Monte-Carlo simulation of the artificial quantum neural network

Oleg Pavlovsky
ITEP & MSU, Moscow, Russia
Programmers / Biologists

Two point of view on neural networks

Programmers

Biologists
Neural network: Biology
How Neuron Works: Biology

The neuron shown in the diagram is a typical neuron with the following parts:

- **Cell body**
- **Dendrites**
- **Axon**

The neuron receives signals through its dendrites and sends them out through its axon. The cell body contains the nucleus.

The graph on the right illustrates the membrane potential over time, showing:

- **Resting Potential**
- **Threshold Potential**
- **Action Potential**
- **Hyperpolarization**

The membrane potential changes from a resting state to an action potential, which is a sudden and brief change in voltage.
Integrate-and-fire

The models of Biological Neuron

$I(t) = C_m \frac{dV_m}{dt}$

Different for Different Models
Main idea of Artificial Neural Network:

Artificial Neural Network (ANN) is an information system that is inspired by the biological nervous systems, such as the brain.

The key element of ANN is a large number of highly interconnected processing elements (neurones) working in unison to solve specific problems.

ANN, like brain, learn by examples or patterns.

Typical ANN problems: pattern recognition, data classification and so on.
Biological (real) neural networks are VERY complicated systems.

Models of neuron – stochastic diff. equations.

So…

Only qualitative analysis is possible now.

Statistical mechanics approach can be used

Idea of the universality classes: biological details may be not so essential in contrast with Symmetries and Topology of the Network.
Conformity in neural networks

Main question is:

What is the mechanism of the big correlation on the neural network?

Statistical mechanics gives the possible answer: Conformity near the phase transition. (Michael A. Buice and Jack D. Cowan, 2008-2009)
Conformity in neural networks

Jack D. Cowan: “Strange and interesting things happen in the neighborhood of a phase transition”

Statistical mechanics gives the possible answer: Conformity near the phase transition. (Michael A. Buice and Jack D. Cowan, 2008-2009)

Progress in Biophysics and Molecular Biology 99 (2009) 53–86

Contents lists available at ScienceDirect

Progress in Biophysics and Molecular Biology

journal homepage: www.elsevier.com/locate/pbiomolbio

Review

Statistical mechanics of the neocortex

Michael A. Buice a, Jack D. Cowan b,*
Neural network as statistical model

3-state model of neuron
If the brain is statistical machine, Why ANN is classical one?
If the brain is statistical machine, Why ANN is classical one?

Artificial Neuron must be stochastic ....
If the brain is statistical machine, Why ANN is classical one?

Artificial Neuron must be stochastic or Quantum.
If $\Sigma (W_1 X_1 + W_2 X_2 + \ldots + W_n X_n) > I_{\text{threshold}}$

then neuron generates spike
Neuron: classical VS stochastic

Simplification

Real neuron potential
Neuron: classical VS stochastic

Simplification

Real neuron potential

Spikes
Neuron: classical VS stochastic

Simplification

Real neuron potential

Under-threshold fluctuations

Spikes
Quantum neuron = Q-neuron

![Diagram of a neuron with inputs and outputs](image)
Quantum neuron = Q-neuron
Quantum neuron

\[ \hat{H}_i = \frac{1}{2} \hat{p}_i^2 + V_0(\phi_i). \]

\[ V_0(\phi_i) = \frac{\Lambda}{4} \left( \phi^2 - \frac{\mu^2}{\Lambda} \right)^2. \]

\[ \phi(x,t) = \frac{m}{\sqrt{\lambda}} \tanh \left( \frac{m}{\sqrt{2}} (x - x_0) \right). \]
Quantum neuron = Q-neuron
Quantum neuron = Q-neuron
Quantum neuron = Q-neuron
Nano-technological realizations

We need in Nano-technological platform for realization of QNN.

One possible way: **quantum double dots**.
Nano-technological realizations

Quantum double dots.
Coupled quantum dots as quantum gates

Guido Burkard* and Daniel Loss†

Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland

David P. DiVincenzo‡

IBM Research Division, Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598
(Received 3 August 1998)
Quantum neural network as quantum many body system

\[ Z = \int \prod_i D\varphi_i(\tau) \exp(-S(\varphi_i(\tau))), \varphi_i(0) = \varphi_i(T) \]

\[ S = \int_0^T d\tau \left[ \sum_i \left( \frac{1}{2} \dot{\varphi}_i^2 + V_0(\varphi_i) \right) + \sum_{i>j} V_{int}(\varphi_i, \varphi_j) \right] \]

\[ \langle \mathcal{O}(\varphi_1, \ldots, \varphi_i) \rangle = \frac{1}{Z} \int \prod_i D\varphi_i(\tau) \mathcal{O}(\varphi_1, \ldots, \varphi_i) \exp(-S(\varphi_i)) \]
“Axon” is output information line from neuron. So neural net is very non-local system.
Role of Synepse is the contact coefficient, the measure of neuron connection.
Excitation connection

\[ S = \int_0^T d\tau \left[ \sum_i \left( \frac{1}{2} \dot{\varphi}_i^2 + V_0(\varphi_i) \right) + \sum_{i>j} V_{int}(\varphi_i, \varphi_j) \right] \]

1. \[ -\mathcal{L}_0 = \frac{1}{2} \dot{\varphi}_i^2 + \frac{\Lambda}{4} \left( \frac{\varphi_i^2}{\Lambda} - \frac{\mu^2}{\Lambda} \right)^2 \]

2. \[ -\mathcal{L}_{int} = \varepsilon_{exc} \varphi_j^2 \left( \frac{\varphi_i^2}{\Lambda} - \frac{\mu^2}{\Lambda} \right)^2 \]
Excitation connection

\[ \mathcal{L}_{int} = \varepsilon_{exc} \varphi_j^2 \left( \varphi_i^2 - \frac{\mu^2}{\Lambda} \right)^2 \]
\[ \langle A \rangle = \frac{\int \mathcal{D}x A(x) e^{iS_m(x)/\hbar}}{\int \mathcal{D}x e^{iS_m(x)/\hbar}} \to \sum_{\text{conf}} A(x) \frac{e^{-S(x)/\hbar}}{\sum_{\text{conf}} e^{-S(x)/\hbar}} = \sum_{\text{conf}} A(x) \mathcal{P}(x) \]

\[ \mathcal{P}(x) = \frac{e^{-S(x)/\hbar}}{\sum_{\text{conf}} e^{-S(x)/\hbar}} \]

\[ \langle A \rangle = \frac{1}{N_{\text{conf}}} \sum_{k=1}^{N_{\text{conf}}} A(x_k) \]
Excitation connection: simple test

\[ Z = \int \prod_{i} D\varphi_i(\tau) \exp(-S(\varphi_i(\tau))), \varphi_i(0) = \varphi_i(T) \]

\[ S = \int_{0}^{T} d\tau \left[ \sum_{i} \left( \frac{1}{2} \dot{\varphi}_i^2 + V_0(\varphi_i) \right) + \sum_{i > j} V_{int}(\varphi_i, \varphi_j) \right] \]
Excitation connection: simple test

Activity of Q-neuron

\[(\varphi_2^2 - 1)^2\]
Excitation connection: 3 Q-neurons transport
Quantum neuron
Quantum neural network logical elements

Logical **AND**

If both Paths are active, Correlation more 70 %
Quantum neural network logical elements

Logical AND

If only one Path is active, Correlation less 20 %
Inhibiting potential

Neuron A

Neuron B

Inhibitory interneuron

Axon

Axon collateral

Image of a hand with a red spot.
Inhibiting potential

\[ \mathcal{L}_{\text{int}} = \varepsilon_{\text{inh}} \left( \phi^2_i - \frac{\mu^2}{\Lambda} \right)^4 \left( \phi^2_j - \frac{\mu^2}{\Lambda} \right)^4 \]
Quantum neural network logical elements

Logical NOT
Quantum neural network logical elements

Logical NOT

Diagram showing logical elements with connections labeled 0.003, 2\times10^{-16}, and 0.002.
Quantum neural network logical elements

Logical **OR**

Logical **exclusive OR (XOR)**
Convolution neural network
Convolution neural network
Convolution neural network

Monte-Carlo
Convolution neural network
Digit recognition

MNIST database
**Digit recognition**

**MNIST database:** MNIST image has a size of $28 \times 28 = 784$ pixels
Digit recognition

Connections from each input to each NN cell
\[ \mathcal{L}_0 = \sum_{i=0}^{784} \left[ \frac{1}{2} \psi_i^2 + \frac{\Lambda}{4} \left( \psi_i^2 - \frac{\mu^2}{\Lambda} \right)^2 \right] + \sum_{j=0}^{10} \left[ \frac{1}{2} \varphi_j^2 + \frac{\Lambda}{4} \left( \varphi_j^2 - \frac{\mu^2}{\Lambda} \right)^2 \right] \]

\[ \mathcal{L} = \mathcal{L}_0 + \sum_{i=0}^{784} \sum_{j=0}^{10} k (\varepsilon_{ij} - b) A_i \varphi_j^2 \left( \psi_i^2 - \frac{\mu^2}{\Lambda} \right)^2 + \]

\[ + 10^{-17} \sum_{k>j}^{10} \sum_{j=0}^{10} \left( \varphi_j^2 - \frac{\mu^2}{\Lambda} \right)^4 \left( \varphi_k^2 - \frac{\mu^2}{\Lambda} \right)^4, \]

\[ Z = \int \prod_i D\varphi_i(\tau) \exp(-S(\varphi_i(\tau))), \varphi_i(0) = \varphi_i(T) \]
Digit recognition

\[
X \in \mathbb{R}^{N \times M} \quad \text{brightness of } j\text{-th pixel in } i\text{-th image.}
\]

\[
S = X \hat{W}
\]

score of \(i\)-th image treated as \(j\)-th number.

\[
p_{ij} = \frac{\exp(S_{ij})}{\sum_{j=0}^{9} (\exp(S_{ij}))}.
\]

\[
\mathcal{L} = -\frac{1}{N} \sum_{i=0}^{N-1} \ln(p_{ij}) \delta_{\text{correct}}^i + \lambda \sum_{i=0}^{M-1} \sum_{j=0}^{9} \max(-W_{ij}, 0), \lambda \to \infty
\]
\[ \hat{\psi}_i = \hat{\psi}(b_i) = \sqrt{\sqrt{b_i \psi^2} - \sqrt{b_i} + 1} \]

\[ \hat{\varepsilon}_{ij} \varphi_j^2 \left( \hat{\psi}_i^2 - 1 \right)^2 = \hat{\varepsilon}_{ij} b_i \varphi_j^2 \left( \psi_i^2 - 1 \right)^2 = \varepsilon_{ij} \varphi_j^2 \left( \psi_i^2 - 1 \right)^2 \]

We can now use \( W \) as connection matrix for \( \varepsilon \) connecting input and output.
Conclusions

The model of an artificial neural network based on double quantum dots was proposed.

Schemes for logical elements in artificial neural network were proposed.

A convolutional scheme for line recognition was proposed.

Digital recognition of numbers by an artificial neural network was studied.