# Deviations of $R^2$ cosmology from the Einstein's General Relativity

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*based on common work with A. Dolgov and R. Singh*

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**WORKSHOP**

New physics paradigms after Higgs and gravitational wave discoveries

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General Relativity (GR):

$$S_{EH} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g}\, R$$

describes basic properties of the universe in very good agreement with observations.

- $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass

Beyond the frameworks of GR:

$$S_{tot} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g}\, [R + F(R)] + S_m$$

$F(R) = -R^2/(6m^2)$:

- $R^2$-term was suggested by V.Ts. Gurovich and A.A. Starobinsky for elimination of cosmological singularity (JETP **50** (1979) 844).
- It was found that the the addition of the $R^2$-term leads to inflationary cosmology. (A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980))

Curvature $R(t)$ can be considered as an effective scalar field (scalaron) with the mass $m$ and with the decay width $\Gamma$.

## I. Cosmological Equations in $R^2$-theory

- The term describing particle production is included as a source into equation for the energy density evolution.

## II. Solution *ab ovo* to $\Gamma t \lesssim 1$, but $mt \gg 1$

1. Solutions at inflationary and post-inflationary epoch
2. Asymptotic solutions at $mt \gg 1$: RD and MD stages

During this time the universe evolution was quite different from the General Relativity one.

## III. Solution at $\Gamma t \gtrsim 1$: approaching to GR cosmology

- GR is recovered when the energy density of matter becomes larger than the energy density of the exponentially decaying scalaron.
- This transition is delayed: not at $\Gamma t \sim 1$, but at $\Gamma t \sim \ln(m/\Gamma) \Rightarrow$ modification of high temperature baryogenesis, variation of the frozen abundances of heavy DM particles, necessity of reconsideration of the formation of PBHs, etc.

## IV. Conclusions

# I. Cosmological Equations in $R^2$-theory

Let us consider the theory described by the action:

$$S_{tot} = -\frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\left(R - \frac{R^2}{6m^2}\right) + S_m$$

- $m$ is a constant parameter with dimension of mass

The modified Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{3m^2}\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} + g_{\mu\nu}D^2 - D_\mu D_\nu\right)R = \frac{8\pi}{m_{Pl}^2}T_{\mu\nu}$$

- $D^2 \equiv g^{\mu\nu}D_\mu D_\nu$ is the covariant D'Alembert operator.

The energy-momentum tensor of matter $T_{\mu\nu}$

$$T^\mu_\nu = diag(\varrho, -P, -P, -P)$$

where $\varrho$ is the energy density, $P$ is the pressure of matter.

The matter distribution is homogeneous and isotropic

$$P = w\varrho$$

- non-relativistic: $w = 0$, relativistic: $w = 1/3$, vacuum-like: $w = -1$

## FRW: $ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta\, d\varphi^2\right]$, $H = \dot{a}/a$

The curvature scalar:

$$R = -6\dot{H} - 12H^2$$

The covariant conservation condition $D_\mu T^\mu_\nu = 0$:

$$\dot{\varrho} = -3H(\varrho + P) = -3H(1+w)\varrho$$

Trace equation:

$$D^2 R + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}\, T^\mu_\mu$$

For homogeneous field, $R = R(t)$, and with $P = w\varrho$:

$$\ddot{R} + 3H\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1-3w)\varrho$$

This is the Klein-Gordon (KG) type equation for massive scalar field $R$, which is sometimes called "scalaron". It differs from KG by the Hubble friction term.

$$\ddot{R} + 3H\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1 - 3w)\varrho$$

This equation:

- does not include the effects of particle production by the curvature scalar;
- is a good approximation at inflationary epoch, when particle production by $R(t)$ is absent, because $R$ is large and friction is large, so $R \to 0$ slowly.

At some stage, when $\boldsymbol{H}$ becomes smaller than $\boldsymbol{m}$, $\boldsymbol{R}$ starts to oscillate efficiently producing particles.

- It commemorates the end of inflation, the heating of the universe, and the transition from the accelerated expansion (inflation) to a de-accelerated one.
- The latter resembles the usual Friedmann matter dominated expansion regime but differs in many essential features.

Particle production: for the harmonic potential can be approximately described by the additional liquid friction term $\Gamma\dot{\boldsymbol{R}}$, where

$$\Gamma = \frac{m^3}{48 m_{Pl}^2}$$

- Ya.B. Zeldovich, A. Starobinsky, JETP Lett. 26, 252 (1977); A. Vilenkin, Phys. Rev. **D32**, 2511 (1985); EA, A. Dolgov, L. Reverberi, JCAP **1202** (2012) 049.

# Particle Production: friction term approximation

Equation for $R$ acquires an additional friction term:

$$\ddot{R} + (3H + \Gamma)\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1 - 3w)\varrho$$

Particle production leads to an emergence of the source term in Eq. for $\varrho$:

$$\dot{\varrho} = -3H(1 + w)\varrho + \frac{mR_{ampl}^2}{1152\pi}$$

where $R_{ampl}$ is the amplitude of $R(t)$-oscillations.

- The state of the cosmological matter depends not only upon the spectrum of the decay products but also on the thermal history of the produced particles.
- Depending on that, the parameter $w$ may be not exactly equal to 0 or 1/3. The equation of state can be not simple $P = w\varrho$ with constant $w$.
- Two limiting values $w = 0$ and $1/3$ are possible simple examples.
- Different values of $w$ would not change the presented results significantly.

# Dimensionless Equations

Dimensionless time variable and dimensionless functions

$$\tau = tm, \quad H = mh, \quad R = m^2 r, \quad \varrho = m^4 y, \quad \Gamma = m\gamma.$$

The system of dimensionless equations

$$h' + 2h^2 = -r/6$$
$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$
$$y' + 3(1 + w)h\, y = S[r]$$

- prime denotes derivative over $\tau$, $\mu = m/m_{Pl}$, $\gamma = \mu^2/48$

The source term is taken as:

$$S[r] = \frac{\langle r^2 \rangle}{1152\pi}.$$

- $\langle r^2 \rangle$ means amplitude squared of harmonic oscillations, $r^2_{ampl}$, of the dimensionless curvature $r(\tau)$.
- For nonharmonic oscillations we approximate $\langle r^2 \rangle$ as $2(r')^2$ or $(-2r''r)$.

# II. Solution *ab ovo* to $\Gamma t \lesssim 1$

- Solution at inflationary epoch

# Duration of Inflation

The initial conditions should be chosen in such a way that at least 70 e-foldings during inflation are ensured:

$$N_e = \int_0^{\tau_{inf}} h\, d\tau \geq 70$$

We can roughly estimate the duration of inflation neglecting higher derivatives in equations for $h$ and $r$.

- A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980); A. De Felice and S. Tsujikawa, Living Rev. Rel. 13, 3 (2010); arXiv:1002.4928

Simplified system to estimate the duration of inflation ($\boldsymbol{y = 0}$, $\gamma \ll \boldsymbol{1}$):

$$\boldsymbol{h^2 = -r/12, \quad 3hr' = -r}$$

Solutions:

$$\sqrt{-r(\tau)} = \sqrt{-r_0} - \tau/\sqrt{3}, \quad h(\tau) = (\sqrt{-3r_0} - \tau)/6, \quad r_0 = r(\tau = 0)$$

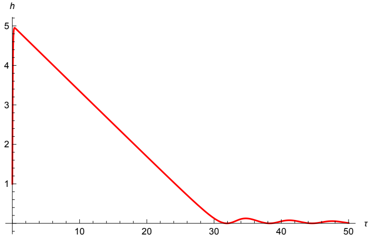
The duration of inflation is roughly determined by the condition $\boldsymbol{h = 0}$, i.e.

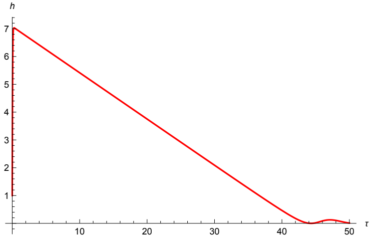
$$\tau_{inf} = \sqrt{-3r_0} \;\Rightarrow\; N_e \approx r_0/4$$

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y$$

## Numerical solutions: Evolution of $\boldsymbol{h(\tau)}$ at the inflationary stage



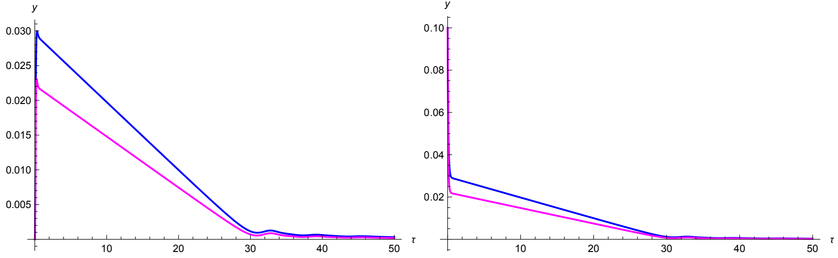

- Initial values of dimensionless curvature $r_0 = 300$ (left) and $r_0 = 600$ (right).
- Initially $h_{in} = 0$, but it quickly reaches the value $h(0) = \sqrt{-r_0/12}$.
- The numbers of e-foldings: $\boldsymbol{N_e \approx r_0/4 = 75}$ (left) and **150** (right).

An excellent agreement with numerical solutions demonstrates high precision of the slow roll approximation and weak impact of particle production at (quasi)inflationary stage.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Evolution of the dimensionless energy density of matter $\boldsymbol{y(\tau)}$ during inflation for $w = 0$ (blue) and $w = 1/3$ (magenta).



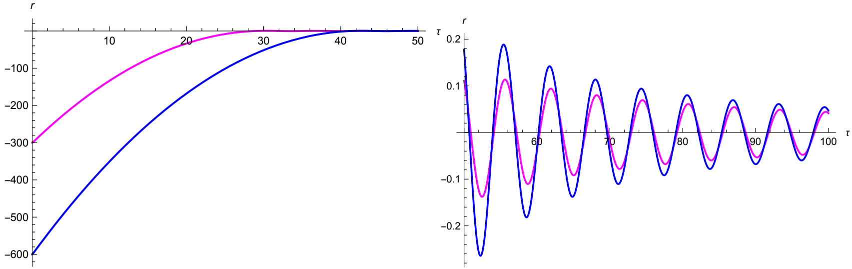

- *Left panel*: initially $y_{in} = 0$. *Right panel*: $y_{in} = 0.1$.

The initial fast rise of $\varrho$ from zero during short time is generated by the $\boldsymbol{S}[\boldsymbol{r}]$-term. The results are not sensitive to the form of $\boldsymbol{S}[\boldsymbol{r}]$, because at inflation $y(\tau)$ quickly vanishes.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Evolution of the dimensionless curvature scalar $\boldsymbol{r(\tau)}$ for $\boldsymbol{r_{in} = -300}$ (magenta) and $\boldsymbol{r_{in} = -600}$ (blue)




- *Left panel*: evolution during inflation.
- *Right panel*: evolution after the end of inflation, the curvature scalar starts to oscillate.

# II. Solution *ab ovo* to $\Gamma t \lesssim 1$

- Solution at inflationary epoch
- Solution at post-inflationary epoch

The behavior of $\boldsymbol{R}$, $\boldsymbol{H}$ and $\varrho$, or dimensionless quantities $\boldsymbol{r}$ , $\boldsymbol{h}$, and $\boldsymbol{y}$ is drastically different at the vacuum-like dominated stage (inflation) and during scalaron dominated stage, which followed the inflationary epoch.

# Numerical solutions at post-inflationary epoch

We will find the laws of evolution of $r(\tau)$, $h(\tau)$, and $y(\tau)$ after inflation till $\gamma\tau \sim 1$, solving the system

$$h' + 2h^2 = -r/6$$
$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$
$$y' + 3(1 + w)h\,y = S[r]$$

The numerical solutions:

- from the end of inflation $\rightarrow$ large $\tau \gg 1$, but not too large
- the numerical procedure for huge $\tau \sim 1/\gamma$ becomes unstable

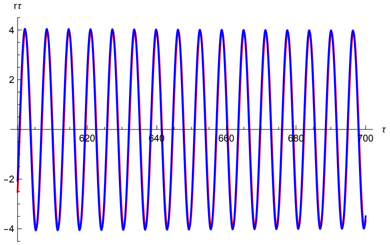
Analytical solutions: asymptotically valid at any large $\tau$ up to $\tau \sim 1/\gamma$.

Very good agreement between numerical and analytical solutions at large but not huge $\tau$ allows to trust asymptotic analytical solution at huge $\tau$.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\, y = S[r] = (r')^2/1152\pi$

$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y, \;\; \mu = m/m_{Pl} = 0.1, \;\; \gamma = \mu^2/48$$

Evolution of the curvature scalar, $\boldsymbol{\tau r(\tau)}$, in post-infationary epoch.



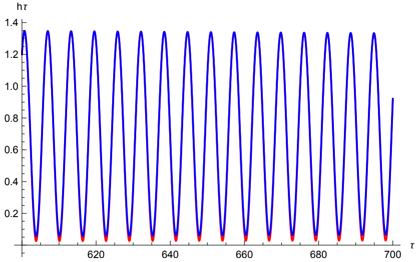

*Left panel* ($\boldsymbol{w = 1/3}$): initially $\boldsymbol{r_{in} = -300}$ (red), $\boldsymbol{r_{in} = -600}$ (blue). There is absolutely no difference between the curves.
*Right panel* ($\boldsymbol{r_{in} = -300}$): $\boldsymbol{w = 1/3}$ (red) and $\boldsymbol{w = 0}$ (blue). The difference is minuscule.

The amplitude $\boldsymbol{r_{ampl}\tau \to const}$. For large $\tau$ the result does not depend upon the initial value of $r$ and very weakly depends on $w$.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y, \;\; \mu = m/m_{Pl} = 0.1$$

Evolution of the Hubble parameter, $h\tau$, in post-inflationary epoch for $w = 1/3$ (red) and $w = 0$ (blue)



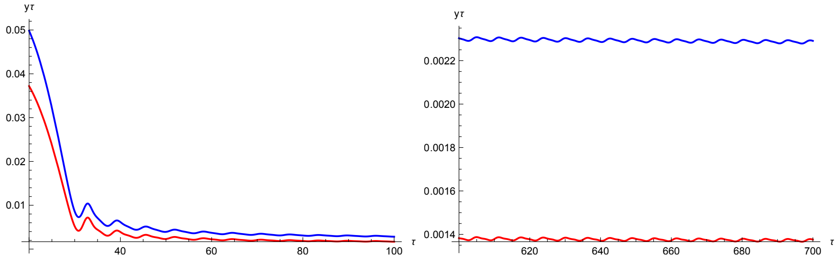

- The dependence on $w$ is very weak, except for small values of $h$.

If $h$ is very close to zero, it may become negative because of numerical error due to insufficient precision.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Energy density of matter as a function of time in post-inflationary (scalaron dominated) epoch for $\boldsymbol{w = 1/3}$ (red) and $\boldsymbol{w = 0}$ (blue)




- Evolution of $\boldsymbol{y\tau}$ at small $\boldsymbol{\tau}$ (left) and at large $\boldsymbol{\tau}$ (right).

The product $\boldsymbol{y\tau \to const}$ with rising $\boldsymbol{\tau}$. It means that $\boldsymbol{\varrho \sim 1/t}$.
This behavior much differs from the standard matter density evolution $\boldsymbol{\varrho \sim 1/t^2}$.

# Asymptotic solution at $\tau \gg 1$, $\gamma\tau \lesssim 1$ and $w = 1/3$

Simple form of numerical solutions at large $\tau$:

- $r$ oscillates with the amplitude decreasing as $1/\tau$ around zero
- $h$ also oscillates almost touching zero with the amplitude also decreasing as $1/\tau$ around some constant value close to 2/3.

In the case $w = 1/3$ we have the system of equations

$$h' + 2h^2 = -r/6, \tag{1}$$

$$r'' + 3hr' + r = 0, \tag{2}$$

$$y' + 4h\, y = \frac{< r^2 >}{1152\pi},$$

We search for the asymptotic expansion of $h$ and $r$ at $\tau \gg 1$ in the form:

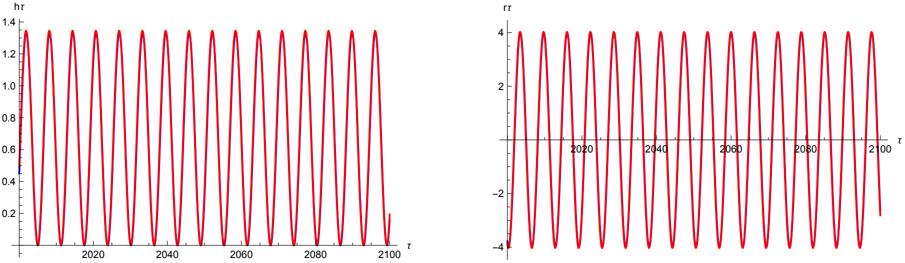
$$r = \frac{r_1 \cos(\tau + \theta_r)}{\tau} + \frac{r_2}{\tau^2}, \quad h = \frac{h_0 + h_1 \sin(\tau + \theta_h)}{\tau}$$

- $r_j$ and $h_j$ are some constant coefficients to be calculated from Eqs.(1)-(2)
- the constant phases $\theta_j$ are determined through the initial conditions and will be adjusted by the best fit of the asymptotic solution to the numerical one

Finally we find:

$$h = \frac{2}{3\tau}\left[1 + \sin(\tau + \theta)\right], \quad r = -\frac{4\cos(\tau+\theta)}{\tau} - \frac{4}{\tau^2}$$

Comparison of numerical calculations with analytical estimates for the adjusted "by hand" phase $\theta = -2.9\pi/4$




- *Left panel*: comparison of numerical solution for $h\tau$ (red) with analytic estimate (blue).
- *Right panel*: the same for numerically calculated $r\tau$.

The difference between the red and blue curves is not observable.

## Equation for energy density: $y' + 4h\,y = < r^2 > /(1152\pi)$

$\langle r^2 \rangle = 16/\tau^2$ is the square of the amplitude of the harmonic oscillations.

Analytical integration gives:

$$y(\tau) = \frac{1}{72\pi} \int_{\tau_0}^{\tau} \frac{d\tau_2}{\tau_2^2} \exp\left[-4 \int_{\tau_2}^{\tau} d\tau_1 h(\tau_1)\right]$$

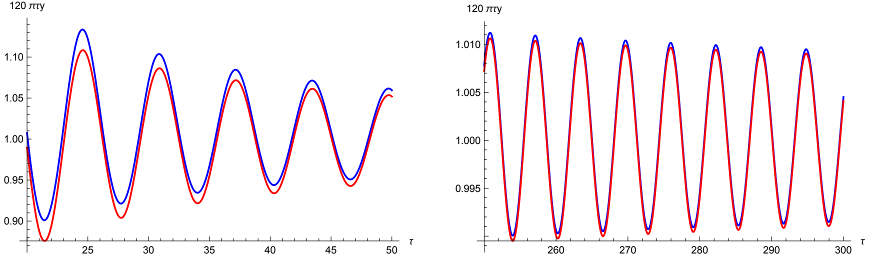
- $\tau_0 \ll \tau$ is some initial value of the dimensionless time.

Taking asymptotical $h(\tau)$ we can find an asymptotic behavior of $y(\tau)$:

$$y_{1/3} = \frac{1}{120\pi\tau} + \frac{1}{45\pi}\frac{\cos(\tau + \theta)}{\tau^2} - \frac{1}{27\pi\tau^2} \int_{\epsilon}^{1} \frac{d\eta_2}{\eta_2^{1/3}} \cos(\tau\eta_2 + \theta)$$

- the subindex (1/3) indicates that $w = 1/3$
- $\epsilon = \tau_0/\tau \ll 1$. The last integral is proportional to $1/\tau^{2/3}$ and is subdominant.

# Asymptotic behavior of the energy density for $w = 1/3$




Comparison of the integral solution

$$y(\tau) = \frac{1}{72\pi} \int_{\tau_0}^{\tau} \frac{d\tau_2}{\tau_2^2} \exp\left[-4 \int_{\tau_2}^{\tau} d\tau_1 h(\tau_1)\right]$$

for the dimensionless energy density $120\pi\tau\, y(\tau)$ with the asymptotic expression $120\pi\tau\, y_{1/3}(\tau)$ for moderately large $\tau$ (*left panel*) and very large $\tau$ (*right panel*).

# Asymptotic solution at $\tau \gg 1$, $\gamma\tau \lesssim 1$ and $w = 0$

For $w = 0$ equations take the form

$$h' + 2h^2 = -r/6$$
$$r'' + 3hr' + r = -8\pi\mu^2 y$$
$$y' + 3h\,y = S[r]$$

$\mu^2 = (m/m_{Pl})^2 \ll 1 \Rightarrow$ the impact of the r.h.s. in Eq. for $r$ is not essential $\Rightarrow$ we can use:

$$h = \frac{2}{3\tau}\left[1 + \sin\left(\tau + \theta\right)\right], \quad r = -\frac{4\cos(\tau+\theta)}{\tau} - \frac{4}{\tau^2}$$

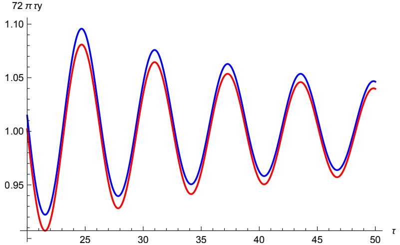
The only essential difference with the $w = 1/3$ case arises in the equation governing the evolution of the energy density, $y(\tau)$.

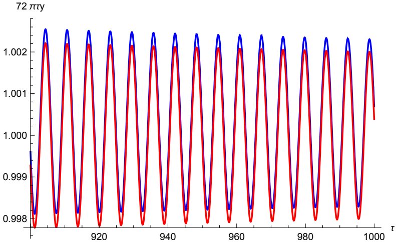
There appears coefficient (-3) in the exponent, instead of (-4):

$$y(\tau) = \frac{1}{72\pi}\int_{\tau_0}^{\tau}\frac{d\tau_2}{\tau_2^2}\exp\left[-3\int_{\tau_2}^{\tau}d\tau_1 h(\tau_1)\right]$$

# Asymptotic behavior of the solution for $w = 0$

$$y_0 = \frac{1}{72\pi\tau} + \frac{\cos(\tau + \theta)}{36\pi\tau^2}$$

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for the dimensionless energy density $72\pi\tau\, y(\tau)$ with the asymptotic expression $72\pi\tau\, y_0(\tau)$ for moderately large $\tau$ (*left panel*) and very large $\tau$ (*right panel*).

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The cosmological evolution in $\boldsymbol{R^2}$-gravity at SD stage is strongly different from the usual FRW-cosmology:

1. The energy density of matter in $R^2$ modified gravity at SD stage drops down as

$$\varrho_{R^2} = \frac{m^3}{120\pi t} \quad \textit{instead of} \quad \varrho_{GR} = \frac{3H^2 m_{Pl}^2}{8\pi} = \frac{3m_{Pl}^2}{32\pi t^2}$$

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2. The Hubble parameter quickly oscillates with time

$$h = \frac{2}{3\tau}\left[1 + \sin\left(\tau + \theta\right)\right]$$

almost touching zero, and it is the same for $w = 1/3$ and $w = 0$.

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3. The curvature scalar drops down as $m/t$ and oscillates changing sign

$$r = -\frac{4\cos(\tau + \theta)}{\tau} - \frac{4}{\tau^2}$$

instead of being proportional to the trace of the energy-momentum tensor of matter, which is identically zero at RD stage and monotonically decreases with time, as $1/t^2$ at MD stage.

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3. The curvature scalar drops down as $m/t$ and oscillates changing sign
$$r = -\frac{4\cos(\tau + \theta)}{\tau} - \frac{4}{\tau^2}$$
instead of being proportional to the trace of the energy-momentum tensor of matter, which is identically zero at RD stage and monotonically decreases with time, as $1/t^2$ at MD stage.
4. It is noteworthy that $R$ is not related to the energy density of matter as is true in General Relativity.

# III. Solution at $\Gamma t \gtrsim 1$

# Solution at $\gamma\tau \gtrsim 1$, $\gamma = \mu^2/48$ and $\mu = m/m_{Pl}$

$$h' + 2h^2 = -r/6$$
$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$
$$y' + 3(1 + w)h\,y = S[r]$$

A straightforward numerical solution of this system quickly becomes unreliable due to very small exponential suppression factor $\exp(-\gamma\tau/2)$, when $\gamma\tau \gg 1$.

The case of relativistic matter, $w = 1/3$:

$$r'' + (3h + \gamma)r' + r = 0$$

Eliminating the first derivative $r'$ by introducing the new function $v$, we find:

$$r = \exp\left[-\gamma(\tau - \tau_0)/2 - (3/2)\int_{\tau_0}^{\tau} d\tau_1 h(\tau_1)\right] v(\tau)$$

where

$$v(\tau) = -4\gamma\cos(\tau + \theta)$$

NB. Curvature $r$ exponentially vanishes at large $\gamma\tau/2 \Longrightarrow$ r.h.s. of Eqs. for $h$ and $y$ tends to $0$ with the same speed, restoring the normal cosmology at RD stage.

## Nonrelativistic dominance: $w = 0$ or some deviations from $w = 1/3$

We study

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$

- non-zero r.h.s. might change the asymptotical exponential decrease of $r$.

Making the same transformation as previously, we find for the curvature scalar:

$$r = r_{hom} + r_{inh}$$

where $r_{hom}$ is a solution of the homogeneous equation:

$$r_{hom} = r_0 \cos(\tau + \theta_r) \exp\left[-\frac{\gamma}{2}(\tau - \tau_0) - \frac{3}{2}\int_{\tau_0}^{\tau} d\tau_2 h(\tau_2)\right]$$

and the inhomogeneous part of the solution is:

$$r_{inh} = -8\pi\mu^2(1 - 3w)\int_{\tau_0}^{\tau} d\tau_1 y(\tau_1)\sin(\tau - \tau_1)\exp\left[-\frac{\gamma}{2}(\tau - \tau_1) - \frac{3}{2}\int_{\tau_1}^{\tau} d\tau_2 h(\tau_2)\right]$$

- the solution of the homogeneous equation, $r_h$, drops down exponentially as $e^{-\gamma\tau/2}$
- the inhomogeneous part does not; integral for $r$ is dominated by $\tau_1$ close to $\tau$.

The value of $(1 - 3w)$ is not yet specified here, we only assume that it is nonzero.

# The transition from the modified $R^2$-regime to GR

Assuming
$$h(\tau) = \frac{h_1 + h_2 \sin(\tau + \theta_h)}{\tau}, \quad y(\tau) = \frac{y_1}{\tau^\beta}$$

we find the asymptotic solution:

$$r_{inh} \approx \frac{16\pi\mu^2(1-3w)\,y_1}{\tau^\beta}\left[\left(\frac{\tau}{\tau_0}\right)^{\beta - 3h_1/2} e^{-\gamma(\tau-\tau_0)/2}\cos(\tau-\tau_0) - 1\right]$$

- $\gamma\tau > 1$: the first term dies down, but the last non-oscillating term survives.
- In this limit the particle production by $R$ vanishes, or strongly drops down.

We have:

$$r = r_{hom} + r_{inh}$$

- $r_{hom}$ decreases as $e^{-\gamma\tau/2}$
- $r_{inh} \sim \mu^2 \ll 1$, but does not drop down exponentially due to the last term in the square brackets

The GR regime is restored when the second term becomes comparable with the exponentially decreasing one.

# The transition from the modified $R^2$-regime to GR

It would be natural to expect the GR regime starting roughly at $\tau \gtrsim 1/\gamma$.

Simple estimate for $w = 0$ :

We have to compare the value of the curvature scalar ($y_1 = 1/72\tau$, $\beta = 1$)

$$r = 2\mu^2/(9\tau)$$

with homogeneous solution for the curvature:

$$r \sim 4\exp\left(-\gamma\tau/2\right)/\tau$$

These expressions become comparable at

$$\gamma\tau \approx 2\ln\left(1/\mu^2\right) \sim \ln\left(m_{Pl}/m\right)$$

Similar arguments cannot be applied to $w = 1/3$, because in this case $R_{GR} \equiv 0$.

In realistic case $w$ differs from zero either due to presence of massive particles in the primeval plasma or because of the conformal anomaly.

# Cosmological history in $R^2$-gravity: 4 distinct epoch

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1. **An exponential (inflationary) expansion:** the universe was void and dark with slowly decreasing curvature scalar $R(t)$. The initial value of $\boldsymbol{R}$ should be quite large, $R > 300m^2$, to ensure sufficiently long inflation ($N_e \geq 70$).

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2. **Scalaron dominated epoch:** $\boldsymbol{R}$ dropped down and started to oscillate as

$$R \sim m\cos(mt)/t$$

The curvature oscillations resulted in the onset of creation of usual matter, which remains subdominant.

The universe expansion is described by unusual law with the Hubble parameter

$$H = (2/3t)[1 + \sin(mt)]$$

Energy density of matter drops down as

$$\varrho_{R^2} = \frac{m^3}{120\pi t} \quad \text{instead of} \quad \varrho_{GR} = \frac{3H^2 m_{Pl}^2}{8\pi} = \frac{3m_{Pl}^2}{32\pi t^2}$$

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4. **After this time we arrive to the cosmology governed by GR.**

# To be continued...

Unusual cosmological evolution during the time $t < 1/\Gamma$ would lead to:

- noticeable modification of the cosmological baryogenesis scenarios
- variation of the probability of formation of primordial black holes
- change of the frozen density of dark matter particles
- etc..

In particular, it opens window for heavy lightest supersymmetric particles to be the cosmological dark matter (afternoon talk at ICNFP 2019):

- EA, A. D. Dolgov and R. S. Singh, "Dark matter in $R + R^2$ cosmology," JCAP **1904** (2019) no.04, 014 [arXiv:1811.05399 [astro-ph.CO]].

The END

Thank You for Your Attention