

Naturally extended Higgs inflation ready for tests

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**Workshop on New physics paradigm
after Higgs and gravitational wave discovering**

New Frontiers in Physics, ICNFP 2019, Kolymbari, Crete

Hot Big Bang within GR and SM: problems

- Dark Matter
- Baryogenesis
- Horizon, Entropy, Flatness, ... problems

$$l_{H_0}/l_{H,r}(t_0) \sim \sqrt{1+z_r} \simeq 30$$

- Singularity at the beginning
- Heavy relics
- Initial perturbations

$$\delta T/T \sim \delta \rho/\rho \sim 10^{-4}, \text{ scale-invariant}$$

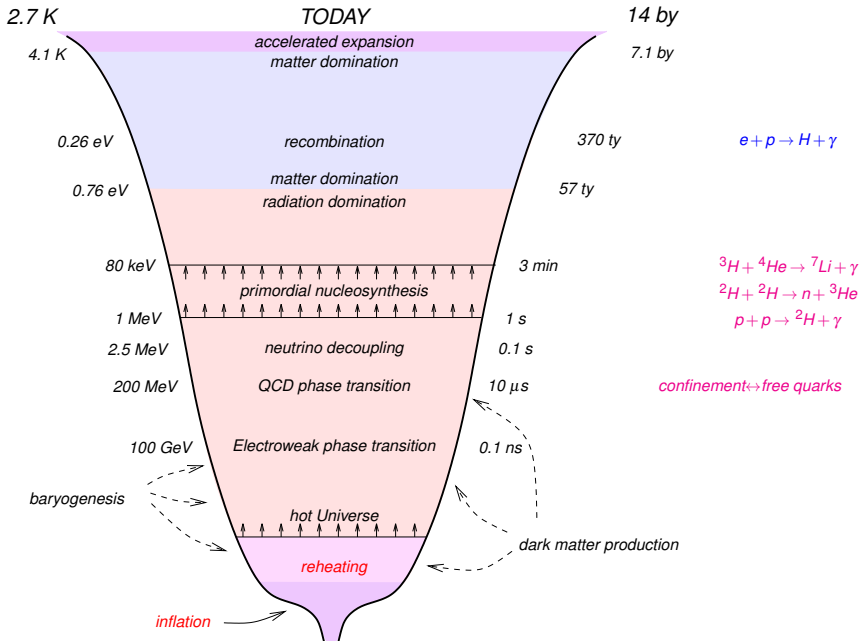
- Dark Energy

$$0 \neq \Lambda \ll M_{Pl}^4 M_W^4 \Lambda_{QCD}^4 \text{ etc?}$$

- Coincidence problems:

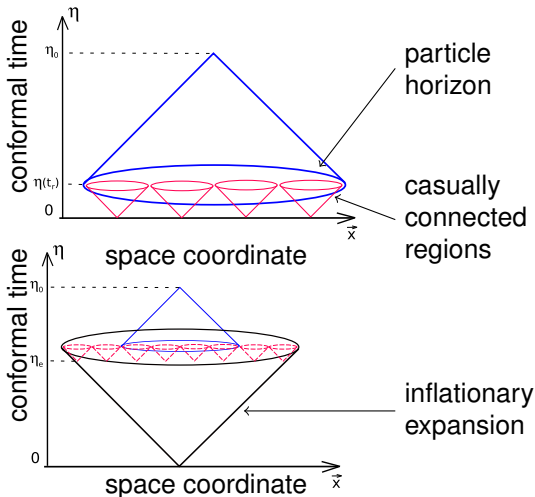
$$\begin{aligned} \Omega_B &\sim \Omega_{DM} \sim \Omega_\Lambda, \\ \eta_B = n_B/n_\gamma &\sim (\delta T/T)^2, \\ T_d^n &\sim (m_n - m_p), \\ &\dots \end{aligned}$$

- Λ CDM tensions: $H_0?$, $\sigma_8?$ dwarfs? cusps? ... (reionization @ $z \simeq 10$, etc)



Inflationary solution of Hot Big Bang problems

- no initial singularity in dS space
- all scales grow exponentially, including the radius of the 3-sphere
the Universe becomes exponentially flat
- any two particles are at exponentially large distances
no heavy relics
no traces of previous epochs!
- no particles in post-inflationary Universe
to solve entropy problem we need post-inflationary reheating



Chaotic inflation at large fields: graceful entrance

in all domains of Planck size
each of the form of inflaton energy
fluctuates similarly

Chaotic inflation, A.Linde (1983), A.Linde (1984)

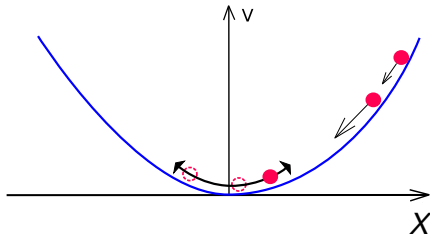
$$\frac{1}{2}\dot{\phi}^2 \sim \frac{1}{2}(\partial_i\phi)^2 \sim V(\phi) \lesssim M_{Pl}^4$$

If $V(\phi)$ dominates by chance

$$\ddot{\phi} - \Delta\phi/a^2 + 3H\dot{\phi} + V'(\phi) = 0$$

for power-law potential at $\phi > M_{Pl}$

$$V \simeq \text{const}$$



“slow roll” solution

$$H^2 = \frac{8\pi}{3M_P^2} V(\phi), \quad a(t) \propto e^{Ht}$$

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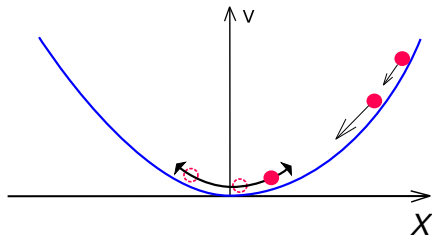
“slow roll” solution

$$H^2 = \frac{8\pi}{3M_P^2} V(\phi), \quad a(t) \propto e^{Ht}$$

valid while

slow roll conditions

$$M_P^2 \frac{V'''}{V} \ll 1, \quad M_P^2 \frac{V'^2}{V^2} \ll 1$$



Inflaton must couple
to Standard Model fields

to reheat the Universe
after inflation

The idea is great,
but is not verifiable
except for the 3d-flatness

Does it make the idea wrong...?

Unexpected bonus: generation of perturbations

- Quantum fluctuations of wavelength λ of a free massless field φ have an amplitude of $\delta\varphi_\lambda \simeq 1/\lambda$
- In the expanding Universe: $\lambda \propto a$

inflation: $l_H \sim 1/H = \text{const}$, so modes "exit horizon"

Ordinary stage: $l_H \sim 1/H \propto t$, $l_H/\lambda \nearrow$, modes "enter horizon"

Evolution at inflation

- inside horizon:** $\lambda < l_H$

$$\lambda \propto a \Rightarrow$$

$$\delta\varphi_\lambda \propto 1/\lambda \propto 1/a$$



- outside horizon:** $\lambda > l_H$

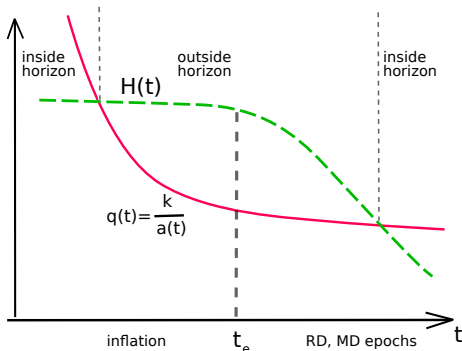
$$\lambda \propto a \Rightarrow$$

$$\delta\varphi_\lambda = \text{const} = H_{infl} !!!$$



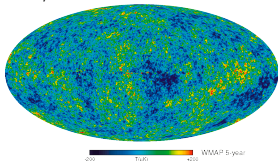
- got "classical" fluctuations:

$$\delta\varphi_\lambda = \delta\varphi_\lambda^{\text{quantum}} \times e^{N_e}$$

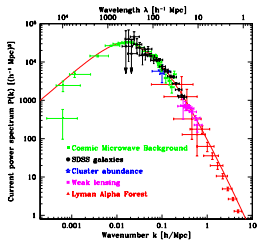


Inflationary solution of Hot Big Bang problems

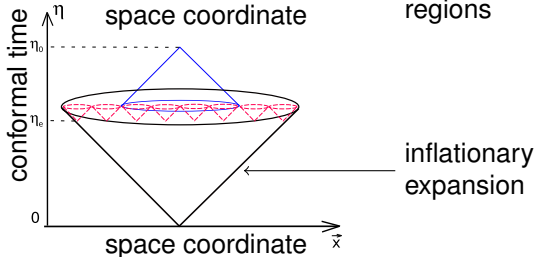
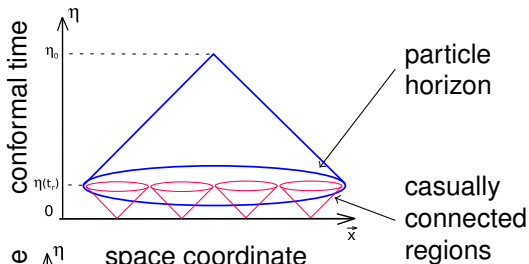
Temperature fluctuations
 $\delta T/T \sim 10^{-5}$



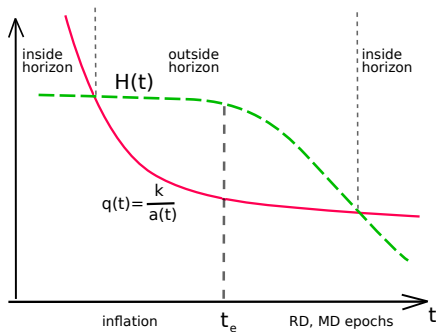
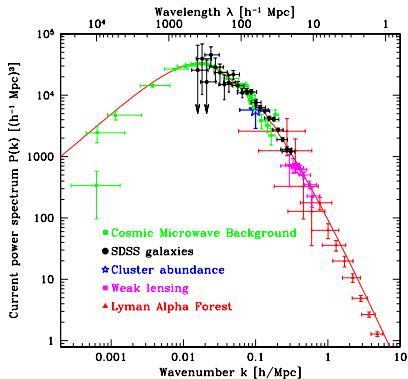
Universe is **uniform!**



$\delta\rho/\rho \sim 10^{-5}$



Probing the matter power spectrum



$$\delta\phi \rightarrow \frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\phi}} \propto \frac{V^{3/2}}{V'}, \quad h \sim \frac{H}{M_{Pl}} \propto V^{1/2}$$

$$A_S \rightarrow \frac{V^{3/2}}{V'}, \quad n_S \rightarrow \frac{V''}{V}, \quad \left(\frac{V'}{V}\right)^2, \quad r \equiv \frac{A_T}{A_S} \rightarrow \left(\frac{V'}{V}\right)^2$$

Chaotic inflation: simple realization

$$S = \int d^4x \sqrt{-g} \left(-\frac{M_P^2}{2} R + \frac{(\partial_\mu X)^2}{2} - \beta X^4 \right)$$

$$\ddot{X} + 3H\dot{X} + V'(X) = 0$$

$$H^2 = \frac{1}{M_P^2} V(X), \quad a(t) \propto e^{Ht}$$

slow roll conditions get satisfied at

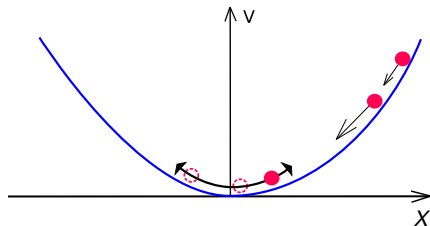
$$X_e > M_{Pl}$$

$$M_P^2 = M_{Pl}^2 / (8\pi)$$

generation of scale-invariant scalar (and tensor) perturbations from exponentially stretched quantum fluctuations of X

⇒

Chaotic inflation, A.Linde (1983)



$$\delta\rho/\rho \sim 10^{-5} \text{ requires } V = \beta X^4 : \beta \sim 10^{-13}$$

We have scalar in the SM! The Higgs field!

In a unitary gauge $H^T = (0, (h+v)/\sqrt{2})$ (and neglecting $v = 246 \text{ GeV}$) $\lambda \sim 0.01 - 1$

$$S = \int d^4x \sqrt{-g} \left(-\frac{M_P^2}{2} R + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda h^4}{4} \right)$$

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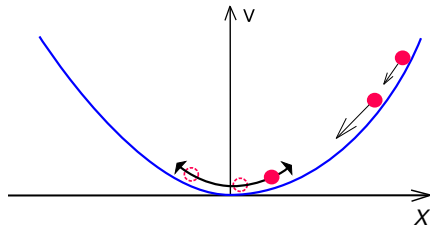
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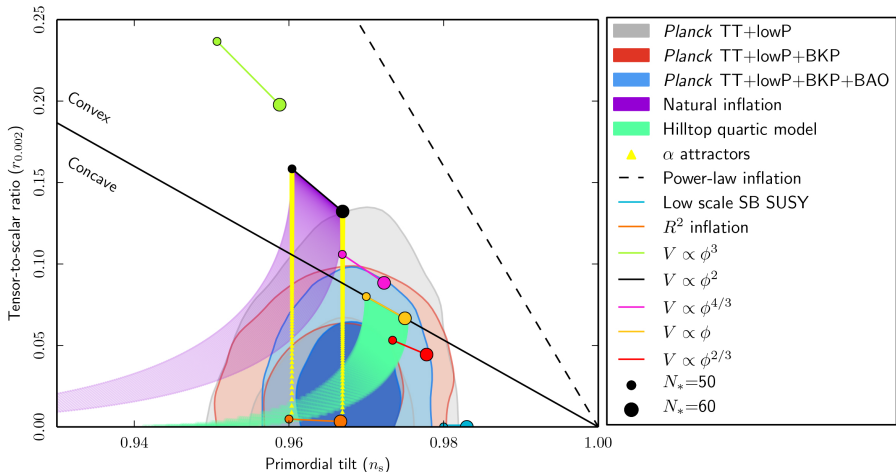
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$$S = \int d^4x \sqrt{-g} \left(-\frac{M_P^2}{2} R + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda h^4}{4} \right)$$

Planck 2015 favors flat inflaton potentials



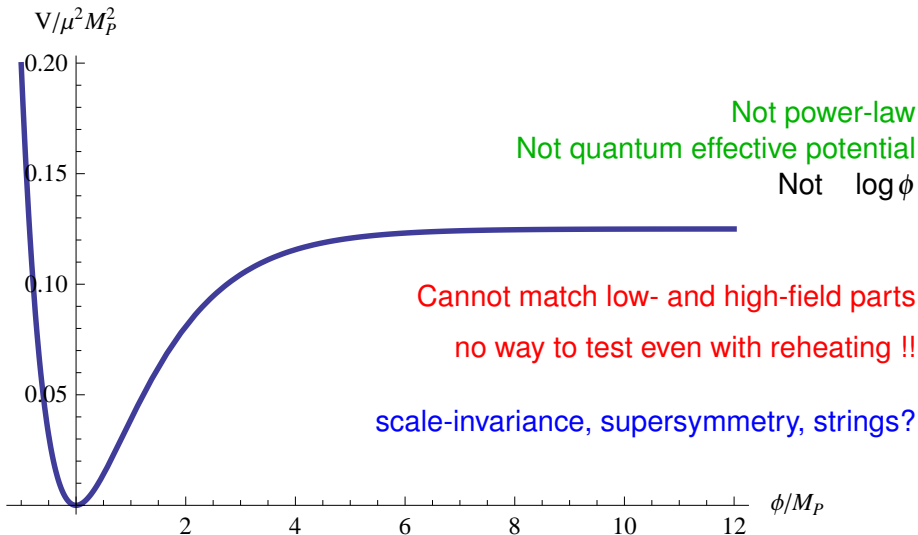
$$r = \frac{A_T}{A_S} \propto \frac{\dot{\phi}^2}{H^2 M_{Pl}^2} \propto \left(\frac{V'}{V} \right)^2 \ll 1$$

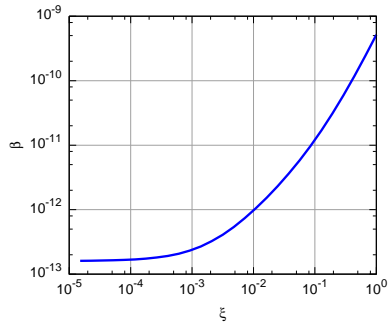
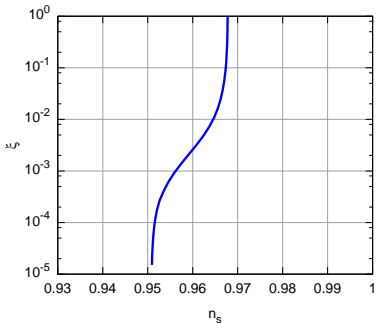
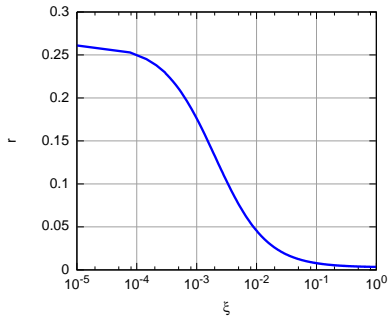
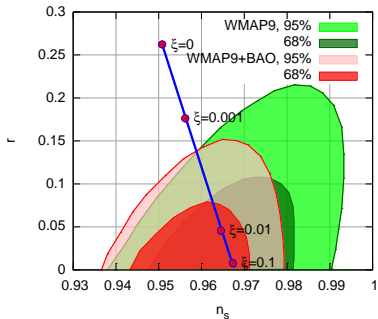
Other ways of testing inflation

- Curvature: the World is flat
not convincing for many
- Relic tensor modes (gravitational waves)
low- l B -mode: well below Galactic foreground
- preheating: $T_{reh} \rightarrow N_e$, GW ?
tiny effects, $n_s, r = f(\log(N_e))$, GW from clumps
- Direct tests: inflaton potential
only in specific models with light inflaton
- Generic for many-field inflation are
isocurvature modes, non-Gaussianity
- Exotic signatures
primordial black holes, GW from oscillons, etc

and the calculations must be reliable

Inflaton potential is apparently nonrenormalizable





F.Bezrukov,
D.G. (2013)

How it
works...

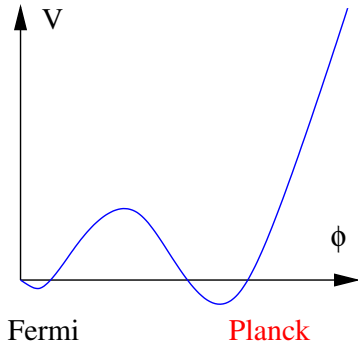
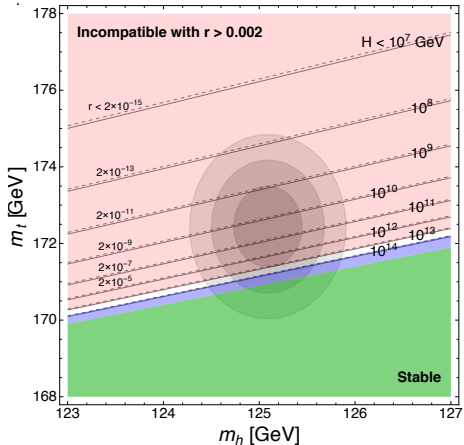
$$V = \beta \phi^4$$

$$\xi \phi^2 R$$

it changes
GR-scale

it changes
kinetic term
hence
changes
scalar
and tensor
spectra

Wrong EW vacuum: $\Phi \sim H/(2\pi)$



1607.00381

Thus we either constrain inflation, $H \lesssim \dots 10^{10} \text{ GeV} \dots$ and hence GW, that is r or just assume we are 2σ -off and $\lambda > 0$

Higgs-driven inflation

F.Bezrukov, M.Shaposhnikov (2007)

$$S^{JF} = \int d^4x \sqrt{-g} \left(-\frac{M_P^2}{2} R - \xi H^\dagger H R + \mathcal{L}_{SM} \right)$$

In a unitary gauge $H^T = (0, (h+v)/\sqrt{2})$ (and neglecting $v = 246$ GeV)

$$S = \int d^4x \sqrt{-g} \left(-\frac{M_P^2 + \xi h^2}{2} R + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda h^4}{4} \right)$$

slow roll behavior due to modified kinetic term even for $\lambda \sim 1$

Go to the Einstein frame:

$$(M_P^2 + \xi h^2) R^{JR} \rightarrow M_P^2 R^{EF}$$

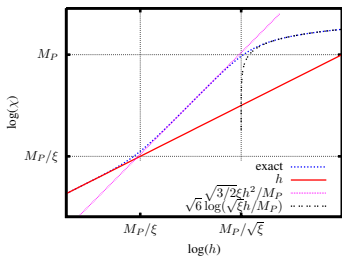
$$g_{\mu\nu}^{JF} = \Omega^{-2} \tilde{g}_{\mu\nu}^{EF}, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

with canonically normalized χ :

interval ds^2 changes !

$$\frac{d\chi}{dh} = \frac{M_P \sqrt{M_P^2 + (6\xi + 1)\xi h^2}}{M_P^2 + \xi h^2}, \quad U(\chi) = \frac{\lambda M_P^4 h^4(\chi)}{4(M_P^2 + \xi h^2(\chi))^2}.$$

we have a flat potential at large fields: $U(\chi) \rightarrow \text{const} \quad @ \quad h \gg M_P/\sqrt{\xi}$



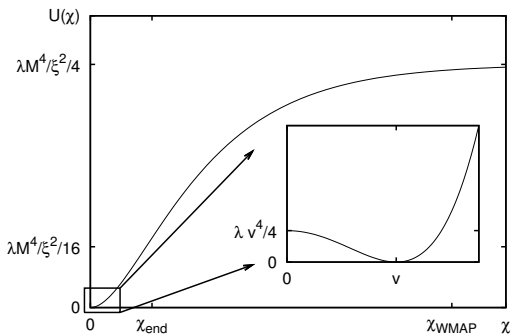
Reheating by Higgs field

after inflation: $M_P/\xi < h < M_P/\sqrt{\xi}$

effective dynamics: $h^2 \rightarrow \chi$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{\lambda}{6} \frac{M_P^2}{\xi^2} \chi^2$$

Advantage: NO NEW interactions
to reheat the Universe
inflaton couples to all SM fields!



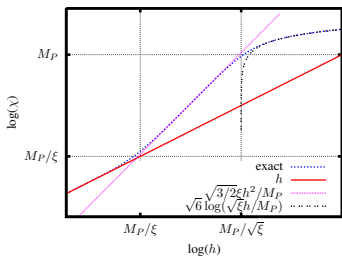
exponentially flat potential! @ $h \gg M_P/\sqrt{\xi}$:

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left(1 - \exp\left(-\frac{\sqrt{2}\chi}{\sqrt{3}M_P}\right) \right)^2$$

NO NEW d.o.f.
Different reheating temperature. . .

0812.3622, 1111.4397

from WMAP-normalization: $\xi \approx 47000 \times \sqrt{\lambda}$



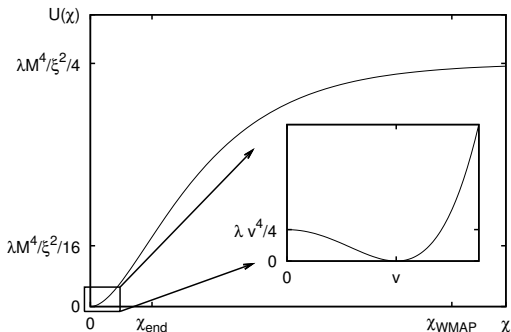
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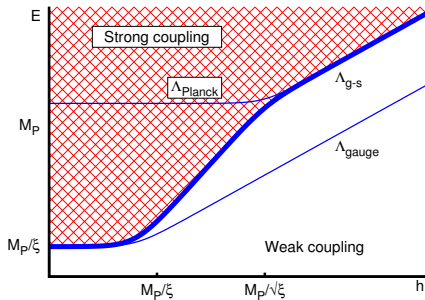
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Strong coupling in Higgs-inflation

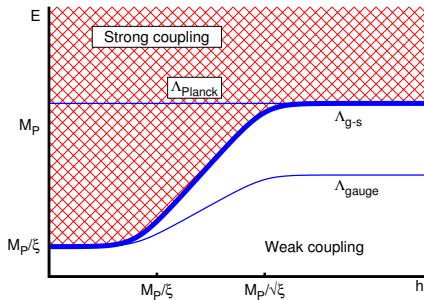
Jordan frame



gravity-scalar sector:

$$\Lambda_{g-s}(h) \simeq \begin{cases} \frac{M_P}{\xi}, & \text{for } h \lesssim \frac{M_P}{\xi}, \\ \frac{\xi h^2}{M_P}, & \text{for } \frac{M_P}{\xi} \lesssim h \lesssim \frac{M_P}{\sqrt{\xi}}, \\ \sqrt{\xi} h, & \text{for } h \gtrsim \frac{M_P}{\sqrt{\xi}}. \end{cases}$$

Einstein frame

gravitons: $\Lambda_{\text{Planck}}^2 \simeq M_P^2 + \xi h^2$

gauge interactions:

$$\Lambda_{\text{gauge}}(h) \simeq \begin{cases} \frac{M_P}{\xi}, & \text{for } h \lesssim \frac{M_P}{\xi}, \\ h, & \text{for } \frac{M_P}{\xi} \lesssim h, \end{cases}$$

We must modify the model to restore the unitarity

Natural completion with R^2

Y.Ema (2017), D.G., A.Tokareva (2018)

 $\xi h^2 R$ induces R^2 -term

hep-th/9510140

$$S_0 = \int d^4x \sqrt{-g} \left(-\frac{M_P^2 + \xi h^2}{2} R + \frac{\beta}{4} R^2 + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right).$$

introduce a Lagrange multiplier L and auxiliary scalar \mathcal{R}

$$S = \int d^4x \sqrt{-g} \left(\frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 - \frac{M_P^2 + \xi h^2}{2} \mathcal{R} + \frac{\beta}{4} \mathcal{R}^2 - L\mathcal{R} + LR \right).$$

integrate out \mathcal{R}

$$S = \int d^4x \sqrt{-g} \left(\frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 + LR - \frac{1}{\beta} \left(L + \frac{1}{2} \xi h^2 + \frac{1}{2} M_P^2 \right)^2 \right)$$

$$\xi \rightarrow \xi^2 / \beta$$

with

$$\beta \gtrsim \frac{\xi^2}{4\pi}$$

everything here look healthy

Further transformations. . .

Y.Ema (2017)

introducing scalaron ϕ with $m = M_P / \sqrt{3\beta}$

$$g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}, \quad \Omega^2 \equiv \frac{2L}{M_P^2}, \quad L \rightarrow \phi \equiv M_P \sqrt{\frac{2}{3}} \log \Omega^2.$$

and setting $M_P = 1/\sqrt{6}$

$$S = \int d^4x \sqrt{-g} \left(-\frac{R}{12} + \frac{1}{2} e^{-2\phi} (\partial h)^2 + \frac{1}{2} (\partial \phi)^2 - \frac{1}{4} e^{-4\phi} \left(\lambda h^4 + \frac{1}{36\beta} (e^{2\phi} - 1 - 6\xi h^2)^2 \right) \right)$$

both gravity and scalar sector are weakly coupled up to M_P with $\beta \gtrsim \xi^2 / (4\pi)$

And one more...

D.G., A.Tokareva (2018)

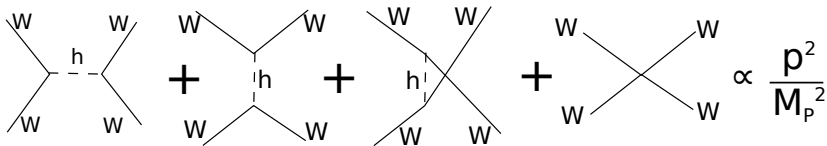
$$h = e^\Phi \tanh H, \quad \phi = e^\Phi / \cosh H,$$

The scalar sector becomes

$$L = \frac{1}{2} \cosh^2 H (\partial\Phi)^2 + \frac{1}{2} (\partial H)^2 - \frac{\lambda}{4} \sinh^4 H - \frac{\lambda}{144\beta} (1 - e^{-2\Phi} \cosh^2 H - 6\xi \sinh^2 H)^2.$$

and the Higgs coupling to gauge bosons, e.g.,

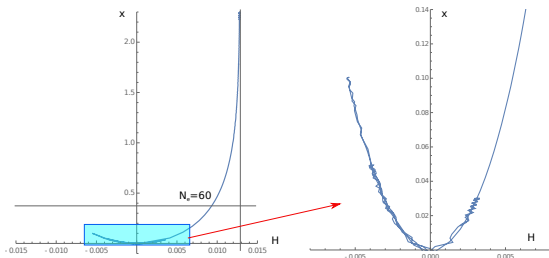
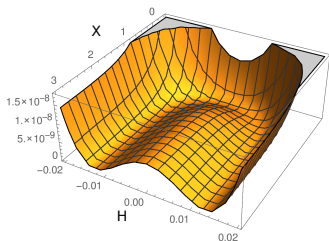
$$L_{\text{gauge}} = \frac{g^2 h^2}{4} e^{-2\phi} W_\mu^+ W_\mu^- = \frac{g^2}{4} \sinh^2 H W_\mu^+ W_\mu^-.$$



$$\mathcal{A} \sim \frac{g^2 p^2}{m_W^2} \left(\frac{4}{g^2} \left(\frac{dm_W(H)}{dH} \right)^2 - 1 \right) \rightarrow \mathcal{A} \propto \frac{p^2}{M_p^2}$$

Cosmological spectra

D.G., A.Tokareva 1807.02392



Scalar perturbations: adiabatic

1701.07665

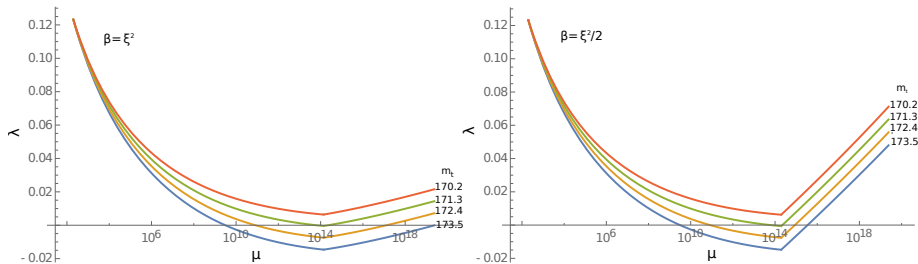
$$\beta + \frac{\xi^2}{\lambda} \simeq 2 \times 10^9$$

At small β like in the Higgs-inflation

heavy scalaron is integrated out

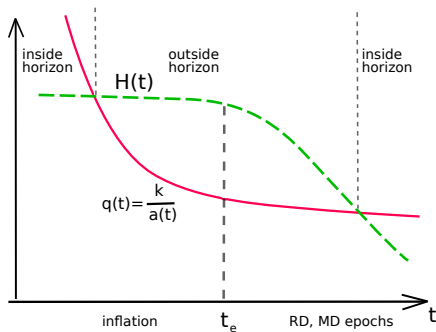
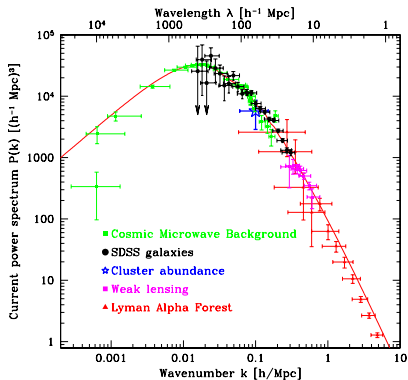
$$\frac{\xi^2}{4\pi} < \beta < \frac{\xi^2}{\lambda} \rightarrow 5 \times 10^{13} \text{ GeV} < m < 1.5 \times 10^{15} \text{ GeV}$$

Bonus: stable for a bit heavier top-quark



We can calculate observables at any energy scale up to Planck

Probing the spectrum at energy scale $\mu = \mu(T_{reh})$



$$\delta\phi \rightarrow \frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\phi}} \propto \frac{V^{3/2}}{V'}, \quad h \sim \frac{H}{M_{Pl}} \propto V^{1/2}$$

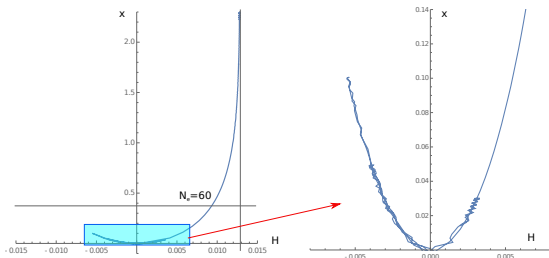
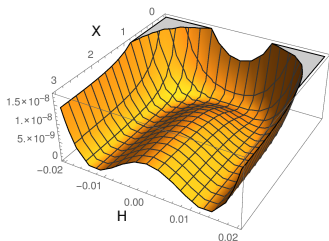
$$A_S \rightarrow \frac{V^{3/2}}{V'}, \quad n_S \rightarrow \frac{V''}{V}, \quad \left(\frac{V''}{V}\right)^2, \quad r \equiv \frac{A_T}{A_S} \rightarrow \left(\frac{V'}{V}\right)^2$$

Reheating. . . all masses depend on oscillating Higgs

- Huge spikes do not reheat !! 1812.10099
- it is a highly nonlinear system
- ω^2 for W_L and Z_L rapidly oscillates and becomes negative for some time
- similar for one of the scalars (a mixture of Higgs and scalaron)
- we expect instant preheating, at least for a region in model parameter space F.Bezrukov, D.G., Ch.Shepherd, A.Tokareva (2019)
- but for precise number the backreaction must be taken into account

Cosmological spectra

D.G., A.Tokareva 1807.02392



Scalar perturbations:

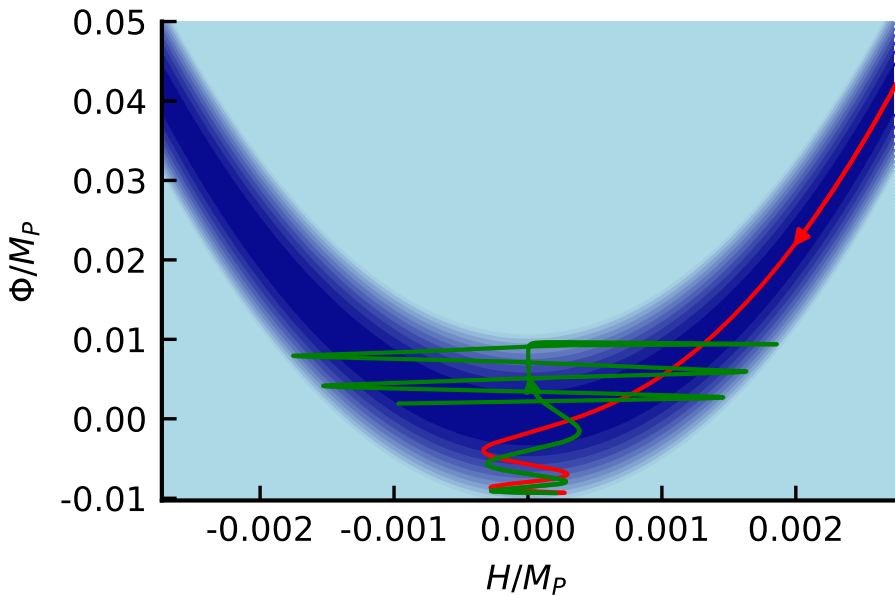
1701.07665

$$\beta + \frac{\xi^2}{\lambda} \simeq 2 \times 10^9$$

At small β like in the Higgs-inflation

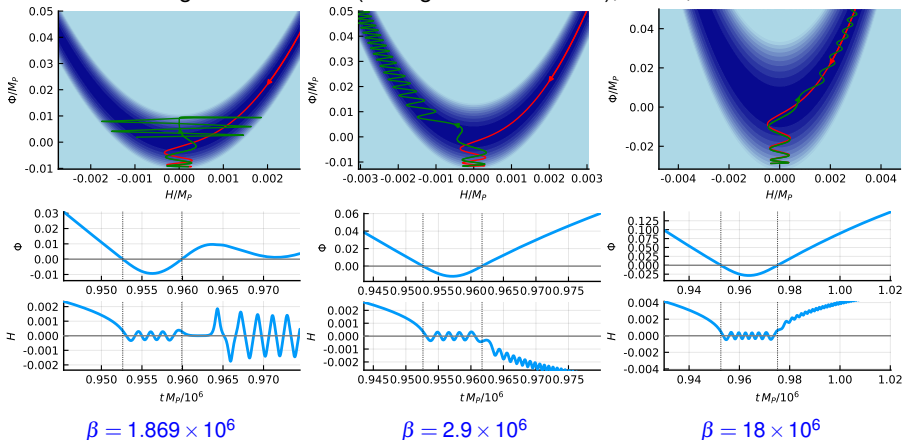
heavy scalaron is integrated out

$$\frac{\xi^2}{4\pi} < \beta < \frac{\xi^2}{\lambda} \rightarrow 5 \times 10^{13} \text{ GeV} < m < 1.5 \times 10^{15} \text{ GeV}$$



Scalaron Φ and Higgs H evolution after inflation

Homogeneous modes (mixing in kinetic sector), $\dot{\Phi} < 0$, $\dot{\Phi} > 0$



$$V(H, \Phi) = \frac{1}{4} \left(\lambda + \frac{\xi^2}{\beta} \right) H^4 + \frac{M_P^2}{6\beta} \Phi^2 - \frac{\xi M_P}{\sqrt{6}\beta} \Phi H^2 + \frac{7}{108\beta} \Phi^4 + \frac{\xi}{6\beta} \Phi^2 H^2 - \frac{M_P}{3\sqrt{6}\beta} \Phi^3$$

Linear equations for gauge bosons

Gauge bosons (e.g. W^\pm)

$$L_g^{(2)} = -\frac{1}{2} \left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ \right) \left(\partial_\lambda W_\rho^- - \partial_\rho W_\lambda^- \right) g^{\mu\lambda} g^{\nu\rho} + \frac{g^2 H_0^2}{4} W_\mu^+ W_\nu^- g^{\mu\nu},$$

transverse modes

$$\ddot{W}_k^T + 3\mathcal{H}\dot{W}_k^T + \frac{k^2}{a^2} W_k^T + m_T^2 W_k^T = 0, \quad m_T \equiv \frac{g}{2} H_0$$

longitudinal modes

$$\ddot{W}_k^L + 3\mathcal{H}\dot{W}_k^L + \omega_W^2(\mathbf{k}) W_k^L = 0.$$

$$\omega_W^2(\mathbf{k}) = \frac{k^2}{a^2} + m_T^2 - \frac{k^2}{k^2 + a^2 m_T^2} \left(\mathcal{H} + 2\mathcal{H}^2 + 3\mathcal{H} \frac{\dot{m}_T}{m_T} + \frac{\ddot{m}_T}{m_T} - \frac{3(\dot{m}_T + \mathcal{H} m_T)^2}{k^2/a^2 + m_T^2} \right).$$

for $k/a \gg m_T$ after inflation

$$\omega_W^2 = \frac{k^2}{a^2} + \frac{g^2}{4} H_0^2 + \frac{\xi}{3\beta} \Phi_0^2 + \left(\lambda + \frac{\xi^2}{\beta} \right) H_0^2 - \frac{\xi\sqrt{2}}{\beta\sqrt{3}} M_P \Phi_0,$$

Linear equations for scalaron and Higgs

A mixture of the two scalars

$$m_{L,H}^2 = \frac{1}{2} (V_{H_0 H_0} + V_{\Phi_0 \Phi_0}) \times \left(1 \pm \sqrt{1 - 4 \frac{V_{\Phi_0 \Phi_0} V_{H_0 H_0} - V_{\Phi_0 H_0}^2}{(V_{H_0 H_0} + V_{\Phi_0 \Phi_0})^2}} \right).$$

after inflation can be approximated as

$$m_{H,L}^2 \approx V_{H_0 H_0} \approx 2 \left(\lambda + \frac{\xi^2}{\beta} \right) H_0^2 + \left(\lambda + \frac{\xi^2}{\beta} \right) H_0^2 - \frac{\sqrt{2}\xi}{\sqrt{3}\beta} M_P \Phi_0$$

Then we calculate the Bogolubov coefficients from the field solutions

$f_{\mathbf{k}}(t) = e^{-i\omega t} / \sqrt{2\omega(\mathbf{k})}$ at $t \rightarrow 0$, which gives for the number density

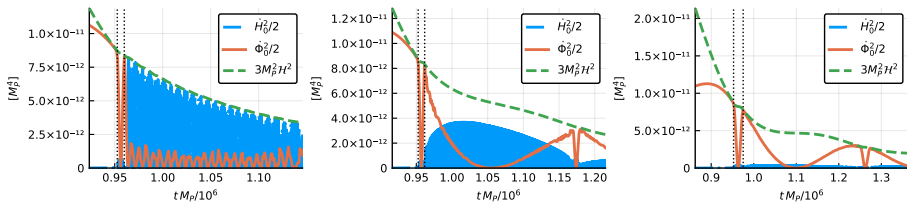
$$n_{\mathbf{k}} = \frac{1}{2} \left| \sqrt{\omega(\mathbf{k})} \dot{f}_{\mathbf{k}} - \frac{i}{\sqrt{\omega(\mathbf{k})}} f_{\mathbf{k}} \right|^2$$

and the physical energy

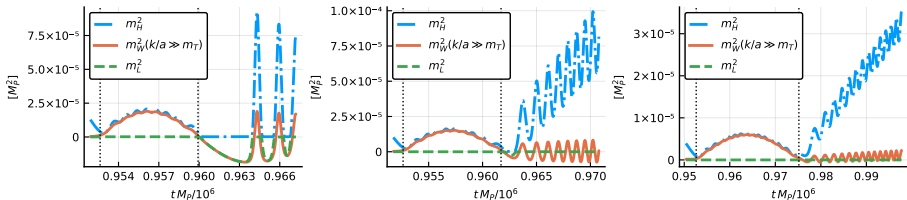
$$\rho = \int \frac{d^3\mathbf{k}}{(2\pi)^3 a^3(t)} \omega(\mathbf{k}) n_{\mathbf{k}}.$$

Numerical results for perturbations

energy distribution over homogeneous modes

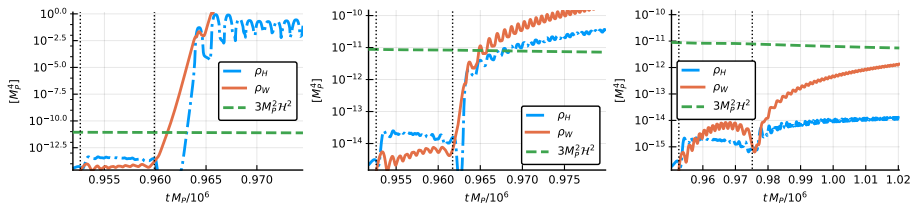


mass squared for the relevant perturbations

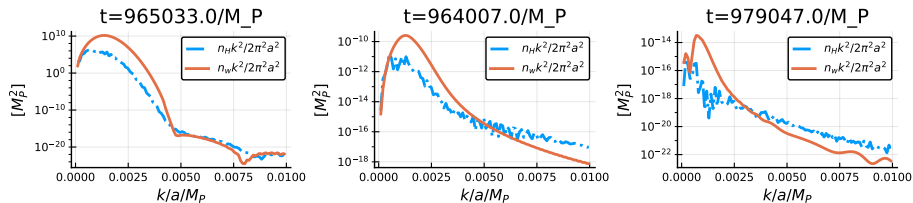


Spectra and energy density of produced particles

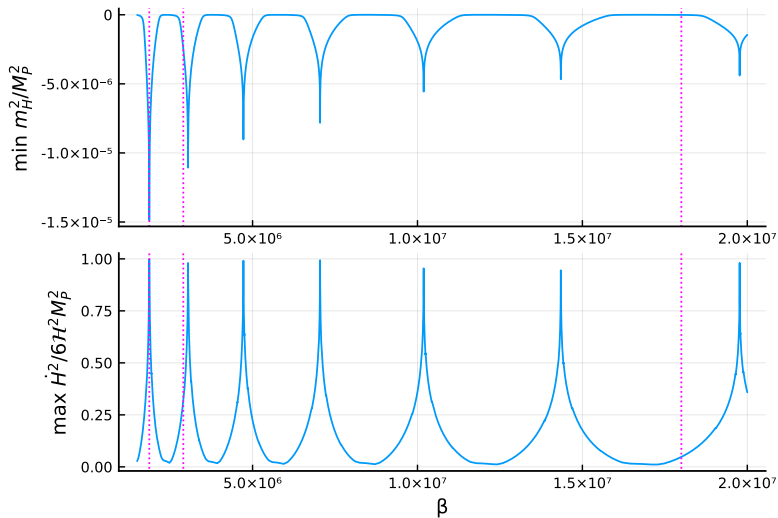
neglecting the adiabaticity conditions... and backreaction



spectra at a reference moment



The resonance positions and energy in Higgs between two zero crossings are correlated



Direct check of the inflation potential

– Higgs frequency is much and scalaron frequency is significantly higher than the expansion rate:

It seems that the reheating is instant

$$N_e = 59, \quad n_s = 0.97, \quad r = 0.0034.$$

– Higgs selfcoupling becomes canonical λ below the scalaron scale $\mu = M_P/\sqrt{3\beta}$

$$V(H, \Phi) = \frac{1}{4} \left(\lambda + \frac{\xi^2}{\beta} \right) H^4 + \frac{M_P^2}{6\beta} \Phi^2 - \frac{\xi M_P}{\sqrt{6\beta}} \Phi H^2 + \frac{7}{108\beta} \Phi^4 + \frac{\xi}{6\beta} \Phi^2 H^2 - \frac{M_P}{3\sqrt{6\beta}} \Phi^3$$

cancellation: $\xi M_P/\beta \times 1/\mu^2 \times \xi M_P/\beta \rightarrow \xi^2/\beta$

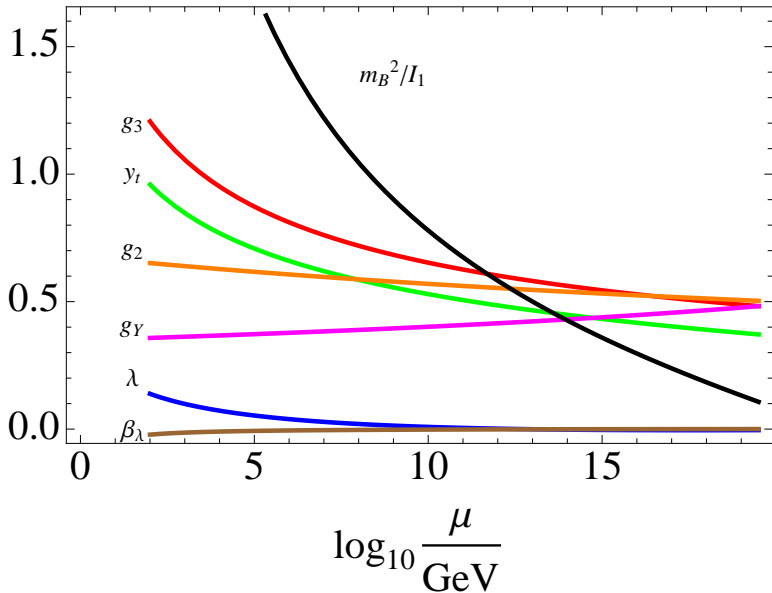
Conclusions

- Higgs inflation is a viable cosmological model unique in minimality:
no new d.o.f., no new interactions to reheat the Universe
- however it suffers from the strong coupling problem:
predictivity $\delta\rho/\rho \leftrightarrow \lambda$ is lost
- R^2 -term with heavy scalaron cures the model:
it seems minimal, natural, Higgs-inflation predictions remain intact
- cosmological and particle physics observables are perturbatively related
- to refine predictions we must study the backreaction at reheating
and improve the accuracy of Y_t , $(m_h, \alpha_s, \text{etc})$ to convince it works indeed
... ILC, FCC, etc

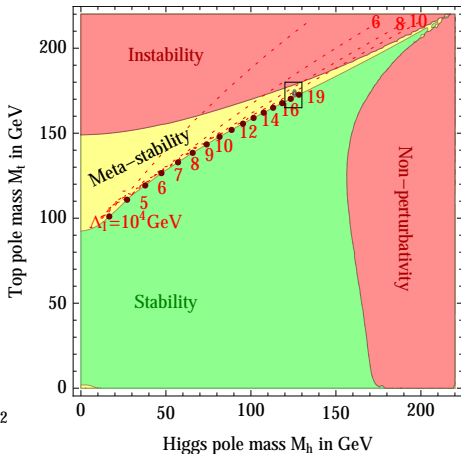
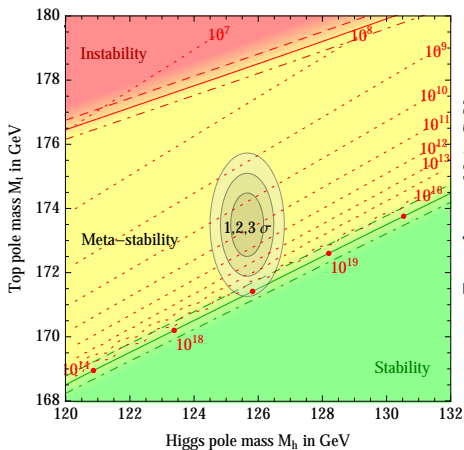
Backup slides

Running of the SM couplings

1305.7055



How weird to live with 125 GeV Higgs. . .



1307.7879

Power spectrum of perturbations

In the Minkowski space-time:

- **fluctuations** of a free quantum field φ are **gaussian** its power spectrum is **defined** as

$$\int_0^\infty \frac{dq}{q} \mathcal{P}_\varphi(q) \equiv \langle \varphi^2(\mathbf{x}) \rangle = \int_0^\infty \frac{dq}{q} \frac{q^2}{(2\pi)^2}$$

We define amplitude as $\delta\varphi(q) \equiv \sqrt{\mathcal{P}_\varphi} = q/(2\pi)$

- In the expanding Universe momenta $q = k/a$ gets redshifted
- Cast the solution in terms $\phi(\mathbf{x}, t) = \phi_c(t) + \varphi(\mathbf{x}, t)$, $\varphi(\mathbf{x}, t) \propto e^{\pm i\mathbf{k}\mathbf{x}} \varphi(\mathbf{k}, t)$
 φ solves the equation

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{k^2}{a^2} \varphi + V''(\phi_c)\varphi = 0$$

- $q = k/a \gg H \Rightarrow$ as in Minkowski space-time
- $q = k/a \ll H \Rightarrow$ for inflaton $\varphi = \text{const}$
- Matching at t_k : $q(t_k) = k/a(t_k) = H(t_k) \equiv H_k$ gives

$$\delta\varphi(q) = \frac{H_k}{2\pi} \Rightarrow \mathcal{P}_\varphi(q) = \frac{H_k^2}{(2\pi)^2}$$

amplification $H_k/q = e^{Ne(k)} !!!$

$H_k \approx \text{const} = H_{\text{infl}}$ hence (almost) flat spectrum

Inflaton parameters and spectral parameters

- Observation of CMB anisotropy gives $\delta T/T$

$$\frac{\delta T}{T} \sim \frac{\delta \rho}{\rho} \Rightarrow \Delta_{\mathcal{R}} \equiv \sqrt{\mathcal{P}_{\mathcal{R}}} = 5 \times 10^{-5}$$

- These are so-called adiabatic perturbations!
Other possibles (isocurvature) modes (e.g. $\delta T = 0$, but $\delta n_B/n_B \neq 0$) are not found.
- $\Delta_{\mathcal{R}} = 5 \times 10^{-5} \Rightarrow$ fixes model parameters, e.g.:

$$V(\phi) = \frac{\beta}{4} \phi^4 \rightarrow \lambda \sim 10^{-13}$$

With such a tiny coupling perturbations are obviously gaussian
So far confirmed by observations

- That's why Higgs boson in the SM does not help!

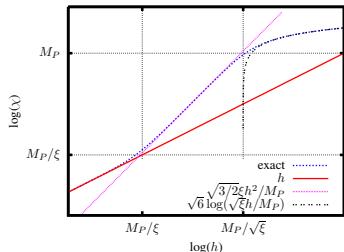
Inflaton parameters and spectral parameters

- In fact, spectra are a bit tilted, as H_{infl} slightly evolves

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}} \left(\frac{k}{k_*} \right)^{n_s - 1}, \quad \mathcal{P}_T(k) = A_T \left(\frac{k}{k_*} \right)^{n_T}.$$

- Measure $\Delta_{\mathcal{R}}$ at present scales $q \simeq 0.002/\text{Mpc}$, it fixes the number of e-foldings left N_e
- For tensor perturbations one introduces:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = \frac{1}{\pi} \frac{M_{Pl}^2 V'^2}{V} = 16\epsilon \rightarrow \frac{16}{N_e} \text{ for } \beta\phi^4$$



$$m_W^2(\chi) = \frac{g^2}{2\sqrt{6}} \frac{M_P |\chi(t)|}{\xi}$$

$$m_t(\chi) = y_t \sqrt{\frac{M_P |\chi(t)|}{\sqrt{6}\xi}} \text{sign } \chi(t)$$

reheating via $W^+ W^-$, ZZ production at zero crossings
then nonrelativistic gauge bosons scatter to light fermions

$$\chi \rightarrow W^+ W^- \rightarrow f \bar{f}$$

Reheating by Higgs field

after inflation: $M_P/\xi < h < M_P/\sqrt{\xi}$

effective dynamics: $h^2 \rightarrow \chi$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{\lambda}{6} \frac{M_P^2}{\xi^2} \chi^2$$

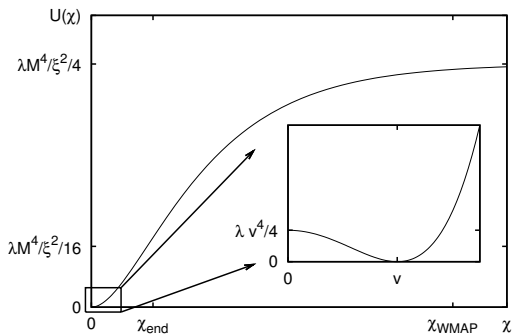
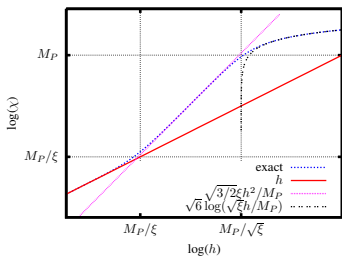
Advantage: NO NEW interactions
to reheat the Universe

inflaton couples to all SM fields!

Hot stage starts almost from $T = M_P/\xi \sim 10^{14}$ GeV:

$$3.4 \times 10^{13} \text{ GeV} < T_r < 9.2 \times 10^{13} \left(\frac{\lambda}{0.125} \right)^{1/4} \text{ GeV}$$

$$n_s = 0.967, r = 0.0032 \text{ F.Bezrukov, D.G.,}$$



$$\frac{d\chi}{dh} = \frac{M_P \sqrt{M_P^2 + (6\xi + 1)\xi h^2}}{M_P^2 + \xi h^2},$$

$$U(\chi) = \frac{\lambda M_P^4 h^4(\chi)}{4(M_P^2 + \xi h^2(\chi))^2}.$$

exponentially flat potential! @ $h \gg M_P/\sqrt{\xi}$:

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left(1 - \exp\left(-\frac{\sqrt{2}\chi}{\sqrt{3}M_P}\right) \right)^2$$

- renormalizable except gravity
frame-dependent renormalization scale
- strong coupling (ϕ -dependent)
save for inflation
but reheating is questionable

F.Bezrukov et al (2008)

$$V_0 \simeq 10^{-12} M_{Pl}^4$$

from WMAP-normalization: $\xi \approx 47000 \times \sqrt{\lambda}$

F.Bezrukov, D.G., M.Shaposhnikov (2008,2011)

And the reheating is almost instant

1609.05209, 1810.01304

We made an error, solving

$$\left(m_W^2(h(t)) + \square \right) W_\nu^\pm = 0$$

while the true equation is

$$W_\nu^\pm m_W^2(h(t)) + \partial_\mu W_{\mu\nu}^\pm = 0$$

the $\partial_\mu W_\mu^\pm = 0$ gauge does not go through the equation...

So, the longitudinal components of vector boson get contributions

$$\omega_L^2 \propto \dot{m}_W, \ddot{m}_W$$

instant reheating well inside the strong coupling domain...