)/\hbar}}{\int \mathcal{D}x e^{iS_m(x)/\hbar}} \rightarrow \frac{\sum_{conf} A(x) e^{-S(x)/\hbar}}{\sum_{conf} e^{-S(x)/\hbar}} = \sum_{conf} A(x) \mathcal{P}(x)$$

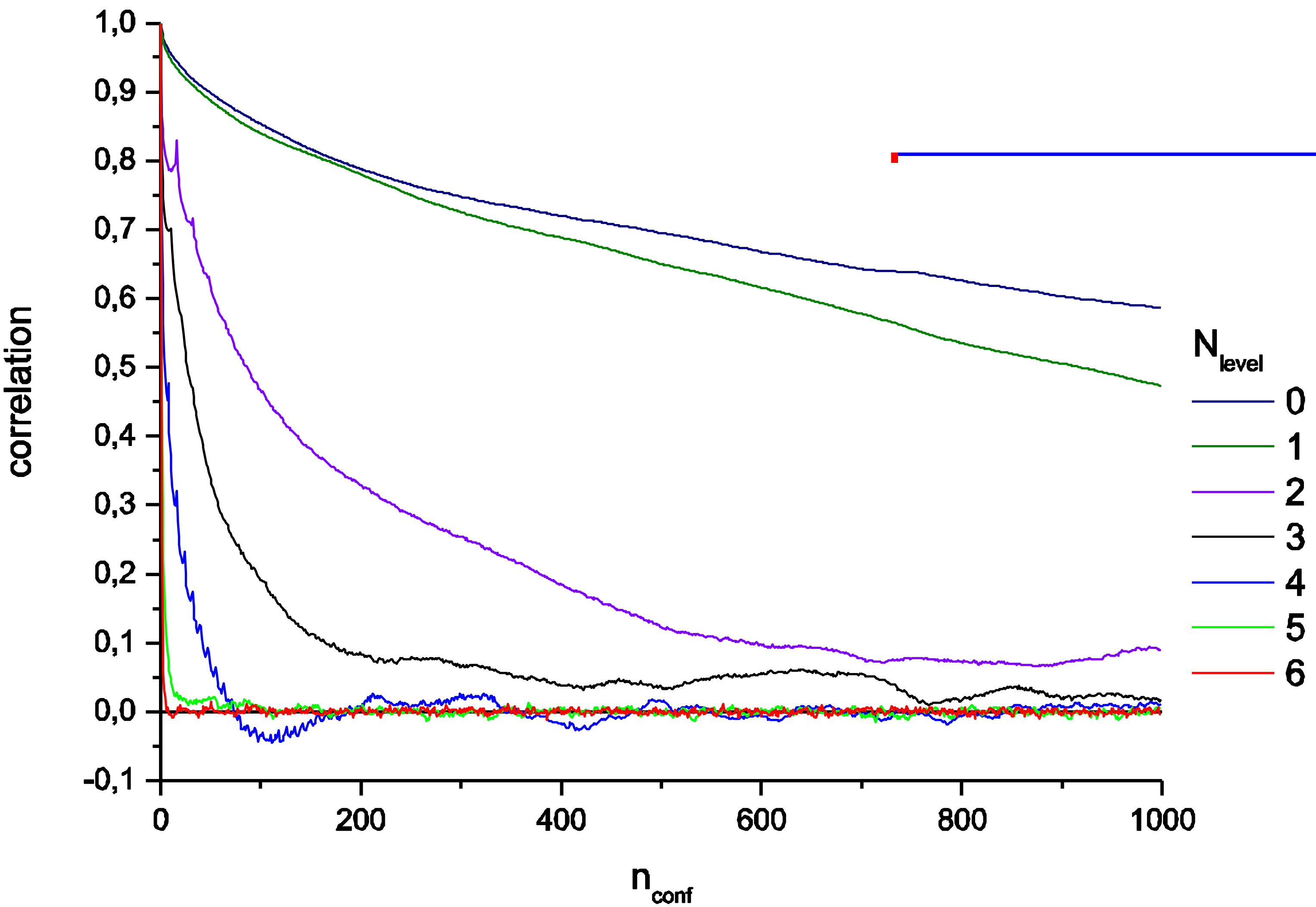
$$\mathcal{P}(x) = \frac{e^{-S(x)/\hbar}}{\sum_{conf} e^{-S(x)/\hbar}}$$

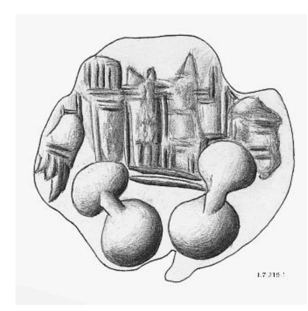



$$\langle A \rangle = \frac{1}{N_{conf}} \sum_{k=1}^{N_{conf}} A(x_k)$$

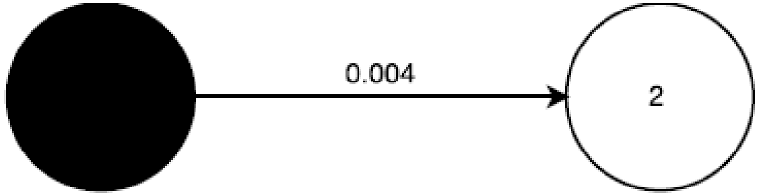


correlation
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
-0,1
0
200
400
600
800
1000
$n_{conf}$



correlation
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
-0,1
0
200
400
600
800
1000
$n_{conf}$
$N_{level}$
0
1
2
3
4
5
6

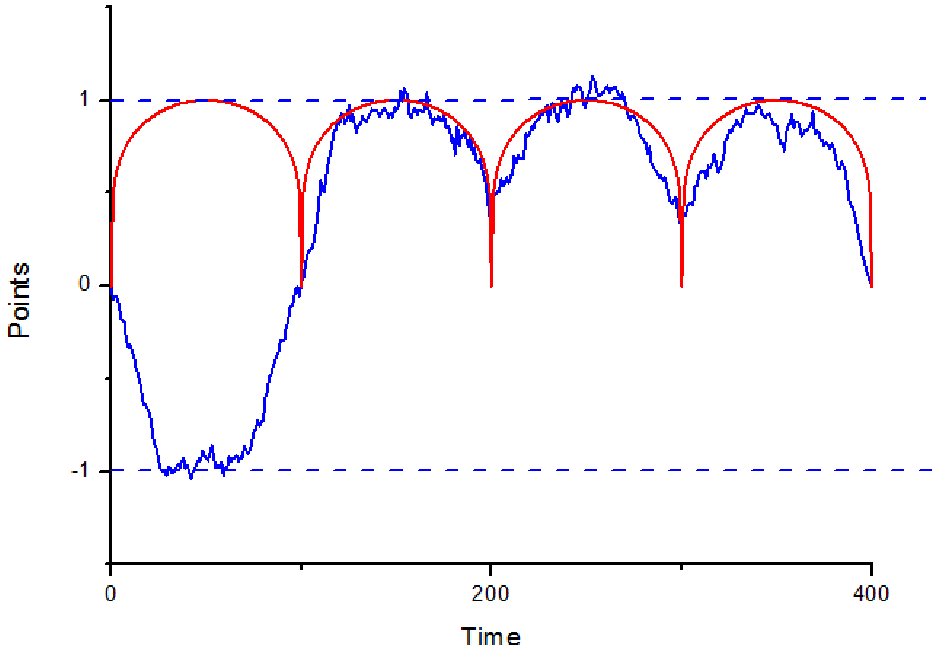


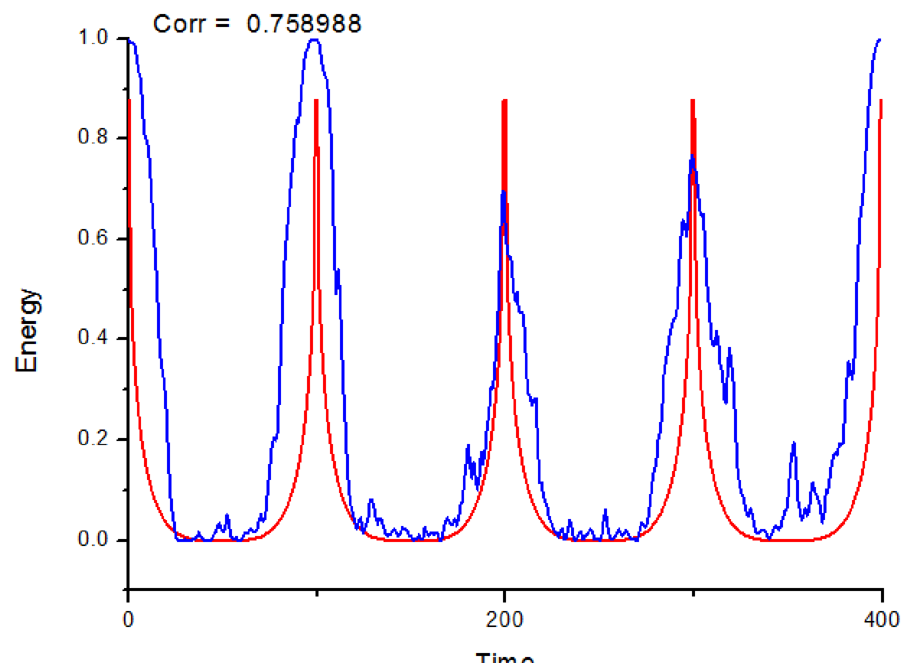
# Excitation connection: simple test



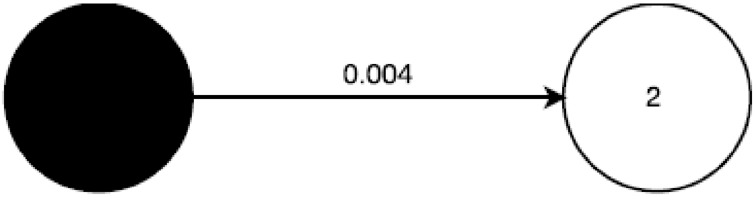

$$Z = \int \prod_i \mathcal{D}\varphi_i(\tau) \exp(-S(\varphi_i(\tau))), \varphi_i(0) = \varphi_i(T)$$

$$S = \int_0^T d\tau \left[ \sum_i \left( \frac{1}{2}\dot{\varphi}_i^2 + V_0(\varphi_i) \right) + \sum_{i>j} V_{int}(\varphi_i, \varphi_j) \right]$$



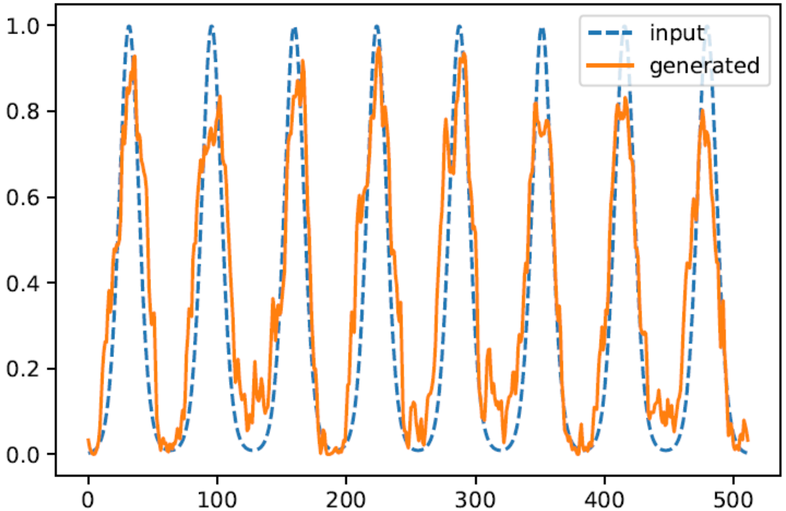



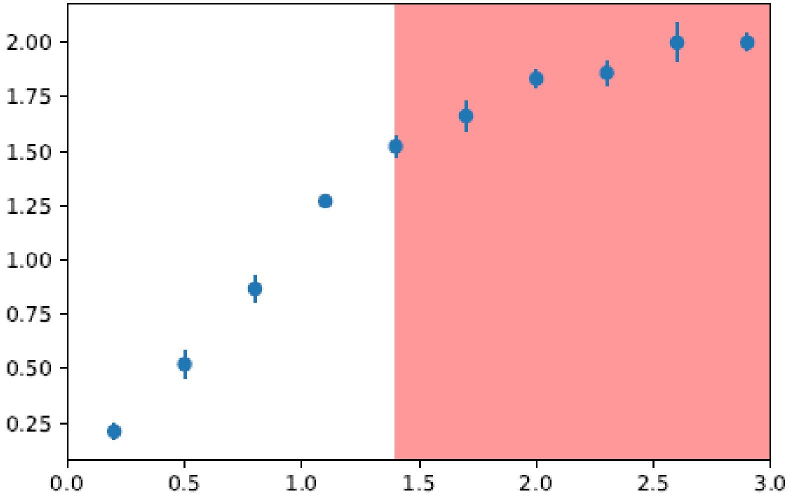
# Excitation connection: simple test



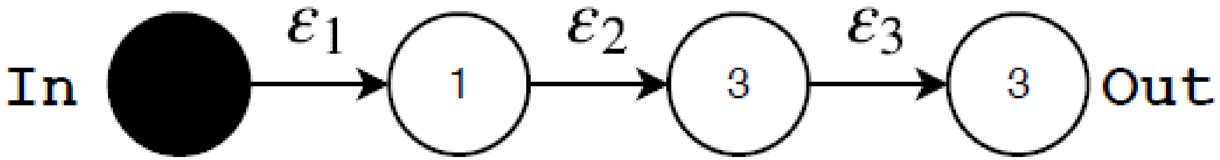

**Activity of Q-neuron**

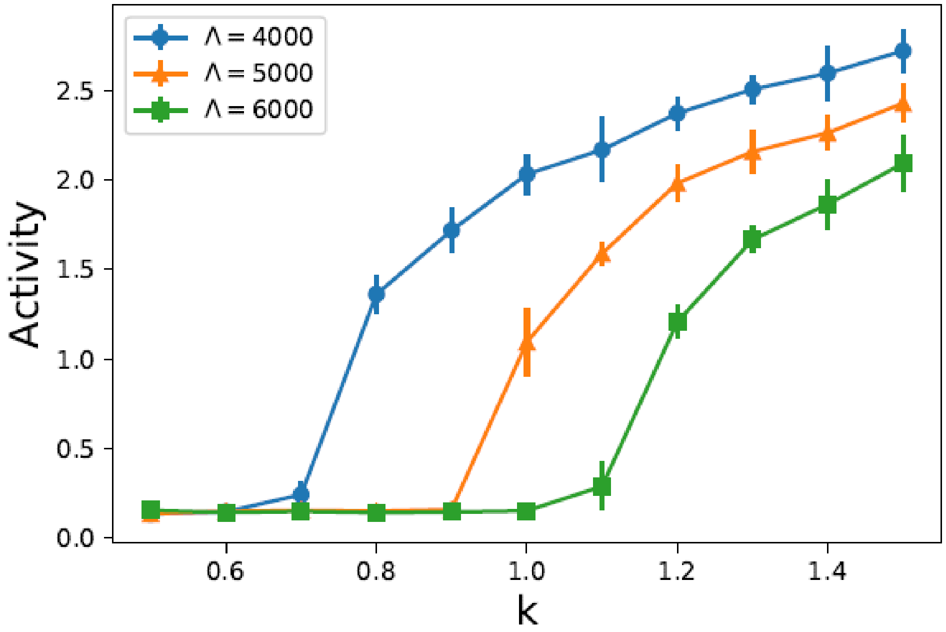
$$\left(\varphi_2^2 - 1\right)^2$$

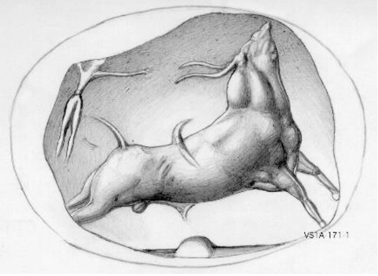


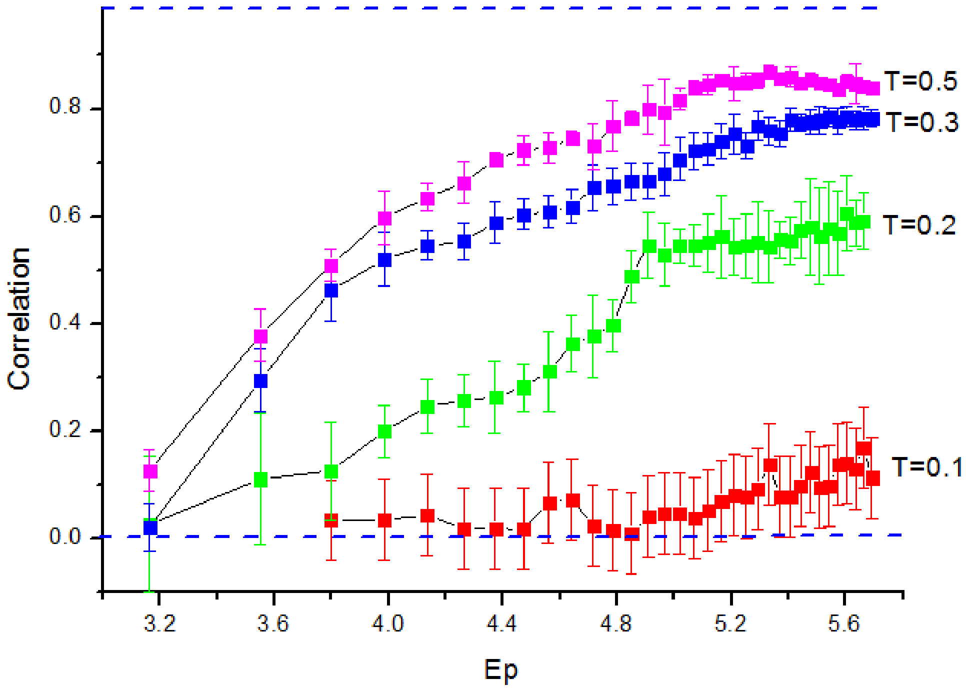
# Excitation connection: 3 Q-neurons transport

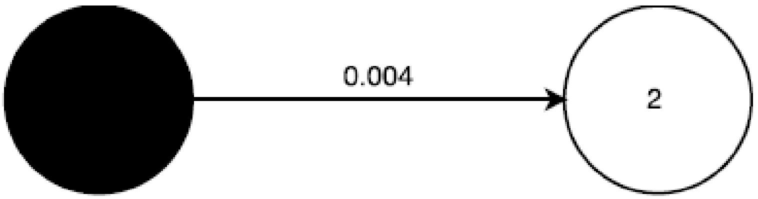


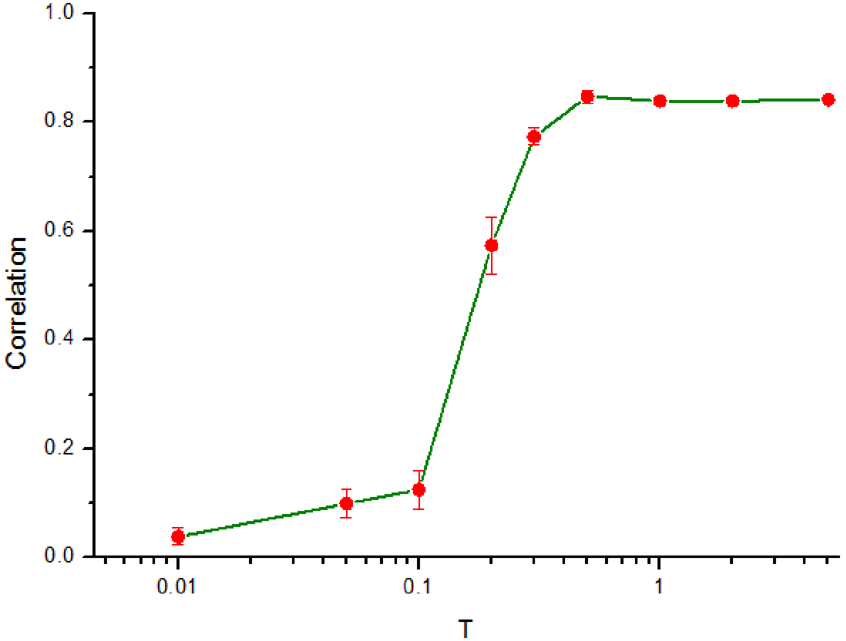



# Quantum neuron



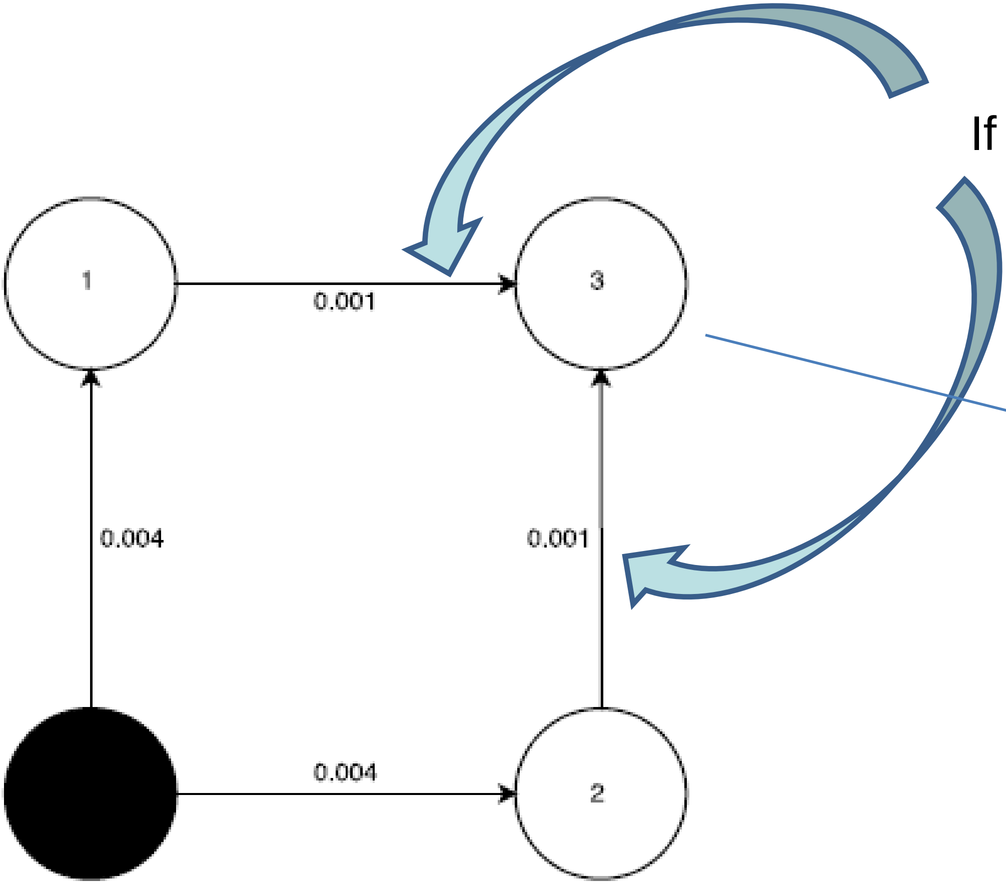



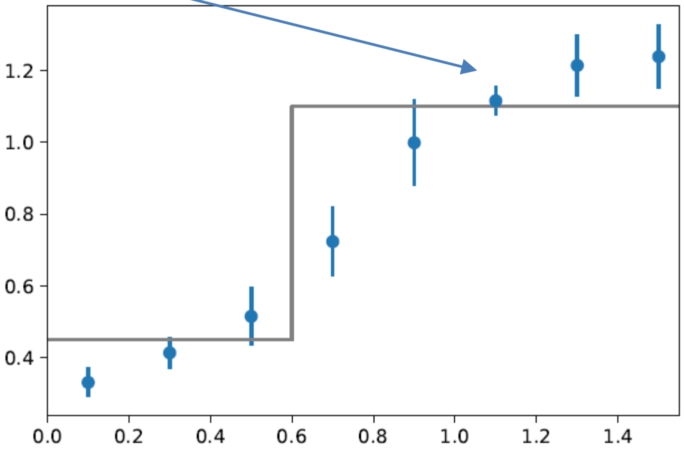



# Quantum neural network logical elements

**Logical AND**

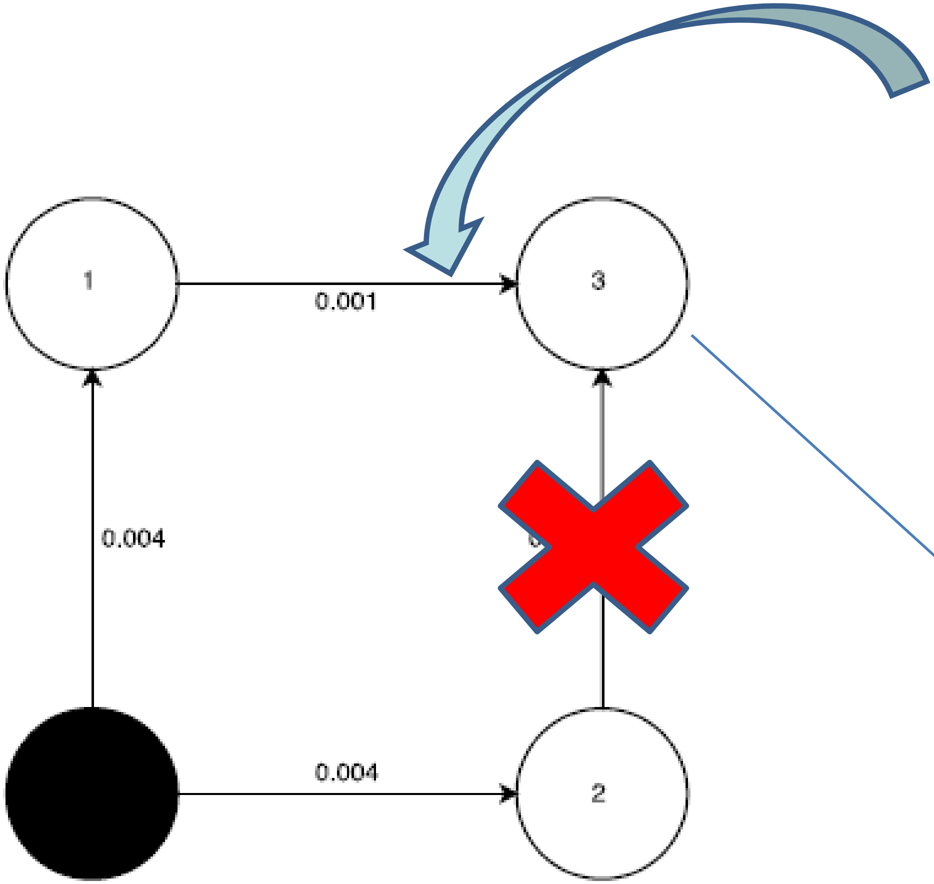
If both Paths are active,
Correlation more 70 %



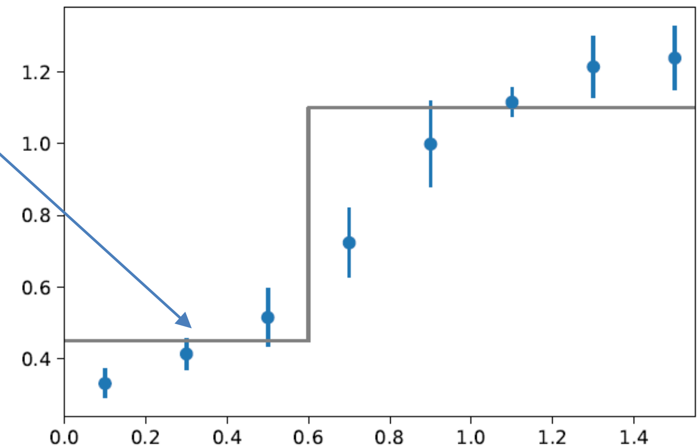



# Quantum neural network logical elements

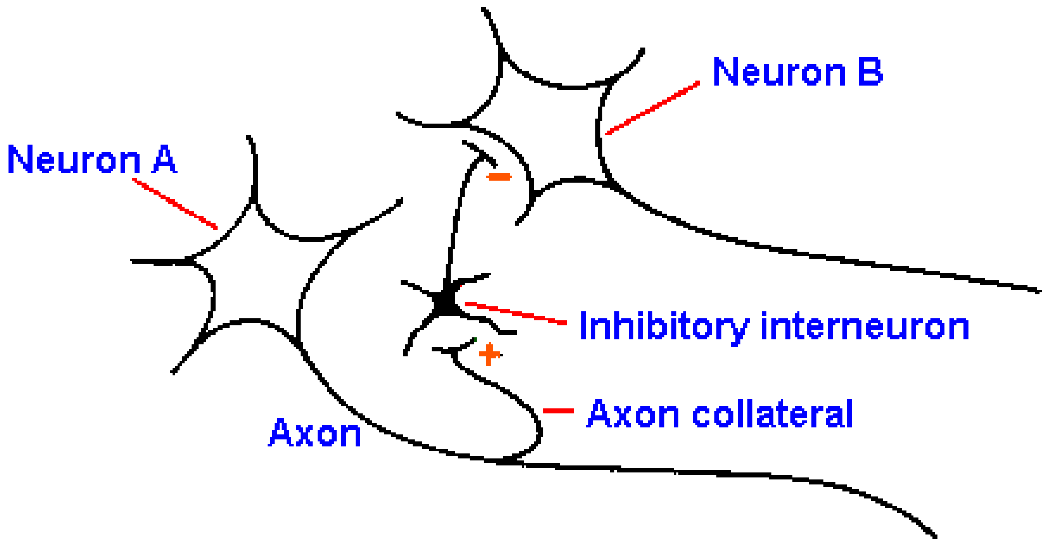
**Logical AND**




If only one Path is active,
Correlation less 20 %

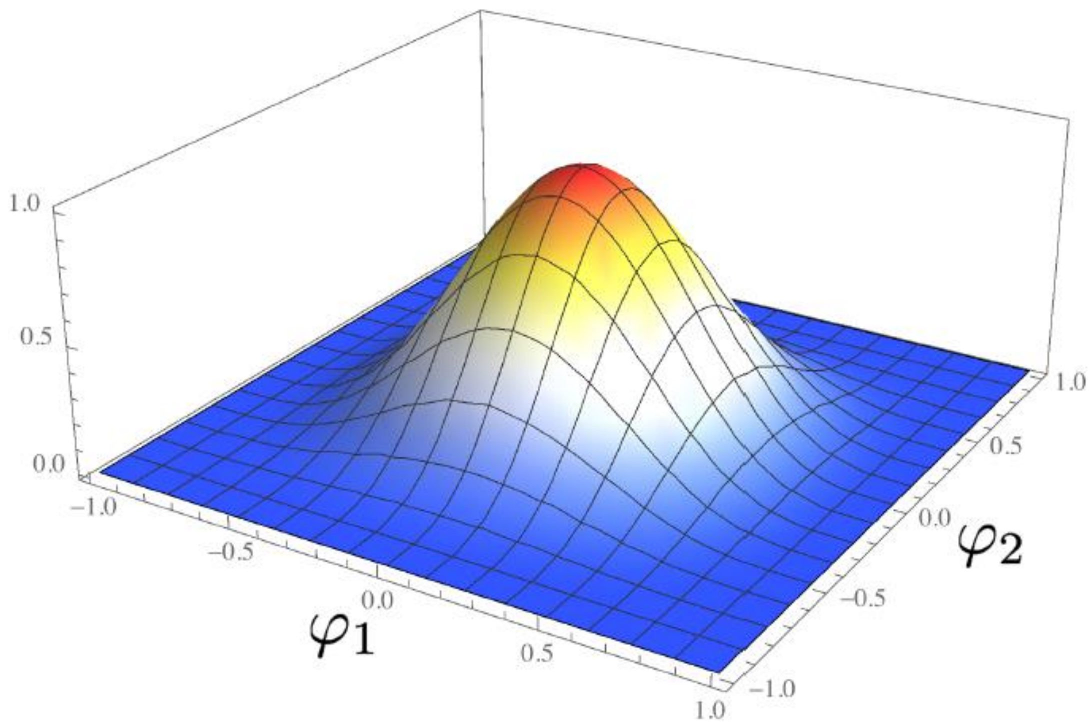


# Inhibiting potential

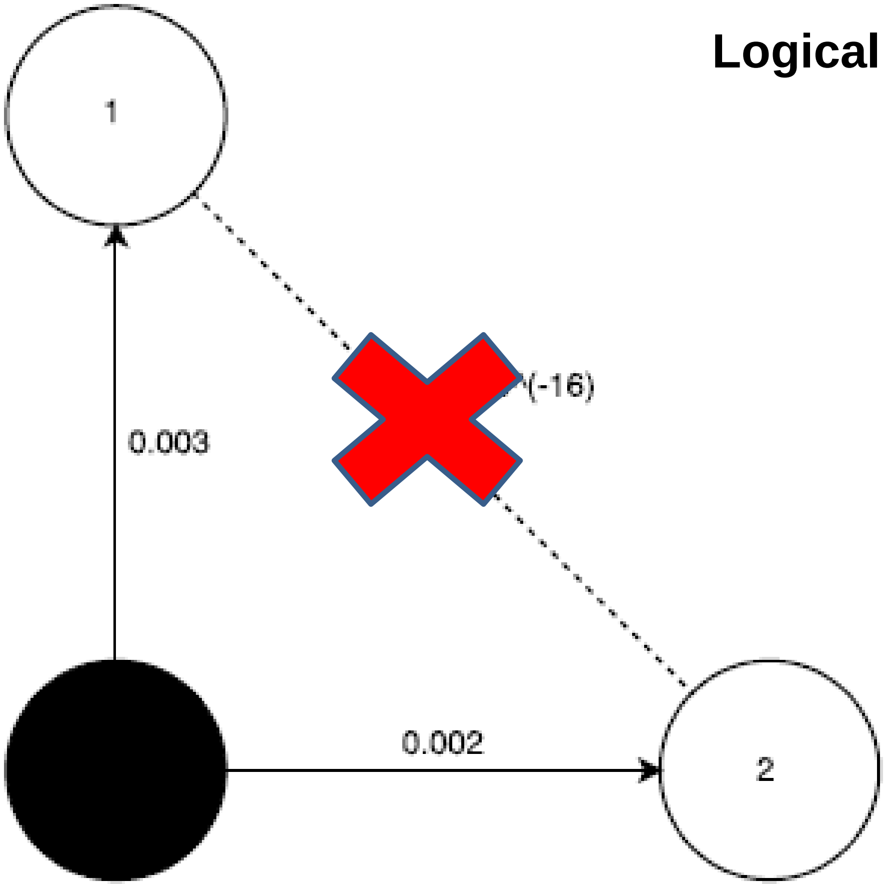



# Inhibiting potential

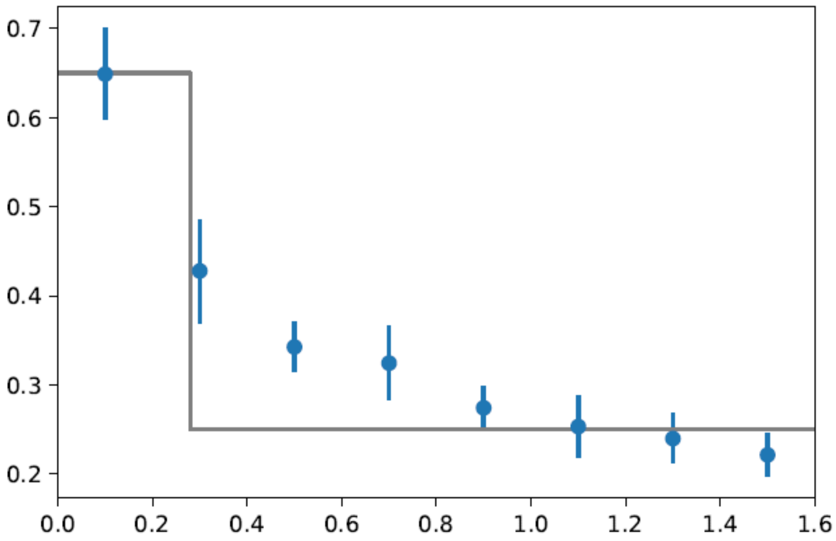

$$-\mathcal{L}_{int} = \varepsilon_{inh}\left(\varphi_i^2 - \frac{\mu^2}{\Lambda}\right)^4\left(\varphi_j^2 - \frac{\mu^2}{\Lambda}\right)^4$$



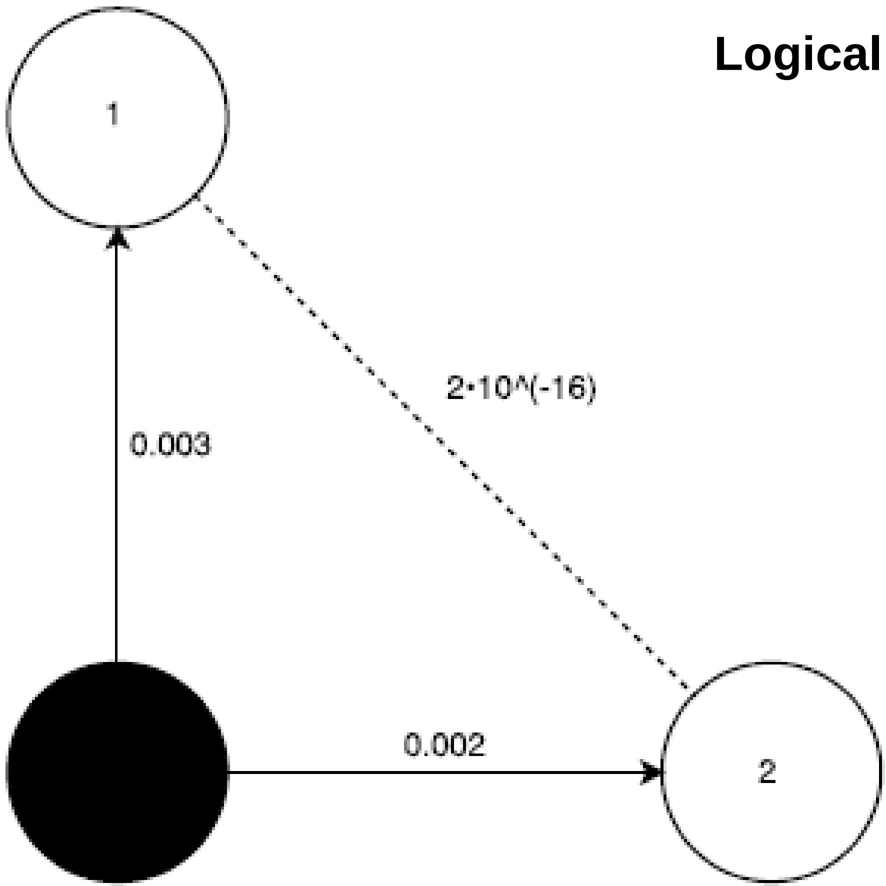
# Quantum neural network logical elements



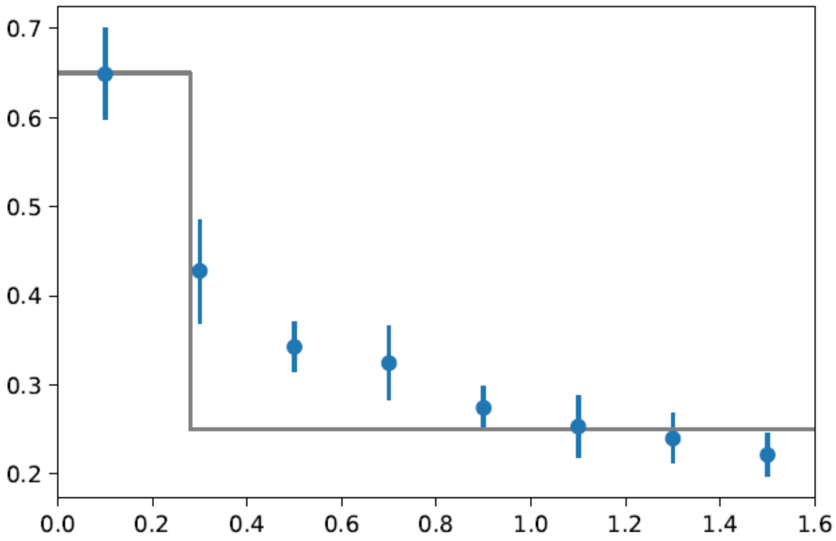

**Logical NOT**



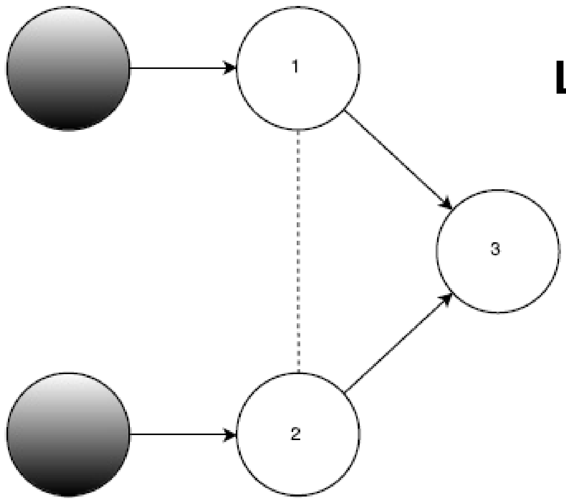
# Quantum neural network logical elements



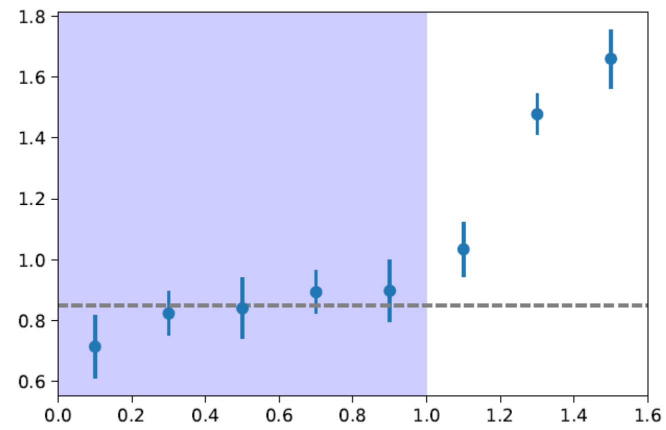

**Logical NOT**



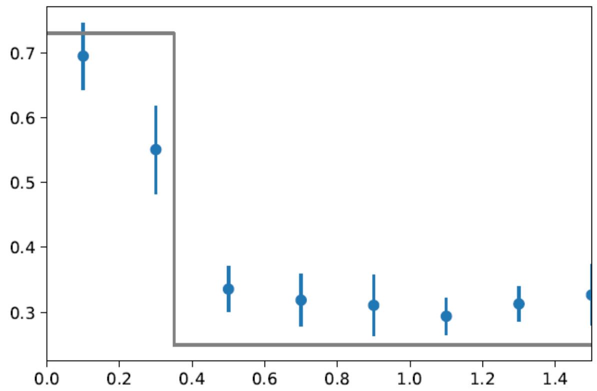
# Quantum neural network logical elements

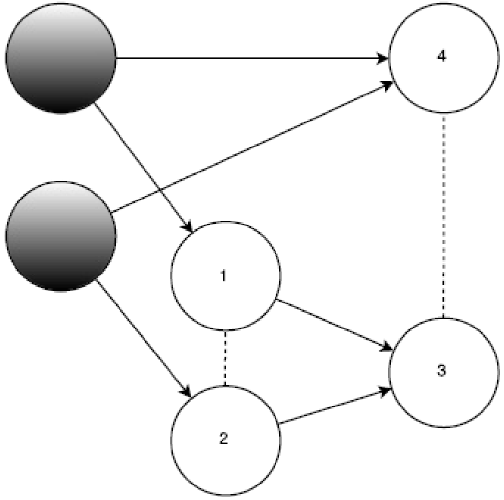



Logical **OR**

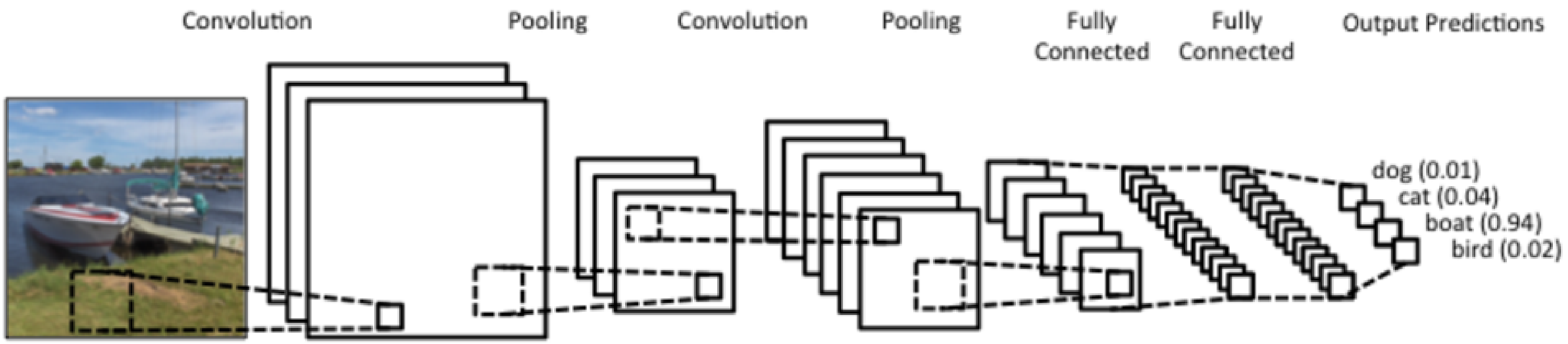


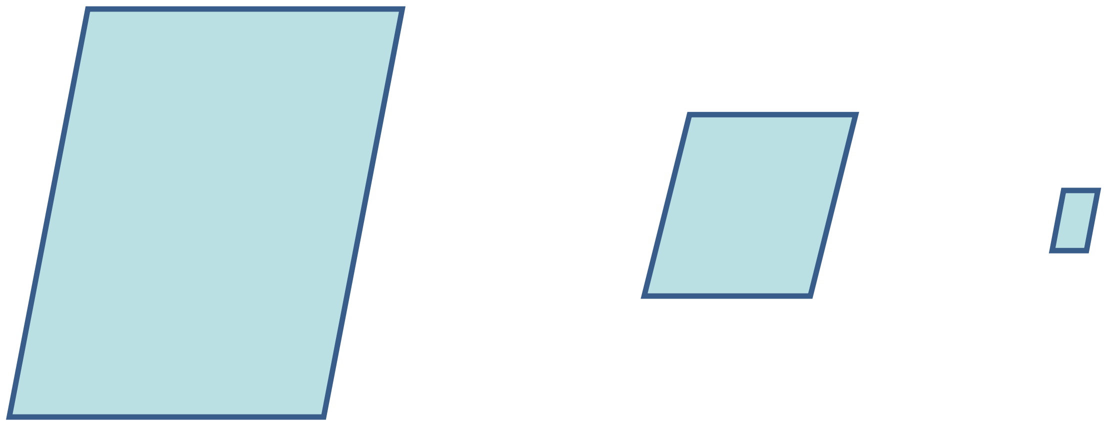

Logical **exclusive OR (XOR)**



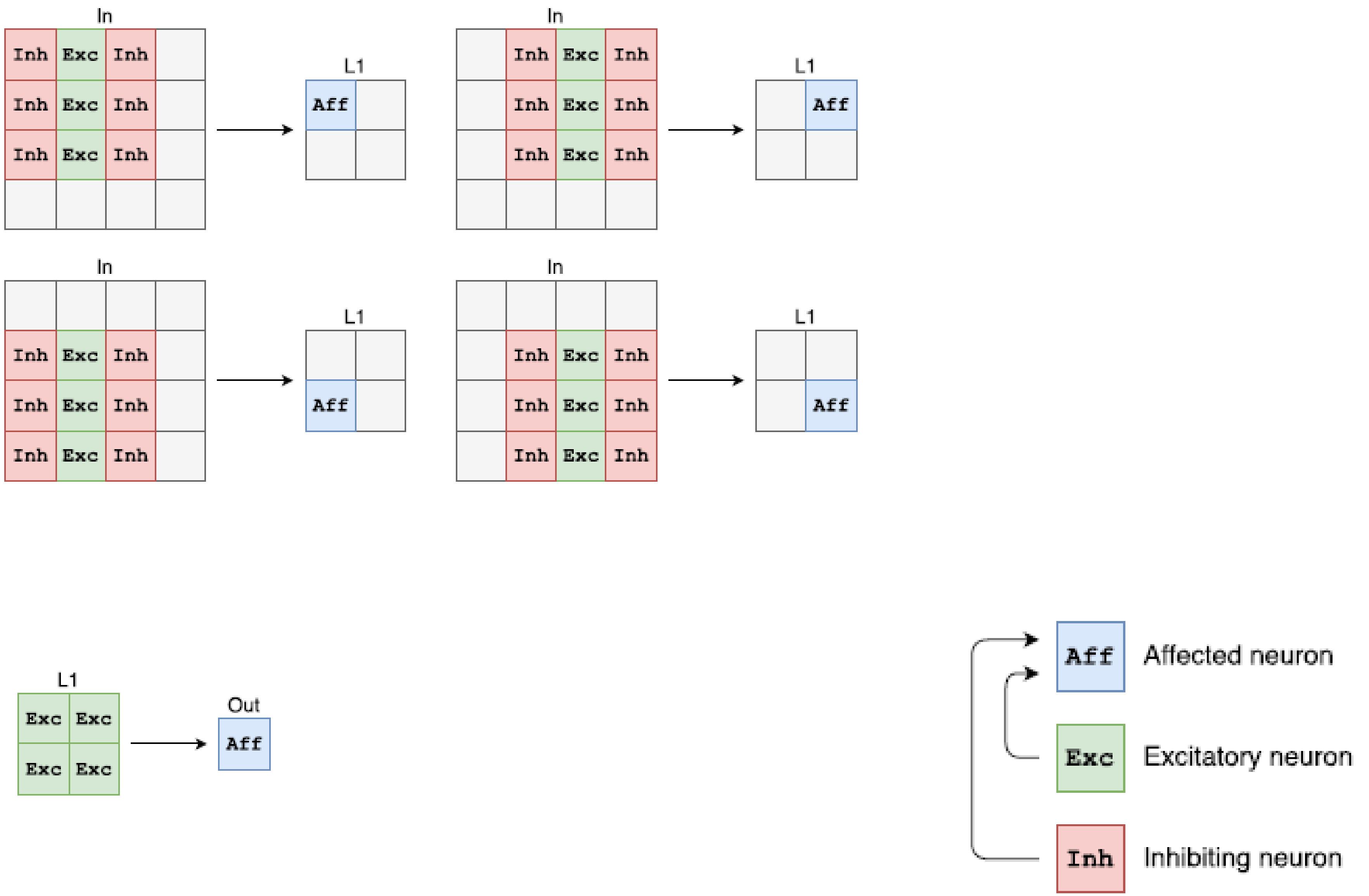



# Convolution neural network

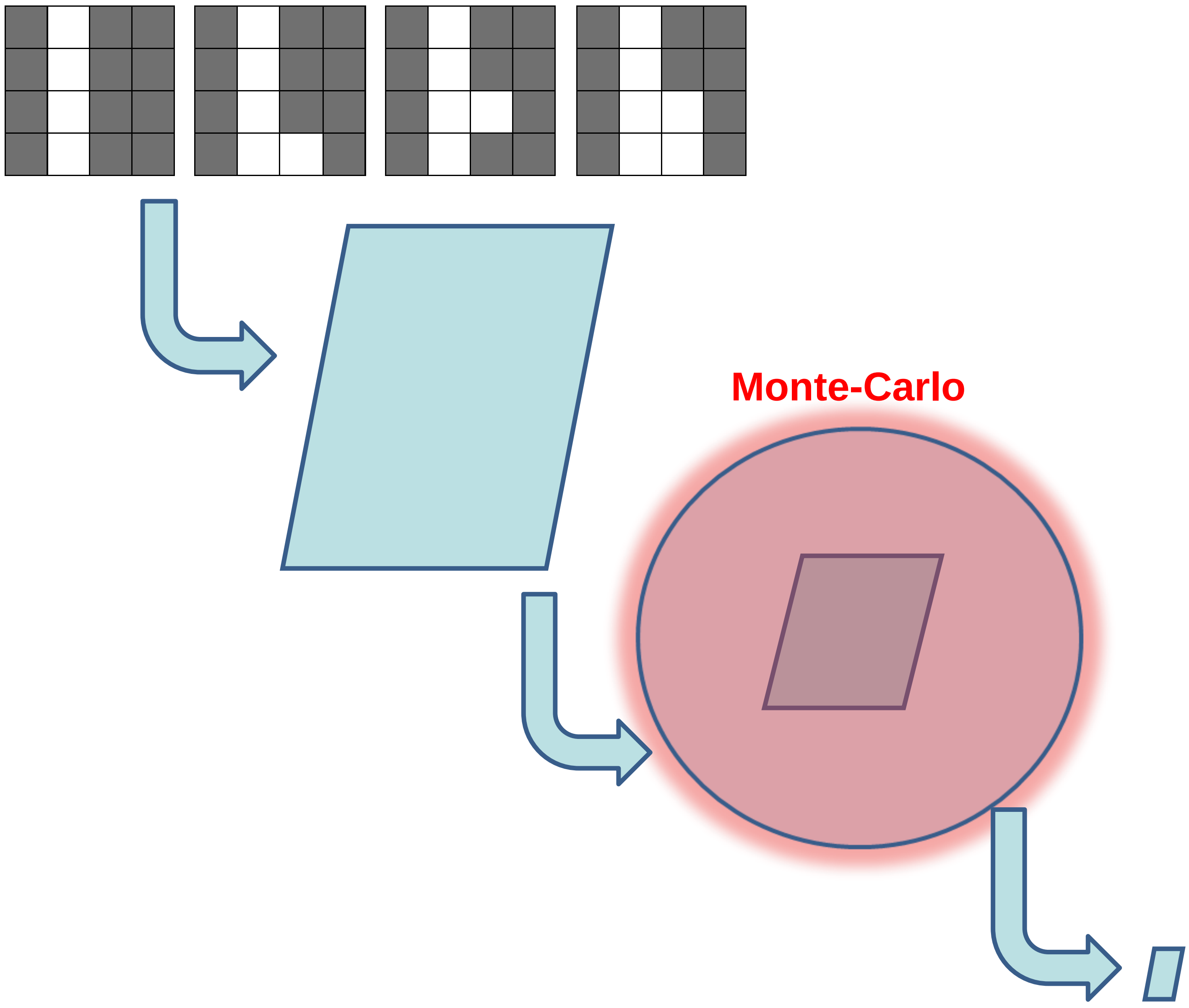


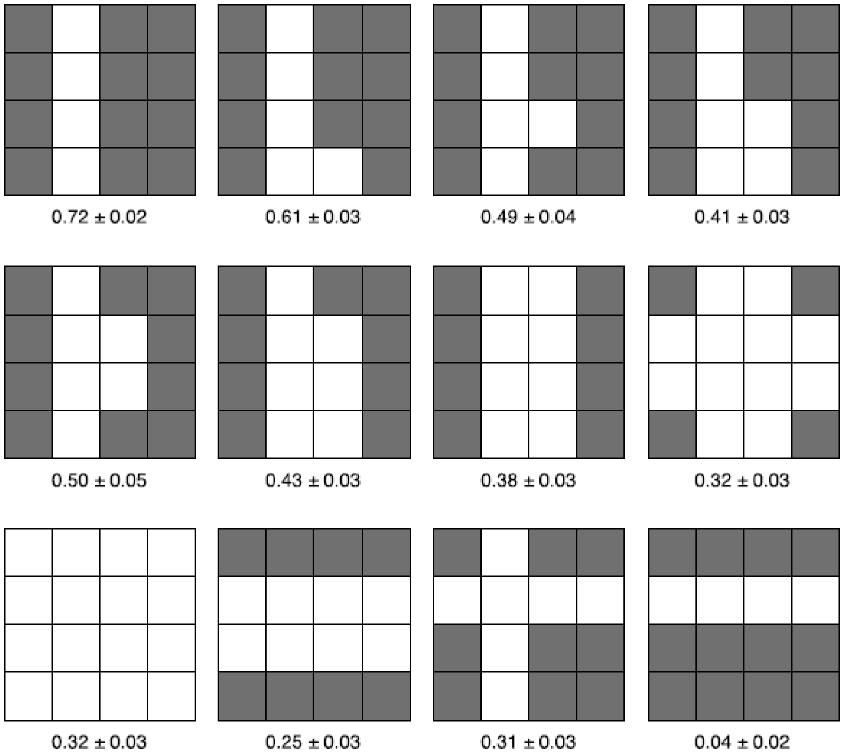
# Convolution neural network

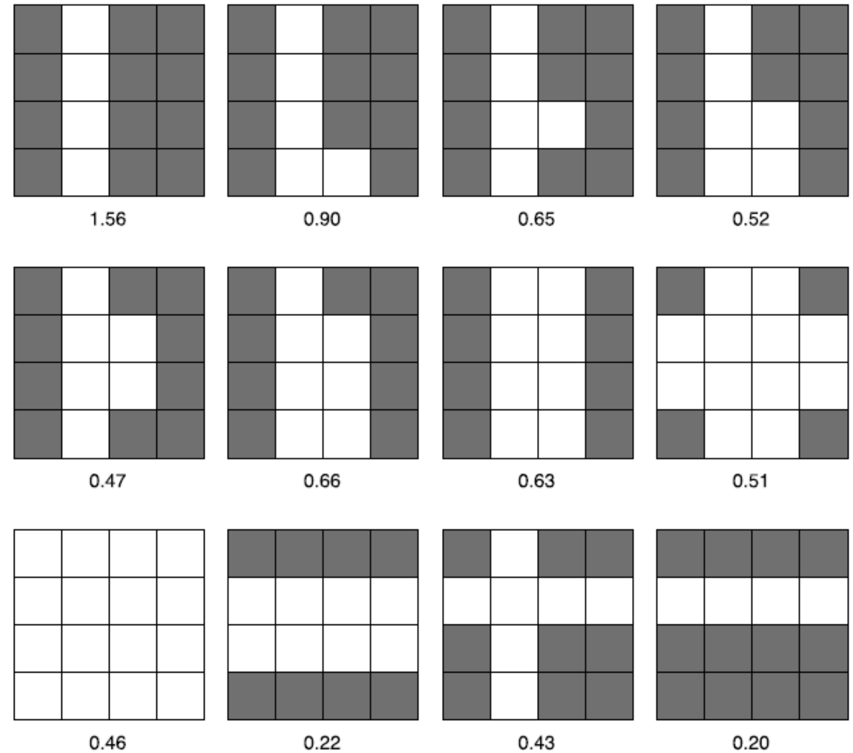


# Convolution neural network



# Convolution neural network

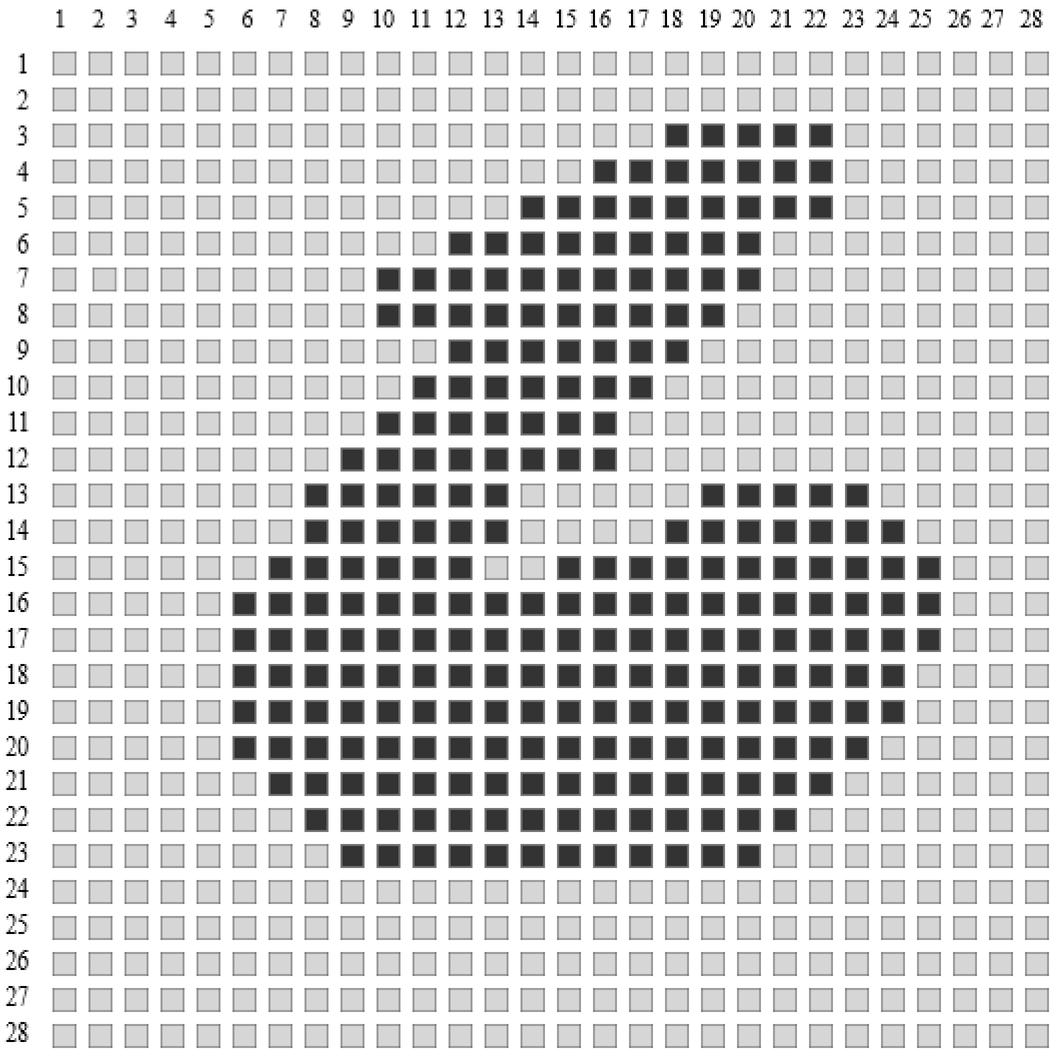


# Digit recognition

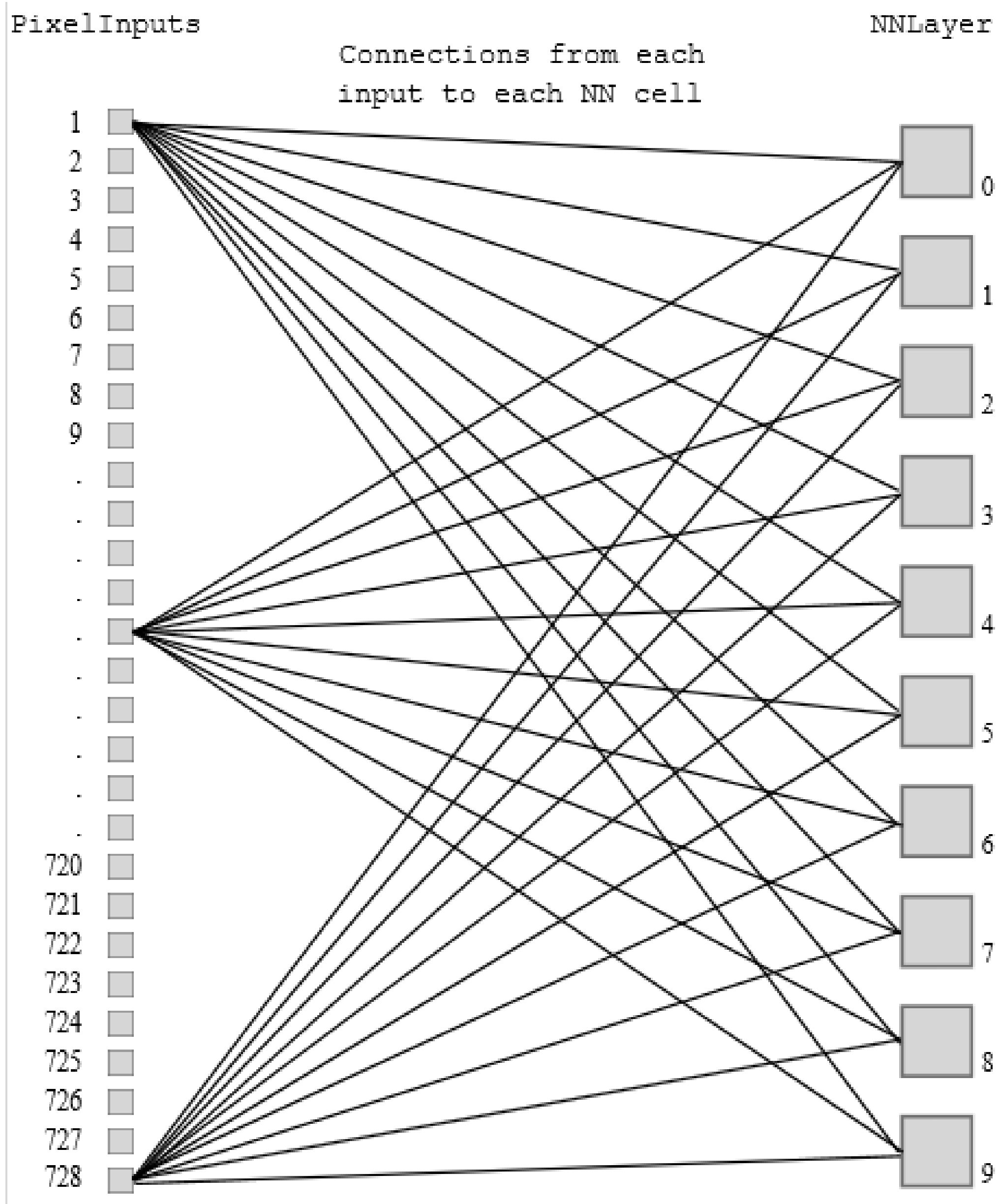
**MNIST database**

# Digit recognition



**MNIST database: MNIST image has a size of 28*28 = 784 pixels**

# Digit recognition



# Digit recognition

$$\mathcal{L}_0 = \sum_{i=0}^{784} \left[ \frac{1}{2}\dot{\psi}_i^2 + \frac{\Lambda}{4}\left(\psi_i^2 - \frac{\mu^2}{\Lambda}\right)^2 \right] + \sum_{j=0}^{10} \left[ \frac{1}{2}\dot{\varphi}_j^2 + \frac{\Lambda}{4}\left(\varphi_j^2 - \frac{\mu^2}{\Lambda}\right)^2 \right]$$

$$\mathcal{L} = \mathcal{L}_0 + \sum_{i=0}^{784}\sum_{j=0}^{10} k\left(\varepsilon_{ij} - b\right) A_i \varphi_j^2 \left(\psi_i^2 - \frac{\mu^2}{\Lambda}\right)^2 + \\ + 10^{-17} \sum_{k>j}^{10}\sum_{j=0}^{10} \left(\varphi_j^2 - \frac{\mu^2}{\Lambda}\right)^4 \left(\varphi_k^2 - \frac{\mu^2}{\Lambda}\right)^4,$$

$$Z = \int \prod_i \mathcal{D}\varphi_i\left(\tau\right) \exp(-S(\varphi_i(\tau))), \varphi_i(0) = \varphi_i(T)$$

# Digit recognition

$X \in \mathbb{R}^{N\times M}$ brightness of $j$-th pixel in $i$-th image.

$$S = X\hat{W}$$

score of $i$-th image treated as $j$-th number.

$$p_{ij} = \frac{exp(S_{ij})}{\sum_{j=0}^{9}(exp(S_{ij}))}.$$

$$\mathcal{L} = -\frac{1}{N}\sum_{i=0}^{N-1} ln(p_{ij})\delta^{i}_{correct} + \lambda \sum_{i=0}^{M-1}\sum_{j=0}^{9} max(-W_{ij}, 0), \lambda \to \infty$$

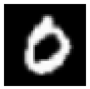
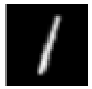
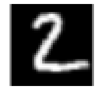
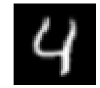
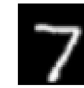
# Digit recognition

$$\hat{\psi}_i = \hat{\psi}(b_i) = \sqrt{\sqrt{b_i}\psi^2 - \sqrt{b_i} + 1}$$

$$\hat{\varepsilon}_{ij}\varphi_j^2\left(\hat{\psi}_i^2 - 1\right)^2 = \hat{\varepsilon}_{ij}b_i\varphi_j^2\left(\psi_i^2 - 1\right)^2 = \varepsilon_{ij}\varphi_j^2\left(\psi_i^2 - 1\right)^2$$

We can now use $W$ as connection matrix for $\varepsilon$ connecting input and output

# Digit recognition

| | | | | |
|---|---|---|---|---|
| P(0) = 0.338608 | P(1) = 0.655741 | P(2) = 0.451795 | P(4) = 0.362327 | P(7) = 0.605863 |
| P(6) = 0.13097 | P(8) = 0.0840482 | P(6) = 0.207845 | P(8) = 0.16814 | P(9) = 0.153816 |
| P(7) = 0.104982 | P(3) = 0.0834241 | P(3) = 0.12158 | P(2) = 0.14839 | P(8) = 0.0527513 |
| P(2) = 0.0962352 | P(2) = 0.0605042 | P(5) = 0.0695778 | P(1) = 0.104967 | P(3) = 0.0501808 |
| P(5) = 0.0873339 | P(7) = 0.0424982 | P(0) = 0.0440336 | P(9) = 0.0852715 | P(1) = 0.0482902 |
| P(4) = 0.0781002 | P(0) = 0.037828 | P(8) = 0.0399262 | P(3) = 0.0759215 | P(0) = 0.0334645 |
| P(9) = 0.0714371 | P(4) = 0.0180404 | P(9) = 0.0267228 | P(5) = 0.0338338 | P(5) = 0.0295938 |
| P(3) = 0.0662971 | P(9) = 0.0130295 | P(7) = 0.0263385 | P(6) = 0.0196095 | P(2) = 0.0167923 |
| P(8) = 0.0260371 | P(6) = 0.00488656 | P(4) = 0.012181 | P(0) = 0.00153889 | P(4) = 0.00924887 |
| P(1) = 0 | P(5) = 0 | P(1) = 0 | P(7) = 0 | P(6) = 0 |

# Conclusions

The model of an artificial neural network based on double quantum dots was proposed.

Schemes for logical elements in artificial neural network were proposed .

A convolutional scheme for line recognition was proposed.

Digital recognition of numbers by an artificial neural network was studied.